



3 1761 08091862 6

VOL. XXXVIII.

PART I.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

No. CCXIII.—JULY 1903.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: 5, RUE LAMARTINE, 5.
BERLIN: CARLSTRASSE 11. MELBOURNE: McCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

PRINTED BY C. & E. LAYTON, FARRINGDON STREET, E.C.
TO WHOM ALL COMMUNICATIONS FOR THE EDITOR SHOULD BE SENT, POST PAID.

Price 2s. 6d.

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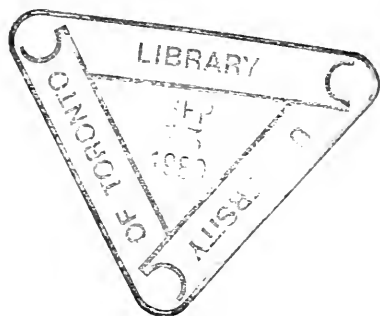
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FARRINGDON STREET.

PARIS 30, RUE LE PELETIER.
BERLIN: CARLSTRASSE 11. MELBOURNE: McCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

1904.



[ENTERED AT STATIONERS' HALL.]

LONDON:
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

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BERLIN: CARLSTRASSE 11. MELBOURNE: McCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

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LONDON :
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGTON STREET.

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Further Remarks on the Valuation of Endowment Assurances in Groups. By GEORGE J. LIDSTONE, F.I.A., *Actuary of the Alliance Assurance Company, Limited.*

[Read before the Institute, 23 February 1903.]

I.—INTRODUCTORY REMARKS.

IN a Paper entitled "Some Remarks on the Valuation of Endowment Assurances in Groups", read before the Institute in January 1898 (*vide J.I.A.*, xxxiv, 61 *et seq.*), the writer described a new method of valuation by which it was claimed the labour of dealing with Endowment Assurances might be reduced to a minimum, and at the same time results might be obtained differing by only an insignificant amount from the exact results. In preparing that Paper it was felt that to encumber it with a mass of technical detail would have been to court the neglect which is the common fate of a lengthy essay, and that the method would be more likely to receive consideration and a fair trial if, in the first instance, no more than an outline of its main principles were given. Since then the author has applied the method, with very satisfactory results, to a volume of Endowment Assurance business much larger than that which was dealt with in the original Paper (*vide J.I.A.*, xxxiv, 510-14), and very gratifying testimony has been received from a number of leading Actuaries who have adopted the plan, and have found

it to work smoothly and efficiently in practice. It is thought, therefore, that the time has arrived when it will be useful to consider the subject in greater detail, and with special reference to some of the points that arise in the application of the method.

(2). It is proposed in this Paper (i) to amplify and extend the discussion of the mathematical theory of the method, and to investigate the origin and extent of the slight theoretical error—insignificant in practice—which it involves; (ii) to consider the application of the method to a valuation by the O^M Table or by the $O^{M.5}$ Table, and to furnish the necessary tables of the function Z ; (iii) to consider the most advantageous ways of actually working out the method in practice.

It has been found impossible to avoid repeating a certain amount of matter contained in the original Paper, and it is hoped that this will be pardoned in the interests of clearness and completeness.

(3). The method has been spoken of above as a “new” one, and perhaps the term is justified by the great accuracy of its results as compared with those of other group-methods which had previously been used: but the fundamental basis of the method—namely, to group together all Policies which will mature in the same year; to assume that all the Policies in a given group mature at a fixed point in that year determined either exactly or approximately so as to give the mean actual duration in the year of maturity; and to use, for each group, valuation factors based upon a mean valuation age—is shared with some of the other plans previously in use. We shall consider later whether or not it will be desirable to make the years of maturity coincide with the financial years of the Company (*vide* pars. 54 *et seq.*). Whatever course may be adopted in this respect, the annuity for valuation of the net premium* will always be of the form

$${}_K|a_{x:\overline{n}} = (1 - \kappa) + a_{x:\overline{n-1+\kappa}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the corresponding assurance value will be

$$A_{x:\overline{n+\kappa}} = 1 - d(1 + a_{x:\overline{n-1+\kappa}}) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or

$$\bar{A}_{x:\overline{n+\kappa}} = 1 - \delta(\frac{1}{2} + a_{x:\overline{n-\frac{1}{2}+\kappa}}) \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

(4). Expressions (1), (2) and (2a) are linear functions of the temporary annuity, and they will remain so when weighted in

* The case of *annual* premiums alone is here dealt with; as to half-yearly and quarterly premiums, see pars. 26, 27 and 29 of the earlier Paper, and notes to column (12) of Schedule I, p. 30.

proportion to the actual net premiums and the actual sums assured, &c. It will therefore be sufficient, in our investigations respecting the mean age, to consider the general form $S_x a_{x:n}^-$, where S_x is the "weight" at age x *; for any given group n will be constant, x and S_x will vary, and the problem is to find y , the mean age for the group, where y is such that

$$a_{y:n}^- [S_x + S_{x+1} + \dots] = a_{x:n}^- S_x + a_{x+1:n}^- S_{x+1} + \dots \quad (3)$$

II.—MATHEMATICAL BASIS OF THE METHOD IN THE CASE OF A TABLE RIGIDLY FOLLOWING MAKEHAM'S LAW.

(5). In developing the mathematical principles of the method it is proposed to proceed on the lines indicated by Mr. G. F. Hardy in the discussion on the earlier Paper (*J.I.A.*, xxxiv, p. 85), and, for the most part, it will be found practicable to avoid the use of the higher mathematics.

(6). In a Mortality Table following Makeham's Law we have $l_x = k s^x g^{c^x}$, whence

$${}_t p_x = l_{x+t} \div l_x = s^t g^{c^x(c^t-1)} = s^t e^{c^x \cdot c^t - 1 \cdot \log_e g}$$

Put $\log_e g = \gamma$, and we have

$${}_t p_x = v^t s^t e^{c^x \cdot c^t - 1 \cdot \gamma}.$$

Expand the last factor by the Exponential Theorem, and we have

$${}_t p_x = v^t s^t \left[1 + c^x(c^t-1)\gamma + \frac{c^{2x}(c^t-1)^2\gamma^2}{2} + \frac{c^{3x}(c^t-1)^3\gamma^3}{3} + \dots \right] \quad (4)$$

Now if $vs = v_0$, $vs c = v_1$, $vs c^2 = v_2$, and so on, this reduces to

$$\begin{aligned} v_0^t + c^x \gamma \cdot [v_1^t - v_0^t] + \frac{c^{2x} \gamma^2}{2} [v_2^t - 2v_1^t + v_0^t] + \dots \\ = v_0^t + c^x \gamma \Delta(v_0^t) + \frac{c^{2x} \gamma^2}{2} \Delta^2(v_0^t) + \dots \end{aligned} \quad (5)$$

Summing for all values of t from 1 to n , both inclusive, and denoting $\sum_1^n v_0$ by a_0 , $\sum_1^n v_1$ by a_1 , and so on, we have

$$\sum_1^n v^t {}_t p_x = a_{x:n} = a_0 + c^x \gamma \cdot \Delta a_0 + \frac{c^{2x} \gamma^2}{2} \Delta^2 a_0 + \dots \quad (6)$$

which may be expressed in the symbolic form

$$e^{c^x \gamma \Delta} a_0 \dots \dots \dots (6a)$$

* *Vide* pars. 29 *et seq.*

where Δ is to be interpreted as a symbol of operation acting on a_0 only.

(7). It will be seen that $a_0, a_1, a_2 \dots$ represent the values of annuities for a uniform term of n years, but calculated at successively diminishing forces of interest; thus we have $\delta_0 = \log_e(1+i) - \log_e s$; $\delta_1 = \delta_0 - \log_e c$; $\delta^2 = \delta_0 - 2 \log_e c$, and so on. At any ordinary rate of interest, $\log_e(1+i)$ is so much smaller than $\log_e c$ that $\delta_1, \delta_2 \dots$ are *negative*, and the effect is that (except when n is small) $a_0, \Delta a_0, \Delta^2 a_0 \dots$ form a rapidly increasing series. This appears from the following statement, in which the normal rate of interest is taken to be 3 per-cent, and the values of s and c are those applicable to the $O^{M(5)}$ Table.

$n=10$					$n=20$				
		Δ	Δ^2	Δ^3		Δ	Δ^2	Δ^3	
a_0	8.271				a_0	14.074			
a_1	13.649	5.378			a_1	23.080			
a_2	24.039	10.390	5.012		a_2	88.51	65.43		
a_3	44.949	20.910	10.520	5.508	a_3	385.71	297.20	231.77	

$n=30$					$n=40$				
		Δ	Δ^2	Δ^3		Δ	Δ^2	Δ^3	
a_0	18.145				a_0	21.001			
a_1	77.632	59.487			a_1	147.34	126.34		
a_2	555.22	477.59	418.10		a_2	2371.1	2223.8	2097.5	
a_3	5351.2	4796.0	4318.4	3900.3	a_3	55572.	53201.	50977.	48880.

(8). Returning to equation (6), it will be found that the successive terms in the expansion of a_{x^n} , so far as they depend on a_0 and its differences, will tend to diverge with increasing rapidity as n increases; and so far as they depend on c^x and its powers, the divergence will be still further increased as x increases; but this tendency to divergence is counteracted by the very rapid diminution in the values of the remaining factors, involving powers of γ . The resultant effect, as will be observed from the following Table, is that the terms converge rapidly for all the ages and terms that occur in practice.

TABLE A.

n	x	(1) a_0	(2) $c^x \gamma \Delta a_0$	(3) $\frac{c^{2x} \gamma^2}{2} \Delta^2 a_0$	(4) $\frac{c^{3x} \gamma^3}{3} \Delta^3 a_0$	(5) (1)+(2)+(3)	(6) $a_{x:n}$	(7) (6)-(5)
10	20	+8.271	- .037	+ .000	- .000	8.234	8.234	...
	25	"	- .059	+ .000	- .000	8.212	8.213	+ 1
	30	"	- .092	+ .001	- .000	8.180	8.180	...
	35	"	- .144	+ .002	- .000	8.129	8.129	...
	40	"	- .226	+ .004	- .000	8.049	8.050	+ 1
	45	"	- .354	+ .011	- .000	7.928	7.928	...
	50	"	- .554	+ .027	- .001	7.744	7.743	- 1
	55	"	- .868	+ .065	- .004	7.468	7.465	- 3
	60	"	- 1.360	+ .160	- .015	7.071	7.058	- 13
20	20	+11.074	- .161	+ .002	- .000	13.915	13.915	...
	25	"	- .252	+ .004	- .000	13.826	13.826	...
	30	"	- .394	+ .010	- .000	13.690	13.689	- 1
	35	"	- .618	+ .023	- .001	13.479	13.479	...
	40	"	- .968	+ .058	- .003	13.164	13.160	- 4
	45	"	- 1.517	+ .141	- .011	12.698	12.687	- 11
	50	"	- 2.377	+ .347	- .042	12.044	12.005	- 39
30	20	+18.145	- .414	+ .010	- .000	17.741	17.741	...
	25	"	- .649	+ .025	- .001	17.521	17.520	- 1
	30	"	- 1.017	+ .061	- .003	17.189	17.186	- 3
	35	"	- 1.593	+ .150	- .012	16.702	16.690	- 12
	40	"	- 2.496	+ .368	- .048	16.017	15.974	- 43
40	20	+21.001	- .880	+ .051	- .003	20.172	20.169	- 3
	25	"	- 1.378	+ .125	- .011	19.748	19.737	- 11
	30	"	- 2.159	+ .306	- .041	19.148	19.111	- 37

(9). Consider now a group of policies in which n is constant but x varies, and let the "weight" at age x be S_x ; then it has been shown in paragraph 4 that we require to find a mean age y , such that

$$a_{y:n} [S_x + S_{x+1} + \dots] = a_{x:n} S_x + a_{x+1:n} S_{x+1} + \dots$$

(10). Now by equation (6) we have

$$a_{x:n} S_x = a_0 S_x + c^x S_x (\gamma \Delta a_0) + c^{2x} S_x \left[\frac{\gamma^2 \Delta^2 a_0}{2} \right] + \dots$$

$$a_{x+1:n} S_{x+1} = a_0 S_{x+1} + c^{x+1} S_{x+1} (\gamma \Delta a_0) + c^{2x+1} S_{x+1} \left[\frac{\gamma^2 \Delta^2 a_0}{2} \right] + \dots$$

.

If we find ages $y_1, y_2 \dots$ such that

$$\Sigma c^x S_x = c^{y_1} (\Sigma S_x) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\Sigma c^{2x} S_x = c^{2y_2} (\Sigma S_x) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

.

we shall have

$$\Sigma(S_x a_{x\bar{n}}) = a_0 \Sigma S_x + (\gamma \Delta a_0) c^{y_1} (\Sigma S_x) + \left(\frac{\gamma^2 \Delta^2 a_0}{2} \right) c^{2y_2} (\Sigma S_x) + \dots \quad (9)$$

If for an approximation we take

$$\begin{aligned} \Sigma(S_x a_{x\bar{n}}) &= (\Sigma S_x) a_{y_1 \bar{n}} \\ &= a_0 \Sigma S_x + c^{y_1} \Sigma S_x (\gamma \Delta a_0) + c^{2y_1} \Sigma S_x \left(\frac{\gamma^2 \Delta^2 a_0}{2} \right) + \dots \quad (10) \end{aligned}$$

(that is, if the mean age y_1 derived from the second term be assumed to apply to all the subsequent terms), the first two terms in (9) will be exactly reproduced, and the error due to subsequent terms will be

$$\begin{aligned} (10) - (9) &= -(\Sigma S_x) \left[\frac{\gamma^2 \Delta^2 a_0}{2} (c^{2y_2} - c^{2y_1}) \right] + \dots \\ &= -(\Sigma S_x) \left[\frac{\gamma^2 \Delta^2 a_0}{2} c^{2y_1} (c^{2y_2 - 2y_1} - 1) \right] + \dots \quad (11) \end{aligned}$$

(11). Now it may be shown generally that, c being > 1 and $S_x, S_{x+1} \dots$ being all positive, $y_1 < y_2 < y_3 \dots$

The error due to the first term in (11) will therefore be negative, and the error due to subsequent terms will obviously be $\Sigma S_x \times$ (a negligible factor), since the terms diminish in value with great rapidity. It follows that the approximate result found by using the mean age y_1 will be too small; we proceed to show that the error will be *very* small, and to find an approximation to its extreme value in ordinary practice.

(12). In order to do this, some assumption must be made as to the distribution of the sums assured according to valuation age. First assume that the main body of the assurances in the principal groups are distributed either

(a) In twenty equal parts at each integral age from x to $x + 19$; or

(b) In four equal parts at quinquennial intervals, *i.e.*, at ages $x, x + 5, x + 10, x + 15$.

Then we shall find, taking $\log_{10} c = .039$,

(a)	(b)
$y_1 - x = 10.96$	$y_1 - x = 8.87$
$y_2 - x = 12.22$	$y_2 - x = 10.04$
$y_2 - y_1 = 1.26$	$y_2 - y_1 = 1.17$

(13). The above assumption is, however, an extreme one, and in practice the sums assured are much more likely to be distributed in a curve such that the sums assured will be greatest in the region near the mean age, and will gradually decrease as the

ages differ more and more, either in excess or defect, from the mean. This condition is favourable to the application of the method, as it reduces the difference between y_2 and y_1 . For example, let the assurances be distributed, as before, at ages x , $x+5$, $x+10$, and $x+15$, but in the proportions of 2, 3, 3, 2. Then we shall find

$$\begin{array}{rcl} y_2 - x & = & 9.69 \\ y_1 - x & = & 8.66 \\ \hline y_2 - y_1 & = & 1.03 \end{array}$$

as compared with 1.17 when the distribution is assumed to be equal.

(14). It seems fair to assume that in any ordinarily distributed business—

- (1) The value of $y_2 - y_1$ will not exceed 1.50,* and will usually be less;
- (2) The average value of y_1 will not exceed $60 - n$, and will usually be less.

Now, if $y_2 - y_1$ be not > 1.50 , $e^{2y_2 - 2y_1} - 1$ will not be $> .3$. Hence, returning to equation (11), the error will not exceed

$$-\Sigma S_x \times \left[.3 \frac{\gamma^2 \Delta^2 a_0}{2} e^{2y_0 - n} \right],$$

disregarding the terms after the first as relatively insignificant. The expression in square brackets represents the error in the mean annuity-value, equal to $.3 \times$ (the third term in the expansion of a_{y_1} according to formula 6), and, by reference to Table A, its value corresponding to different values of n will be found to be as follows :

	Mean Error in Annuity-value $a_{60-n} \bar{n}$	Corresponding Mean Error in Assurance-value $A_{60-n} \bar{n}-1$
$n=10$	-.008	+ .0002
$n=20$	-.017	+ .0005
$n=30$	-.018	+ .0005

(15). These results may be confirmed in an interesting manner by an entirely different line of investigation, which was adopted in the former paper, and which will be found useful when we come to deal with Tables which do not follow Makeham's Law.

* This value, which is higher than those brought out in the preceding paragraphs, has been selected in order to allow for the effect of cases falling outside the assumed range of 20 years.

(16). It has been shown (*J.I.A.*, xxxiv, 63-4) that

$$\frac{d}{dx} a_{x:\overline{n}|} = -Bc^x \int_0^n v^t (c^t - 1)_t p_x \cdot dt.$$

Now if the expression under the integral sign had a constant value, $\frac{d}{dx} \bar{a}_{x:\overline{n}|}$ and $\Delta a_{x:\overline{n}|}$ (n remaining constant and x varying) would form a G.P. with the common ratio c per annum, and in this case the mean age y_1 (*vide* par. 10) would be exactly correct. But, in fact, the integral *decreases* in value as x increases; hence $\Delta \bar{a}$ and Δa *increase at a slower rate than c per annum*, say at r per annum, and the mean age should therefore be somewhat $< y_1$. By finding an approximate value for r we may measure the error involved in y_1 , and hence the error in the mean annuity-value $a_{y_1:\overline{n}|}$. An approximate value of r may be found as follows:

Let y be the mean valuation age.

Let t be a term of years such that the bulk of the business falls within a range of t years on either side of the mean valuation age for the group, *i.e.*, between $y+t$ and $y-t$. Then an approximate average value of r will be

$$\left[[a_{y:\overline{n}|} - a_{y+t:\overline{n}|}] \div [a_{y-t:\overline{n}|} - a_{y:\overline{n}|}] \right]^{\frac{1}{t}}$$

(17). Using suitable assumed values of t and y , we shall find the following values of r for the $O^{M.51}$ Table, 3 per-cent interest, while for other rates of interest the values of r will be very little different.

TABLE B.—*Values of r . $O^{M.51}$ Table.*

n	ASSUMED VALUE OF			Resulting Value of r	$c-r$
	$y+n$	t	$2t = \text{range}$		
10	60	10	20	1.084	.010
20	"	"	"	1.081	.013
30	"	"	"	1.081	.013
40	65	5	10	1.077	.017

(18). It was remarked in par. 13-14 that 1.50 might usually be taken as the maximum value of $y_2 - y_1$, where

$$y_2 = \text{the mean age derived from } \sum S_x c^{2x} \text{ (common ratio } c^2 = 1.197)$$

$$y_1 = \text{ " " " " " } \sum S_x c^x \text{ (" " " " } c = 1.094)$$

The difference $y_1 - y_r$ (where y_r is the mean age derived on the assumption that the common ratio is r) will be roughly proportional to the differences in the common ratios, *i.e.*, nearly equal to

$$1.5 \times \frac{c-r}{c^2-c} = 1.5 \times \frac{1.094-r}{.103}$$

Applying this formula to the values of r given in the preceding paragraph, we get

n	r	$y_1 - y_r$	$a_{y_1\bar{n}} - a_{y_r\bar{n}}$
10	1.084	.15	-.607
20	1.081	.19	-.015
30	1.081	.19	-.019

where the figures given in the last column are deduced from Table A on the assumption (as in par. 14) that y_1 equals $60 - n$.

(19). It will be seen that the mean errors in the annuity-values agree very fairly with those previously deduced in par. 14 by a totally different process, but on the same assumptions as to range of ages, &c.* These assumptions were purposely chosen so as to give an extreme value of the mean error, and it will probably be within the mark to say that over the whole business the mean error in the annuity-value will not exceed .015, and the mean error in the assurance-value will not exceed .0004. As the approximate annuity-value is slightly too *small*, and the approximate assurance-value slightly too *large*, the small error in the reserves will be on the side of stringency; it may therefore be said that the method gives practically exact results, with a very small "factor of safety", when the Mortality Table rigidly follows Makeham's Law.

III.—APPLICABILITY OF THE METHOD TO TABLES NOT FOLLOWING MAKEHAM'S LAW.

(20). In Part II it has been shown by analytical methods, illustrated numerically by reference to the $O^{M(5)}$ Table,† that in the case of a Mortality Table which strictly follows Makeham's

* The assumptions have been based on the author's observations in respect of Endowment Assurances in his own office. It is thought that they will apply with sufficient accuracy to most Endowment Assurance business in Great Britain, but any Actuary wishing to calculate the extreme mean error on different assumptions can easily do so by means of Table A. [Cf. Mr. Thomson's remarks in the discussion and the author's comments thereon, p. 47.]

† If the usual range of Makeham's constants be considered, it will be found that the general results must be very similar in any ordinary Mortality Table following Makeham's Law.

Law, a very close approximation will be secured by grouping the Policies as mentioned in par. 3, and valuing all the Policies in any group by the annuity-value and assurance-value corresponding to the mean age found as described above. If, however, the Table does *not* follow Makeham's Law, the analysis does not hold, and special investigation will be required, in each case, to determine how far the method will be applicable.

(21). There are two principal ways of making such an investigation, and it will be convenient to exemplify them by an application to the exceedingly important case of the O^M Table.

(i) A Table which does not follow Makeham's Law through the whole of its course may yet follow it sufficiently closely for practical purposes in that part which is mainly required in connection with Endowment Assurances—say from ages 25–65. If, then, this part of the Table can be successfully re-graduated by Makeham's Law, the resulting value of c will evidently give the means of applying the method with good results.

(ii) An investigation on the lines of pars. 16–18 will furnish an independent means of obtaining a common ratio r by which to calculate the mean age, and the closeness of the agreement between the various values of r derived from annuities of different terms will be a measure of the suitability of the method.

(22). For the purposes of this Paper the author has made two re-graduations (following Makeham's Law) of the portion of the O^M Table from age 25 to age 64.

In the first place the well-known "Aggregate Method" (*vide Text-Book*, Part II, chap. VI) was used, and the following values were determined for A , B and c where $\mu_x = A + Bc^x$.

AGGREGATE METHOD.

$$A = \cdot 003492$$

$$B = \cdot 0002612$$

$$c = 1\cdot 0784782$$

$$\log_{10} c = \cdot 03281139$$

These constants give the following values of μ_x as compared with the original O^M μ 's.

TABLE C.—Values of μ_x .

x	O ^M	Re-graduation by Aggregate Method	Difference	x
25	·00473	·00522	+·00049	25
30	·00584	·00601	+·00017	30
35	·00725	·00717	-·00008	35
40	·00900	·00885	-·00015	40
45	·01132	·01132	...	45
50	·01473	·01491	+·00018	50
55	·02000	·02015	+·00015	55
60	·02824	·02780	-·00044	60
65	·04121	·03896	-·00225	65

(23). A second re-graduation was made by a modification of Karl Pearson's Method of Moments (vide *J.I.A.*, xxxiii, 531), similar to that employed with such brilliant success by Mr. G. F. Hardy, in his graduation of the British Offices Life Tables. The function operated on was $\text{colog}_{10} \cdot p_x$, which, in a Table following Makeham's Law, is of the form

$$\cdot 43429448 \times \left[A + B \frac{c-1}{\log_e c} \cdot c^x \right] = a + \beta c^x, \text{ say,}$$

where, as before, $\mu_x = A + Bc^x$.

Putting $\Sigma^1 = \Sigma \text{ colog}_{10} (p_x)$ for ages 25-64 inclusive

$$\Sigma^2 = \Sigma(\Sigma^1) \quad \text{,,} \quad \text{,,} \quad \text{,,}$$

$$\Sigma^3 = \Sigma(\Sigma^2) \quad \text{,,} \quad \text{,,} \quad \text{,,}$$

the values of A, B and c were determined subject to the condition that they should exactly reproduce the values of Σ^1 , Σ^2 and Σ^3 deduced from the O^M Table. This is practically equivalent to reproducing the 1st, 2nd and 3rd "moments" respectively of the function $\text{colog}_{10} \cdot p_x$, and leads to the following equations:

$$\Sigma^1 = 40a + \beta c^{40} \cdot \frac{c^{40} - 1}{c - 1}$$

$$\Sigma^2 = \frac{40 \times 41}{2} a + \beta c^{40} \cdot \frac{40c^{40} - \frac{c^{40} - 1}{c - 1}}{c - 1}$$

$$\Sigma^3 = \frac{40 \times 41 \times 42}{2 \times 3} a + \beta c^{40} \cdot \frac{\frac{40 \times 41}{2} c^{40} - \frac{40c^{40} - \frac{c^{40} - 1}{c - 1}}{c - 1}}{c - 1}$$

(24). The results are as follow :

METHOD OF MOMENTS.

$$\begin{aligned} a &= \cdot 001510 \\ \beta c^{25} &= \cdot 0007370 \\ A &= \cdot 003477 \\ B &= \cdot 0002329 \\ c &= 1\cdot 0809873 \\ \log_{10} c &= \cdot 0338205 \end{aligned}$$

TABLE D.—*Values of μ_x .*

x	O^M	Re-graduation by Method of Moments	Difference	x
25	·00473	·00511	+ ·00038	25
30	·00584	·00589	+ ·00005	30
35	·00725	·00704	— ·00021	35
40	·00900	·00873	— ·00027	40
45	·01132	·01123	— ·00009	45
50	·01473	·01491	+ ·00018	50
55	·02000	·02036	+ ·00036	55
60	·02824	·02839	+ ·00015	60
65	·04121	·04025	— ·00096	65

(25). It will be seen that the two adjustments give almost identical values of A ; but the second gives a lower value of B , coupled with a slightly higher rate of increase (c). Both graduations give an abnormally low value of c , and both sensibly twist the curve out of its course, showing (as might have been expected) that the O^M Table is not really suited to a Makeham graduation. But the results are sufficiently close to the original O^M values at the principal ages to justify the anticipation that the method of valuation which is the subject of this Paper will give satisfactory results if the mean ages be based on a value of c equal to about 1·08.

(26). Let this result be tested by the second plan referred to in par. 21. Taking the same values of y and t as previously (*vide* par. 17), the resulting values of r will be as follow, the calculations being based on annuity-values at 3 per-cent interest.

TABLE E.—*Values of r . O^M Table.*

n	ASSUMED VALUE OF			Resulting Value of r	$c - r$ taking $c = 1.08$
	$y + n$	t	$2t = \text{range}$		
10	60	10	20	1.082	-.002
20	"	"	"	1.070	+.010
30	"	"	"	1.057	+.023
40	65	5	10	1.055	+.025

(27). Comparing the above values of r with those given for the O^{M.5} Table in Table B (*vide* par. 17), it will be seen at once that the O^M values change much more rapidly than the O^{M.5} as n , the term of the annuity, increases. It follows that if the mean age be in every case based upon the uniform ratio c , the differences between the *true* mean age and *approximate* mean age will be less uniform for the O^M Table than for the O^{M.5}.

In the latter case the constant ratio c and the mean age y are *always slightly too high*; while in the former case these quantities are very slightly *too low* for the shorter terms, slightly *too high* for the medium terms, and increasingly *too high* as the terms become long. Thus instead of the method producing reserves which are slightly too high throughout, there will be a certain balance of error, and it is believed that in an ordinarily-constituted business, the method as applied to the O^M Table should give results nearly as satisfactory in the aggregate as those produced in the case of the O^{M.5} Table. It is probable, however, that the small excess of reserve produced by the use of the method will be slightly greater for the O^M Table than for the O^{M.5}, especially if there be a large proportion of Policies having a long unexpired term. (See remarks on p. 49.)

(28). It is possible that in some instances slightly improved results might be obtained, in the case of the O^M Table, by basing the Z's on the common ratio, 1.075, instead of 1.08; but the difference would be very small and the higher ratio is on the side of safety, and for this reason it has been adopted in preparing the Table of Z.

IV.—CALCULATION OF THE MEAN AGES: THE FUNCTION Z.

(29). It will be seen from par. 10 that the mean valuation age, y , in any group will be found from the equation

$$S_x c^x + S_{x+1} c^{x+1} + \dots = (S_x + S_{x+1} + \dots) c^y \quad . \quad . \quad (12)$$

$$\text{or} \quad c^y = \frac{\sum S_x c^x}{\sum S_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\text{or} \quad c^{y+n} = \frac{\sum S_x c^{x+n}}{\sum S_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where, for each separate valuation age in the group, S_x represents the quantity which is to be valued, *i.e.*, the Sum Assured, Bonus Additions, Office Premium, Net Premium, or Reduction of Premium as the case may be. In theory the method requires that a separate mean age be determined in each case, and we shall discuss later whether this is necessary in practice. Using S_x in its widest sense, it will be seen from equation (12) that if the product $S_x c^x$ be formed for each age, and the sum of the products, $\sum S_x c^x$, be divided by $\sum S_x$, the quotient will give c^y ; whence y (the mean age required) may be found by entering inversely a Table of c^y or directly from the equation $y = \frac{\log (\text{Quotient})}{\log c}$.

(30). But this process, although the most obvious, is not the most convenient, since it renders it necessary to form the product $S_x c^x$ afresh at each valuation. Equation (14) shows that instead of $S_x c^x$ we may use $S_x c^{x+n}$, where n is a quantity which is independent of x , but is not necessarily the same at each valuation; then $\sum S_x c^{x+n} \div \sum S_x$ gives c^{y+n} where $y+n$ is the valuation age n years hence. Put n equal to the number of years from the valuation date to the end of the financial year in which the assurance matures (or is assumed by the valuation formulæ to mature). Then $y+n$ will give what may be called the "maturity age", say M , which will evidently be constant during the whole currency of the Policy, and the value of $S \times c^M$ may therefore be found and recorded once and for all at the inception of the Policy. It will evidently make no difference if $S_x \times c^M$ be multiplied by a constant factor, say c^κ , giving $S_x \times c^M \times c^\kappa$, provided the value of M be determined by entering inversely a Table of the same function; and it will be desirable to do this in order to reduce the size of the quantities to be operated on. Let us then put $\kappa = -55$, and write Z_M for $S \times c^{M-55}$. Then we shall have

$$\text{Mean value of } Z \text{ per unit assured} = \frac{\sum Z_M}{\sum S_M}$$

$$y = \text{mean valuation age} = (\text{mean value of } M) - n.$$

Here we write S_M instead of S_x , because the quantities S and Z are now considered as functions of M (the maturity age) only.

(31). Extended Tables of Z_M , and of its difference for tenths of a year of age, have been prepared for all values of M from 35 to 75, on the basis of the H^M , O^M and $O^{M(5)}$ Tables of Mortality; the first will be found in the *J.I.A.*, xxxiv, 80-1, and the second and third are appended to this Paper. It will be useful to give a practical example of their application.

(32). Suppose that we are valuing a group of Policies which will mature exactly 10 years hence, so that $n=10$. Let the sums assured be 1,000 at valuation age 40; 2,000 at valuation age 45; 2,500 at valuation age 50; and 1,500 at valuation age 55. Then, inserting the values of Z from the prepared Tables, we shall have the following results.

Valuation Age x	$x+n$ $=M$	S_M	$Z_M(H^M)$	$Z_M(O^M)$	$Z_M(O^{M(5)})$
40	50	1,000	633	681	638
45	55	2,000	2,000	2,000	2,000
50	60	2,500	3,948	3,673	3,918
55	65	1,500	3,738	3,239	3,682
...	...	7,000	10,319	9,593	10,238

	H^M	O^M	$O^{M(5)}$
$\Sigma Z_M \div \Sigma S_M$	1.474	1.370	1.462
Corresponding Value of M	59.2	59.1	59.2
„ „ „ $y=M-n$	49.2	49.1	49.2

(33). The value of M is found by inspection of the Table. Thus, taking the $O^{M(5)}$ values, we have $\Sigma Z \div \Sigma S = 1.462$, corresponding to 1,462 in the column headed 1,000 in the Table of Z . The next lower tabular value in that column is the one corresponding to age 59, namely, 1,432; deducting this from 1,462 we have the remainder 30, and the nearest tabular difference (on the line for age 59) being that corresponding to the column 2, we have finally $M=59.2$.

(34). The quotient $\Sigma Z \div \Sigma S$ can be calculated, with sufficient accuracy for all practical purposes, by means of the invaluable Crelle's Tables, or the similar Tables of Cotsworth. These will give by inspection three figures in the quotient, so that the greatest error will be 5 in the fourth figure, corresponding to about $\frac{1}{20}$ of a year of age in the Table of Z .

(35). It will be seen that in the above example the mean age is precisely the same for the H^M and $O^{M(5)}$ Tables, and having regard to the close agreement in the respective values of $\log_{10} c$ (namely, $\cdot 03966$ for the H^M and $\cdot 039$ for the $O^{M(5)}$), it is evident that the mean ages can never sensibly differ. It follows that the H^M Z's can be safely used for an $O^{M(5)}$ Valuation (thus avoiding the necessity for recalculating the Z's on a change of Valuation Table), *provided they be used throughout*; but it would obviously be incorrect to use *mixed* values of H^M and $O^{M(5)}$ Z's in the same group, for in that case there would be no fixed standard of reference by which to deduce the mean age.

(36). While the H^M and $O^{M(5)}$ mean ages agree very closely, the O^M value is sensibly less than either. Thus the effect of using H^M or $O^{M(5)}$ Z's for an O^M Valuation would be to bring out too high a mean age, and therefore too high a reserve, but the difference would appear to be slight.

V.—THE QUESTION OF EMPLOYING SEPARATE MEAN AGES FOR SUMS ASSURED, NET PREMIUMS, &c.

(37). It has been remarked in par. 29 that in strictness the mean valuation age should be determined separately for each of the quantities which fall to be valued, namely:—

- (a) Sums Assured.
- (b) Bonus Additions.
- (c) Reductions of Premiums.
- (d) Office Premiums.
- (e) Net Premiums.

It is clear, however, that if this had actually to be done the advantage of the method would be greatly reduced, even if it were not entirely forfeited, and it is satisfactory to be able to state that in practice it is usually found that the several mean ages differ so slightly that the one which is derived from the Sums Assured may safely be used throughout. This appears plainly from a comparison between the results of a detailed valuation (age by age) and a group valuation by the Z method based on one set of mean ages (*vide J.I.A.*, xxxiv, 66, and 510–4, and the Appendix hereto); but in the absence of any ascertainable law as to the relative volume of business for different entry ages and maturity ages, the result cannot be established by a rigorous theoretical investigation. The following remarks will, it is hoped, throw some light on the question.

(38). Consider the mean age (say Y) derived from the Net Premiums, as compared with that based on the Sums Assured, which has been denoted by y ; for any given group the latter will be determined (par. 29) from the equation

$$S_x c^x + S_{x+1} c^{x+1} + \dots = (\Sigma S_x) c^y$$

or

$$S_x c^{-(y-x)} + S_{x+1} c^{-(y-x-1)} + \dots + S_{y-1} c^{-1} + S_y + S_{y+1} c + \dots = \Sigma S_x$$

(15)

where S_x represents the total Sums Assured at valuation age x in the group. If π_x represent the average rate of net premium on the policies included in S_x , so that the total net premiums corresponding to S_x will be $S_x \pi_x$, the mean age Y will be determined by the similar equation

$$\Sigma (S_x \pi_x) = S_x \pi_x c^{-(Y-x)} + S_{x+1} \pi_{x+1} c^{-(Y-x-1)} + \dots$$

$$+ S_y \pi_y + S_{y+1} \pi_{y+1} c + \dots \quad (16)$$

(39). It is evident that if π_x have a constant value, Y and y will be absolutely identical; and if π_x be subject to merely casual variation (so that at any age a value in *excess* of the mean is as likely to occur as a value in *defect*), it will be highly improbable that y and Y will (on an average of cases) differ sensibly. But if π_x systematically $\frac{\text{increase}}{\text{decrease}}$ as we pass from the youngest to the oldest age, it is clear that the effect on Y will be the same as if the proportionate amount assured at the oldest ages were $\frac{\text{increased}}{\text{decreased}}$ (leaving π_x constant); that is to say, Y will be $\frac{\text{greater}}{\text{less}}$ than y .

(40). It thus appears that the extent to which Y will differ from y depends upon the direction and extent of the change in π_x as x increases. Let us take now a particular case, say unexpired term 20 years, and analyze the composition of the business. For the purpose of illustration it will be sufficient to deal only with quinquennial ages and durations, though in practice the intervening ages and durations will also occur.

Unexpired Term, 20 years.

Age at Entry	Term Completed	Valuation Age	Maturity Age	Term of Assurance from Entry to Maturity.	Average Rate of Premium 11½ 3 per-cent
20	0	20	40	20	·03646
20	5	25	45	25	·02930
25	0	25	45	20	·03870
20	10	30	50	30	·02364
25	5	30	50	25	·02985
30	0	30	50	20	·03947
20	15	35	55	35	·01988
25	10	35	55	30	·02431
30	5	35	55	25	·03076
35	0	35	55	20	·04046
20	20	40	60	40	·01733
25	15	40	60	35	·02070
30	10	40	60	30	·02540
35	5	40	60	25	·03199
40	0	40	60	20	·04190

(41). It will be seen from the above scheme that if the sums assured were the same for each possible combination of entry ages and durations, the policies on lives aged 40 would have, on the average, a longer term of assurance (from entry to maturity), and a smaller net premium than those on lives aged 35; these again would have a longer term and a smaller net premium than policies on lives aged 30; and so on: thus the average rate of premium would tend to *decrease* as the valuation age increased; and *Y* would tend to be slightly lower than *y*, as shown in par. 39. This tendency, however, may be checked, if not counteracted, by other considerations.

- (i) It is probable that the proportion of Policies effected, and the average amount of each Policy, will be greater for the middle ages at entry than for the young ages.
- (ii) For any given valuation age the Policies on lives of the middle entry ages must have been a shorter time in force, and will, therefore, have been reduced to a smaller extent by deaths, surrenders and discontinuances than the Policies on lives of the youngest entry ages.

- (iii) * The continued increase, which for many years past has taken place in the amount of Endowment Assurance business transacted, will still further proportionately increase the assurances which have the shortest duration, and therefore the highest entry-ages.

(42). All these conditions act in the same direction, and their effect is to counteract *pro tanto* the fall in the average rate of premium, which (as we have seen in par. 41) might otherwise be expected to occur as the valuation age increased. Theory does not enable one to say what the resultant effect of these conflicting forces will be, but enough has been said to show generally that no very marked change in the average rate of premium need generally be anticipated, and that, accordingly, Y and y should agree very closely, as they are found to do in practice. It will, however, be prudent for each Office to test the question for a few representative groups. All that is necessary is (in each of such groups) to calculate the average rate of premium at each valuation age: if no marked systematic progression in the rates be observed it will be safe to use the same mean age y for Sums Assured and Net Premiums; if, on the other hand, such a progression be detected, a separate mean age Y should be calculated, and this will at once show whether the difference between y and Y is of any practical importance.

(43). It seems hardly necessary to consider specially the question of a separate mean age for the Office Premiums. The percentage of loading changes so slowly, and within such comparatively narrow limits, that the average rates of Office Premiums will evidently follow a curve closely resembling that of the Net Premiums, and therefore the same mean age Y can safely be used for the Net Premiums and the Office Premiums, whether this be identical with y (the mean age for the Sums Assured) or not.

(44). Passing to the Bonus Additions we cannot do better than quote the luminous remarks of Mr. G. F. Hardy (*J.I.A.*, xxxiv, 86), who said, in the discussion on the earlier Paper—

“If a separate calculation were not made in respect
“of the Premiums a better result would then be obtained

* In the case of a Company which has been transacting a fairly uniform Endowment Assurance for a long period this feature will be absent, and it will then be specially desirable to institute a test such as that referred to in par. 42.

“ by using Sums Assured only [in calculating the mean age], and not Sums Assured plus Bonus Additions. The reason for that was that, given Policies having a uniform term to run, those Policies which had been longest on the books, and which, therefore, would have the largest reversionary bonuses, would, of necessity, have the smallest premiums, and, as the effect of the Bonuses acted in the opposite direction, there would be a counteracting influence, and the error in computing the ages would be less by taking simply the sums Assured than by taking the Sums Assured and the Bonuses.”

It is thought that, unless the Bonus System be a very special one, it can but rarely be necessary to use a separate mean age for the Bonuses, or to find a mean age for the Sums Assured *plus* Bonuses, and similar remarks apply to Reductions of Premiums, unless these be very considerable. But should it be thought desirable in any particular case, the question can easily be investigated by a process similar to that described above in relation to the Net Premiums, the only difference being that the ratio [Bonus Additions at age x] : [Sums Assured at age x] will take the place of π_x .

VI.—PRACTICAL APPLICATION OF THE METHOD.

(45). We now proceed to consider the best mode of working out the details of the method in practice. And first as to the computation of the auxiliary function Z . The most convenient plan will be found to be as follows :

A. By means of a prepared Table, such as those appended to this Paper, calculate the Z for each individual Policy at its inception (in the same way as the net premium), and record it on the valuation card once and for all. In the great majority of cases this can be done at sight, without any calculation whatever. When the Policies are scheduled for valuation, a column will be provided in which the Z 's will be entered, side by side with the sums assured, and they will be totalled with the rest of the particulars.

Then we shall have ΣSA and ΣZ , by means of which the mean age may be determined.

(46). The computation of the Z's by means of the Table is such a simple matter that a very large number of Policies may be dealt with in a short time. If there are a few "fashionable" maturity ages, such as 45, 50, 55, 60 and 65, embracing the bulk of the business, it will be found convenient for checking purposes to sort the cards into groups according to the maturity ages, and to subdivide each group according to the sums assured. Each subdivision will then have a constant value of Z, and thus the checking may be done very rapidly by simple inspection. The author's practice is to deal with a year's business at a time, except in the valuation year, when of course the computation and checking of the Z's should be done at frequent intervals so as to keep the cards well up to date.

(47). The alternative plan is as follows :

B. Calculate the Z's after the Policies have been scheduled for valuation, and with this view subdivide the Policies in each group into sub-groups, arranged according to the maturity ages. Find the total sums assured in each sub-group and calculate the Z corresponding to each sub-total, in one aggregate sum. For example, if in any sub-group the total sum assured for maturity age 50 be 13,850, the corresponding $O^{M.5} Z$ will be $13,850 \times .638, = 8,837$, which will be inserted in the Schedule.

(48). The advantages of plan A are—

- (i) The work is so simple that it can be done by any intelligent clerk though unskilled in actuarial matters, and the checking (which it will be prudent to entrust to an actuarial clerk) can be spread over the inter-valuation period.
- (ii) The Z's being all calculated and checked before the Policies are scheduled, the Policies in any group can be entered in numerical order, irrespective of the maturity age. This avoids all the difficulty of spacing out the valuation book, which, by method B, is rather troublesome when it is desired to commence the writing-up of the particulars before the actual valuation date or when a continuous Classification Register is kept.

- (iii) When the entry of the particulars has been finally completed, the schedules can be handed out to be cast without any special instructions, whereas the sub-grouping of plan B is a likely source of error, unless very carefully scrutinized.
- (iv) The plan is particularly suited to a continuous classification, as every policy that comes in or goes off carries with it the appropriate Z.

(49). On paper these advantages may not appear to be great, but in practice anyone who has much experience in supervising and controlling the work of valuation will recognize the importance of simplifying the procedure so that the greatest use may be made of unskilled labour, in order to reduce the pressure on the actuarial clerks and free them for more important work. Usually there is an ample supply of clerks who—while incapable of performing any actuarial work—may safely be entrusted with the scheduling and casting of particulars, provided there are no complications of sub-grouping, &c., &c.

(50). The disadvantage of method A, is, that any errors in the Z's that may remain undetected will endure for the whole currency of a Policy. With proper care, however, they should be few and unimportant, and in a large mass of Policies, a few isolated errors, even if they remain undetected, can have but very little, if any, sensible effect on the mean valuation age.

(51). Should it be considered necessary to determine the mean age separately for the net premiums (*vide* pars. 37-44), a separate auxiliary function, which we may call Ω_M , will have to be calculated—where Ω_M is strictly analogous to Z_M and will be equal to (net premium) $\times e^{M-55}$. Here again methods A and B will be available, and the same remarks will apply as to the relative advantages of these methods. If method A be adopted, the best plan will be to construct Tables of Ω for the principal maturity ages, and for each age at entry, rather than to operate directly on the actual net premiums for each Policy. The construction of such tables, once and for all, is a very simple matter, and when this has been done the calculation of the Ω 's will be precisely similar to that of the net premiums. It will, however, be amply sufficient to record the Ω 's correct to one decimal place, and if the mass of business be large, the nearest integer will suffice.

(52). If the mean age is also to be determined separately for

the bonus additions (which it is thought can only be necessary in special cases), method B is practically the only one available, for the labour of forming a special bonus- Z for each Policy at each valuation would be prohibitive, as prepared Tables would not, in this case, be of any great assistance.

(53). It has been suggested that the function Z_M need not be specially calculated, as the mean valuation age can equally well be determined by scheduling the product (sum assured \times the force of mortality at the maturity age). This of course is perfectly true, and obviously the use of any other function of the form $A + Bc^{x+t}$ (where A , B and t may have any constant values whatever) would produce the same results. But it does not appear that any saving of labour would be secured by using the μ 's instead of the Z 's. If the mortality table follows Makeham's Law (and as the method is based on this assumption it cannot in any other case be safely used without special investigation), the value of c is known and the labour of calculating c^x is perfectly inconsiderable, while an extended table for different sums assured is equally required if the μ 's be employed instead of the Z 's to find the approximate mean age. In the absence of any saving of labour it seems most natural to use Z 's based on the simplest function, namely c^x , that will answer the purpose, rather than to use μ , which involves two other quantities, A and B , which do not affect the mean age in any way. Moreover, the Z based on c^x alone has the important advantage that at some particular maturity age (which may be selected arbitrarily and will most conveniently be one of those for which the greatest number of Policies are issued), *the sums assured and the Z 's are identical*; so that for the whole of such Policies (probably a considerable proportion of the whole) the Z 's are known without either calculation or reference to a table.

Methods of grouping the Policies and of Calculating the Maturity Age M.

(54). The main principle which governs the method of grouping is stated in par. (3), but the particular plan to be used in any given case will depend partly on the form of the Policy and partly on other considerations. It will be desirable to consider the principal cases that occur in practice, and to give a detailed statement of the appropriate expressions for the maturity age " M ." It will be assumed that the valuation is made as at

31 December. Whatever method be used to find the valuation ages, the phrase "office year of birth" will be used to denote the year at the end of which the valuation age was 0, *i.e.*, the year which, deducted from the valuation year, will give the valuation age.

(55). First consider the most modern form of Policy, namely, that in which the assurance becomes payable on the Policy anniversary next preceeding the actual day on which the life assured attains the specified age; *i.e.* (in the case of Policies subject to annual premiums), one year after the last annual premium falls due.

I. The simplest method of classification (and, on the whole, the most convenient) is to group the Policies according to the *calendar year* in which the Policies mature; *e.g.*, all the Policies maturing from 1 January to 31 December (both inclusive) in the year 1920 will form one group; those maturing from 1 January to 31 December 1921 will form the next group, and so on. Then M (the maturity age) denoting the valuation age at the end of the calendar year of maturity, we shall have—

$$M = [\text{Calendar year of maturity}] - [\text{Office year of birth}]$$

$$n \left\{ \begin{array}{l} = \text{Number of annual} \\ \text{premiums to be paid} \\ = \text{curtate unexpired} \\ \text{term} \end{array} \right\} = [\quad \quad \quad] - [\text{Year of valuation} + 1]$$

$$y = \text{Mean valuation age} = [\text{Mean value of } M] - [n + 1]$$

$$\text{Annuity-value} = {}_k|a_{y\overline{n}|} = (1 - k) + a_{y\overline{n-1+k}|}$$

$$\text{Assurance-value} = A_{y\overline{n+k}|} = 1 - d(1 + a_{y\overline{n-1+k}|})$$

where k = average interval to next anniversary.

(56). If the valuation ages be the *nearest integral ages* on 31 December in the valuation year, we shall have—

Office year of birth = [calendar year of birth] if the birthday fall between 1 July and 31 December (both inclusive).

Office year of birth = [calendar year of birth - 1] if the birthday fall between 1 January and 30 June (both inclusive).

If the office year of birth be found by this method and recorded on the cards, M will be found at once from the above formula. It may then be checked by the following process, which is so perfectly different from the first that a reliable clerk can safely be allowed to check his own work by the use of the alternative method. In the following scheme $E.A.$ represents the ordinary endowment age, *i.e.*, the age *n.b.d.* at entry + the term of the endowment.

Birthday falling in	Day of Entry falling	Value of M
First half of calendar year .	<i>Before</i> birthday .	$E.A. + 1$
„ „ „	<i>After</i> birthday .	$E.A.$
Second half „ „	<i>Before</i> birthday .	$E.A.$
„ „ „	<i>After</i> birthday .	$E.A. - 1$

(57). If the valuation ages be calculated on the same assumption as that made in deducing the H^M Mortality Experience, namely, that the age next birthday at entry is attained, on the average, at the end of the year of entry, it will be found that M is in all cases equal to $E.A.$ This furnishes an exceedingly convenient practical rule, and it is thought that it may be safely adopted.*

(58). II. Another method of classification is to group the Policies according to the *nearest integral unexpired term*; so that all Policies maturing from 1 July 1920 to 30 June 1921 (both inclusive) will form one group assumed on the average to mature on 31 December 1920, and 1920 may be called the “office year of maturity.” This method will be found fully described (with illustrative schedules) by Mr. H. A. Thomson, *J.I.A.*, xxxiv, 6, 28-9. Taking M as the valuation age on the assumed date of maturity, we shall have—

* It probably understates the valuation age very slightly, but this may be set against the fact that the Z method in itself tends to slightly overstate the age. An investigation of a complete year's endowment assurance business, transacted subject to all the modern conditions of dating back and crowding in business at the end of the year, showed the following results:

Average age by “nearest birthday” method was *greater* than true average age by .019 year.

Average age by method of par. 57 was *less* than true average age by .090 year.

$$M = [\text{Office year of maturity}] - [\text{Office year of birth}]$$

$$n = [\quad , \quad , \quad , \quad] - [\text{Year of valuation}] = \text{mean unexpired term}$$

$$y = \text{mean valuation age} = [\text{Mean value of } M] - n$$

$$\text{Annuity-value} = \begin{cases} 1 + a_{y\overline{n}|} & \text{for Policies maturing in first half of cal. year} \\ a_{y\overline{n}-1|} & \text{,, ,, ,, second ,, ,,} \end{cases}$$

$$\text{Assurance-value} = A_{y\overline{n}}$$

(59). If the valuation ages be the nearest integral ages, we may, as before, find M by two distinct processes, as shown in the following scheme :

Birthday falling in	Day of Entry falling in	Value of M	Value of M by alternative process
First half of cal. year	Second half of cal. year	E.A.	$[\text{Cal. year of maturity}] - [\text{Cal. year of birth}] + 1$
Second half of cal. year	First half of cal. year	E.A. - 1	$[\text{Cal. year of maturity}] - [\text{Cal. year of birth}] - 1$
Both in same half year, the date of entry being <i>earlier</i>		E.A.	} $[\text{Cal. year of maturity}] - [\text{Cal. year of birth}]$
Both in same half year, the date of entry being <i>later</i>		E.A. - 1	

If the valuation ages are found by the plan mentioned in par. 57 we shall have—

$M = \text{E.A.}$ when date of entry falls in second half of calendar year.

$M = \text{E.A.} - 1$ when date of entry falls in first half of calendar year.

(60). A classification in Form II is somewhat less convenient than one in Form I, but, on the other hand, Form II has the great advantage of automatically making a very accurate allowance for the possible unequal distribution of the renewal dates and maturity dates over the calendar year. It will be seen at once that as the renewal dates by Form II are centred round 31 December, and assumed, on the average, to fall *on that date*, the abnormal amount of business which is commonly transacted at the close of the year will not have any disturbing effect as it does in Form I. The great power of the method

is shown by the following figures, deduced from 889 Policies, forming a complete year's new endowment assurance business in the case of a progressive Office.

Average Interval (k) from 31 December to Policy Anniversary.

	True Value	Assumed Value by Form II
Average value of k unweighted	·619	·623
Average value of k weighted in proportion to net premiums	·631	·638

(61). Here it is evident that the policy anniversaries are not at all evenly distributed over the calendar year, yet Form II gives (without any calculation) almost precisely the correct average interval from the valuation date to the anniversary. In using Form I the value of k enters into the calculation, and it is necessary either to determine it separately for each group or to find an average value (usually assumed to be $\frac{1}{2}$) which can be used over the whole business. In practice the value of k will generally be a little greater than $\frac{1}{2}$ because of the large proportion of new business effected in the closing months of the year. It must, however, be observed that the change in the reserve consequent on a deviation from the true value of k (*the valuation age and unexpired term being fixed independently of k*) will be much smaller for Endowment Assurances than for Whole-Life Assurances. In the case of the latter an increase in k will always increase the reserve, whereas the effect on Endowment Assurances will be to *increase* the reserve for Policies recently effected, but to *decrease* the reserves for Policies which have been long in force. An investigation of this question will be found in pars. 24, 25 of the earlier paper (*J.I.A.*, xxxiv, 72, 73), and it is unnecessary here to discuss the subject further.*

(62). The form of Policy referred to in par. 55 has been discussed in full detail partly because it is rapidly superseding other forms, and partly because it is the natural theoretical standard to which others may be conveniently referred. The other form most commonly met with is as follows:—

* An independent investigation by Mr. H. A. Thomson will be found *J.I.A.*, xxxiv, 8-9; but it should be noted that this is based on a different assumption (namely, that the valuation age changes with k).

Sum assured payable on the actual attainment of a given age (*i.e.*, on a specified birthday), not on the preceding Policy anniversary. Number of years' premiums limited to the difference between the Endowment age and the age next birthday at entry.

If t be the average interval between the birthday and the preceding Policy anniversary, effect may be given to the difference between this form and that previously discussed, by using the Assurance Value $A_{y\overline{n+k+t}}$ in place of $A_{y\overline{n+k}}$ if Classification I be used; * or by using $A_{y\overline{n+t}}$ instead of $A_{y\overline{n}}$ in the case of Classification II. For the purpose of grouping the Policies, an *assumed* date of maturity (being the date on which the Policies would mature if issued in the form stated in par. 55) should be entered in the valuation cards: if Classification I be adopted, it will only be necessary to record the assumed *year* of maturity. For example, take a Policy effected in February 1903 on a life aged 30 next birthday (born January 1874) maturing at age 50, *i.e.*, in January 1924. The assumed date of maturity will be February 1923.

In par. 28 of the earlier paper (*J.I.A.*, xxxiv, 74) reasons are given for thinking that the most satisfactory plan is to value the Policies as though they were actually payable on the anniversary preceding the birthday.

Forms of Valuation Schedule.

(63). Since every Office has to deal with its own particular conditions, it is impossible to give forms of schedules which will answer the requirements of every case, but it may be useful to give specimen forms which will be suitable under ordinary conditions, and can easily be adapted to meet special circumstances. If the business is extensive, it will be found convenient to have two forms—one for recording the particulars of individual Policies, and the other for summarizing and valuing the totals of each group. Specimens of each form are given on the following pages, with explanatory notes.

(64). If it is desired to record the aggregate amount of premiums paid from the commencement, an additional column

* If it be assumed (as in par. 57) that the age next birthday at entry is attained (on the average) at the end of the calendar year of entry, we shall have, by Classification I, $A_{y\overline{n+k+t}} = A_{y.n+1}$, which is very convenient for numerical calculation.

should be added to each schedule for the purpose. It may, however, be worth while to point out that in a number of cases the Board of Trade have recently agreed to dispense with this information in respect of Endowment Assurance Policies (though it is required by Schedule VI of the Life Assurance Companies Act, 1870), provided that the remainder of the particulars required by the Schedule be subdivided into groups arranged according to the years of maturity, and that the mean valuation age be given for each group. This can, of course, be done without the smallest difficulty when the valuation has been made by the Z method.

(65). It may be added that although the Board of Trade would now find it difficult to refuse to permit this modification of the Return, in view of the number of precedents, yet they apparently wish their permission to be applied for in each individual case, and they have not up to the present made any general pronouncement on the subject.

[For Schedules, *see* pp. 30 and 31.]

SCHEDULE I.—PRELIMINARY SCHEDULE OF PARTICULARS.

Maturing from 19 to 19 years.
*Unexpired term

Policy No.	Age at Entry	Maturity Age	M	Sum Assured	Z_M	Total Bonus Additions	Annual Office Premium	Total Annual Reduction of Premium	Extra Annual Premium	Annual Net Premium	k	Outstanding Instalments of Net Premium for Current Policy Year.	l (No. of Years' Pre-miums paid during 5 years ending 31 Dec. 19)	l Years' Bonus Addition at 1 per-cent per annum on Sum Assured
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	

NOTES.

- Col. (2). The "Age at Entry" will be the age to which the Net Premiums correspond, *i.e.*, the age next birthday, or age nearest birthday as the case may be.
- " (3). *vide* par. 29 to 36.
- " (5). If the business be considerable, the Z 's may safely be cut down by one figure, but care must be taken to make the necessary adjustment in calculating $\Sigma Z \div \Sigma S$.
- " (11). This is only required if the value of k (*vide* par. 60-1) is to be calculated separately for each group. If it be thought proper to weight k in proportion to the Sums Assured or Net Premiums, the appropriate heading should be substituted.
- " (12). *vide* par. 26-27 of the earlier paper, *J.L.A.*, xxxiv, 73-4, where it is pointed out that, in the case of half-yearly and quarterly Premiums, practically correct results may be obtained by valuing the yearly Net Premiums by the annuity-value which would be used in a yearly case and increasing the result by the actual amount of the unpaid portion of the full Net Premium for the current policy year.
- " (13, 14). If the Profits are allotted according to the Uniform Reversionary Bonus Plan (Simple or Compound) this column affords the means of calculating the cost of a Bonus at any given rate (see notes to columns 22 and 23 of the Valuation Schedule on the next page). If a different system be adopted, the appropriate quantity should be tabulated in place of Column (14).
- The same result will be obtained by selecting in Column (14) the amount by which the New Bonus *will fall short of 5 years' Bonus*, and deducting the total from $\cdot 05 \times$ the total of Column (4). In this case no entry will be required for Policies which rank for 5 years' Profits.

* This will be the *entire* unexpired term by Classification No. 1 (*vide* pars. 55-7) or the *nearest integral* unexpired term of Classification No. 11 (*vide* pars. 58-9).

SCHEDULE II.—SUMMARY OF PARTICULARS AND VALUATION SCHEDULE.

Un- expired Term Policy	No. of Policies	Total Sums Assured	Total Bonus Additions	A	Total Z	(6) (3)	Mean Maturity Age	Mean Valua- tion Age	Total Annual Office Premiums	Total Annual Reduction of Premiums	Total Extra Annual Premiums	Total Annual Net Premiums	a	Average value of £ (Propor- tion of year to next annu- ity year)	Total standing amounts of Net Pre- miums for current Policy Year	Value of Total Sums Assured	Value of Total Bonus Additions	Value of Total Office Premiums	Value of Total Reduc- tions Premiums	Value of Total Net Premiums	New Bonus Additions at 1 per-cent per annum on Sums Assured	Value of New Bonus Additions at 1 per-cent per annum on Sums Assured
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)

NOTES.

Columns 2-4, 6, 10-13, 16 and 22 will be the transcribed totals of the Preliminary Schedule of Particulars.

" 5 and 14 . . . will contain the mean assurance-value and annuity-value respectively, corresponding to the mean age in col. (9).

" 7-9 . . . , *Vide* pars. 54 *et seq.* ; also numerical example in pars. 32-4.

" 15 . . . , Col. (11) in Preliminary Schedule ; number of policies in group. (If k be weighted in proportion to sums assured or Net Premiums, the divisor will be ΣSA or Σ Net Premiums.)

" 16 . . . , See note to col. 12 of Preliminary Schedule.

" 17-18 . . . , Product of (5) by (3) and (4) respectively.

" 19-21 . . . , Product of (14) by (10), (11), and (13) respectively.

" 22 . . . , Total of col. (14) in Preliminary Schedule.

" 23 . . . , Product of (5) by (22).

Should the value of a *Compound Reversionary Bonus* of 1 per-cent per annum (*i.e.*, 5 per-cent for the quinquennium) be required, col. (23) must be increased by $\cdot 05 \times$ col. (18), assuming that all the Policies which have already participated will now receive five years' bonus.

Elimination of Special Policies.

(66). As a precautionary measure it is usual to exclude, and to deal separately with, what may be called "Abnormal" Policies, which might disturb the accuracy of the group valuation if included with the rest. The extent to which this is necessary will depend upon the size and distribution of the business, and must be a matter for individual judgment. Experience and an analysis of a few special cases will quickly enable the Actuary to detect at a glance the Policies which should be excluded, and when this faculty has been acquired he may boldly and safely include Policies which he might at first decide to eliminate. No precise rules can be laid down, but the following considerations may afford some guidance.

(67). A Policy may be "abnormal" in consequence of

- (a) The sum assured being very much larger than the average.
- (b) The maturity age being much higher or much lower than the mean for the group—say over 70 or under 40.
- (c) The rate of premium being much higher or much lower than the average for the group.

The question of inclusion or exclusion will depend upon the *degree* of abnormality. If any two, or all three, of these abnormal features occur concurrently and to any considerable extent, the Policy should almost certainly be excluded; but the effect of any *one* existing singly will be much less serious. For example, a Policy having a very much higher sum assured than the average may safely be included if the maturity age and the rate of premium do not differ much from their respective mean values, and the nearer they are to the mean the higher the sum assured which can be safely included; while, obviously, if they absolutely *coincided* with the mean values, no sum assured, however large, could lead to any disturbance. Similarly, a Policy, abnormal in respect of (b) $\frac{\text{and}}{\text{or}}$ (c), can produce very little error if the sum assured be small; and the smaller the sum assured the greater the abnormality which can safely be permitted in (b) $\frac{\text{and}}{\text{or}}$ (c).

In the same way a Policy abnormal in respect of (c) only can safely be included if the maturity age do not differ much from the mean, and the closer it is to the mean, the greater the

abnormality which can be permitted in respect of (*c*) ; while if the maturity age exactly *coincided* with the mean, no abnormality (however great) in the average rate of premium would necessitate the exclusion of the Policy. [This will be seen at once by inspection of Equation (16), par. (38)].

(68). In the above remarks it is assumed that the same mean valuation age is used for the sum assured and the net premiums : if *separate* ages are used (*c*) ceases to be an "abnormality."

(69). In this Paper the author has endeavoured to examine as fully and impartially as possible the principal problems which present themselves in regard to the theoretical basis of the method, its suitability and adaptation to different conditions, and its practical working-out. It is hoped that no real difficulty has been shirked, and if, on the other hand, it be thought that in some places too much detail has been given, it may be explained that the object has been twofold—*first* to smooth the path for those who are studying the subject without much experience to guide them ; and *second* to lighten the work of the practising Actuary by placing on record—in a form easy of reference—a clear and connected statement of the most important points to be considered, and the principal formulæ, &c., to be used, in the application of the method. If he has succeeded in attaining that object he will feel amply repaid for the not inconsiderable labour which the Paper has involved.

The author desires to express his sincere thanks to Mr. G. W. Richmond, F.I.A., of the Scottish Widows' Fund (London Office) for his very kind and valuable assistance in checking much of the numerical work.

APPENDIX.

By the courtesy of the Actuary of a large Scottish Office, the author has received permission to publish the following particulars of a valuation of practically the whole of the Participating Endowment Assurance business of that Office by the method discussed in this Paper, as compared with a strict valuation, and it will be of interest to exhibit the results.

"The object of the application of Lidstone's method was . . . to find out what would be the actual amount of the *difference* in total value if the Policies, in place of being valued individually, were valued in blocks or groups depending on the term still to run and on the number of premiums remaining. It was therefore sought, as far as possible, to keep upon the same lines and to make the same assumptions as had been adopted in the original valuation. The same arrangement of Policies under years of maturity was accordingly made use of. The premiums and ordinary vested bonuses were valued at the same mean valuation ages as the sums assured."

In one particular Deferred Bonus Class, the mean valuation age for the bonuses had to be determined separately owing to the special character of the bonus system in that class.

The following is a summary of the results (Valuation basis, H^M 3 per-cent) :

	Amount	Present Value by direct Valuation	Present Value by Group Valuation	Error in Group Valuation
	£	£	£	£
Sums Assured ...	1,272,368	829,580	829,594	+ 14
Bonus Additions ...	77,719	53,095	53,086	- 9
Net Premiums <i>less</i> } Bonus Reductions }	43,406	454,146	454,048	- 98
Total Net Liability ...	-	428,529	428,632	+ 103

The net error in the total reserve, namely, £103 in excess, is thus less than $\frac{1}{40}$ of 1 per-cent of the total reserves, or about $\frac{1}{120}$ of 1 per-cent of the total sums assured, a result which must be considered eminently satisfactory.

DISCUSSION.

Mr. H. ARCHER THOMSON remarked that the paper, especially the part of it relating to the application of Mr. Lidstone's "Z method" to valuations by the O^M and $O^{M(5)}$ Tables made a very timely appearance. The O^M and $O^{M(5)}$ Tables were evidently becoming the standard tables for life office valuations, and while they would have expected the method to be successful in the case of the latter table, as it followed Makeham's Law, the doubtful question whether it would apply satisfactorily in the case of the O^M Table must have presented itself to the minds of several actuaries,

0^m MORTALITY.

Proportional Parts (relating to Sum Assured 1,000) for differences of tenths of a Year of Age									Maturity Age
1	2	3	4	5	6	7	8	9	M
2	3	5	7	9	10	12	14	15	35
2	4	6	7	9	11	13	15	17	36
2	4	6	8	10	12	14	16	18	37
2	4	6	9	11	13	15	17	19	38
2	5	7	9	12	14	16	19	21	39
3	5	8	10	13	15	18	20	23	40
3	5	8	11	14	16	19	22	24	41
3	6	9	12	15	18	21	24	26	42
3	6	10	13	16	19	22	25	29	43
3	7	10	14	17	21	24	27	31	44
4	7	11	15	18	22	26	30	33	45
4	8	12	16	20	24	28	32	36	46
4	9	13	17	22	26	30	35	39	47
5	9	14	19	23	28	33	37	42	48
5	10	15	20	25	30	35	40	45	49
5	11	16	22	27	33	38	44	49	50
6	12	18	24	29	35	41	47	53	51
6	13	19	25	32	38	44	51	57	52
7	14	21	27	34	41	48	55	62	53
7	15	22	30	37	44	52	59	67	54
8	16	24	32	40	48	56	64	72	55
9	17	26	35	43	52	60	69	78	56
9	19	28	37	47	56	65	75	84	57
10	20	30	40	50	60	71	81	91	58
11	22	33	44	54	65	76	87	98	59
12	24	35	47	59	71	82	94	106	60
13	25	38	51	63	76	89	102	114	61
14	27	41	55	69	82	96	110	123	62
15	30	44	59	74	89	104	118	133	63
16	32	48	64	80	96	112	128	144	64
17	35	52	69	86	104	121	138	155	65
19	37	56	75	93	112	131	149	168	66
20	40	60	81	101	121	141	161	181	67
22	44	65	87	109	131	152	174	196	68
24	47	71	94	118	141	165	188	212	69
25	51	76	101	127	152	178	203	228	70
27	55	82	110	137	164	192	219	247	71
30	59	89	118	148	178	207	237	266	72
32	64	96	128	160	192	224	256	288	73
35	69	104	138	173	207	242	276	311	74
37	75	112	149	186	224	261	298	336	75
1	2	3	4	5	6	7	8	9	M

EXTENDED TABLE OF Z_N

ON MORTALITY

[illegible]

OM(5) MORTALITY.

Proportional Parts (relating to Sum Assured 1,000) for differences of teuths of a Year of Age									Maturity Age
1	2	3	4	5	6	7	8	9	M
2	3	5	6	8	9	11	12	14	35
2	3	5	7	8	10	12	14	15	36
2	4	6	7	9	11	13	15	17	37
2	4	6	8	10	12	14	16	18	38
2	4	7	9	11	13	16	18	20	39
2	5	7	10	12	15	17	20	22	40
3	5	8	11	13	16	19	21	24	41
3	6	9	12	15	18	20	23	26	42
3	6	10	13	16	19	22	26	29	43
3	7	10	14	17	21	24	28	31	44
4	8	11	15	19	23	27	31	34	45
4	8	13	17	21	25	29	33	38	46
5	9	14	18	23	27	32	37	41	47
5	10	15	20	25	30	35	40	45	48
5	11	16	22	27	33	38	44	49	49
6	12	18	24	30	36	42	48	54	50
7	13	20	26	33	39	46	52	59	51
7	14	22	29	36	43	50	57	65	52
8	16	24	31	39	47	55	63	71	53
9	17	26	34	43	52	60	69	77	54
9	19	28	38	47	56	66	75	85	55
10	21	31	41	51	62	72	82	92	56
11	23	34	45	56	68	79	90	101	57
12	25	37	49	62	74	86	98	111	58
13	27	40	54	67	81	94	108	121	59
15	29	44	59	74	88	103	118	132	60
16	32	48	64	81	97	113	129	145	61
18	35	53	70	88	106	123	141	159	62
19	39	58	77	96	116	135	154	173	63
21	42	63	84	105	126	148	169	190	64
23	46	69	92	115	138	161	184	208	65
25	50	76	101	126	151	177	202	227	66
28	55	83	110	138	166	193	221	248	67
30	60	91	121	151	181	211	242	272	68
33	66	99	132	165	198	231	264	297	69
36	72	108	145	181	217	253	289	325	70
40	79	119	158	198	237	277	316	356	71
43	86	130	173	216	259	303	346	389	72
47	95	142	189	237	284	331	378	426	73
52	104	155	207	259	311	362	414	466	74
57	113	170	226	283	340	396	453	509	75
1	2	3	4	5	6	7	8	9	M

EXTENDED TABLE OF Z_M

OM(6) MORTALITY

Male Age	Value of Z_M from A-9000 to (A-9000) ¹⁰ Z_M corresponding to the underlined and same assumed										Proportion of Parts (relative to same Assumed Log) for difference of lengths of a Year of Age										Male Age
	M	100	200	300	400	500	600	700	800	900	1,000	1	2	3	4	5	6	7	8	9	M
25	17	33	50	66	83	100	116	133	149	166	182	2	3	5	6	8	9	11	12	14	35
26	18	36	54	71	91	109	127	145	163	182	200	2	3	5	7	8	10	12	14	15	36
27	20	40	60	79	99	119	139	159	179	199	219	2	4	6	7	9	11	13	15	17	37
28	22	43	64	87	109	130	152	174	196	217	238	2	4	6	8	10	12	14	16	18	38
29	24	46	71	95	119	143	166	190	214	238	262	2	4	7	9	11	13	16	18	20	39
30	26	52	78	104	130	156	182	208	234	260	286	2	5	7	10	12	14	17	20	22	40
31	28	57	85	114	142	171	199	228	256	284	312	3	5	8	11	13	16	19	21	24	41
32	31	62	93	124	157	191	218	249	280	311	343	3	6	9	12	14	18	20	23	26	42
33	34	66	102	136	170	204	238	272	306	340	374	3	6	10	13	16	19	22	26	29	43
34	37	74	112	149	186	223	261	298	335	372	410	3	7	10	14	17	21	24	28	31	44
35	41	81	122	163	204	244	285	326	367	407	448	4	8	11	15	19	23	27	31	34	45
36	45	89	134	178	223	267	312	352	391	430	469	4	8	12	16	20	24	28	32	36	46
37	49	98	146	195	244	295	341	380	419	458	497	5	9	13	17	21	25	29	33	38	47
38	53	107	160	213	267	320	371	417	460	503	545	5	10	14	18	22	26	30	34	39	48
39	58	117	175	233	292	350	408	467	525	583	641	5	11	16	22	27	33	38	44	49	49
40	64	128	191	255	319	385	447	511	574	638	701	6	12	18	24	30	36	42	48	54	50
41	70	140	209	279	349	419	489	559	628	698	767	7	13	20	26	33	39	46	52	59	51
42	76	153	229	309	382	458	535	611	687	764	841	8	14	22	29	36	43	50	57	64	52
43	84	167	251	334	415	501	585	666	745	825	904	8	16	24	31	39	47	55	63	71	53
44	91	181	274	366	457	547	640	731	821	914	1,000	9	17	26	34	43	52	60	69	77	54
45	100	200	300	400	500	600	700	800	900	1,000	1,000	9	19	28	38	47	56	66	75	85	55
46	109	219	328	438	547	656	766	875	985	1,094	1,203	10	21	31	41	51	62	73	82	92	56
47	120	239	359	479	598	718	837	957	1,077	1,197	1,316	11	23	34	45	56	68	79	90	101	57
48	131	262	393	524	654	784	914	1,044	1,174	1,304	1,434	12	25	37	49	62	74	86	98	111	58
49	143	290	430	573	716	859	1,003	1,146	1,289	1,432	1,575	13	27	40	54	67	81	94	108	121	59
50	157	313	470	627	783	940	1,097	1,254	1,410	1,567	1,723	15	29	44	59	74	88	101	118	132	60
51	171	343	514	687	857	1,028	1,200	1,371	1,541	1,714	1,886	16	32	48	64	81	97	111	129	144	61
52	187	375	562	750	937	1,125	1,312	1,500	1,688	1,876	2,064	18	35	51	70	88	105	123	141	160	62
53	203	419	618	820	1,023	1,228	1,433	1,641	1,848	2,054	2,261	19	38	56	77	96	116	135	154	173	63
54	218	449	673	898	1,122	1,345	1,571	1,795	2,020	2,244	2,468	21	42	61	84	105	126	148	169	190	64
55	235	491	736	987	1,227	1,473	1,718	1,964	2,209	2,454	2,699	23	46	69	92	114	138	161	184	206	65
56	250	527	806	1,074	1,343	1,611	1,880	2,148	2,417	2,686	2,955	25	50	76	101	126	151	177	202	227	66
57	268	588	881	1,175	1,469	1,767	2,056	2,345	2,634	2,923	3,212	28	54	82	110	138	166	193	221	248	67
58	321	643	984	1,285	1,607	1,908	2,209	2,510	2,811	3,112	3,413	30	60	91	121	151	181	211	241	272	68
59	352	703	1,053	1,406	1,726	2,046	2,366	2,686	3,006	3,326	3,646	32	66	99	132	163	193	223	253	283	69
60	385	769	1,144	1,538	1,923	2,308	2,692	3,077	3,461	3,846	4,230	36	72	105	145	181	217	253	289	324	70
61	421	844	1,265	1,681	2,104	2,524	2,943	3,363	3,782	4,201	4,620	40	79	119	158	198	237	277	316	356	71
62	460	928	1,381	1,841	2,301	2,727	3,152	3,577	4,002	4,427	4,852	45	86	135	172	213	250	291	330	369	72
63	504	1,024	1,511	2,044	2,478	2,921	3,364	3,807	4,250	4,693	5,136	52	95	145	189	232	274	317	358	400	73
64	551	1,123	1,642	2,221	2,754	3,305	3,856	4,407	4,958	5,509	6,060	62	104	155	202	249	291	332	374	416	74
65	601	1,225	1,808	2,410	3,013	3,615	4,218	4,820	5,422	6,024	6,626	72	113	170	220	268	316	364	412	460	75
M	100	200	300	400	500	600	700	800	900	1,000	1,000	1	2	3	4	5	6	7	8	9	M

and was of equal importance. Mr. Lidstone now came forward and answered this question for them. He (Mr. Thomson) was not in a position to make what was perhaps the most useful commentary that could be made on the paper, namely, to quote statistics (similar to those in the appendix) showing how small was the deviation from the truth brought out by the application of the Z method in actual practice. It was to be hoped, however, that succeeding speakers would be able to furnish such statistics. One of the great beauties of the Z method was its transparent simplicity, both in the main principle underlying it and in its practical application. Until Mr. Lidstone launched his Z method about five years ago, the method of valuing endowment assurances in groups most usually adopted, was to classify them in groups according to the unexpired term, and then employ for each group valuation factors based on the mean age derived from weighting the individual valuation ages with the sums assured, a method generally associated with the name of Mr. Manly. It had been principally used, he believed, for check valuations, and for this limited purpose it had proved extremely useful. The Z method differed from Mr. Manly's in two respects (*a*) a difference in principle, which made the numerical results much more accurate, (*b*) a difference in the practical working, which did not affect the numerical results. The first difference consisted of employing the product $S_x \times c^x$ instead of $S_x \times x$ in the process for obtaining the mean valuation age. Mr. Lidstone proved by mathematical analysis that, subject to certain limitation, the difference between the mean age so found, namely, y , given by the equation

$$c^y = \frac{\sum S_x \times c^x}{\sum S_x}$$

and the true mean age, namely, y , given by the equation

$$a_{y:n} = \frac{\sum S_x \times a_{x:n}}{\sum S_x}$$

was small, the necessary consequence being that the error in the reserves brought out by the method must be insignificant. The second distinction between the two methods consisted in dealing with the maturity age instead of the valuation age. That ingenious modification saved a good deal of labour, for each product, $S \times c^n$, when once formed, held good throughout the duration of a policy; whereas, if one dealt with the valuation ages, it was necessary to form a fresh product at each successive valuation. Of course, such a modification could be applied to Mr. Manly's or to any similar method; it would not affect the numerical results of such method, but would save labour. The paper contained such a wealth of suggestive detail that in making a selection for comment one suffered from *embarras de richesses*. Some members might think that the "actuarial microscope" had been somewhat freely employed in the paper, but in actuarial investigation, as in other scientific research, the microscope was an extremely useful instrument. It gave a clearer insight into the constitution of the elements dealt with, and so enabled one to draw more accurate and more

reliable general conclusions. In par. 6 one found the expression $e^{\gamma \Delta} a_0$; at the first blush one hardly recognized in that symbolic expression their old friend the temporary annuity $a_{x:n}$; but by carrying one step further Mr. G. F. Hardy's powerful resolution of the temporary annuity into its constituent elements (*J.I.A.*, xxxiv, p. 85), the author had arrived at this very striking and attractive expression. In pars. 12 to 14 Mr. Lidstone, passing from the stern logic of mathematical analysis, had tried the effect of various tentative assumptions, and would doubtless expect them, before implicitly accepting his conclusions, to make assumptions of their own and see the results. First, as to the nature of the error involved in the method; *ceteris paribus*, the greater the "range" of valuation ages in a group (or of maturity ages, for that comes to the same thing) the greater would be the error: also, *ceteris paribus*, the higher the mean valuation age (or maturity age) in a given group, the greater would be the error. In par. 13 Mr. Lidstone had assumed a distribution of assurances in the proportions 2, 3, 3, 2 at ages x , $x+5$, $x+10$, $x+15$, giving a "range" of 15 years. This distribution would suitably represent policies maturing at the favourite ages 50, 55, 60 and 65 (a few at intermediate ages would not materially affect the argument). That brought out $y_2 - y_1 = 1.03$, and mean $M = 58.66$, both well within Mr. Lidstone's proposed maxima in par. 14, and the resulting errors in the annuity-value would be about one-half of those given in par. 14. It seemed advisable in testing the method to adopt a wider range than 15 years. If they prefixed 1 (representing policies maturing at 45) to the series 2, 3, 3, 2, the "range" was increased from 15 to 20 (resulting in an increase of $y_2 - y_1$ from 1.03 to 1.3), but the mean M was reduced from 58.66 to 57.9. As already pointed out, the former would increase the error and the latter would diminish it, and the net result in this case was to leave it practically unchanged. If they now added 1 at the end of the series—that is, assumed a distribution of assurances at maturity ages 45, 50, 55, 60, 65, 70 in the proportion 1, 2, 3, 3, 2, 1, the "range" was increased from 20 to 25 years (resulting in an increase of $y_2 - y_1$ from 1.3 to 1.9), and the mean M was increased from 57.9 to 59.6. Both these changes increased the error, and their combined effect practically doubled it, the results being as follows (and practically identical with those found in par. 14):—

n	Mean Error in Annuity-Value.
10009
20018
30019

The conclusions he had drawn were threefold: (1) that, although the first assumption in par. 14 (namely, that $y_2 - y_1$ would not exceed 1.5) might bear a little stretching, no reasonable assumption as to distribution would bring out a higher value than 2; and the second assumption, namely, that y_1 would not exceed 60— n , *i.e.*, that mean M would not exceed 61, was sufficiently stringent to counteract any possible weakness in the first; (2) that in any ordinarily distributed business the mean error in the annuity-value (and reserve) must

necessarily be small; (3) that in the event of policies maturing at high ages (above 65 or 70) becoming popular the method would have to be applied with caution. For instance, if they were to assume an even distribution of assurances over maturity ages 50-80 (a most unlikely distribution to be found in practice) $y_2 - y_1$ became 2.5, and the mean M became 68.4, and the resulting errors

\bar{n}	Mean Error in Annuity Value.
10057
20123
30131
40109

These might well cause an error of over 2 per-cent in the reserve. In pars. 22 to 24 it was very interesting to compare the graduations of the O^M Table by the "Aggregate Method", and by the powerful Method of Moments, and it was satisfactory to have the author's opinion that his "Z method" of valuation would apply to the O^M almost as satisfactorily as the O^{M(5)} Table. From par. 37 *et seq.* it appeared that, in strict theory, a separate mean age should be calculated for the sum assured, bonuses, reductions of premiums, office premiums, and net premiums. It was fortunate that in practice, as the author pointed out, that seemed unnecessary, for if it were necessary to calculate five Z's and five mean ages for each group, he feared that that would be sufficient to condemn the method. In pars. 54 *et seq.* the methods of calculating M (maturity age), according to the various valuation hypotheses, were given in great detail; but it was just at that point that detail was necessary. When an office adopted the Z method for the first time care had to be exercised in order to make the calculation of M exactly fit the valuation hypotheses. When once that had been done, in succeeding valuations the method worked almost automatically. Of the various cases specified, certainly that given in par. 57 appeared the simplest, namely, to calculate the valuation age by the "Institute Method"; in that case M was always equal to the endowment age, and one was saved all the troublesome details involved in the other methods. The author mentioned that he had found this method to understate the age by .09 of a year; if they could regard the case as typical the small error was a blessing in disguise, for the Z method involved a small error in the other direction, and thus the two errors tended to neutralize one another. In pars. 66 to 68 the author indicated some of the cases in which it was necessary to be on the look-out in applying the method. If the method had a weak spot it lay in its comparative inapplicability to policies maturing at late ages. For instance, if there were a large number of policies which had been converted into endowment assurances from whole-life policies by the application of bonus, those would necessarily mature mostly at late ages, 65, 75, 80, and so on. In such a case the Z method would not, he thought, be applicable. Assuming, for instance, an even distribution over maturity ages 55-85, the errors in the annuity-value would be e^{10} times those given above for ages 50-80, *i.e.*, about $2\frac{1}{2}$ times as large. A progressive assurance institution in one of the colonies had

recently ceased granting whole-life policies altogether, and now granted instead endowment assurances at 80. He did not know how far that was to be taken as a sign of the times, or an indication of an incipient demand on the part of the public for late maturing endowment assurances; at present, fortunately, they had few such policies on their books. It seemed, however, rather captious criticism of an excellent method to say that though it met the needs of the moment it might not do so in future. For endowment assurance business, as at present constituted, he thought the Z method of valuation was *facile princeps*.

Mr. GEORGE KING said he felt it a privilege to be allowed to say a few words of appreciation of Mr. Lidstone's work. Formerly he (Mr. King) had, with many others, experienced a feeling of oppression on account of the overwhelming mass of endowment assurances; but now, even with such vast numbers of policies, he thought endowment assurances were one of the easiest classes on the books to value. Therefore, Mr. Lidstone's paper of five years ago marked an epoch in this department of actuarial activity. That was emphasized by the present paper, in which the author again expounded scientific principles which were most brilliantly conceived, and worked them out to the most minute details in a perfectly practical manner. He was himself a convert by Mr. Lidstone to the valuation of endowment assurances in groups. Formerly he objected strongly to the use of group methods for valuation purposes. He had advocated them for check purposes, and he had suggested that a statement of endowment assurances should be given in groups with the average age for the Board of Trade returns, so that the outside actuaries might be able to form a rough idea of the reserves of a company, which they could not do from the forms which, up to quite lately, were insisted upon by the Board of Trade. He had used them to check the actual official valuation he had made, and they had given good results in the particular cases to which they had been applied, but there was no scientific principle underlying them, and one could not tell what would be the amount of the error, nor even in what direction it would lie. Therefore, formerly, for an official valuation he had no confidence in making only a group valuation. Now, however, the case was different. The reason was that Mr. Lidstone's method was based upon scientific principles, so that one could feel sure that it gave accurate results. By it one knew the direction of the error, and could prove how small the error was. Moreover, the method was easy to apply. The present paper, the second on the subject by Mr. Lidstone, was of great service to actuaries, if for no other reason, because points originally left somewhat obscure were now explained. For instance, in the former paper there were certain expressions used without definition, such as "office year of birth", but they were now explained and were perfectly clear. One of the most important matters in the present paper was the discussion of the O^M Table as a representative table which did not follow Makeham's Law, and it was very valuable to have Mr. Lidstone's analysis of the question. But to convince his own mind of the applicability of the method to such a table as the O^M he did not really require all the proof which Mr. Lidstone had

supplied. Reference to Mr. Lidstone's first paper would show that he happened on the method empirically in the first instance. He took the temporary annuity for lives of different ages, but for a uniform term; and he found that practically the first difference ran as a geometrical progression. Then, applying a mathematical analysis, he found that the ratio 1.085, which he had arrived at empirically, was really an approximation to Makeham's constant c , which was the upper limit of the ratio he had sought. Therefore, it was quite clear, seeing that the empirical ratio 1.085 gave very good results, that one did not require to have a table rigidly following Makeham's Law in order to make the method apply. But he would go further, and say that the constant c ran almost uniform throughout most of the tables known to actuaries; in fact, throughout all the known tables which at all closely represented human mortality. It was shown in the *Text-Book*, chap. vi, article 67, that such was the case, that c was an almost uniform quantity for all tables; and in the *Text-Book* Table one was able to carry c down to age 10, and although at age 28 it was necessary to change the other constants, the change taking the form of a duplication of the constants at the younger ages; yet the constant c remained uniform throughout the whole Table, and gave perfect results. In the present paper Mr. Lidstone had investigated the constant c for a portion of the O^M Table, and had found what he called an abnormally low value—1.07848. But if he had taken the constant c for the whole table he would have found that that was not so. The constant c for the whole O^M Table from age 15 to age 94, inclusive, came out at 1.09127, as against $O^{M(5)}$ 1.09396, and H^M 1.09561. Makeham's constants were so curiously blended, that even a considerable change in the value of c would produce but little change in the force of mortality in a section of a table if the other constants were calculated for that section. Therefore, he thought it would be found, if one took for the constant c the value 1.09127 for the O^M Table, and calculated the other constants for the section from age 15 to age 64, one would arrive at a force of mortality very similar to that brought out by the author. It was true that there would require to be a change in the other constants somewhere about age 60, but that scarcely affected endowment assurances. It would also result that if one took the ratio 1.09127 for the O^M Table, one would arrive at the identical maturity age for the O^M Table that was found for the H^M and $O^{M(5)}$ Tables. It was a curious fact, which had been noticed by many, that when there were twin tables, such as the O^M and the $O^{M(5)}$, or the H^M and the $H^{M(5)}$, if one followed Makeham's Law the other did not. But in a conversation he had had with Mr. Lidstone a few days previously that gentleman pointed out the reason. The twin tables ran into each other at the older ages when new entrants no longer came in, and at the older ages both tables were based on the same observations, so that at the older ages the tables must have identical constants. Therefore, if one table can be represented by the same constant throughout its whole length, the other table must have a change in its constant. Passing to minor points, it was a stroke of genius on the part of Mr. Lidstone to transfer the mean age from the valuation age to the maturity age, because the maturity

age was the same for all valuations until the policy was paid off, whereas the valuation age changed at each valuation. Thus, by that very ingenious device, one could use the same mean age throughout the whole history of a policy, and great simplicity was introduced into the practical application of the method. It was extremely interesting, also, to have the explanation given by the author of the various ways in which the method might be applied, and the advantages attaching to each. He would like to point out that an advantage attaching to one of them, that discussed in par. 58, extended also to whole-life assurances, and, in fact, to assurances of every class. Those premiums falling due in the first half of the financial year could be valued by $1+a$, and those in the second half simply by a , and thus a complete correction could be introduced in all classes for any irregular distribution of premium income, as shown for endowment assurances by the author in par. 60.

Mr. C. D. HIGHAM said that his own walk in life had not led him to those places where endowment assurances became the incubus under which others appeared to be staggering, and if he interposed in the debate it was not because he wished to criticize either the mathematics or the methods of the skilful and practical paper before them, but because he desired to add a personal note to those congratulations which had been offered to Mr. Lidstone that evening. Mr. Lidstone truly said, and others agreed with him, that long before his time it had been observed that, in the ordinary case, the value of endowment assurances depended much more on the term than on the age. He was intending to claim for Mr. Manly the credit of having been the first to group endowment assurances according to the temporary annuity, but he had been forestalled. The more excellent way, however, in which Mr. Lidstone found his average age and his very convenient Z , marked a great step in advance; nor did he think Mr. Lidstone had made enough of the advantage of using the calendar year for his grouping, because it was so handy to know at the beginning of the calendar year which policies were going to mature, so that they could be ready for the claims.

Mr. T. G. ACKLAND wished to mention the class of joint endowment assurances where two lives were involved, the sum assured being payable on their jointly surviving a fixed term or on the earlier death of either. It was not very common for a large number of that special class of endowment assurances to be included on the books of any individual office, but an instance came under his notice recently where some hundreds of cases of that class had to be valued, and they were peculiarly troublesome and difficult, because there were two ages involved, and the unexpired term would in many cases vary. The usual plan had been to take the equivalent equal ages and then classify those according to the maturity terms, and value the cases in detail. He was curious to ascertain how far Mr. Lidstone's method would apply to the case of joint endowment assurances. He had not had time to fully explore the mathematical analysis of the matter, but it appeared to be clear at the outset that it was not quite on the lines set out in the earlier part of Mr. Lidstone's paper, because for joint lives one had to deal with the constant s , which had to be squared, and the constant e , which

also had to be squared. The squaring of the latter was dealt with by taking two equivalent ages, but the effect of introducing s^2 in the formula was not clear, although it would probably be found that the error was almost equally small with that ascertained by Mr. Lidstone in the case of a single life. At all events, the practical result of valuing some 300 cases might be comparatively stated to indicate that the method did undoubtedly apply in practice. The sums assured were just above £100,000, and he might briefly state the value of the sums assured, the value of the net premiums, and the net liability, as ascertained by the method of equal pairs of ages in detail, and by the pair of average equal ages, on Mr. Lidstone's method. The value of the sums assured by the detailed method was £67,856; by Mr. Lidstone's method it was £67,854—a difference of £2 in 300 cases. The value of the net premiums was £46,497 by the detailed method, and by Mr. Lidstone's method £46,500—a difference of £3. Therefore, the net liability varied by about £5 over £20,000 liability and £100,000 of sums assured. He only wished to make one other remark, upon a point which interested him towards the close of the paper. Mr. Lidstone referred to the fact that the Board of Trade had in some cases dispensed with the return of the total amount of premiums paid in the endowment assurance class (and also occasionally in some other classes), where what might be termed a *quid pro quo* had been given by the office in the way of additional particulars not required by the Act, which would assist the actuary in valuing the cases tabulated. He thought that in some cases the maturity years only had been given, but it seemed desirable that the averages should also be given as employed in the valuation. He believed there were only six or seven offices to which the concession referred to had been granted, but no doubt it was, as Mr. Lidstone had said, in the direction of a precedent, and one that the Board of Trade might be expected to follow in the future. In some returns the classes of joint lives and limited premiums had been set out in considerable detail by the offices, and in those instances also the Board of Trade had seen its way to dispense with the particulars as to premiums paid under the schedule.

Mr. W. P. ELDERTON ventured to offer his congratulations to Mr. Lidstone on the paper he had presented. He thought it might not be generally known that Professor Karl Pearson had himself given a solution of the application of the Makeham hypothesis to a mortality table, and arrived at the result on different lines from those mentioned by Mr. Lidstone. The solution he referred to appeared in *Biometrika*, Vol. I., p. 268, and he believed that volume had recently been added to the Library. In par. 32 Mr. Lidstone stated that the mean maturity ages did not differ very much from one table to another, and he thought this fact might be helpful in rough valuations. Not long ago he took about 2,000 endowment assurances, and writing maturity ages across the top and the unexpired terms down the side of the paper, he tabulated the number of policies in the body of the table. He then weighted the numbers by Mr. Lidstone's Z 's, and on working out the table he found the age at maturity could be represented as $57.595 + 1.200$ times the unexpired term. If the unexpired term were ten years, the average maturity age for the group could be taken

as 58.8. He valued the groups given in Mr. Lidstone's previous paper, and found the error was less than 2 per-cent. Considering that he never saw the internal arrangement of Mr. Lidstone's grouping, and only valued the figures in 5-year groups, he thought the result was fairly good. Not long ago Julius Altenburger wrote to the *Journal* mentioning a Continental method, which he (Mr. Elderton) had had an opportunity of seeing applied to 3,700 with-profit endowment assurances, 400 with-profit limited payment policies, and smaller numbers of without-profit assurances. The calculation of the constants was a very simple matter, and had been done by a person having no actuarial knowledge, but checked, as Mr. Lidstone stated was desirable, with the Z constants, by someone having such knowledge. To show that the labour was not, as some supposed, prohibitive, he might mention that a yearly valuation, including a calculation of the death strain, was made.

Mr. T. E. YOUNG wished to express his admiration of the essay, which amply secured the distinctive title one felt compelled to apply to the system of classification, namely "Lidstone's method." The paper appeared to him to afford a genuine type of what such a contribution should be. There was the provision of an adequate mathematical basis, without those excessive refinements which so frequently disfigured actuarial professional work, and bestowed an appearance of demonstration upon it to which it had inherently no pretension whatever. On that sound mathematical foundation the author had erected a practical method which possessed both the merit of exhaustiveness, and also the further significant merit of enabling the actuary to wield with efficiency, and with certainty, age valuations and cumbrous modes of endowment assurances, which now unhappily constituted a serious portion of actuarial business.

Mr. H. W. MANLY said that he had very early adopted a system of grouping according to the year of maturity. As had been well said by Mr. Higham, that plan had one great advantage, namely, that one knew exactly when one had to pay endowments; it was known by that means how many were going to mature in the present year, and how many in the next year, and so on. At the same time, he was very glad that Mr. Elderton had referred to Mr. Altenburger's method, because that method really produced correct results. But then it had the inconvenience, so far as he could see, that when the policies were grouped according to present age one could not see exactly when the payments were going to occur. If, as he believed was done on the Continent, the figures were mixed up with the ordinary whole-life policies, there was, on the one hand, a very great advantage in being able to get at the probable death-rate and the death strain; but, on the other hand, when it was necessary, as in this country, to make certain Board of Trade Returns, the figures would have to be picked out very carefully afterwards, and summarized, in order to be able to compile those returns, in addition to which the number of premiums paid would have to be calculated upon each policy, and added up. That would involve an immense amount of labour. He had not taken to that method, nor did he at present intend to try it, being quite satisfied with the shorter and better method as laid down by Mr. Lidstone.

Mr. S. G. WARNER, in closing the discussion, remarked that

it would perhaps be permissible to begin by referring to that upon which there could be no discussion—because there was no difference of opinion,—namely, the great indebtedness felt by all to Mr. Lidstone for his masterly and exhaustive paper. One naturally thought of it in connection with his first paper, and read the two together. When one heard the first paper of Mr. Lidstone, proceeding as it did from the happy discovery that the first differences of temporary annuities formed roughly a geometrical progression, one felt at once that there was a difference, not so much of degree as of kind, between that and the many other attempts which had been made towards the solution of the grouping problem—that here there was actually something new, something which contained in itself the elements of a definite solution. One obtained, in fact, a sense of finality. Mr. King had not said too much in describing the new method as epoch-making. Its chief excellencies appeared to be three in number. First of all, there was the mathematical principle upon which it depended, which had already been dealt with sufficiently, and was brilliantly and exhaustively worked out in the paper before them. Then there was the very ingenious device by which the maturity age was made the predominating factor in *Z* instead of the present age. The third advantage in the method was that any error was on the side of safety. It had been felt that previous methods of approximation rather tended to give the error on the other side, and although it might be small, the fact that it was in the wrong direction gave a feeling of discomfort. It was very re-assuring to feel that any error which might be involved would add to the reserve rather than diminish it. One of the most interesting parts of the present paper was that which dealt with the *O^M* Table. The fact that there had been completed another great and important investigation into mortality made it necessary that the *Z* Tables should be brought up to date. He had thought that that evening one would have heard a little more of practical experience in the application of the method. It was known that it had been extensively used. He would like to quote his own practical experience, though he confessed to feeling some awe of Mr. Lidstone, because that gentleman became so critical about the degree of approximation that it might be expected, unless the difference were in the ratio of about £50 to a reserve of half-a-million, Mr. Lidstone might consider it unsatisfactory. All he could say was, that the application of the method of which he had experience took place very soon after the publication of Mr. Lidstone's first paper, and was made during the stress of a quinquennial valuation. An individual valuation of endowment assurances was on the point of being completed when the new method came before the actuarial world. It was felt that it was of great interest and importance, and that a good opportunity would be afforded, by applying the method to the facts, and comparing notes with the individual valuation, to test its value. There was not very much time to attend to minor refinements, and therefore it was not to be expected that such close results would be obtained as had been shown in other cases quoted by Mr. Lidstone. Still, there was an agreement to within something like $\frac{1}{2}$ per-cent on a reserve of about £200,000, which was considered very satisfactory—so satisfactory

that since that time Mr. Lidstone's method of grouping had been adopted without question. He had very little doubt that if they had had time to harmonize exactly all the assumptions as to valuation age, maturity age, &c., in the individual and group valuations a large part of the difference would have been accounted for. But still, as it stood, he considered that a method which brought out a difference of only $\frac{1}{2}$ per-cent on a reserve of £200,000, and that on the side of safety, was eminently satisfactory; and that any office was justified in adopting a method which gave such good results.

The CHAIRMAN (Mr. R. P. Hardy) said that before asking those present to vote their thanks to the author, he wished to say a few words on his own account, the subject being one which the circumstances of his life had forced him to pay considerable attention to. If Mr. Warner would forgive him, he would say that he thought sufficient evidence had been produced, and upon a sufficient scale, that the results Mr. Lidstone's method brought out were so singularly close to those obtained by current methods, that one might safely accept the principle upon which it rested. It appeared, and the excellent schedules attached to his paper abundantly confirmed it, that neither the grouping nor the insertion of the Z function required any trained skill, and therefore that the clerical labour involved, up to the point of valuation, did not exceed that necessary under the ordinary forms, but that the summary of the quantities prepared for valuation could be reduced to the limits of a single sheet, and the work of valuation brought down to that of a few hours only. It must be admitted that that, standing by itself, was no inconsiderable achievement, and one which all actuaries received with gratitude. But when one looked to see how powerfully the necessities of the case had been grasped in their true bearing, with what scientific skill the centre of gravity had been determined, measuring alike the large range of age and the great variety of amounts at risk, it was difficult to say whether one ought to admire most the beauty of the method or the elegance of the mind which conceived it. Up to this point, one had not heard what that interesting person the *advocatus diaboli* might have to say upon the subject. He invited members to consider the only suggestion which that amiable personage might make. He would possibly say that actuaries purchase that undoubted convenience at a certain cost, the arrangement of the data precluding all enquiry into the mortality and rate of lapse experienced; neither could one account for any change in the financial incidence in the period of valuation. That might, or might not, be important; but he (the Chairman) had his own opinion upon the point. However, it was just a consideration to make one pause before abandoning some of the traditional methods of classification. It was pertinent to ask: Did there exist a method in which the materials or data would be still preserved in a convenient shape, so as to enable the actuary to produce a valuation the results of which no one could challenge upon any single point? There was such a method, not to be compared in point of beauty to that of Mr. Lidstone, nor, perhaps, one so easily worked, but one which did retain those advantages, and brought out results which he thought all in the room would admit were unchallengeable. His (Mr. Hardy's) attention

was directed to a letter that appeared in the *Insurance Record* of 26 May 1871. signed by Mr. John Karup. In that letter the writer pointed out that by adding two columns of permanent constants to the usual valuation schedules a result would be brought out arithmetically identical with that which would follow from the old-fashioned seriatim valuation. Being very much troubled with a mass of endowment assurances, his mind recently recurred to this idea, knowing that if the method was applicable to whole-term assurances it was *à fortiori* to endowment assurances; but it could not be conveniently worked, except for those cases where the sums were payable at the quinquennial ages 40, 45, 50, 55, &c. It would give some trouble where the periods of payment were 10, 15, 20 years. But, subject to that, the method was one which he would commend to the attention of his colleagues. It certainly was less lofty in its aims, but none the less it possessed some distinct qualifications for impartial consideration. He hoped that in bringing that matter before his brethren Mr. Lidstone would not think he wished to challenge, or to criticize, the beautiful method of that gentleman, which he so much admired.

The vote of thanks to Mr. Lidstone was carried by acclamation.

MR. LIDSTONE, in reply, said he would be indeed difficult to satisfy if he were not—as in fact he was—deeply grateful to the members for the extremely cordial way in which they had received his paper, and for the vote of thanks so kindly proposed by the Chairman. He was particularly indebted to those gentlemen who had joined in what he believed everyone would admit was a most interesting discussion. It was of course a peculiar satisfaction to him to hear some of the leaders of actuarial thought express publicly their approval of the method, and that in the most practical shape, by saying that they used it themselves. For his own part he might say his opinion was that the more one looked into it the more one found it was readily adaptable to a variety of different circumstances; and that with proper care it would produce results which were amply sufficient for all practical purposes. Like all other actuarial methods, it had to be controlled by a skilled hand: it would not do to lash up the helm and go full speed ahead without keeping watch or ever taking an observation. At such a late hour he could scarcely deal with all the points which had been raised in the discussion; but his first duty was to pass on to somebody else the kind things which had been said about the selection of the maturity age as the basis of calculating the Z's. In his previous paper he said in a footnote: "It should be stated that the particular form here given to the function Z, has been adopted (in accordance with a suggestion made to the Author) as an improvement upon the form originally proposed in the paper." The form originally proposed was to reckon the Z's according to the *year of birth minus 1800*. It was Mr. Todhunter (who, at the time, specially requested that his name should not be mentioned) to whom the suggestion was due. He (Mr. Lidstone) was glad the remarks which had been made that evening had made it possible, indeed imperative upon him, to give Mr. Todhunter the tribute which at that time he refused to take. Reference had been made to other methods, particularly

Herr Altenburger's. He had examined that with some care, and he was bound to say he would rather leave Mr. Elderton to use it than use it himself. It would be interesting to hear if that gentleman's opinion of it had changed when he had applied it to two or three successive valuations, and found he had to calculate fresh auxiliary functions for bonus additions and reductions of premiums at each valuation. Apart from that, which was a very practical objection, he did not care for a method which put the whole of the policies into a kind of hotch-potch without giving one the means of testing the accuracy of the results in a broad way. Mr. King spoke with such high authority that it was always with great hesitation one differed from him. At the same time he would not like it to go forth without question that the ratio c , to be used in applying the method to the O^M Table, was 1.09127. Mr. King said that was the value of c which was brought out if one attempted to graduate the whole O^M Table by Makeham's method. But he thought if Mr. King pushed that enquiry a little further and noticed what deviations would ensue between the raw material and the adjusted values, he would agree with him (Mr. Lidstone) that the lower value of c , which he brought out as applying to the section which was most used in endowment assurance business, namely, 1.080, was better. The method was based upon a uniform value of c , and the other constants did not enter into it. He was inclined, therefore, to think that the value of c given in the paper was the proper one to adopt in the case of the O^M Table. Mr. Elderton had referred to Professor Karl Pearson's application of the method of moments to the graduation of the H^M Table in *Biometrika*. He had read that paper with very great interest, but, for practical purposes, thought the method adopted by himself, following Mr. George Hardy, was quite as effective, and it certainly was very much simpler. Mr. Ackland's remarks on the application of the method to joint-life endowment assurances were very important and interesting, and, as he had pointed out, the results were ridiculously close. He (the author) thought that, with very slight modifications, the demonstrations given in Part II. of the paper could be made to apply to joint-life endowment assurances.* The Chairman had referred to a particular method, and had pointed out the advantage it certainly had—if it really were an advantage—of enabling the actuary to test the mortality experience with greater completeness. By the Z method it was possible to calculate the expected claims and expected death strain in the aggregate. By the other method, which was similar to that usually adopted in the past, one could deal with each age separately. Whether that was a great advantage, considering that one was working with aggregate tables of mortality, which did not correctly represent the expected mortality, he would not at the moment say, but he did not think he would sacrifice the simplicity and ease of the Z method merely on that account.

* See remarks on next page.

[Mr. Lidstone has sent us the following further remarks for publication.—ED. J.I.A.].

Mr. Thomson's remarks, as to the caution which must be exercised in applying the method to policies which have been converted from Whole-Life Assurances to Endowment Assurances, by the application of successive Bonuses, appear to be fully justified. From the nature of the case, many of the maturity ages of such policies must be very advanced, and these advanced maturity ages will coincide with very long unexpired terms, thus throwing upon the method a doubly heavy strain which it can hardly be expected to bear with entire success. The method could not be considered suitable in such circumstances, unless after special investigation some simple and effective modification were found to be practicable. In the case of an Office granting Endowment Assurances maturing at a very advanced age—say 80 or 85—in place of Whole-Life Assurances, the proper course would undoubtedly be to value such policies as a class by themselves, and not to attempt to include them with the ordinary Endowment Assurances in a "Z" valuation.

The interesting examples, given by Mr. Thomson, of the effect of various assumptions as to the range of maturity ages and their respective weights, are strikingly similar to experiments which were made by the author when preparing the Paper, and incidentally they illustrate the ease with which various cases may be dealt with by means of the analysis in the Paper. The general conclusions which Mr. Thomson enunciates are entirely in accordance with the author's own views. It may be worth while to point out that for the higher values of n , the range of ages will be automatically reduced, since the youngest maturity ages are then practically non-existent. For example, take Mr. Thomson's assumption of maturity ages 45, 50, 55, 60, 65, 70. Since a very small proportion of business is effected under age 20, hardly any policies maturing at or under age 45 will be found in groups having a longer unexpired term than 25 years; hardly any maturing at or under 50 will be found in groups having a longer unexpired term than 30 years; and so on, the minimum maturity age being practically $n+20$. There is thus a kind of automatic compensation tending to reduce the error for the longest unexpired terms, that is precisely at the point where the error is theoretically most sensible.

The *rationale* of the method in the case of Endowment Assurances on two joint lives (*vide* Mr. Ackland's remarks in the discussion) may be thus exhibited. As in par. 6 of the Paper, it may be shown that

$$v^t \cdot t p_{uv} = v^t s^{2t} e^{(c^u-1-c^v+c^v)\gamma}.$$

Put $c^u + c^v = c^{ic}$, which is equivalent to $Z_{u|} + Z_{v|} = Z_{ic}$, (where (v) denotes the maturity age corresponding to v , and so on). Then we shall have

$$v^t \cdot t p_{uv} = v^t s^{2t} e^{(c^{ic}-1)c^v\gamma}$$

Summing from $t=1$ to $t=n$, we have $a_{uv|} = a'_{ic|}$, where the accent denotes that the annuity-value is calculated at a rate of interest such that $\delta' = \delta - \log_e s$. Thus for each policy, the joint-life annuity-value may be replaced by a single-life annuity-value at a special (but uniform) rate of interest. To these substituted annuity-values, the analysis of

the Paper will apply unchanged, so that the mean annuity-value for the group will be $a'_{y_1\bar{n}}$, where y_1 is defined by the equation

$$M_{(y_1)} \text{ per unit} = \frac{\Sigma Z_{(u)}}{\Sigma S.A.} = \frac{\Sigma(Z_{(u)} + Z_{(v)})}{\Sigma S.A.}$$

But $a'_{y_1\bar{n}}$ (calculated at the special rate of interest) may now be replaced by $a_{x\bar{n}}$ (calculated at the ordinary valuation rate); where x is defined by the equation

$$2c^x = c^{y_1}, \text{ or } M_{(x)} = \frac{1}{2} M_{(y_1)} = \frac{\frac{1}{2} \Sigma(Z_u + Z_v)}{\Sigma S.A.}$$

Thus the mean annuity-value and assurance-value for the group will correspond to the two joint lives of equal ages $x:x$, and in order to find these equal ages, all that is necessary is *to record the ordinary Z for each of the lives assured by every policy*, and to divide the sum of the Z's by two before determining, by the usual process, the mean Z and thence the mean equal ages.

The error in the mean annuity-value $a_{x:x;\bar{n}}$ will be the same as that in the equivalent mean single-life annuity $a'_{y_1\bar{n}}$, which will not differ much from the error in $a_{y_1\bar{n}}$, the change in the rate of interest being small.

Since $c^{y_1} = 2c^x$, we have $2 = c^{y_1-x}$, whence

$$y_1 - x = 7\frac{3}{4} \text{ for the H}^M \text{ Table and for the O}^{M(5)} \text{ Table,} \\ \text{or} = 9 \quad \quad \text{O}^M \quad \quad$$

so that the difference between y and x is constant for any given table. *Thus the error will be nearly the same as in the case of a group of assurances on single lives having a mean age y , exceeding the mean equal ages $x:x$ by a constant number of years.*

From particulars which Mr. Ackland has kindly furnished, it would appear that the remarkably close approximation in the case referred to by him is no doubt partly accidental, but is largely due to the following causes:

- (1) The unexpired terms were comparatively short and the maturity ages comparatively low.
- (2) Separate mean ages were found for the Sums Assured and the Net Premiums. (The latter ages were found to be on the average about $\cdot 2$ of a year higher than the former.)
- (3) The Z's were based on the common ratio 1.085 (instead of e), which might be expected to bring out very close results on the average.

With the object of illustrating the remarks in pars. 27-8 of the Paper, and of comparing the application of the method to the O^M and $O^{M(5)}$ Tables respectively, the following table has been computed, showing for various terms of years (and on two different assumptions as to the distribution of the maturity ages) the amount of the deviation between the true mean annuity-value and the annuity-value corresponding to the mean age brought out by the Z method. The

assumptions as to the maturity ages were as follows, subject, however, to the condition that no life of less valuation age than 19 was included:

DISTRIBUTION A.—Equal amounts maturing at ages 50, 55, 60 and 65.

DISTRIBUTION B.—Amounts proportionate to the numbers 1, 2, 3, 3, 2, 1, maturing at ages 45, 50, 55, 60, 65, and 70 respectively.

The valuation ages were taken to be the mean maturity ages *minus* ($n+1$).

TABLE.

	<i>n</i>	MEAN VALUATION AGE BASED ON Z 's		O^M 3 PER-CENT		O^M 5 3 PER-CENT	TRUE MEAN AGE DEDUCED FROM MEAN ANNUITY-VALUE	
		Using $O^{M(5)}$ Z 's x	Using O^M Z 's z	$a_{x:n}$ <i>minus</i> True Mean Annuity	$a_{z:n}$ <i>minus</i> True Mean Annuity	$a_{x:n}$ <i>minus</i> True Mean Annuity	O^M 5 x'	O^M z'
Distribution A	5	52.9	52.7	-.005	+.002	-.001	52.8	52.8
	10	47.9	47.7	-.007	...	-.005	47.7	47.7
	15	42.9	42.7	-.015	-.005	-.009	42.7	42.6
	20	37.9	37.7	-.029	-.016	-.011	37.7	37.5
	25	32.9	32.7	-.043	-.027	-.011	32.7	32.4
	30	27.9	27.7	-.049	-.032	-.011	27.7	27.3
	35	24.7	24.6	-.025	-.015	-.004	24.6	24.4
	40	21.8	21.7	-.014	-.003	-.005	21.7	21.7
Distribution B	5	53.6	53.3	-.002	-.003	-.002	53.5	53.5
	10	48.6	48.3	-.011	+.001	-.007	48.4	48.3
	15	43.6	43.3	-.023	-.006	-.012	43.4	43.2
	20	38.6	38.3	-.041	-.020	-.016	38.4	38.0
	25	33.6	33.3	-.060	-.036	-.016	33.4	32.9
	30	29.3	29.1	-.060	-.041	-.014	29.1	28.7
	35	25.7	25.5	-.048	-.027	-.011	25.6	25.2
	40	23.0	22.9	-.031	-.020	-.010	22.9	22.7

Looking first at the $O^{M(5)}$ figures, it will be seen that there is in every case a small *negative* deviation, which confirms the general results deduced in pars. 11-19 for tables following Makeham's Law.

In the case of the O^M annuities, based on the corresponding O^M Z 's, there is a small *positive* deviation for terms of 10 years and under: a small *negative* deviation (less than the corresponding deviation in the $O^{M(5)}$ figures) for term 15 years; while for terms of 20 years and over, the negative deviation is considerably in excess of the $O^{M(5)}$ deviation. This is precisely what was predicted from theoretical considerations in pars. 27-8. Over the whole area of an ordinarily distributed business, the mean O^M deviation should not differ much from the $O^{M(5)}$, but the O^M would probably be somewhat the greater. This could be counteracted, if desired, by using a lower value of c than 1.08 in deducing the O^M Z 's.

The effect of using O^M Z's (or H^M Z's, which produce practically identical results) for an O^M valuation is to produce a considerable increase in the negative deviation, so that in this case (as remarked in par. 36) the excess of reserve would be considerably greater. The most satisfactory course would appear to be to change the Z's when passing from an H^M to an O^M valuation, or in the alternative to make a small graduated deduction from the mean ages brought out by the H^M Z's.

G. J. L.

MR. LIDSTONE'S METHOD OF VALUING ENDOWMENT ASSURANCES.

CONTINUOUS TEMPORARY ANNUITIES.

For the practical application of Mr. Lidstone's method of valuing endowment assurances, tables of the values of continuous temporary annuities are, in many cases, required. Such tables for the H^M Experience accompanied his former paper (see *J.I.A.*, vol. xxxiv, pp. 82-4). Messrs. S. G. Dunn, F.I.A., and Basil May, F.I.A., have kindly contributed the accompanying tables, based upon the O^M Experience at $2\frac{1}{2}$ per-cent and 3 per-cent interest. The values of n , the term, are given at the side, and the values of M , the maturity age, at the head of the columns; and the column headed "Diff." gives the difference between two successive values *in the same row* corresponding to an increase in M , while n remains constant. The tables were formed from the temporary annuities given in the published O^M Table by the approximate formula—

$$\bar{a}_{xn} = \frac{1}{2}(1 + a_{xn-1} + a_{xn}).$$

The maturity age has been taken as equal to $1 + (\text{valuation age}) + (\text{unexpired term})$, thus conforming to classification No. 1 in paragraphs 55 and 56 of the paper. *This must be borne in mind, and the necessary modifications made, if any other classification be adopted.*—EDITOR *J.I.A.*

$M \ 2\frac{1}{2}\%$

Continuous Temporary Annuities.

 $O^M \ 2\frac{1}{2}\%$

$$\bar{a}_{M-\overline{n+1}:n}$$

MATURITY-AGE, i.e., M

MATURITY-AGE, i.e., M														n
35	Diff.	36	Diff.	37	Diff.	38	Diff.	39	Diff.	40	Diff.	41	Diff.	
.984	0	.984	0	.984	0	.984	0	.984	0	.984	0	.984	0	1
1.939	0	1.939	1	1.938	1	1.937	0	1.937	1	1.936	1	1.935	0	2
2.864	1	2.863	1	2.862	1	2.861	1	2.860	2	2.858	1	2.857	2	3
3.763	2	3.761	2	3.759	2	3.757	2	3.755	3	3.752	2	3.750	3	4
4.635	3	4.632	3	4.629	3	4.626	3	4.623	4	4.619	4	4.615	4	5
5.483	4	5.479	4	5.475	5	5.470	5	5.465	5	5.460	5	5.455	5	6
6.307	5	6.302	6	6.296	6	6.290	6	6.284	6	6.278	7	6.271	7	7
7.107	6	7.101	7	7.094	8	7.086	7	7.079	8	7.071	8	7.063	9	8
7.886	8	7.878	8	7.870	9	7.861	9	7.852	9	7.843	10	7.833	11	9
8.644	9	8.635	10	8.625	10	8.615	11	8.604	11	8.593	12	8.581	12	10
9.381	10	9.371	11	9.360	12	9.348	13	9.335	13	9.322	13	9.309	14	11
10.098	11	10.087	13	10.074	13	10.061	13	10.048	15	10.033	15	10.018	16	12
10.797	13	10.784	14	10.770	14	10.756	15	10.741	16	10.725	17	10.708	18	13
11.477	14	11.463	15	11.448	16	11.432	17	11.415	17	11.398	19	11.379	19	14
12.138	15	12.123	15	12.108	17	12.091	18	12.073	19	12.054	21	12.033	21	15
12.783	16	12.767	17	12.750	17	12.733	19	12.714	20	12.694	22	12.672	23	16
13.411	16	13.395	18	13.377	19	13.358	20	13.338	22	13.316	23	13.293	24	17
14.022	17	14.005	18	13.987	20	13.967	21	13.946	23	13.923	24	13.899	26	18
14.616	17	14.599	19	14.580	20	14.560	22	14.538	24	14.514	25	14.489	27	19
...	...	15.178	20	15.158	20	15.138	23	15.115	24	15.091	27	15.064	28	20
...	15.721	21	15.700	23	15.677	25	15.652	27	15.625	29	21
...	16.248	23	16.225	26	16.199	28	16.171	29	22
...	16.758	26	16.732	28	16.704	30	23
...	17.252	29	17.223	31	24
...	17.728	31	25
...	26
...	27
...	28
...	29
...	30
...	31
...	32
...	33
...	34
...	35
...	36
...	37
...	38
...	39
...	40
...	41
...	42
...	43
...	44
...	45
35	Diff.	36	Diff.	37	Diff.	38	Diff.	39	Diff.	40	Diff.	41	Diff.	

$0^M 2\frac{1}{2}\%$

Continuous Temporary Annuities.

 $0^M 2\frac{1}{2}\%$

$$\bar{a}_{M-\overline{n+1}:n}$$

MATURITY-AGE, i.e., M

<i>n</i>	42	Diff.	43	Diff.	44	Diff.	45	Diff.	46	Diff.	47	Diff.	48	Diff.
1	.984	1	.983	0	.983	0	.983	0	.983	1	.982	0	.982	1
2	1.935	1	1.931	1	1.933	1	1.932	1	1.931	1	1.930	1	1.929	1
3	2.855	1	2.854	2	2.852	2	2.850	2	2.848	2	2.846	2	2.844	2
4	3.747	3	3.744	3	3.741	3	3.738	3	3.735	3	3.732	4	3.728	4
5	4.611	4	4.607	4	4.603	4	4.599	5	4.594	5	4.589	5	4.584	6
6	5.450	5	5.445	6	5.439	6	5.433	6	5.427	7	5.420	8	5.412	8
7	6.264	8	6.256	7	6.249	8	6.241	8	6.233	9	6.224	9	6.215	10
8	7.054	9	7.045	9	7.036	9	7.027	11	7.016	11	7.005	11	6.994	12
9	7.822	11	7.811	11	7.800	11	7.789	12	7.777	13	7.764	14	7.750	15
10	8.569	13	8.556	13	8.543	14	8.529	14	8.515	15	8.500	16	8.484	17
11	9.295	15	9.280	15	9.265	16	9.249	17	9.232	17	9.215	18	9.197	19
12	10.002	17	9.985	17	9.968	18	9.950	19	9.931	20	9.911	20	9.891	22
13	10.690	19	10.671	19	10.652	20	10.632	21	10.611	22	10.589	23	10.566	24
14	11.360	20	11.340	22	11.318	23	11.295	23	11.272	24	11.248	25	11.223	26
15	12.012	22	11.990	23	11.967	24	11.943	26	11.917	27	11.890	28	11.862	29
16	12.649	25	12.624	25	12.599	26	12.573	28	12.545	29	12.516	30	12.486	32
17	13.269	26	13.243	27	13.216	29	13.187	29	13.158	31	13.127	32	13.095	34
18	13.873	27	13.846	29	13.817	30	13.787	32	13.755	33	13.722	35	13.687	36
19	14.462	29	14.433	30	14.403	32	14.371	34	14.337	35	14.302	36	14.266	39
20	15.036	29	15.007	32	14.975	34	14.941	35	14.906	37	14.869	39	14.830	40
21	15.596	31	15.565	32	15.533	35	15.498	37	15.461	39	15.422	41	15.381	42
22	16.142	32	16.110	34	16.076	36	16.040	38	16.002	40	15.962	43	15.919	44
23	16.674	33	16.641	35	16.606	37	16.569	39	16.530	41	16.489	44	16.445	46
24	17.192	33	17.159	35	17.124	38	17.086	40	17.046	43	17.003	45	16.958	47
25	17.697	33	17.664	36	17.628	39	17.589	41	17.548	43	17.505	46	17.459	48
26	18.189	33	18.156	36	18.120	39	18.081	42	18.039	44	17.995	47	17.948	50
27	18.635	36	18.599	39	18.560	42	18.518	45	18.473	48	18.425	50
28	19.066	39	19.027	43	18.984	45	18.939	48	18.891	51
29	19.481	42	19.439	45	19.394	48	19.346	52
30	19.883	45	19.838	49	19.789	52
31	20.270	48	20.222	53
32	20.643	52
33
34
35
36
37
38
39
40
41
42
43
44
45
	42	Diff.	43	Diff.	44	Diff.	45	Diff.	46	Diff.	47	Diff.	48	Diff.

Continuous Temporary Annuities.

 $0^M 2\frac{1}{2}\%$

$$\bar{a}_{M-n+1:n}$$

MATURITY-AGE, i.e., M

MATURITY-AGE, i.e., M														n
49	Diff.	50	Diff.	51	Diff.	52	Diff.	53	Diff.	54	Diff.	55	Diff.	
.981	0	.981	0	.981	0	.981	1	.980	0	.980	1	.979	1	1
1.928	1	1.927	2	1.925	1	1.924	2	1.922	2	1.920	2	1.918	2	2
2.842	3	2.839	3	2.836	3	2.833	3	2.830	4	2.826	4	2.822	4	3
3.724	4	3.720	5	3.715	5	3.710	5	3.705	6	3.699	7	3.692	7	4
4.578	6	4.572	7	4.565	7	4.558	8	4.550	9	4.541	10	4.531	10	5
5.404	8	5.396	9	5.387	10	5.377	11	5.366	11	5.355	12	5.343	14	6
6.205	11	6.194	11	6.183	12	6.171	13	6.158	15	6.143	15	6.128	17	7
6.982	13	6.969	14	6.955	15	6.940	16	6.924	17	6.907	19	6.888	21	8
7.735	15	7.720	16	7.704	18	7.686	19	7.667	20	7.647	23	7.624	24	9
8.467	18	8.449	19	8.430	20	8.410	22	8.388	25	8.364	25	8.339	27	10
9.178	20	9.158	22	9.136	23	9.113	25	9.088	26	9.062	29	9.033	31	11
9.869	23	9.846	24	9.822	26	9.796	27	9.769	29	9.740	32	9.708	35	12
10.542	26	10.516	27	10.489	28	10.461	30	10.431	32	10.398	34	10.361	38	13
11.197	29	11.168	29	11.139	31	11.108	33	11.075	36	11.039	38	11.001	40	14
11.833	30	11.803	32	11.771	34	11.737	36	11.701	38	11.663	40	11.623	44	15
12.454	33	12.421	34	12.387	36	12.351	39	12.312	41	12.271	44	12.227	46	16
13.061	36	13.025	37	12.988	39	12.949	41	12.908	44	12.864	46	12.818	50	17
13.651	38	13.613	40	13.573	42	13.531	43	13.488	46	13.442	50	13.392	52	18
14.227	40	14.187	42	14.145	44	14.101	47	14.054	49	14.005	52	13.953	54	19
14.790	43	14.747	44	14.703	47	14.656	49	14.607	51	14.556	54	14.502	58	20
15.339	44	15.295	47	15.248	49	15.199	52	15.147	54	15.093	57	15.036	60	21
15.875	47	15.828	48	15.780	51	15.729	54	15.675	57	15.618	59	15.559	62	22
16.399	48	16.351	51	16.300	53	16.247	56	16.191	59	16.132	62	16.070	65	23
16.911	50	16.861	53	16.808	55	16.753	58	16.695	61	16.634	65	16.569	67	24
17.411	52	17.359	55	17.304	57	17.247	59	17.188	63	17.125	66	17.059	70	25
17.898	53	17.845	55	17.790	58	17.732	62	17.670	65	17.605	68	17.537	72	26
18.375	54	18.321	57	18.264	60	18.204	63	18.141	66	18.075	70	18.005	73	27
18.840	54	18.786	58	18.728	61	18.667	64	18.603	68	18.535	71	18.464	75	28
19.294	55	19.239	58	19.181	62	19.119	65	19.054	69	18.985	73	18.912	76	29
19.737	55	19.682	59	19.623	62	19.561	66	19.495	70	19.425	74	19.351	78	30
20.169	55	20.114	59	20.055	63	19.992	66	19.926	70	19.856	75	19.781	79	31
20.591	55	20.536	59	20.477	63	20.414	67	20.347	71	20.276	75	20.201	80	32
21.003	56	20.947	58	20.889	63	20.826	68	20.758	71	20.687	75	20.612	80	33
...	...	21.349	59	21.290	62	21.228	67	21.161	72	21.089	76	21.013	80	34
...	21.682	62	21.620	67	21.553	71	21.482	76	21.406	81	35
...	22.003	67	21.936	70	21.866	76	21.790	81	36
...	22.310	70	22.240	75	22.165	80	37
...	22.606	74	22.532	80	38
...	22.889	79	39
...	40
...	41
...	42
...	43
...	44
...	45
49	Diff.	50	Diff.	51	Diff.	52	Diff.	53	Diff.	54	Diff.	55	Diff.	

$0^M 2\frac{1}{2}\%$

Continuous Temporary Annuities.

 $0^M 2\frac{1}{2}\%$

$$\bar{a}_{M-\overline{n+1}: \overline{n}_1}$$

MATURITY-AGE, i.e., M														
n	56	Diff.	57	Diff.	58	Diff.	59	Diff.	60	Diff.	61	Diff.	62	Diff.
1	·978	0	·978	1	·977	0	·977	1	·976	1	·975	2	·973	1
2	1·916	2	1·914	2	1·912	3	1·909	3	1·906	3	1·903	4	1·899	4
3	2·818	5	2·813	5	2·808	6	2·802	6	2·796	7	2·789	7	2·782	8
4	3·685	8	3·677	8	3·669	9	3·660	10	3·650	11	3·639	12	3·627	13
5	4·521	11	4·510	11	4·499	14	4·485	15	4·470	15	4·455	17	4·438	18
6	5·329	15	5·314	15	5·299	17	5·282	19	5·263	22	5·241	22	5·219	24
7	6·111	19	6·092	20	6·072	21	6·051	24	6·027	26	6·001	29	5·972	30
8	6·867	22	6·845	24	6·821	26	6·795	29	6·766	31	6·735	33	6·702	37
9	7·600	25	7·575	29	7·546	31	7·515	33	7·482	36	7·446	39	7·407	43
10	8·312	30	8·282	33	8·249	35	8·214	38	8·176	41	8·135	44	8·091	48
11	9·002	33	8·969	36	8·933	39	8·894	43	8·851	46	8·805	49	8·756	55
12	9·673	37	9·636	40	9·596	43	9·553	46	9·507	51	9·456	55	9·401	59
13	10·326	40	10·286	44	10·242	47	10·195	51	10·144	55	10·089	59	10·030	64
14	10·961	44	10·917	47	10·870	50	10·820	55	10·765	60	10·705	63	10·642	70
15	11·579	47	11·532	50	11·482	54	11·428	59	11·369	63	11·306	68	11·238	73
16	12·181	50	12·131	53	12·078	58	12·020	62	11·958	66	11·892	72	11·820	78
17	12·768	53	12·715	56	12·659	61	12·598	65	12·533	70	12·463	76	12·387	81
18	13·340	56	13·284	59	13·225	63	13·162	68	13·094	73	13·021	80	12·941	85
19	13·899	58	13·841	63	13·778	67	13·711	71	13·640	76	13·564	81	13·483	89
20	14·444	62	14·382	64	14·318	69	14·249	74	14·175	79	14·096	86	14·010	91
21	14·976	63	14·913	68	14·845	72	14·773	77	14·696	82	14·614	87	14·527	94
22	15·497	67	15·430	70	15·360	74	15·286	79	15·207	85	15·122	90	15·032	96
23	16·005	68	15·937	73	15·864	77	15·787	81	15·706	87	15·619	92	15·527	100
24	16·502	70	16·432	75	16·357	79	16·278	84	16·194	90	16·104	94	16·010	102
25	16·989	73	16·916	77	16·839	82	16·757	86	16·671	91	16·580	97	16·483	104
26	17·465	75	17·390	79	17·311	84	17·227	88	17·139	93	17·046	100	16·946	105
27	17·932	77	17·855	82	17·773	85	17·688	91	17·597	95	17·502	102	17·400	108
28	18·389	79	18·310	84	18·226	88	18·138	92	18·046	97	17·949	103	17·846	110
29	18·836	81	18·755	85	18·670	90	18·580	94	18·486	99	18·387	105	18·282	111
30	19·273	82	19·191	86	19·105	91	19·014	96	18·918	101	18·817	106	18·711	113
31	19·702	83	19·619	88	19·531	93	19·438	97	19·341	103	19·238	108	19·130	114
32	20·121	84	20·037	89	19·948	93	19·855	99	19·756	104	19·652	110	19·542	115
33	20·532	85	20·447	90	20·357	94	20·263	100	20·163	105	20·058	111	19·947	117
34	20·933	85	20·848	90	20·758	95	20·663	101	20·562	106	20·456	112	20·344	118
35	21·325	85	21·240	90	21·150	95	21·055	101	20·954	107	20·847	113	20·734	119
36	21·709	85	21·624	90	21·534	96	21·438	101	21·337	107	21·230	114	21·116	119
37	22·085	85	22·000	90	21·910	95	21·815	102	21·713	108	21·605	114	21·491	119
38	22·452	85	22·367	90	22·277	95	22·182	101	22·081	108	21·973	113	21·860	120
39	22·810	84	22·726	89	22·637	96	22·541	101	22·440	107	22·333	113	22·220	119
40	23·159	83	23·076	88	22·988	95	22·893	100	22·793	106	22·687	113	22·574	120
41	23·419	88	23·331	93	23·238	100	23·138	106	23·032	112	22·920	118
42	23·666	93	23·573	98	23·475	105	23·370	111	23·259	118
43	23·902	98	23·804	103	23·701	110	23·591	117
44	24·127	103	24·024	109	23·915	116
45	24·340	108	24·232	114
	56	Diff.	57	Diff.	58	Diff.	59	Diff.	60	Diff.	61	Diff.	62	Diff.

$2\frac{1}{2}\%$

Continuous Temporary Annuities.

 $0^M 2\frac{1}{2}\%$

$$\bar{a}_{M-\overline{n+1}:n}$$

MATURITY-AGE $i.e., M$

MATURITY-AGE <i>i.e.</i> , M															<i>n</i>
63	Diff.	64	Diff.	65	Diff.	66	Diff.	67	Diff.	68	Diff.	69	Diff.	70	
·972	1	·971	1	·970	1	·969	1	·968	2	·966	2	·964	2	·962	1
1·895	4	1·891	5	1·886	5	1·881	5	1·876	6	1·870	6	1·864	7	1·857	2
2·774	8	2·766	10	2·756	11	2·745	11	2·734	12	2·722	13	2·709	14	2·695	3
3·614	14	3·600	15	3·585	17	3·568	18	3·550	19	3·531	22	3·509	23	3·486	4
4·420	20	4·400	22	4·378	24	4·354	26	4·328	27	4·301	30	4·271	33	4·238	5
5·195	26	5·169	29	5·140	32	5·108	33	5·075	37	5·038	39	4·999	43	4·956	6
5·942	33	5·909	36	5·873	38	5·835	42	5·793	46	5·747	50	5·697	53	5·644	7
6·665	40	6·625	43	6·582	47	6·535	50	6·485	54	6·431	59	6·372	63	6·309	8
7·364	46	7·318	49	7·269	54	7·215	59	7·156	63	7·093	69	7·024	73	6·951	9
8·043	52	7·991	58	7·933	61	7·872	66	7·806	72	7·734	77	7·657	83	7·574	10
8·701	58	8·643	64	8·579	68	8·511	74	8·437	79	8·358	86	8·272	92	8·180	11
9·342	65	9·277	68	9·209	76	9·133	81	9·052	87	8·965	94	8·871	101	8·770	12
9·966	70	9·896	76	9·820	81	9·739	88	9·651	95	9·556	101	9·455	109	9·346	13
10·572	74	10·498	81	10·417	87	10·330	94	10·236	101	10·135	109	10·026	117	9·909	14
11·165	80	11·085	85	11·000	93	10·907	100	10·807	107	10·700	115	10·585	124	10·461	15
11·742	84	11·658	90	11·568	97	11·471	105	11·366	113	11·253	122	11·131	130	11·001	16
12·306	88	12·218	95	12·123	102	12·021	109	11·912	118	11·794	127	11·667	136	11·531	17
12·856	91	12·765	98	12·667	107	12·560	114	12·446	122	12·324	133	12·191	141	12·050	18
13·394	95	13·299	101	13·198	110	13·088	119	12·969	127	12·842	136	12·703	146	12·560	19
13·919	98	13·821	105	13·716	112	13·604	122	13·482	131	13·351	141	13·210	150	13·060	20
14·433	101	14·332	108	14·224	116	14·108	125	13·983	134	13·849	144	13·705	154	13·551	21
14·936	104	14·832	111	14·721	119	14·602	128	14·474	136	14·338	147	14·191	157	14·034	22
15·427	106	15·321	113	15·208	122	15·086	130	14·956	140	14·816	149	14·667	160	14·507	23
15·908	108	15·800	115	15·685	125	15·560	132	15·428	142	15·286	152	15·134	162	14·972	24
16·379	110	16·269	118	16·151	125	16·026	135	15·891	144	15·747	154	15·593	164	15·429	25
16·841	112	16·729	120	16·609	128	16·481	136	16·345	146	16·199	155	16·044	166	15·878	26
17·292	113	17·179	121	17·058	130	16·928	138	16·790	147	16·643	157	16·486	168	16·318	27
17·736	116	17·620	123	17·497	131	17·366	139	17·227	149	17·078	158	16·920	169	16·751	28
18·171	118	18·053	125	17·925	132	17·796	140	17·656	150	17·506	160	17·346	169	17·177	29
18·598	120	18·478	126	18·352	134	18·218	142	18·076	151	17·925	161	17·764	170	17·594	30
19·016	121	18·895	127	18·768	135	18·633	144	18·489	152	18·337	161	18·176	171	18·005	31
19·427	122	19·305	129	19·176	137	19·039	144	18·895	153	18·742	162	18·580	172	18·408	32
19·830	123	19·707	130	19·577	137	19·440	146	19·294	154	19·140	163	18·977	172	18·805	33
20·226	124	20·102	131	19·971	138	19·833	147	19·686	155	19·531	164	19·367	173	19·194	34
20·615	125	20·490	132	20·358	140	20·218	147	20·071	155	19·916	164	19·752	174	19·578	35
20·997	126	20·871	133	20·738	140	20·598	148	20·450	155	20·295	165	20·130	174	19·956	36
21·372	126	21·246	134	21·112	140	20·972	149	20·823	156	20·667	165	20·502	174	20·328	37
21·740	127	21·613	134	21·479	140	21·339	149	21·190	157	21·033	165	20·868	174	20·694	38
22·101	127	21·974	134	21·840	141	21·699	149	21·550	157	21·393	165	21·228	174	21·054	39
22·454	126	22·328	134	22·194	141	22·053	149	21·904	157	21·747	164	21·583	174	21·409	40
22·802	127	22·675	133	22·542	140	22·402	149	22·253	157	22·096	164	21·932	174	21·758	41
23·141	125	23·016	133	22·883	140	22·743	148	22·595	156	22·439	164	22·275	173	22·102	42
23·474	125	23·349	131	23·218	139	23·079	147	22·932	156	22·776	164	22·612	172	22·440	43
23·799	123	23·676	130	23·546	138	23·408	146	23·262	155	23·107	163	22·944	171	22·773	44
24·118	122	23·996	129	23·867	137	23·730	145	23·585	153	23·432	161	23·271	171	23·100	45
63	Diff.	64	Diff.	65	Diff.	66	Diff.	67	Diff.	68	Diff.	69	Diff.	70	

OM 3%.

Continuous Temporary Annuities.

OM 3

$$\bar{a}_{M-n+1:\overline{n}|}$$

MATURITY-AGE, i.e., M														
n	35	Diff.	36	Diff.	37	Diff.	38	Diff.	39	Diff.	40	Diff.	41	Diff.
1	.982982982982982	1	.981981	...
2	1.930	1	1.929	1	1.928	...	1.928	1	1.927	...	1.927	1	1.926	1
3	2.844	1	2.843	1	2.842	2	2.840	1	2.839	2	2.837	1	2.836	1
4	3.728	2	3.726	2	3.724	2	3.722	3	3.719	2	3.717	3	3.714	2
5	4.581	3	4.578	3	4.575	3	4.572	4	4.568	3	4.565	4	4.561	4
6	5.406	4	5.402	4	5.398	5	5.393	4	5.389	5	5.384	5	5.379	4
7	6.204	5	6.199	6	6.193	5	6.188	6	6.182	6	6.176	7	6.169	7
8	6.977	7	6.970	7	6.963	7	6.956	7	6.949	8	6.941	8	6.933	8
9	7.724	8	7.716	8	7.708	9	7.699	9	7.690	9	7.681	9	7.672	11
10	8.447	8	8.439	10	8.429	10	8.419	10	8.409	11	8.398	12	8.386	11
11	9.148	10	9.138	11	9.127	11	9.116	12	9.104	12	9.092	13	9.079	14
12	9.827	12	9.815	11	9.804	13	9.791	13	9.778	14	9.764	15	9.749	15
13	10.484	12	10.472	13	10.459	14	10.445	15	10.430	16	10.414	16	10.398	17
14	11.121	13	11.108	14	11.094	15	11.079	16	11.063	17	11.046	18	11.028	18
15	11.738	14	11.724	15	11.709	16	11.693	17	11.676	18	11.658	19	11.639	21
16	12.336	14	12.322	17	12.305	16	12.289	18	12.271	20	12.251	21	12.230	21
17	12.916	16	12.900	16	12.884	18	12.866	19	12.847	20	12.827	22	12.805	24
18	13.477	15	13.462	18	13.444	18	13.426	21	13.405	21	13.384	23	13.361	24
19	14.021	16	14.005	17	13.988	20	13.968	20	13.948	23	13.925	23	13.902	25
20	14.532	18	14.514	19	14.495	21	14.474	23	14.451	26	14.425	26
21	15.024	20	15.004	22	14.982	23	14.959	25	14.934	27
22	15.499	22	15.477	24	15.453	25	15.428	29
23	15.955	24	15.931	26	15.905	27
24	16.395	26	16.369	29
25	16.818	27
26
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45
	35	Diff.	36	Diff.	37	Diff.	38	Diff.	39	Diff.	40	Diff.	41	Diff.

3%.

Continuous Temporary Annuities.

0^M 3%.

$$\bar{a}_M - \bar{a}_{n+1:n}$$

MATURITY-AGE, i.e., M

n

42	Diff.	43	Diff.	44	Diff.	45	Diff.	46	Diff.	47	Diff.	48	Diff.	n
.981981981981	1	.980980	1	.979	...	1
1.925	1	1.924	...	1.924	2	1.922	...	1.922	1	1.921	1	1.920	1	2
2.835	2	2.833	2	2.831	1	2.830	2	2.828	2	2.826	2	2.824	3	3
3.712	2	3.710	4	3.706	3	3.703	3	3.700	4	3.696	3	3.693	4	4
4.557	4	4.553	3	4.550	5	4.545	5	4.540	5	4.535	5	4.530	6	5
5.375	6	5.369	6	5.363	6	5.357	7	5.350	6	5.344	8	5.336	7	6
6.162	6	6.156	8	6.148	7	6.141	8	6.133	9	6.124	9	6.115	10	7
6.925	9	6.916	9	6.907	9	6.898	10	6.888	11	6.877	11	6.866	12	8
7.661	10	7.651	11	7.640	11	7.629	11	7.618	13	7.605	14	7.591	14	9
8.375	13	8.362	12	8.350	13	8.337	15	8.322	14	8.308	15	8.293	16	10
9.065	13	9.052	16	9.036	15	9.021	15	9.006	18	8.988	18	8.970	18	11
9.734	16	9.718	16	9.702	18	9.684	19	9.665	18	9.647	20	9.627	21	12
10.381	18	10.363	18	10.345	19	10.326	20	10.306	21	10.285	22	10.263	24	13
11.010	20	10.990	21	10.969	21	10.948	22	10.926	24	10.902	24	10.878	25	14
11.618	21	11.597	22	11.575	24	11.551	24	11.527	25	11.502	26	11.476	28	15
12.209	23	12.186	24	12.162	26	12.136	26	12.110	27	12.083	29	12.054	30	16
12.781	24	12.757	26	12.731	26	12.705	29	12.676	29	12.647	31	12.616	31	17
13.337	25	13.312	27	13.285	29	13.256	30	13.226	31	13.195	33	13.162	35	18
13.877	28	13.849	28	13.821	30	13.791	32	13.759	33	13.726	34	13.692	36	19
14.399	28	14.371	30	14.341	31	14.310	32	14.278	35	14.243	37	14.206	38	20
14.907	29	14.878	30	14.848	33	14.815	35	14.780	36	14.744	37	14.707	40	21
15.399	29	15.370	32	15.338	33	15.305	35	15.270	38	15.232	40	15.192	41	22
15.878	31	15.847	32	15.815	34	15.781	37	15.744	38	15.706	41	15.665	43	23
16.340	30	16.310	33	16.277	35	16.242	37	16.205	40	16.165	41	16.124	44	24
16.791	31	16.760	34	16.726	35	16.691	38	16.653	40	16.613	43	16.570	44	25
17.226	31	17.195	33	17.162	36	17.126	38	17.088	41	17.047	42	17.005	47	26
...	...	17.618	33	17.585	36	17.549	38	17.511	41	17.470	45	17.425	46	27
...	17.995	36	17.959	39	17.920	41	17.879	44	17.835	46	28
...	18.356	38	18.318	41	18.277	44	18.233	47	29
...	18.704	41	18.663	43	18.620	48	30
...	19.039	45	18.994	47	31
...	19.358	46	32
...	33
...	34
...	35
...	36
...	37
...	38
...	39
...	40
...	41
...	42
...	43
...	44
...	45
42	Diff.	43	Diff.	44	Diff.	45	Diff.	46	Diff.	47	Diff.	48	Diff.	

O^M 3%.

Continuous Temporary Annuities.

O^M :

$$\bar{a}_{M-\overline{n+1}:n}$$

n	MATURITY-AGE, I.C., M													
	49	Diff.	50	Diff.	51	Diff.	52	Diff.	53	Diff.	54	Diff.	55	Diff.
1	.979979	1	.978978	1	.977977977	1
2	1.919	1	1.918	2	1.916	2	1.914	1	1.913	2	1.911	2	1.909	2
3	2.821	3	2.818	2	2.816	3	2.813	3	2.810	4	2.806	4	2.802	5
4	3.689	4	3.685	4	3.681	6	3.675	5	3.670	6	3.664	7	3.657	6
5	4.524	6	4.518	7	4.511	7	4.504	8	4.496	7	4.489	10	4.479	10
6	5.329	8	5.321	8	5.313	10	5.303	11	5.292	11	5.281	12	5.269	13
7	6.105	10	6.095	11	6.084	12	6.072	13	6.059	15	6.044	14	6.030	17
8	6.854	13	6.841	14	6.827	14	6.813	15	6.798	17	6.781	19	6.762	19
9	7.577	15	7.562	15	7.547	18	7.529	18	7.511	20	7.491	21	7.470	24
10	8.277	18	8.259	19	8.240	19	8.221	22	8.199	22	8.177	25	8.152	26
11	8.952	19	8.933	21	8.912	23	8.889	23	8.866	26	8.840	28	8.812	29
12	9.606	22	9.584	23	9.561	25	9.536	27	9.509	28	9.481	30	9.451	34
13	10.239	24	10.215	26	10.189	27	10.162	29	10.133	32	10.101	33	10.068	35
14	10.853	27	10.826	28	10.798	30	10.768	32	10.736	33	10.703	36	10.667	39
15	11.448	29	11.419	31	11.388	32	11.356	34	11.322	37	11.285	39	11.246	42
16	12.024	31	11.993	33	11.960	35	11.925	37	11.888	38	11.850	41	11.809	44
17	12.585	35	12.550	34	12.516	37	12.479	40	12.439	41	12.398	44	12.354	47
18	13.127	35	13.092	38	13.054	39	13.015	41	12.974	45	12.929	46	12.883	49
19	13.656	38	13.618	40	13.578	42	13.536	43	13.493	46	13.447	50	13.397	51
20	14.168	40	14.128	41	14.087	44	14.043	46	13.997	49	13.948	50	13.898	54
21	14.667	42	14.625	44	14.581	45	14.536	49	14.487	50	14.437	54	14.383	56
22	15.151	43	15.108	46	15.062	48	15.014	49	14.965	53	14.912	55	14.857	59
23	15.622	45	15.577	48	15.529	48	15.481	53	15.428	54	15.374	58	15.316	60
24	16.080	46	16.034	48	15.986	52	15.934	53	15.881	57	15.824	59	15.765	62
25	16.526	48	16.478	50	16.428	53	16.375	55	16.320	57	16.263	62	16.201	64
26	16.958	48	16.910	51	16.859	54	16.805	57	16.748	59	16.689	63	16.626	66
27	17.379	49	17.330	52	17.278	55	17.223	57	17.166	61	17.105	64	17.041	67
28	17.789	51	17.738	52	17.686	56	17.630	59	17.571	61	17.510	66	17.444	69
29	18.186	50	18.136	54	18.082	55	18.027	60	17.967	63	17.904	66	17.838	70
30	18.572	50	18.522	53	18.469	57	18.412	60	18.352	64	18.288	66	18.222	71
31	18.947	50	18.897	54	18.843	57	18.786	60	18.726	63	18.663	68	18.595	72
32	19.312	50	19.262	53	19.209	57	19.152	61	19.091	64	19.027	68	18.959	71
33	19.665	49	19.616	53	19.563	57	19.506	60	19.446	65	19.381	68	19.313	72
34	19.960	53	19.907	56	19.851	60	19.791	64	19.727	68	19.659	72
35	20.241	56	20.185	60	20.125	63	20.062	67	19.995	72
36	20.511	59	20.452	63	20.389	68	20.321	71
37	20.768	62	20.706	66	20.640	71
38	21.014	66	20.948	70
39	21.249	69
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	49	Diff.	50	Diff.	51	Diff.	52	Diff.	53	Diff.	54	Diff.	55	Diff.

3%.

Continuous Temporary Annuities.

OM 3%.

$$\bar{a}_{M-\overline{n+1}:n}$$

MATURITY-AGE, i.e., M

MATURITY-AGE, i.e., M														n
56	Diff.	57	Diff.	58	Diff.	59	Diff.	60	Diff.	61	Diff.	62	Diff.	
976	1	975	...	975	1	974	1	973	...	973	1	972	1	1
1907	2	1905	3	1902	2	1900	3	1897	3	1894	4	1890	4	2
2797	4	2793	5	2788	6	2782	6	2776	6	2770	7	2763	8	3
3651	8	3643	8	3635	10	3625	9	3616	11	3605	12	3593	13	4
4469	10	4459	12	4447	13	4434	15	4419	15	4404	16	4388	19	5
5256	15	5241	15	5226	17	5209	19	5190	21	5169	21	5148	24	6
6013	18	5995	20	5975	21	5954	23	5931	25	5906	28	5878	29	7
6743	22	6721	24	6697	25	6672	28	6644	30	6614	33	6581	36	8
7446	25	7421	27	7394	30	7364	32	7332	35	7297	39	7258	41	9
8126	29	8097	31	8066	34	8032	37	7995	39	7956	43	7913	47	10
8783	32	8751	36	8715	37	8678	41	8637	45	8592	48	8544	52	11
9417	35	9382	38	9344	42	9302	44	9258	49	9209	52	9157	58	12
10033	39	9994	41	9953	46	9907	49	9858	52	9806	57	9749	62	13
10628	42	10586	45	10541	48	10493	52	10441	57	10384	61	10323	65	14
11204	43	11161	48	11113	52	11061	56	11005	60	10945	64	10881	71	15
11765	48	11717	51	11666	54	11612	58	11554	63	11491	70	11421	72	16
12307	50	12257	53	12204	57	12147	62	12085	66	12019	71	11948	78	17
12834	52	12782	56	12726	60	12666	65	12601	69	12532	74	12458	80	18
13346	55	13291	58	13233	63	13170	66	13104	72	13032	78	12954	82	19
13844	57	13787	61	13726	65	13661	69	13592	75	13517	79	13438	86	20
14327	59	14268	63	14205	67	14138	72	14066	76	13990	83	13907	87	21
14798	62	14736	65	14671	69	14602	74	14528	78	14450	84	14366	90	22
15256	63	15193	67	15126	72	15054	76	14978	80	14898	86	14812	92	23
15703	67	15636	69	15567	73	15494	77	15417	83	15334	88	15246	93	24
16137	67	16070	71	15999	76	15923	79	15844	85	15759	89	15670	96	25
16560	68	16492	74	16418	76	16342	82	16260	85	16175	92	16083	97	26
16974	72	16902	74	16828	78	16750	83	16667	88	16579	92	16487	99	27
17375	71	17304	76	17228	81	17147	84	17063	89	16974	94	16880	100	28
17768	73	17695	78	17617	81	17536	86	17450	90	17360	96	17264	101	29
18151	75	18076	78	17998	83	17915	87	17828	92	17736	96	17640	102	30
18523	74	18449	79	18370	84	18286	88	18198	93	18105	98	18007	104	31
18888	77	18811	80	18731	84	18647	89	18558	94	18464	99	18365	103	32
19241	75	19166	81	19085	85	19000	89	18911	95	18816	100	18716	105	33
19587	77	19510	80	19430	85	19345	91	19254	94	19160	101	19059	106	34
19923	77	19846	81	19765	85	19680	89	19591	96	19495	101	19394	106	35
20250	75	20175	81	20094	86	20008	90	19918	95	19823	101	19722	107	36
20569	76	20493	80	20413	85	20328	90	20238	95	20143	100	20043	107	37
20878	74	20804	80	20724	84	20640	90	20550	95	20455	100	20355	106	38
21180	75	21105	78	21027	84	20943	88	20855	95	20760	100	20660	106	39
21472	72	21400	78	21322	83	21239	88	21151	93	21058	100	20958	104	40
...	...	21685	77	21608	81	21527	87	21440	93	21347	98	21249	105	41
...	21888	81	21807	86	21721	91	21630	97	21533	104	42
...	22079	84	21995	90	21905	96	21809	101	43
...	22260	87	22173	95	22078	100	44
...	22432	92	22340	99	45
56	Diff.	57	Diff.	58	Diff.	59	Diff.	60	Diff.	61	Diff.	62	Diff.	

0^M 3%.

Continuous Temporary Annuities.

0^M 3

$$\bar{a}_{M-\overline{n+1}:n}$$

MATURITY-AGE, i.e., M															
n	63	Diff.	64	Diff.	65	Diff.	66	Diff.	67	Diff.	68	Diff.	69	Diff.	70
1	.971	2	.969	1	.968	2	.966	1	.965	2	.963	1	.962	3	.959
2	1.886	4	1.882	5	1.877	4	1.873	6	1.867	6	1.861	6	1.855	7	1.848
3	2.755	9	2.746	9	2.737	11	2.726	11	2.715	11	2.704	14	2.690	14	2.676
4	3.580	13	3.567	16	3.551	15	3.536	19	3.517	19	3.498	21	3.477	23	3.454
5	4.369	20	4.349	21	4.328	24	4.304	24	4.280	29	4.251	29	4.222	32	4.190
6	5.124	26	5.098	28	5.070	31	5.039	33	5.006	36	4.970	39	4.931	42	4.889
7	5.849	33	5.816	34	5.782	39	5.743	42	5.701	43	5.658	49	5.609	52	5.557
8	6.545	38	6.507	42	6.465	45	6.420	49	6.371	51	6.317	57	6.260	62	6.198
9	7.217	45	7.172	49	7.123	51	7.072	58	7.014	61	6.953	67	6.886	71	6.815
10	7.866	51	7.815	55	7.760	60	7.700	64	7.636	69	7.567	75	7.492	81	7.411
11	8.492	56	8.436	61	8.375	66	8.309	71	8.238	78	8.160	82	8.078	89	7.989
12	9.099	62	9.037	66	8.971	72	8.899	79	8.820	84	8.736	90	8.646	97	8.549
13	9.687	66	9.621	73	9.548	78	9.470	84	9.386	90	9.296	98	9.198	105	9.093
14	10.258	73	10.185	76	10.109	83	10.026	90	9.936	97	9.839	104	9.735	112	9.623
15	10.810	75	10.735	82	10.653	88	10.565	94	10.471	103	10.368	110	10.258	118	10.140
16	11.349	80	11.269	87	11.182	92	11.090	99	10.991	108	10.883	115	10.768	124	10.644
17	11.870	83	11.787	89	11.698	97	11.601	104	11.497	111	11.386	121	11.265	129	11.136
18	12.378	86	12.292	93	12.199	100	12.099	108	11.991	116	11.875	124	11.751	133	11.618
19	12.872	89	12.783	96	12.687	104	12.583	111	12.472	119	12.353	128	12.225	138	12.087
20	13.352	91	13.261	99	13.162	106	13.056	114	12.942	123	12.819	131	12.688	141	12.547
21	13.820	94	13.726	101	13.625	108	13.517	118	13.399	124	13.275	135	13.140	143	12.997
22	14.276	97	14.179	103	14.076	110	13.966	119	13.847	128	13.719	136	13.583	147	13.436
23	14.720	98	14.622	106	14.516	113	14.403	120	14.283	129	14.154	139	14.015	149	13.866
24	15.153	100	15.053	107	14.946	115	14.831	123	14.708	131	14.577	140	14.437	150	14.287
25	15.574	101	15.473	109	15.364	116	15.248	123	15.125	133	14.992	142	14.850	151	14.699
26	15.986	103	15.883	110	15.773	117	15.656	126	15.530	134	15.396	142	15.254	153	15.101
27	16.388	105	16.283	111	16.172	119	16.053	126	15.927	135	15.792	144	15.648	154	15.494
28	16.780	106	16.674	112	16.562	120	16.442	127	16.315	135	16.180	146	16.034	153	15.881
29	17.163	107	17.056	114	16.942	120	16.822	128	16.694	137	16.557	144	16.413	155	16.258
30	17.538	109	17.429	114	17.315	122	17.193	129	17.064	136	16.928	146	16.782	155	16.627
31	17.903	108	17.795	116	17.679	122	17.557	130	17.427	138	17.289	145	17.144	155	16.989
32	18.262	111	18.151	116	18.035	124	17.911	129	17.782	137	17.645	147	17.498	155	17.343
33	18.611	111	18.500	117	18.383	123	18.260	131	18.129	138	17.991	146	17.845	155	17.690
34	18.953	111	18.842	117	18.725	125	18.600	131	18.469	138	18.331	147	18.184	154	18.030
35	19.288	112	19.176	118	19.058	124	18.934	132	18.802	138	18.664	146	18.518	156	18.362
36	19.615	112	19.503	118	19.385	124	19.261	132	19.129	139	18.990	146	18.844	154	18.690
37	19.936	112	19.824	119	19.705	125	19.580	131	19.449	139	19.310	146	19.164	155	19.009
38	20.249	113	20.136	117	20.019	125	19.894	132	19.762	138	19.624	147	19.477	153	19.324
39	20.554	111	20.443	118	20.325	124	20.201	131	20.070	139	19.931	145	19.786	154	19.632
40	20.854	112	20.742	117	20.625	124	20.501	130	20.371	139	20.232	145	20.087	152	19.935
41	21.144	109	21.035	117	20.918	123	20.795	130	20.665	137	20.528	145	20.383	152	20.231
42	21.429	109	21.320	116	21.204	121	21.083	130	20.953	136	20.817	143	20.674	152	20.522
43	21.708	110	21.598	113	21.485	122	21.363	127	21.236	135	21.101	143	20.958	150	20.808
44	21.978	107	21.871	114	21.757	119	21.638	127	21.511	134	21.377	140	21.237	149	21.088
45	22.241	104	22.137	112	22.025	119	21.906	125	21.781	132	21.649	140	21.509	147	21.362
	63	Diff.	64	Diff.	65	Diff.	66	Diff.	67	Diff.	68	Diff.	69	Diff.	70

Further Notes upon the application of Mr. Lidstone's Method to the case of Joint Endowment Assurances. By THOMAS G. ACKLAND, F.I.A.

PROCEEDING upon the lines of §§ (5)–(10) of Mr. Lidstone's paper, but taking two joint lives, (w) and (z), we have

$$v^t {}_t p_{w,z} = v^t s^{2t} g^{(c^w + c^z)(c^t - 1)} = v^t s^{2t} e^{c^w + c^z \cdot c^t - 1} \gamma$$

(where $\gamma = \log_e g$)

$$= v^t s^{2t} \left[1 + (c^w + c^z)(c^t - 1)\gamma + \frac{(c^w + c^z)^2 (c^t - 1)^2 \gamma^2}{2} + \frac{(c^w + c^z)^3 (c^t - 1)^3 \gamma^3}{3} + \frac{(c^w + c^z)^4 (c^t - 1)^4 \gamma^4}{4} + \dots \right]$$

Let $rs^2 = r_0$, $rs^2c = r_1$, $rs^2c^2 = r_2$, and so on; then

$$v^t {}_t p_{w,z} = r_0^t + (c^w + c^z)(r_1^t - r_0^t)\gamma + \frac{(c^w + c^z)^2 (r_2^t - 2r_1^t + r_0^t)\gamma^2}{2} + \frac{(c^w + c^z)^3 (r_3^t - 3r_2^t + 3r_1^t - r_0^t)\gamma^3}{3} + \frac{(c^w + c^z)^4 (r_4^t - 4r_3^t + 6r_2^t - 4r_1^t + r_0^t)\gamma^4}{4} + \dots$$

Summing t for all values from 1 to n inclusive,

$$\sum_{t=1}^{t=n} (v^t {}_t p_{w,z}) = a_{w,z,\overline{n}} = a_0 + (c^w + c^z) \Delta a_0 \gamma + \frac{(c^w + c^z)^2}{2} \Delta^2 a_0 \cdot \gamma^2 + \frac{(c^w + c^z)^3}{3} \Delta^3 a_0 \cdot \gamma^3 + \frac{(c^w + c^z)^4}{4} \Delta^4 a_0 \cdot \gamma^4 + \dots$$

Let now $(c^w + c^z) = 2c^x$, then

$$a_{w,z,\overline{n}} = a_{x,x,\overline{n}} = a_0 + 2c^x \Delta a_0 \cdot \gamma + \frac{4c^{2x}}{2} \Delta^2 a_0 \cdot \gamma^2 + \frac{8c^{3x}}{3} \Delta^3 a_0 \cdot \gamma^3 + \frac{16c^{4x}}{4} \Delta^4 a_0 \cdot \gamma^4 + \dots (1)$$

which may be symbolically expressed as

$$e^{2c^x \cdot \gamma} \Delta a_0$$

where Δ is to be interpreted as a symbol of operation, acting on a_0 only.

Taking the values of s and c from the $O^{M(5)}$ Table, and interest at 3 per-cent per annum, we deduce the values of a_0, a_1, \dots , and the successive differences of a_0 , as under, for different values of n :—

	$n=10$	$n=20$	$n=30$	$n=40$
a_0	8.023	13.330	16.840	19.161
a_1	13.181	34.582	69.329	125.744
a_2	23.120	115.261	482.486	1,946.001
a_3	43.070	464.427	4,586.624	44,914.627
a_4	84.520	2,114.242	50,857.510	1,221,413.882
Δa_0	5.158	21.252	52.489	106.583
$\Delta^2 a_0$	4.781	59.427	360.668	1,713.674
$\Delta^3 a_0$	5.230	209.060	3,330.313	39,434.695
$\Delta^4 a_0$	6.259	823.102	35,145.454	1,052,947.565

The following Table shows the computation of the annuity-values on two lives aged x , by the employment of the above formula, and the comparison of the resulting values with those deduced by the ordinary formula :—

n	x	a_0	$2c^x\gamma\Delta a_0$	$\frac{4c^{2x}\gamma^2\Delta^2 a_0}{2}$	$\frac{8c^{3x}\gamma^3\Delta^3 a_0}{6}$	$\frac{16c^{4x}\gamma^4\Delta^4 a_0}{24}$	$\frac{(3)+(4)}{+(5)+(6)} + (7)$	$a_{x,x,\bar{n}}$	$(9)-(8)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
10	20	8.023	— .072	+ .000	— .000	+ .000	7.951	7.951	...
	30	...	— .176	+ .003	— .000	+ .000	7.850	7.850	...
	40	...	— .433	+ .017	— .001	+ .000	7.606	7.606	...
	50	...	— 1.063	+ .101	— .008	+ .000	7.053	7.054	+ .001
	60	...	— 2.609	+ .611	— .113	+ .017	5.929	5.927	— .002
20	20	13.330	— .296	+ .006	— .000	+ .000	13.040	13.039	— .001
	30	...	— .727	+ .035	— .001	+ .000	12.637	12.636	— .001
	40	...	— 1.784	+ .209	— .021	+ .002	11.736	11.735	— .001
	50	...	— 4.378	+ 1.261	— .305	+ .062	9.970	9.959	— .011
30	20	16.840	— .731	+ .035	— .001	+ .000	16.143	16.141	— .002
	30	...	— 1.795	+ .211	— .022	+ .002	15.236	15.235	— .001
	40	...	— 4.405	+ 1.270	— .328	+ .073	13.450	13.436	— .014
40	20	19.161	— 1.485	+ .166	— .018	+ .002	17.826	17.826	...
	30	...	— 3.644	+ 1.002	— .263	+ .060	16.316	16.305	— .011

It will be seen that the series, though still rapidly convergent, is not so much so as in the case of single lives. The term involving fourth differences, the value of which is significant at the older ages, has therefore been introduced.

Assuming now that the sums assured (in a group where n is constant), are so distributed at the equated ages $(x-15)(x-15)$, $(x-10)(x-10)$, $(x-5)(x-5)$, $(x)(x)$, $(x+5)(x+5)$, $(x+10)(x+10)$, $(x+15)(x+15)$, as to be in the proportions of the numbers 1, 2, 3, 4, 3, 2, 1, then we have for y_1 ,

$$32c^{y_1} = c^x [2(c^{-15} + c^{15}) + 4(c^{-10} + c^{10}) + 6(c^{-5} + c^5) + 8] = 40.89c^x$$

whence $c^{y_1-x} = 1.278$, and $y_1 - x = 2.732$.

Similarly, for y_2 ,

$$64c^{2y_2} = c^{2x} [4(c^{-30} + c^{30}) + 8(c^{-20} + c^{20}) + 12(c^{-10} + c^{10}) + 16] \\ = 159.32c^{2x}$$

whence $c^{2y_2-2x} = 2.489$, and $2y_2 - 2x = 10.155$.

Again, for y_3 ,

$$128c^{3y_3} = c^{3x} [8(c^{-45} + c^{45}) + 16(c^{-30} + c^{30}) + 24(c^{-15} + c^{15}) + 32] \\ = 823.50c^{3x}$$

whence $c^{3y_3-3x} = 6.434$, and $3y_3 - 3x = 20.730$.

Finally, for y_4 ,

$$256c^{4y_4} = c^{4x} [16(c^{-60} + c^{60}) + 32(c^{-40} + c^{40}) + 48(c^{-20} + c^{20}) + 64] \\ = 5,024.48c^{4x}$$

whence $c^{4y_4-4x} = 19.627$, and $4y_4 - 4x = 33.154$.

From the above results we deduce

$$2y_2 - 2y_1 = 4.691 \qquad c^{2y_2-2y_1} = .524$$

$$3y_3 - 3y_1 = 12.534 \qquad c^{3y_3-3y_1} = 2.08$$

$$4y_4 - 4y_1 = 22.226 \qquad c^{4y_4-4y_1} = 6.36$$

and, taking the average maturing age ($y_1 + n$) as 60, the amount of the errors in the terms involving second, third, and fourth differences in formula (3) are as under:—

n	Second Differences	Third Differences	Fourth Differences	Total Error
10	-.053	+.017	-.000	-.036
20	-.110	+.044	-.013	-.079
30	-.111	+.046	-.013	-.078
40	-.087	+.037	-.013	-.063

Proceeding now upon the lines of the alternative method given in §§ (16)–(18), we have

$$\frac{d}{dx} a_{x, x+n} = -2Bc^x \int_0^n v^t (c^t - 1) t p_{x, x} \cdot dt$$

and for the value of r , the assumed rate of increase of the expression under the integral sign, we have, approximately,

$$[(a_{y,y,\bar{n}} - a_{y+t,y+t,\bar{n}}) \div (a_{y-t,y-t,\bar{n}} - a_{y,y,\bar{n}})]^{\frac{1}{t}}$$

and taking, as before, $(y+n)=60$, and $t=10$, we deduce for different values of n :

n	y	r	$c-r$
10	50	1.074	.020
20	40	1.070	.024
30	30	1.071	.023

Then we have

Common ratio on which y_2 is based $= c^2 = 1.197$

„ „ „ y_1 „ „ $= c = 1.094$

Difference in common ratios $= .103$

whence, approximately (assuming the same distribution of cases as before),

$$y_1 - y_r = (y_2 - y_1) \frac{c-r}{c^2-c} = 2.345 \times \frac{1.094-r}{.103}$$

which becomes, for the several values of n ,

n	$y_1 - y_r$
10	.455
20	.546
30	.524

that is to say, the assumed average age is, for values of n between 10 and 30, about half a year greater than the true average age.

If now the true joint annuity-values be computed, taking (y_1+n) as $=60$, and interpolating for the fractional reduction of age as above deduced, we obtain the following values for the error in the annuity:—

n	$a_{y_r,y_r,\bar{n}} - a_{y_1,y_1,\bar{n}}$
10	.035
20	.067
30	.065

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values which do not differ very materially from those previously deduced.

It may be convenient to add a table by the use of which the value of the equated ages $x.x$ can readily be obtained to the nearest integer, in respect of any two lives (w) and (z):—

DEDUCTION TO BE MADE FROM THE AGE OF THE ELDER OF TWO JOINT LIVES (w) AND ($w+t$), IN ORDER TO OBTAIN THE EQUIVALENT AGES $x.x$ TO THE NEAREST INTEGER.

$$x = (w+t) - \Delta; \quad c^{w+t} + c^w = 2c^{w+t-\Delta}; \quad \therefore 1 + c^{-t} = 2c^{-\Delta}$$

DIFFERENCE (t) BETWEEN AGES OF JOINT LIVES		Deduction (Δ) from Age of Elder Life
H ^M Table (Text-Book)	OM(5) Table	
1	1	(take even age)
2— 3	2— 3	1
4— 5	4— 5	2
6— 8	6— 8	3
9— 12	9—12	4
13— 17	13—16	5
18— 24	17—24	6
25— 52	25—44	7
53—102	45—92	8

In the case of the H^M Table, the above results are not quite exact, where either of the lives is younger than 28, but within the ordinary limits of age the above table may be taken as correct.

In the practical valuation of joint endowment assurances, I have found it convenient to deduce, once for all in respect of each policy, the equated entry ages, by the employment of the above table. The joint endowment assurances may then be separately scheduled, precisely as if effected on single lives of the equated age, and the values of Z_M taken out, M being equal to $(x+n)$, where x is the equated entry age, and n the full term of the assurance. The average value of Z_M being then found by the usual process, the aggregate cases are valued at the average age by formulæ applicable to joint endowment assurances. This process differs from that suggested by Mr. Lidstone in his supplementary note on page 47, of taking out the values of Z_M at the *true* maturing ages in respect of each life, and dividing the sum of the Z 's, thus deduced, by 2.

The following example of the valuation of a group of five assurances will explain the process now suggested:—

*Joint Endowment Assurances.**Unexpired term=10 years.* $O^{M(5)}$ 3 PER-CENT.

	True Entry Ages		Equated Entry Ages		Original Term of Years	Equated Maturing Ages	Sum Assured	Z_M $O^{M(5)}$	Net Premiums O^M 3 per-cent	VALUATION AT TRUE AGES		
	<i>w</i>	<i>z</i>	<i>x</i>	<i>x</i>	<i>n</i>	<i>M</i>	£			Sums Assured £	Net Premiums £	Net Liability £
(a)	20:25		23:23		40	63:63	100	205	2.326	78.56	17.12	61.44
(b)	25:35		31:31		30	61:61	500	857	16.205	390.96	121.35	269.61
(c)	30:40		36:36		20	56:56	1,000	1,004	47.810	773.25	372.20	401.05
(d)	35:40		38:38		15	53:53	200	167	12.638	153.99	99.83	54.16
(e)	40:50		46:46		10	56:56	400	438	39.524	300.30	307.69	1.61
							2,200	2,761	118.503	1706.06	918.19	787.87

Approximate valuation :

$$\frac{\sum Z_M}{\sum S} = \frac{2.761}{2.200} = 1.25 = Z_{57.5}$$

$$a_{47.5:47.5,10} = 7.714, \quad A_{47.5:47.5,10} = 77532.$$

$$\text{True average annuity-value} = \frac{918.19}{118.50} = 7.748.$$

$$\text{Error in approximate annuity-value} = -.034$$

$$\text{Value of sums assured} = 77532 \times 2,200 = \text{£}1,705.70$$

$$,, \text{ net premiums} = 7.714 \times 118.503 = 914.13$$

$$\text{Approximate net liability} = 791.57$$

$$\text{Amount of error} = + 3.70$$

Taking now the average value of M as $=57$ (to give effect to the approximate excess of half a year in the average age as demonstrated above), we have

$$a_{47:47,10} = 7.741, \quad A_{47:47,10} = 77453.$$

$$\text{Value of sums assured} = 77453 \times 2,200 = \text{£}1,703.97$$

$$,, \text{ net premiums} = 7.741 \times 118.503 = 917.33$$

$$\text{Net liability} = 786.64$$

$$\text{Amount of error} = - 1.23$$

The correction of half a year in the value of M will, on the average, be given effect to by neglecting the fraction of age in taking out the final value of Z ; that is, by taking the average maturing age, in all cases, at the *next lower integer*.

By Mr. Lidstone's suggested method, the process would be as follows:—

	True Maturing Ages (M)	Sums Assured £	Z^M $O^{M(5)}$
(a)	60)	100	157
	65)		245
(b)	55)	500	500
	65)		1,227
(c)	50)	1,000	638
	60)		1,567
(d)	50)	200	128
	55)		200
(e)	50)	400	255
	60)		627
		2,200	5,544

$$\frac{\Sigma Z_M}{\Sigma S} = \frac{5,544}{2,200} = 2.52 = 2Z_{57.5}$$

and, valuing as before at this average maturing age, results would be arrived at identical with those deduced by the other method. It need hardly be added that, in the valuation at the average maturing age, *joint-life* factors must be employed; no practical difficulty arises, however, as to this, as D and N columns for *equal* joint ages are alone required, and are readily computed. The assumption of a *single* life maturing at the average age would introduce a material error (equivalent to an increase in the rate of interest i of '006 $(1+i)$ approximately), as the principle of uniform seniority does not apply under such an assumption.

*Methods of Valuation and Distribution of Profits in the United Kingdom.** By GERALD H. RYAN, F.I.A., General Manager and Actuary of the British Empire Mutual Life Assurance Company.

SYNOPSIS.

1. LIFE OFFICE VALUATIONS, 1776-1870.—The Equitable Assurance Society.—Northampton Table and System of Valuation.—Criticism of the latter.—A "True" Table advocated by F. Baily, B. Gompertz, A. de Morgan and others.—Origin of the Net Premium System of Valuation: its merits and general adoption.—The "Re-assurance" System of Valuation.
2. DISTRIBUTION OF SURPLUS, 1776-1870.—The plan of the Equitable Society.—Methods in use in 1850.—The "Loading" Method.—Method of distribution "in Proportion to the difference between the Accumulated Premiums and the Reserve."—The Compound and Uniform Reversionary Bonus Methods.—Sheppard Homan's "Contribution" Method.
3. LIFE OFFICE VALUATIONS, 1870-1900.—Life Assurance Companies Act, 1870.—Net Premium System general.—Valuation by Select Tables: by H^M and $H^{M(3)}$ Tables.—Table giving bases of Valuation of 52 Offices in 1870 and 1900.—Increased stringency of Reserves.—Use of "Model" Offices.—Estimated Increase in Reserves £20,000,000.—Dr. Sprague's "Select" Tables and their applicability.—Modifying the "Net Premium" in a Net Premium Valuation.
4. DISTRIBUTION OF SURPLUS, 1870-1900.—Comparison of methods used by 52 Offices.—Popularity of the Compound and Uniform Reversionary Bonus Methods.
5. GENERAL OBSERVATIONS.—The fall in the Rate of Interest as affecting methods of Valuation and Distribution.—Interest not essential to Life Assurance.—Tables of Premiums and Reserves at 0 $\frac{1}{2}$ % (that is, assuming no Interest earned).—Should Rates of Premium be increased as Interest falls?—The Net Premium system of Valuation as affected by the said decline.—A conjecture as to the future method of Valuation.—Criticism of the "Contribution" methods of Distribution.—Conclusion.

Appendix, with references.

In order to define the object and scope of the present contribution, it may be well to state at the outset that it puts forth no pretensions to be a complete survey of a subject which embraces in its wide sweep almost the entire field of practical Life Assurance. The author's design is merely to give a brief sketch of the development, from early times, of the systems and practice of Life Offices in ascertaining their liability under Policies of Assurance, and distributing the surpluses brought out by such valuations among those entitled to them; and the idea has been consistently held in view that it is mainly to assist foreign students that each country is asked to give a record and state the lessons of its past experience. By the combined aid of many writers,

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representing the various countries of the world where the business of life assurance has become firmly established, a complete store of information, a universal text-book, may thus be prepared, to which all who are interested in the work of life assurance may be referred for an account of the history of the subject and the evolution of modern practice in other countries than their own. In regard to similar matters of great professional importance, the value of an International Congress has already been established upon a firm footing; and it may be hoped that in regard to the *Valuation and Distribution of Profits*—a subject of conspicuous interest—past experience will be fully corroborated.

1. LIFE OFFICE VALUATIONS, 1776-1870.

The need of a reserve fund in a Life Assurance Company, and consequently of a system of valuation (or, in other words, of a means of testing the sufficiency of such reserve fund) arose at the time when rates of premium were first graduated according to age at entry. This, as is well known, coincides with the establishment of the Equitable Assurance Society in 1762. Prior to that time, premiums were fixed on the basis of a yearly contract with little regard to the age of the assured, and (except in the case of the Amicable Society) the right of renewal was not vested in the assured.¹ * The year's premiums were looked to for payment of the year's claims—for which indeed they were beyond question adequate—and over and above a reserve for unexpired risks, following the practice of fire and marine insurance, no fund was required to be accumulated. The introduction of the scientific system of Life Assurance, through the adoption by the "Equitable" of a scale of premiums increasing with the age, changed all this; and an early difficulty with the management of that historical institution was the determination of the principles according to which the adequacy of their accumulated fund should be ascertained. This and the succeeding stages of the development of our theory have been concisely explained in a most interesting pamphlet² prepared by Mr. H. W. Manly, a Past-President of the Institute of Actuaries, and the present Actuary of the Equitable Society. Passing by the first valuation of that Society (as to which there is some uncertainty in regard to the principles adopted), the valuation methods of to-day are seen to have sprung from the system employed by the Equitable Society which involved the use of the Northampton Table of Mortality with 3 per-cent interest, the full office premiums (which were likewise based upon the Northampton Table at 3 per-cent) being valued as an asset. This method merits some attention at our hands, since it is well we should comprehend how the foundation stone of modern practice was laid. Now it is well to bear in mind that the scientific advisers of the Equitable Society in its early days were in search of an absolutely safe, rather than a theoretically exact, system of valuation. Hence we find them employing a mortality-table which even then was known to exaggerate the death-rates, on the ground that if they over-estimated their claims they must be on the safe side. Hence, again, the use of 3 per-cent interest, a rate much below that yielded even by British Government

* These reference numbers relate to the Appendix.

Securities at the time. Thus the principles adopted in these early valuations of the Equitable Society were:—First, the assumption that the mortality experience among the lives assured would follow the incidence of the Northampton observations, a course known to exaggerate the death-rates; and secondly, that interest would be earned upon the funds at the rate of 3 per-cent only, an admitted under-estimate of the yield from this source. In this way was safety sought; but another protection against miscalculation or misadventure was secured by retaining in hand one-third of the surplus shown by valuation, before any distribution among the members, an expedient which has never since been altered or abandoned by the Society. Experience has amply confirmed the wisdom and prudence of the policy thus inaugurated at a momentous period by the eminent man, Mr. W. Morgan, who so ably guided the affairs of the Society.

Now it is not to be denied that the critical skill of actuaries has proved the "Northampton" method of valuation to be open to objection in several respects. In the first place, it is no longer maintained that a high rate of mortality means high reserves, inasmuch as policy-values are now known to depend not upon the *rate of mortality* shown by the table on which they are based, but upon the *rate of increase* in the mortality rates. And investigations have proved that, apart from theoretical considerations, the policy-values by the Northampton Table are actually smaller than those by the Tables usually adopted which profess to represent a true or normal mortality rate. How then, it may be asked, did the Equitable Society flourish and prosper, if its reserves were always too small? The reply is that, whereas the values of the policies, as originally effected, were smaller than those of a normal table, the values of the bonus additions were greater (as they obviously must be), and greater to such an extent as to turn the scale in favour of the stringency of the Northampton valuation as against that of the valuation by a normal table. And besides this, the low rate of interest used was an important safeguard.

But it is probably the case that opposition had set in to the use of the Northampton (or any other "false" or "hypothetical") Table long before mathematical analysis had proved that high rates of mortality might produce low policy-values. The novel experiment of the Equitable in seeking to give a permanent scientific basis to life assurance quickly attracted, as it was bound to attract, the notice and curiosity of mathematicians; and from them we might expect to issue objections, strongly expressed, to measuring liabilities by a confessedly false standard.

This, then, brings us to the second line of attack against the "Northampton" method of valuation, which sought to demolish the idea that a result scientifically sound can be produced by false premises. On this point, the opinion of Francis Baily will carry great weight and authority. Warm admirer as he was of the Equitable Society, he must have foreseen at an early date the trouble and error into which the use of the Northampton Table and system of valuation would lead an expert in the solution of general problems. Thus, in the Preface to his epoch-making treatise on "*The Doctrine of Life Annuities and Assurances*" (published in 1813) he says:

"Much of my time is taken up in answering questions, which are

"laid before me for solution, relative to Annuities and Assurances . . .
 "The theorems, from which my practical rules are deduced, are
 "strictly mathematically demonstrated in the course of the present
 "work: and in the numerical enunciation of those rules . . . I
 "discard the indiscriminate use of the Life Annuity Tables deduced
 "from the Northampton observations; so generally adopted by the
 "different Assurance Offices, and so much recommended by their
 "immediate supporters."

Baily appears to have preferred the use of "the probabilities of life deduced from observations in Sweden" for assurance problems, and the table of De Parcieux for annuity problems. Again (on page 506) he states: "Experience has . . . proved to us that the Northampton Table is by no means a correct index of the rate of human mortality amongst the members of such a Society (the Equitable)"; and (on page 508) "Now since the rate of human mortality, amongst the members assured in such Societies, is by no means correctly indicated by the Northampton table of observations, but approximates more nearly to the observations made in Sweden, or to those made by M. De Parcieux" . . . We need not spend time in considering whether Francis Baily was judicious in the choice of his standard; but he was clearly in search of a "true" table to advocate in opposition to the Northampton, and in his attitude I think we can discern the germ of the revolt against the use of a false standard, which eventually, though not very quickly, met with such complete success.

More attention is rightly bestowed upon the pioneer of a movement of this importance than upon those who follow in his train. And so we need say little more of the successors of Baily than that, in the persons of Joshua Milne³ Benjamin Gompertz⁴ Augustus de Morgan⁵ Charles Babbage⁶ and Griffith Davies⁷, we recognize the same strenuous endeavour to dislodge, from its position of influence and authority, the Northampton Table as a standard for measuring mortality among assured lives. As evidence of the growing adoption of a true table, I find in a footnote to the report of the Actuary of the Commercial Union Assurance Company in 1868 (*J.I.A.*, xiv, 402) the following lines: "The Carlisle Table, I am aware, has been very widely adopted by Companies in their valuations; and I observe Mr. Thomson, the Actuary of the Standard Life Office, has recently stated that his Company had adhered to this table for 30 years, and that the leading Offices had, one by one, been led to adopt it."

The third vulnerable point in the Northampton method of valuation consisted in the fact that it brought into account, and treated as an asset, the present value of the full office, or gross, premium payable under the contract. Thus no provision was specifically made for the expenses of carrying on the business, which were in fact left to be met out of the current profits of the Society. A fatal though totally unforeseen error which lent the weight of great authority and honourable tradition to many malpractices in the future, when the like course of bringing into account the full premium was followed by certain companies, *in conjunction with the use of a true table of mortality!* For it must not be overlooked that the Northampton system of valuation was in reality a

net premium method, notwithstanding the fact that the premiums valued were the full office premiums; since it valued the mathematical premiums according to the mortality table by the annuities strictly deduced from the same basis, and satisfied the essential condition that at the moment of assurance the value of the policy was zero.

With all its inherent defects, the Northampton system continued to be used (with the safeguards already mentioned) by the Equitable Society down to our own times, and by a majority of the leading British Companies for fully half the present century. But for a large part of this period, a fierce controversy was carried on between the advocates of the old method and those of the scientific net premium method based on a true mortality table. The profession of actuary at this time can scarcely be said to have existed. True, the established Life Offices had expert advisers in their service whose attention had perforce been devoted to the principles and practice of Life Assurance, and whose ability and knowledge it would, in many instances, be presumptuous to call in question. But as a calling in life, to which persons not connected with insurance offices might turn their thoughts and direct their studies, the profession of actuary was practically unknown. And yet, as time went on, there was an increasing number of men of scientific attainments whose gaze was turned towards this field of investigation; and it is to these men, who very naturally regarded the subject from a somewhat different standpoint from that taken up by the trained insurance official, that changes and improvements in the scientific processes applied to life assurance were due. As often happens, improvement came from without. Francis Baily has already been named in this connection, and we need but add the names of Benjamin Gompertz, John Finlaison, Thomas Galloway and Augustus de Morgan as further instances in point; though it would be unfair not to pay a tribute of admiration to the fine work of actuaries holding official positions in the insurance world—of W. Morgan, Joshua Milne, Griffith Davies and David Jones, to mention but a few of the most prominent members of this class. It is a reasonable inference that the leading actuaries of, let us say, 1840 (to choose a date convenient for the purpose of marking development) were themselves alive to the strength of outside criticism of the ruling system of valuation, since we find a Committee appointed in 1838 to make preparations for collecting observations of the mortality among assured lives from the principal Life Offices—in other words, to deduce from actual experience a true mortality table. This movement, as all actuarial students now know, led to the construction of the “Seventeen Offices’ Experience”, the forerunner of those collective experiences, in Great Britain and other countries, from which so much has been learnt and such invaluable aid given to the progress of life assurance.

The popularity and vogue of the Northampton system of valuation, nevertheless, lasted long and died hard. A prominent writer already quoted (Mr. Manly) gives 1845 as the probable date when the principles of a true net premium valuation^s first became advocated and formally supported. The actual date is involved in obscurity, but I take leave, with great respect, to differ from this authority, and should ascribe the formulation of the principles of the net premium system of valuation to a much earlier date than 1845. A most

instructive fund of information on this subject will be found in the "*Report of the Parliamentary Committee on Assurance Associations* (1853)", to which students of the early history of the valuation of Life offices may be referred for a variety of most curious and interesting facts and opinions. The evidence of John Finlaison, J. A. Higham, C. Jellicoe and W. T. Thomson may be cited as showing the remarkable extent to which a belief in the net premium system as the best and most scientific method of estimating the liabilities of a Life Office, was even at that time entertained. And in face of this testimony, it seems to me impossible to affirm that the origin of the system could have been of so recent a date as 1845. Confirmatory evidence to this effect is, indeed, found in the writings of more than one contemporary author. J. H. James, in his "*Treatise on Life and Fire Assurance*", published in 1851 (p. 178), states, "The Valuation of the Assets and Liabilities of an Assurance Company, at the periodical investigations for Bonus or Profit, is conducted, upon the same principle as the valuation of Life Policies made upon a transfer for a continuance, except that in the estimation of the present value of the premiums, the net amount, and not the gross amount is taken. That is—all marginal additions or surplusage laid on the original premium for commission, expenses, or fluctuation of mortality, are rejected."

The point, interesting perhaps historically, is however of trifling importance. What is more to the purpose is that many years had yet to pass by before the Northampton system was generally discarded. It is probable that the period from 1855 to 1865 saw the net premium system adopted by many, but not the oldest and most conservative, companies. In Scotland, I have reason to think, the latter year rather than the former marked the point of change. But nearly all the companies had submitted to the development of enlightened opinion in this direction before the passing of the Life Assurance Companies Act of 1870; and this date may accordingly be considered as an epoch of the greatest importance in the evolution of the theory and practice of Life Office valuations.

Pausing for a moment to review the situation, we shall be right in saying, first, that the problem of the proper valuation of the liabilities of a Life Office, had as a result of one hundred years' consideration, passed through the various stages from the experimental to a well-established and scientific basis; that the labours of trained minds and experienced investigators, by slow but sure process, had laid the foundations of a sound and trustworthy solution; and that the elements of the problem had come to be correctly understood, and the tendencies and effects of opposing methods fully analyzed and disclosed. This century of the history of Life Assurance will always remain incontestably the most important in the annals of our subject. For henceforth we shall find the development perhaps slower and acting more in the direction of modifying the application of principles, already ascertained and established, than of vitally changing those principles themselves.

But to appreciate the situation thus brought about, we must consider more in detail the leading events and achievements of this period. First, we find the Northampton system of valuation invented; then, gradually adopted, until it became the ruling and accepted

standard for the measurement of the liabilities of a Life Office ; and finally, criticized and condemned by the experts of a new school, until it lost favour and yielded ground to its victorious opponent, the net premium system. This result, due largely in the first instance to the attacks of outside opinion, was materially assisted by the investigations, both scientific and practical, of men whose names have since become household words in the actuarial world—of Dr. Sprague, the late W. P. Pattison, and Mr. H. W. Manly, to name only three of the most prominent writers. By the contributions of such experts, the true effect of the Northampton method, the superior merits of the net premium system, the consequences, immediate and eventual, of the use of a “strong” valuation compared with a weak one, of the adoption of a low rate of interest in place of a high one, were clearly shown, and the “permanent way” of Life Office valuations skilfully and firmly laid. It is unnecessary, in these days, to devote much time to explaining what the principles of the net premium system are ; but for the completeness of this brief historical sketch it may be placed on record that they are based on the essential assumptions :

1. That a normal, or what has been termed a “true” Mortality Table is adopted for the probabilities of life, and combined with interest in valuing future payments, whether of sums assured or premiums ; and

2. That the mathematical or net premium (picturesquely termed by Mr. John Finlaison in his evidence before the Parliamentary Committee of 1853 the “dry risk premium”) according to these elements of mortality and interest, be alone brought into account and valued as an asset (or reduction of gross liability in respect of the sum assured), thus leaving intact for future expenses, contingencies and profits, the margin or loading, representing the difference between the gross or office premiums actually charged and the mathematical or net premiums.

This is, of course, the net premium system in its barest outline and reduced to its simplest terms.

Before closing this section of the present essay, it may be well to point out briefly that other methods of valuation besides those here examined were in occasional use by Life Offices within the period passed in review. There was the “re-assurance” or “hypothetical” method, supported with so much skill and ingenuity by the late R. Tucker⁹ and criticized with equal ability and success by Dr. Sprague.¹⁰ Again, there was a system of using net-values for the valuation units in combination with a percentage of the gross premium—in some shameful cases, even the entire gross premium—instead of the net premium. A consideration of these methods need not detain us at this point, though references to the principal Papers in the *Journal of the Institute of Actuaries* dealing with them, in defence and attack, are given for the student who may wish to push his enquiries further in this direction.

11.—THE DISTRIBUTION OF SURPLUS, 1776-1870.

It will be convenient, as the two branches of our theme are so closely connected, if we break the narrative at this point, and refer to the methods employed by the various companies in dividing the ascertained surpluses among their members. The earliest history of the subject is again the record of the practice of the Equitable

Society. At the first distribution of profits made by this Office in 1776, a cash return (the only cash bonus ever declared by the Society) of ten per-cent of all the premiums paid by the members was made to them. Thenceforth the bonus took the form of a uniform addition per-cent on the sum assured for each premium paid from the commencement of the assurance, and this system has since remained a distinguishing feature of the Society; and the large profits realized, aided so materially by the prudent expedient of retaining undivided at each distribution one-third of the entire reported surplus, have permitted the allotment of handsome bonus additions to policies of relatively short duration while giving magnificent results to the older contracts. Where everyone has been so generously treated there is no ground for objection on the score of principle, though under less favourable conditions the equity of giving, at one division of profits, £10 per £100 assured to a policy of 10 years' duration, and £50 to one of 50 years' duration, might not have been so tacitly admitted.

There is little or no evidence readily available to show that the competitors of the Equitable Society, though adopting its system of valuation, followed its identical method of distribution in these early times. One company indeed—The Rock Life Insurance Company—did so, but no general custom seems to have existed to this effect. Trustworthy information upon this head of our subject is, however, not easily obtained. Life Offices reserved such facts concerning their inner working as they chose to make known at all, for their own clients; and the literature of the subject was meagre in the extreme. In an editorial article under the heading of "The Determination and Division of Surplus", to be found in the first volume of the *Assurance Magazine* (now called the *Journal of the Institute of Actuaries*), on page 22, the following words appear:

"Considering the undeniable importance of a conformity to "correct principles in carrying out the processes which must be "resorted to for the purposes above mentioned, it is not a little "remarkable that no work exclusively on the subject has hitherto "appeared. If we except Mr. De Morgan's and Mr. Sang's Essays, "and the observations scattered through the writings of Mr. Babbage, "Mr. David Jones, Mr. Samuel Brown, and one or two others, we "believe it may be truly said that *scarcely any allusion to it is to "be found.*"

This paragraph, and particularly the words I have set in italics, shows us very plainly the position of matters in 1850. The authorities here mentioned should be carefully consulted by students of this period of life assurance history, and be made to yield aught that they can contribute to the known facts of the time. For the present object in view no such exhaustive digging and delving is necessary.

It will be sufficient to gather from the early numbers of the *Journal of the Institute of Actuaries*, that the principal methods hitherto employed by Life Offices in the distribution of profits (in addition to what we may call the "Equitable" plan) were:

To divide the surplus:

1. In proportion to the premiums paid since the last distribution, accumulated at interest;
2. In proportion to the difference between the amount of premiums (accumulated at interest) and the respective values of the policies.

3. By way of uniform reversionary addition in proportion to the sum assured (and, in some cases, existing bonus additions).

Besides these, there were several fantastic methods in use, and others confined to, and a special characteristic of, individual offices; and for a full description and analysis of these, reference may be made to the writings of C. Jellicoe, Dr. Sprague, and W. P. Pattison. But it is important to bear in mind that from about this year, 1850, the fairness and scientific basis of the several methods in use became a common subject of critical examination. Going back to first principles, the authors above mentioned, and others, maintained with irresistible force, that surplus should only be divided among those who had contributed to it, and in proportion to their several contributions as closely as they could be ascertained with due regard to practical considerations. Here was a line of attack which, from the first, was bound to succeed. No system at variance with its principles could hope to withstand a steady fire delivered from so strong and commanding a position. Untenable methods—such as several of those we have simply alluded to, without description—were swept away one by one and became things of the past.

How did it fare, in the conflict, with the three methods specifically named? Of these, the second in order was employed by several strong and old-established Life Offices, who were conservative by instinct and slow to move. Nevertheless, objections of practice and principle were brought against it which admitted of no satisfactory rebuttal. In practice it was shown to be cumbrous as involving the separate valuation of each policy: in essence, to be “contrary to equity and to include an assumption opposed to the fundamental principles of Life Assurance”,¹¹ inasmuch as it allotted the surplus in the proportion of contributions to profits and *claim-payments*. The effect of this method was that it gave very large bonus additions to old lives, which was assumed to be the same thing as to old policies; and this increasing character of its results made it popular with offices whose merits had to stand comparison with those of the “Equitable”, in which the like feature was much more prominently shown. It was also, no doubt, thought to possess the advantages which we now attach to the modern “contribution methods”, where the profit from loading, mortality, and interest enters into the scale of distribution. Nevertheless, before the advancing tide of enlightened criticism, this method disappeared. Popular, and in vogue at one time with the strongest institutions, it does not now find a place among the many systems in use by British offices. Its importance in the past fully merits the attention it has received; but it is a piece of history and that alone.

For a considerable period, the first of the three systems named succeeded the foregoing plan in favour. To divide the surplus of a quinquennium (or other term of years) in proportion to the premiums paid therein, was a reasonable and logical expedient, based on the rough idea that the existence of a surplus proved the premiums to have been unnecessarily high and that the excess could most fairly be returned in the ratio of the premiums paid. Accordingly we find that in 1860, this was “the most general mode of distribution”; and in one form or another it survived, until quite recent times, as one of the leading and more frequently used methods, and may even now, though rarely, be met with.

Round this method criticism played with less effect at first, owing to the fact that the companies using it had either chosen it to fit in with their valuation system or selected the latter to harmonize with this particular bonus system. But the rigid application of the general principles enunciated when we were considering Method 2, soon showed that this plan had inherent defects. First, it was demonstrated that the fundamental idea of the plan required that the "loading" should form a uniform percentage of the net premium, and this proving not to be the case with the rates of office premiums current, Mr. Jellicoe substituted the loading for the gross premium in applying the method. Next it came to be seen that the excess of the premium charged over the net premium, while generally constituting the chief source of surplus, by no means included all the contributions to profit; and according as these latter contributions were of small value or important magnitude, so the method was entitled to support or the reverse. Moreover, the method had the practical disadvantage of making the scale of distribution virtually constant at successive distributions, a result which, on the assumption of a steady profit-earning power, led to reversionary bonuses diminishing with the duration of a policy: a thing not perhaps wrong in itself, but apt to be misunderstood by policyholders before whose eyes the dazzling inducements of the increasing bonuses produced by the older systems were held out in competition. Self-interest as well as a growing scientific spirit of enquiry were alike interested in probing and testing the soundness of this method, and to such effect was their work done that, towards the end of the period we are considering (1870) its foundations had been greatly weakened. The causes that led to this result may be briefly summed up as arising, first, from the growing tendency with well-established offices to strengthen their basis of valuation by the use of a lower rate of interest than that at which their funds were actually accumulating, and secondly, from the true appreciation of the importance as a source of profit of excess interest—by which term I mean the difference between the actual yield from interest of the fund, and the necessary hypothetical yield on the basis of the rate of interest assumed in the valuation of the liabilities. Now, the incidence of the interest profit is very different from that of the loading profit, as every actuarial student knows. Take as an illustration the case of a company increasing largely the stringency of its reserves by a reduction in the valuation rate of interest: here the loading profit would be considerably diminished and the interest profit simultaneously increased, and as the former is a function of the current premium which does not vary throughout the contract, while the latter is a function of the reserve-value of the policy which continually increases, it is clear that the two sources of profit operate with different force, and affect a policy's right to surplus in varying measure. So long, therefore, as a Company valued its obligations at a rate of interest approximating within narrow limits to that which it earned, so long, that is to say, as its interest-profit formed but a small portion of the total surplus, the "loading" system of distribution could be defended. In the generality of instances, this condition had ceased to be fulfilled in 1870; since then it has become practically obsolete. Stronger

valuations have diminished the margin of premiums and increased the margin of interest. As a source of profit, the former has become smaller and smaller, while the latter is relatively, if not absolutely, larger than before. Hence the "loading" system which rested on the former and neglected the latter, was doomed to disappear, and we are not likely, under present conditions, ever to see its revival.

We are now brought to a discussion of the third of the methods proposed to be examined, namely, that which includes what are so familiar to us under the name of the Uniform Reversionary Bonus and the Compound Reversionary Bonus plans. It is easy to discern their origin. The example of the Equitable Society was always a potent influence in the early days of life assurance; and the Equitable, as we have seen, adopted the device of allotting its surplus in the form of a uniform reversionary addition for each year from the commencement of the assurance. To pass from this to a plan based on the same lines, but taking account of the time, not from the original date of the policy but from the last division, was at the most a simple step for those offices who were not burdened with the immense resources and unrivalled surplus of the Equitable; while the modifying expedient of making the scale of reversionary addition apply to existing bonuses as well as the contract sum assured was doubtless introduced to give to the results that increasing character which was very general and, we need hardly add, very popular. At any rate this method approached the question from an essentially different point of view from the first method, as it disregarded the premium payable, or the loading contribution, and considered the "stake in the company", or in other words the policy moneys payable, as the fairer measure of participation in surplus.

The search-light of scientific criticism was turned upon this as upon the other systems. It was denounced as being arbitrary; as neglecting the sources of profits and the respective contributions of members; as liable to be thrown out of gear by change of mortality table or rate of interest in the basis of valuation. But it was conceded that it might be made fairly equitable by specially calculating or adjusting the rates of premium chargeable, in which case the last-mentioned objection would alone remain. It cannot be said that up to 1870 any preponderating weight of opinion or custom favoured the Uniform or Compound Reversionary Bonus methods; and it is highly probable that neither their advocates nor their hostile critics, at this time, anticipated that a majority of the leading companies would in years to come adopt one or other of these forms of apportionment of profit in substitution for systems then more popular; in short, that these methods should win their way to the first place among the many plans utilized by Life Offices for the division of surplus.

We are, however, anticipating in some degree the course of events. More must be said upon the merits of the Uniform and Compound Reversionary Bonus systems, when we come to deal with the subject in its next stage of development. For the present, restricting our view to 1870 and the preceding years, it is enough to summarize the growth of practice and opinion by saying that:

Method I (in proportion to premiums paid or loadings) was very freely used but was slowly losing ground.

Method II (in proportion to the difference between the accumulated premiums and the reserve), though still adopted by several leading companies, was very generally condemned on grounds of principle, and was doomed to disappear; and

Method III (in proportion to Sum Assured, or Sum Assured and existing Bonus additions) had influential adherents and was gaining ground.

With these words, we may leave the matter for the present, though it would be neglectful not to make a brief reference to the suggestions of the late Sheppard Homans which rightly fall within the period we have been considering. Sheppard Homans' proposal was that the profit should be ascertained from its separate sources, namely, the loading, the interest and the mortality profit; and that these respective contributions should be taken into account in estimating the measure of participation among the various beneficiaries. It is a system that has had an immense following in America, and of which the Congress will no doubt hear more exhaustively from an American writer. But, at the same time, its re-publication in the *Journal of the Institute*¹² gives it an honoured place in the British literature of the subject; its influence on expert opinion was undoubtedly great; and we may fairly acclaim it as the origin and harbinger of the modern scientific "contribution methods" of which it will behove us to speak later on.

3. LIFE OFFICE VALUATIONS, 1870-1900.

The enactment of the Life Assurance Companies Act, 1870, brought about many important changes in the position and working of Life Offices. Henceforth their affairs were to be submitted to the light of publicity through the annual publication of revenue accounts and balance sheets, and the periodical publication of valuation statements. Since that date, accordingly, the historian or student has had a comparatively easy task. All the necessary rough material for his enquiries has been available in a form both trustworthy and, on the whole, of adequate fulness. The methods of valuation since then employed by Life Offices can be ascertained from official public sources, and the development and trend of practice in this direction investigated with as much thoroughness as may be desired. If this were the only good achieved by the Life Assurance Companies Act, it would probably be entitled to greater regard and respect than the bulk of measures that have been placed upon the Statute-book; though, as a matter of fact, this is by no means its sole public service.

From this point we pass out of the stage of development as regards the form in which the liabilities of a Life office are computed, for by this time the system of using a normal or true mortality table and valuing net premiums only—in other words, the net premium system—was adopted by a sweeping majority of the soundest companies. But progress was not suspended: it merely followed other lines. The substitution of stronger standards of reserve in respect of mortality and interest took the place of changes of system; and the steady improvement in the financial condition of Life Assurance institutions proceeded at an undiminished pace. This circumstance is of advantage in that it enables us not merely to record with accuracy the changes brought about, but (thanks to

the investigations of contributors to the *Journal*¹³ among whom we may mention Mr. H. W. Manly, Dr. Sprague, Mr. G. King, and the late W. Sutton and J. Valentine) to measure the effect of such changes, and to show the extent to which the reserves of our Life Offices have been strengthened. The path is thus cleared for many interesting statistical enquiries, the data for which are to be found, in the rough, in the Parliamentary Returns to the Board of Trade made in compliance with the Life Assurance Companies Act.

But before turning to account this valuable element of information, one important modification of the net premium system remains to be considered. The work of the late J. A. Higham and Dr. Sprague in connection with the influence of "selection"¹⁴ and the duration of insurance upon mortality rates and policy-values, proved conclusively that the use of an "aggregate" table—that is, one deduced from observations on assured lives collated without distinction of duration of the contract—under-estimates the reserves shown to be required when allowance is made for selection. Such is particularly and increasingly the case as the duration, or time elapsing since entry, becomes greater. This could have been put right, in strict accordance with the principles of the net premium method, by employing a complete set of tables giving for each age at entry (or certain groups of ages at entry) a full mortality table from which the corresponding valuation units and net premium could be deduced. But not only would the labour of such a process be almost prohibitory, but the statistics collected by the 20 British Offices up to 1863 in date (and published in 1869) were too slender to justify this minute subdivision and to support this vast superstructure. While, further, the rapid increase in the number of endowment assurances granted by the offices pointed to the logical necessity of a full canon of temporary annuities and endowment assurance-values for the entire (or selected) ages at entry. This was obviously a hopeless task to enter upon. Accordingly the architects of the Twenty-Offices' Table, called the Healthy Males or H^M Table (when confined to observations on male assured lives) sought to meet the difficulty by assuming that the light mortality due to medical selection wore off at the end of five years (more strictly speaking $4\frac{1}{2}$ years) from entry, and that the mortality after that period might be measured by a new table termed the $H^{M(5)}$, constructed from the observations of all assured lives which had been at least 5 (or $4\frac{1}{2}$) years on the books. And some influence was given to the effect of selection by the contrivance of using the H^M Table for all policies under 5 years in force, and the $H^{M(5)}$ Table for those of longer duration, the H^M net premium being valued in both cases. This formed a composite and rather complicated system of valuation which was, however, found to give, *en bloc*, a rough approximation to the reserves under pure select tables; and, owing to its greater stringency, compared with the other simple tables in use, it rapidly became accepted as the highest standard for the reserves of a Life Office. Later on something may be said in criticism of this now authoritative method of valuation, though, as that may concern the future rather than the period under review, our remarks on this head may be postponed.

An attempt may now be made to gain an idea of the degree to which the reserves of Life Offices have been increased in amount

FIRST VALUATION AFTER 1870			
No. of Company	Date	Basis	Liability
1	31 Dec. 1873	Carlisle 4 % (reserving £24,950 in addition)	£ 861
2	31 Dec. 1869	Northampton 3 % and Special Table. Gross Premiums	1,372
3	31 Dec. 1872	Carlisle 3 %, Bonuses 3½ % Net	464
4	13 May 1871	" 3 % " " " "	324
5	20 Jan. 1874	" 3½ %, Bonuses 4 % 3 % " " " "	567
6	31 May 1871	Special Table derived from Premiums and Carlisle 3 %	1,532
7	30 June 1871	Carlisle 3 % Net Premiums	1,468
8	31 Dec. 1872	17 Offices 3 % and India Madras Military Fund " "	222
9	30 June 1872	Carlisle 4 %; 76 % W. P., 88 % N. P., and 85 % Gtd. Bonus, of Gross Premiums	2,667
10	31 Dec. 1873	Special Table 3½ % Net Premiums	2,478
11	31 Mar. 1871	20 Offices, 1863, 3½ %, and Carlisle 3 % Specials " "	725
12	25 Dec. 1870	Carlisle 3 % " "	535
13	1 Nov. 1879	Northampton 3 % (½ surplus undivided) Gross Premiums	2,415
14	31 Dec. 1874	H ^M & H ^M (5) 3 % Net	752
15	20 Nov. 1872	Special Table 3 % Gross	771
16	31 Dec. 1872	Equitable Experience, 1834, and Carlisle 4 % Net	344
17	25 Dec. 1874	H ^M 3 % and Contingent Carlisle 3 % " "	1,398
18	31 Dec. 1871	Davies' Equitable 4 % Gross	306
19	31 Jan. 1871	17 Offices 4 %, Specials Carlisle 3 % Net	706
20	31 Dec. 1872	Carlisle 3½ % " "	217
21	31 Dec. 1869	Northampton 3 %, W. P. and Davies' Equitable 3½ %, N. P. Gross Premiums	3,910
22	30 Nov. 1874	Carlisle 3 % Net	245
23	31 Dec. 1871	17 Offices 3 % " "	1,27
24	31 Dec. 1873	Carlisle, Guaranteed Bonus 4 %, N. P. 3 % " "	1,59
25	31 Dec. 1870	17 Offices 4 %, and Govt. Annuities, 1829, 3 % " "	1,12
26	31 Dec. 1872	Carlisle 3 % " "	7
27	31 Dec. 1874	Carlisle 3 % " "	17
28	20 Nov. 1872	Special Table derived from Premiums 3 % Gross	2,65
29	31 Dec. 1870	Carlisle 3½ %, Specials 3 % Net	1,82
30	31 Dec. 1870	Carlisle 3 % " "	68
31	30 June 1871	Equitable Experience 3½ %, Specials Carlisle. 82 % W. P. 88 %, N. P. of Gross Premiums	1,11
32	30 June 1875	H ^M 3½ % Net	88
33	31 Dec. 1872	Northampton and Special Table 3 % W. P. Gross Premiums. Carlisle 3 % N. P. Net Premiums ½ Surplus reserved	1,39
34	31 Dec. 1872	Carlisle 3 % Net Premiums	42
35	31 Dec. 1871	Carlisle 3 %; 80 % W. P., 90 % N. P. of Gross	26
36	20 Aug. 1875	Northampton 3 % " "	1,45
37	31 Dec. 1869	Davies' Equitable 3 % and Carlisle Specials Net	85
38	31 Dec. 1870	Northampton 3 % and Equitable 4 % Gross	1,48
39	31 Dec. 1873	Carlisle 3½ %, 82-313 % of " "	2
40	31 Dec. 1874	English Life Table 1, Carlisle (joint lives) 4 % Northampton, (for Bonuses) Net Premiums	1,34
41	1 Mar. 1873	Carlisle 3 %, Bonuses 3½ % " "	1,87
42	31 Dec. 1873	H ^M 4 % Carlisle Specials " "	1,71
43	31 Dec. 1873	Carlisle 3 % " "	4,36
44	15 Nov. 1870	Carlisle 3 %, English Life 1 and Special Table " "	3,14
45	31 Dec. 1873	Carlisle 3 %; 80 % W. P., 85·7 % N. P. of Gross	97
46	24 June 1872	Carlisle 4 %, Bonuses 3 % Net	1,09
47	30 June 1872	H ^M 3 %, Carlisle Specials " "	64
48	31 Dec. 1870	Carlisle 3 %; 80 % W. P., 89 % N. P. of Gross	98
49	31 Dec. 1870	Special Table derived from Premiums, 3 % " "	55
50	1 May 1870	Table upon which Premiums are calculated, 3 % " "	77
51	31 Dec. 1871	Carlisle 3 % " "	28
52	1 Mar. 1870	Table from Premiums, 3 %, and Carlisle 3 % Net	34

£59,68 35

y 52 British Offices.

LAST VALUATION PRIOR TO 1900

Year	Company	Basis	Liability	No. of Company
			£	
1898	HM & HM ⁽⁵⁾	3 % Net. Carlisle for Contingent Assurances	2,591,135	1
1894	HM & HM ⁽⁵⁾	2½ % with 3 % Net Prem. Carlisle Contingent	1,288,251	2
1896	HM	3½ % Net Premiums (Canada 3½ %)	1,759,713	3
1897	HM	3 % Net Premiums and Carlisle (Specials)	1,250,864	4
1899	HM	3 % " " " "	2,169,757	5
1896	HM & HM ⁽⁵⁾	2½ % With Profits and 3 % Non-Profit Net Prems.	3,417,791	6
1896	HM & HM ⁽⁵⁾	2½ % Net Premiums, Special Reserve £60,485	2,863,543	7
1897	HM	3 % " " " £37,000	1,685,445	8
1897	HM	3 % " " Carlisle (Specials)	2,175,587	9
1898	HM	3 % " " " " " "	3,472,834	10
1897	HM	3 % " " " " " "	2,570,641	11
1895	HM & HM ⁽⁵⁾	3 % " " Carlisle (Contingent)	1,512,362	12
1889	Northampton	3 % (½ surplus undivided) Gross Premiums	2,497,939	13
1894	HM & HM ⁽⁵⁾	2½ % Net Premiums	2,261,476	14
1897	Special Table	3 % " " (91 % Office)	1,963,458	15
1897	HM	3 % " " and Specials Carlisle 3½ %	1,449,050	16
1894	HM & HM ⁽⁵⁾	3 % " " and Contingent Carlisle 3 %	2,176,571	17
1898	HM	3 % Gross " HM 2 % net exdgd. by £272,323	2,160,950	18
1896	HM & HM ⁽⁵⁾	3 % W. P. 3½ % N.P. Net Prems. Contingent Carl.	1,951,829	19
1894	HM	3½ % Net Premiums	869,754	20
1894	HM	3 % " " " " " "	3,378,789	21
1895	HM	3 % " " and Contingent Carlisle.	879,478	22
1896	HM	2½ % Premiums 82 % W. P. and 90 % N. P. Gross	2,301,576	23
1898	HM & HM ⁽⁵⁾	3 % Net Premiums	3,196,587	24
1895	HM	3 % " " " " " "	1,706,341	25
1897	HM	3½ % Canada, 4 % Net Prems., Cont. Carlisle 3 %	1,138,583	26
1894	HM & HM ⁽⁵⁾	3 % Net Premiums	496,825	27
1897	HM	3 % " " " " " "	4,391,590	28
1895	HM	3 % " " " " " "	6,206,097	29
1895	HM & HM ⁽⁵⁾	3 % " " " " " "	2,526,204	30
1896	HM	3½ % " " New Bonus HM ⁽⁵⁾ & £90,000 rsrvd.	2,661,604	31
1895	HM	3 % " " " " " "	1,014,604	32
1897	HM	3 % " " and ½ surplus reserved	2,500,908	33
1897	HM	3 % " " " " " "	1,630,805	34
1897	HM	3 % " " " " " "	13,631,985	35
1895	HM	3 % W. P. D. P. 3½ % N. P.	1,428,635	36
1894	HM & HM ⁽⁵⁾	3 % Net Premiums, Carlisle Contingent	4,256,883	37
1895	HM & HM ⁽⁵⁾	3 % " " " " " "	1,743,059	38
1898	HM & HM ⁽⁵⁾	3 % " " " " " "	670,991	39
1895	HM & HM ⁽⁵⁾	2½ % " " HM 3½ %	3,087,304	40
1898	HM	3 % " " " " " "	3,588,902	41
1894	HM	3½ % " " " " " "	7,353,479	42
1894	HM & HM ⁽⁵⁾	3½ % " " (Special Reserve, £445,850).	10,010,441	43
1895	(Reg. Genl.)	English Life 1, Carlisle 3 % & 3½ %	6,211,108	44
		Net Premiums		
1995	HM	3 % Net Premiums	4,166,043	45
1896	HM & HM ⁽⁵⁾	3 % " " " " " "	2,887,586	46
1897	HM	3 % " " and Carlisle (Specials)	1,975,203	47
1895	HM & HM ⁽⁵⁾	2½ % " " " " " "	4,821,109	48
1898	HM	3 % " " " " " "	754,825	49
1895	HM & HM ⁽⁵⁾	3 % " " and Carlisle (Specials)	771,464	50
1896	HM	3 % " " " " " "	518,873	51
1895	HM & HM ⁽⁵⁾	3 % " " " " " "	528,121	52
			£144,524,982	

during the period 1870-1900. For this purpose a schedule has been prepared of 52 companies in which are given the basis of valuation and assurance fund shown by the first return made under the Act of 1870, and the last return prior to 1900. The several companies have been distinguished by numerals only, with a view to giving the statement a general and impersonal character; and the facts here collected will doubtless be considered interesting for the light they throw on the progress made within these years.

No uniformity in the standard of reserves is to be looked for among Life Offices whose ages, circumstances of birth, and social and commercial connections are, indeed, too various to be compatible with a common measure of prosperity. Nevertheless, in the 52 chosen cases, representing leading companies of established position, it is easy to see a general tendency towards the adoption of identical principles and bases of valuation; so much so at any rate that, neglecting alike the extremes of strength and weakness, we may select the Carlisle 3 per-cent basis as affording the most generally accepted test for the liabilities of Life Offices in 1870, and the H^M and $H^{M(b)}$ 3 per-cent basis in 1900. And it is a fair assumption that the increase in the reserves generally is to be measured by the difference between the results obtained by these respective valuations, inasmuch as, with few exceptions, the Companies whose reserves were in excess of the normal standard of 1870 are in excess of that of 1900, and *vice versa*.

From the beginning of the century our actuaries have probably been greatly interested in the financial result of a change in the basis of valuation, and for the last thirty years we have been able, with a tolerable degree of accuracy to gauge the effect of such a course. An instrument suited to this purpose was devised by Mr. Manly¹³ in 1868, and since that date other investigators have made improvements upon the original which a larger body of available statistics rendered feasible. Mr. Manly constructed a table (given in *J.I.A.*, vol. xiv, p. 292) "showing the assumed amounts of policies taken "out at various ages, and remaining in force at the end of stated "periods, for the purpose of exemplifying the different results "obtained by the use of different data in the valuation of the "liabilities of an office." In this, he assumed, for the sake of simplicity that all entrants entered at the beginning of the year, and at intervals of five years; and that the amounts assured were reduced periodically by claims, lapses, surrenders, &c., to a normal extent. Upon this basis was constructed a table exhibiting the reserve made at the end of each quinquennial period by each mortality table, each rate of interest, and each method of valuation. Similarly, Mr. G. King¹³ some ten years later, calculated another such table from improved data. The object in each case was to produce a series of amounts assured, of various durations and at various ages, which was free from the incidental characteristics of any single company's business, and might be said to give the general, rather than the individual, effect of a change of valuation method or basis. Such assumed results have been variously termed a "model office", a "hypothetical office", and (perhaps best of all) an "average office." It would be highly interesting to learn whether any such compilations

have been made in European or other Foreign countries; and, if not, what means have there been adopted for testing and foreshadowing the effects of changes in the principles and basis of valuation.

Now the tabular statement given above shows that the funds of the 52 offices in 1870 amounted to £59,683,735, whereas their funds at the close of the period, or, as we have shortly put it, in 1900, were £144,524,982. A rough approximation to the increase in the reserves of the above companies brought about by the change of standard from the Carlisle 3 per-cent to the H^M and $H^{M(5)}$ 3 per-cent may be obtained from the comparative reserves calculated by Mr. G. King (see *Journal of the Institute*, vol. xx, pp. 268-272), on the strength of which it is reasonable to conclude that at least 10 per-cent has been added to the reserves in this way. In other words, the continued use of the old standard of 1870 would have shown a net liability of something like £132,000,000, the difference of £13,200,000, or 10 per-cent of the total amount, representing the increase in the reserves brought about by strengthening the standard of valuation. And if an equal improvement took place among the other companies which have, for one reason or another, been omitted from our short table, whose aggregate net liability may be taken to be about £70,000,000, the valuation reserves of the whole group of Life Offices in the space of about a generation will have increased by no less a sum than £20,000,000. This result, so creditable to the caution and foresight of the Actuaries of Great Britain, has been achieved by continuous stages. Step by step, the companies have improved their financial positions and made due preparations for the altered conditions under which their business is carried on and on which their valuations are dependent. By such means, a great improvement has been effected without marked, or at least violent, disturbance in the bonus return to participating policyholders. It is no idle thing to say that British Life Offices are in a position of strength unknown at any other epoch in the history of assurance, and probably unequalled in any other country of the world—though it is fair to remark that a company, not of British constitution, proudly avers in public advertisement itself to be “the strongest life office in the world”! Few matters of greater interest to British actuaries will probably be dealt with by contributors to the proceedings of this Congress than the movement in the reserves of Foreign Life Companies in stated periods.

That the business of Life Assurance has progressed in a remarkable degree, as regards its financial soundness, admits therefore of no dispute. But, save for one important undertaking, to which a fuller reference will be made, it cannot be said that the development of the principles and methods of valuation made any great headway during the period we are now considering. The net premium method made its position secure, and the changes have been in the direction of employing the Institute of Actuaries' Tables in lieu of the other older standards, and in a reduction of the rate of interest. The schedule given on pages S2 and S3 will give a good idea of the extent to which matters moved in both respects. The introduction of the $H^{M(5)}$ Table in what is termed a valuation by the “combined Tables of the Institute of Actuaries” was, indeed, an innovation of principle to which attention should rightly be called; but with this exception the records of our subject show no

fundamentally novel system to have been adopted by any office of established repute.

The literature of our subject does, however, contain an elaborate contribution in which was constructed and most fully explained a new instrument for the valuation of a Life Office. We refer to the monumental papers by Dr. Sprague on "Select Mortality Tables" (*J.I.A.*, xxi, 229; xxii. 391 and 407), which may be said, without exaggeration, to be one of the chief works of that distinguished writer on actuarial subjects and to mark out a new and important line of development. These tables were designed to be used in connection with the $H^{M(5)}$ Table, and to show the true reserve that should be made for a life-policy of any duration and age at entry. They also proved conclusively that the H^M Table did not supply a proper basis for office premiums, and that the net H^M premiums commonly used in valuations were not scientifically sound. Previous investigations into the value of selection among assured lives had been made in earlier years: a most enlightened paper was written by the late J. A. Higham as far back as March 1850 (see *J.I.A.*, xx, 1.), noteworthy for the grasp of the subject and true insight shown, as well as for its clearness and precision of statement. But Dr. Sprague carried the enquiry much further than any of his predecessors, and produced a series of tables which would permit of a valuation being made of all the (whole life) policies on the books of a company with strict regard to the age at entry and duration in every case. No Life Office has, it is true, ever faced the formidable task of employing Dr. Sprague's Select Tables in computing its reserves; though it is not unreasonable to assume that, had the conditions of Life assurance remained as they were when his papers were produced, some scheme of classifying policies in groups of age at entry, and then applying the select tables, would have been devised. The greater variety of modern policies of assurance in form and method has, however, enormously increased the labour that would be involved by the use of select tables in computing reserves. Whole-Life policies, which twenty years ago formed the overwhelming majority of assurance contracts, can no longer absorb the attention of actuaries in their plans for carrying out the machinery of valuation. Endowment assurances are rapidly becoming a large proportion of the total; and special assurances, of one sort and another, certainly tend to increase. Thus the process of valuation is made unavoidably more complicated and the adoption of highly elaborate methods which would be workable in the case of one homogeneous class is not practicable. For this reason, it is scarcely probable that Life Offices will see their way to use select tables in a complete manner; though select reserves will doubtless continue to be regarded as the criterion and standard by which the results produced by the use of general (or "aggregate") tables must be constantly tested.

Before closing this section of our brief historical survey, a feature that marks out, as worthy of special note, certain recent valuation returns of some of the strongest Life Offices may be lightly touched upon. This consists in the use of a different net premium to that which the valuation basis would prescribe in the undeviating application of the net premium system. It takes various forms—sometimes "a special reserve is made to bring the loading reserved" up to a

minimum percentage; in another case, the net premium has been taken at a higher rate of interest than is used in the valuation units. But, alike in all instances, the desire has been to augment the margin available for the necessary expenses of carrying on the business of the office, and to forestall the criticism of the ignorant or unthinking, that a valuation is unsound which fails to leave intact a sufficient loading to meet expenses. This view we shall later on endeavour to prove to be utterly fallacious. The difficulty thus sought to be met is, however, a consequence of strengthening reserves without any concurrent reconsideration or modification of the office premiums at which assurances are granted. It is, moreover, a testimony to the fact, that there can be no fixed and unimpregnable principles in a business which is carried on under constantly varying conditions. The unassailed tenets of a generation ago may need to be revised in the light of existing practice and circumstance. In this direction, we may, at any rate, say that the net premium system seems likely to encounter criticism once more, with what results the future will show.

4. THE DISTRIBUTION OF SURPLUS, 1870-1900.

The schedule of 52 selected companies given on pages 82 and 83 has been turned to account for the purpose of arriving at an estimate of the changes that have taken place in the systems adopted in the apportionment of the surplus among those policies entitled to participate therein. A separate statement was compiled showing the first and last valuation falling due in the 30 years, respectively. The full details are not now given, as it is thought a brief summary containing the number of offices employing each of the well-known systems will be sufficient. This is accordingly appended.

Comparison of Methods of Distribution of Profits in use by 52 British Offices.

Method	First distribution after 1870 No. of Companies	Last distribution prior to 1900 No. of Companies
1. Premium (or loading) system	11	5
2. Re-assurance method	7	2
3. (a) Uniform Reversionary Bonus	11	17
(b) Compound	9	13
4. Contribution methods	4	7
6. Other methods	10	8
	52	52

The chief lesson to be drawn from these figures is the growing popularity of the Uniform and Compound Reversionary Bonus Methods. The former now shows 17 adherents, as against 11, and the latter 13, as against 9. The older systems, discussed in an earlier section of this paper, have lost ground, and the scientific "Contribution Method" has not succeeded in adding materially to its votaries, owing, no doubt, to the difficulties of application. It will be curious to see what has led to the adoption by the majority of

offices of the uniform and compound reversionary methods. Great simplicity in its application is perhaps its chief claim to favour; a smoothness or regularity in its results, the ease with which its basis and principles can be explained to the non-expert and comprehended by them; these advantages also count for something. But they are in themselves not enough; and the methods would never have become so generally adopted had it not been that the investigations of trained actuaries had proved that they could be defended on grounds of equity, having regard to the proper relation that should always exist between benefit and contribution. Years back, W. P. Pattison, Dr. Sprague, and many other writers: more recently Mr. A. Hewat, the late A. W. Sunderland, the late H. J. Rothery, Mr. G. J. Lidstone and Mr. H. W. Andras, showed that the uniform and compound reversionary bonus plans could be made equitable by calculating or adjusting premiums with that special end in view, and indeed, that current rates of premium were not, speaking generally, inconsistent with the fairness of these systems. It is convenient to deal with the two systems together, in historical argument, as the actual difference in the premium required to provide the respective benefits is trifling, and the greater appropriateness of one or the other will usually depend rather on the sources of profit in any individual office than upon its scale of rates of premium. A great point was gained in popularizing these methods when it was established that it applied with tolerable consistency to the case of endowment assurance policies, in which respect one of the highly scientific modern "Contribution Methods" produced results which it would be difficult to justify or account for on broad grounds. Thus we may say that the practical requirements of business and a due regard to scientific considerations combined to make the uniform and compound reversionary bonus methods the staple systems for the division of surplus among participating policies: and it is a circumstance worthy of note that not a single company which has, in the course of the last thirty years, once adopted these methods has abandoned them in favour of any other method involving a different principle, though a few changes from the uniform to the compound, and *vice versa*, may be discovered. Against these facts, the legitimate charge of inelasticity, of failure to take account of changes of valuation basis or of the origin and incidence of the surplus, has been ineffectual.

5. SUMMARY AND CONCLUSION.

Having endeavoured to trace the development of our subject in bare outline—the great difficulty having, in fact, been to refrain from dealing with the many fascinating matters of interest which lay so close to the main theme, but which have been left untouched in the view that they were episodes in the story whose plot and leading incidents we were alone here concerned with—we now ask permission to devote some attention to the tendency of modern practice and theory, and the outlook for the future.

In no respect have the many grave questions involved in the valuation of the liabilities of a Life Office and the distribution of surplus among its members, been more vitally affected than by the fall in the rate of interest yielded by investments suitable for

assurance funds. It has forced all companies to reconsider the standard of their reserves, and many to recast their systems of distribution. The remarkably low yield of British Government securities has been held in some quarters—without sufficient authority, we think,—to indicate the pitch to which the rate of interest may decline in a not far distant time. Calculations have been made of the reserves for Life policies on the assumption that the rate of interest, starting from its present level, will steadily diminish in a given proportion; and the results have not tended to allay anxiety, inasmuch as they prove that the future rates affect reserves much more materially than the present. (See *J.I.A.*, xxxi, 330; xxxii, 272.)

Now the first thing to bear in mind in this connection is that the business of Life Assurance could be carried on perfectly well if interest were to disappear altogether¹⁵. The transitional period would, of course, deal hardly with companies in respect of existing contracts upon which premiums were payable which had been calculated on the basis that interest at a certain rate would be realized; and companies would need to husband all their resources and strengthen their reserves to the maximum. But if we assume that no interest is earned, and that all the expenses of the business can be provided for out of the miscellaneous profits, life assurance might be just as useful and beneficent a means of thrift and of equalizing the chances of death as ever, *provided that the premiums were duly adjusted*. Companies would then have to charge net premiums based on the probabilities of life alone; and it may be worth while to see what they would be and how they compare with present standards. The following table gives such rates, on the basis of H^M mortality, termed 0 per-cent rates, which are contrasted with the 3 per-cent rates by the same table, together with some specimen endowment-assurances rates on both assumptions.

TABLE I.

Whole-Life Assurances.

Comparison of Net Premiums H^M 0 per-cent and H^M 3 per-cent.

Age	ANNUAL PREMIUM PER-CENT		Difference
	H ^M 0%	H ^M 3%	
10	1·968	1·064	·904
15	2·138	1·227	·911
20	2·350	1·427	·923
25	2·570	1·625	·945
30	2·843	1·850	·962
35	3·173	2·193	·980
40	3·582	2·589	·993
45	4·117	3·114	1·003
50	4·805	3·801	1·004
55	5·727	4·725	1·002
60	6·978	5·987	·991
65	8·688	7·705	·983
70	11·111	10·148	·963
75	14·535	13·585	·950
80	19·157	18·174	·983

Endowment Assurances.

Comparison of Net Premiums H^M 0 per-cent and H^M 3 per-cent.

Age	DEATH OR 50			DEATH OR 55			DEATH OR 60			DEATH OR 65		
	H^M 0 %	H^M 3 %	Diff.	H^M 0 %	H^M 3 %	Diff.	H^M 0 %	H^M 3 %	Diff.	H^M 0 %	H^M 3 %	Diff.
20	3.726	2.521	1.205	3.280	2.131	1.149	2.959	1.864	1.095	2.728	1.682	1.0
25	4.416	3.156	1.260	3.784	2.587	1.197	3.351	2.212	1.139	3.048	1.963	1.0
30	5.452	4.133	1.319	4.493	3.244	1.249	3.877	2.694	1.183	3.464	2.342	1.1
35	7.154	5.762	1.392	5.539	4.231	1.308	4.600	3.366	1.234	4.010	2.844	1.1
40	10.516	9.021	1.495	7.255	5.873	1.382	5.667	4.373	1.294	4.762	3.547	1.2
45	20.524	18.825	1.699	10.654	9.169	1.485	7.425	6.060	1.365	5.880	4.609	1.2
50				20.681	18.986	1.695	10.863	9.389	1.474	7.691	6.348	1.3
55							20.920	19.232	1.688	11.198	9.746	1.4
60										21.327	19.648	1.6

A very curious feature of these figures is that for all ages the difference between the two whole-life scales is *practically constant at 1 per 100 assured*. Thus, allowing for the fact that modern rates bear a fair margin on the H^M 3 per-cent net premiums, we should have to add only 1 per 100 assured to existing rates if interest were to disappear altogether! As large a charge as this is already made to compensate for specially hazardous occupations, or residence in the tropics; and no one could venture to say that life assurance would be impracticable under such a tariff.

Of course, the reserves to be held in hand under the imaginary conditions would be very much larger than those in which the liabilities are discounted at a given rate of interest over a long period of time. Before dismissing the illustration, a few specimens of the 0 per-cent reserves may be given, side by side with the 3 per-cent reserves.

TABLE II.

Comparison of Values of Policies H^M 0 per-cent and H^M 3 per-cent Whole-Life Assurances.

Age at Entry	NO. OF YEARS IN FORCE							
	5		15		30		50	
	0 %	3 %	0 %	3 %	0 %	3 %	0 %	3 %
20	8.58	4.36	25.94	15.00	51.22	35.35	78.85	66.77
30	10.40	6.14	30.96	20.48	59.17	46.16	85.16	77.27
40	13.00	8.71	37.46	27.96	67.76	57.88	89.76	84.93
50	16.10	12.10	44.88	36.78	74.92	68.16
60	19.68	16.18	51.99	46.05	80.04	75.62
Endowment Assurance payable at death or 60								
20	11.70	6.80	35.67	23.93	72.76	61.17	„	„

Such figures as these have naturally no value in the present day, nor probably ever will have. But they lend point to the argument (whose strength and cogency we are strongly convinced of) that any marked decline in the rate of interest should form the occasion for a reconsideration of the premiums charged by companies. Outside the British Isles this principle is indeed already acted upon. In Canada, for instance, the decision of the Dominion Legislature to introduce $3\frac{1}{2}$ per-cent instead of $4\frac{1}{2}$ per-cent into the Government standard of reserve for Life Offices has led the companies concerned to combine for the purpose of increasing their rates all round. But in Great Britain the basis of valuation and the basis appropriate for the calculation of premiums for new assured persons have materially changed without the companies thinking any increase of premiums expedient or incumbent. One case only occurs to memory in which the office scale of premiums has been increased in late years, though it should be mentioned that in several instances the rates derived from the whole-life premiums by commutation—such as single premiums and limited-payment premiums—have been put on a higher level by the use of a lower rate of interest in the conversion. On the other hand, several companies have decreased their non-profit premiums, notwithstanding the poorer outlook as regards the future yield of investments and the large augmentation that has, in view of this circumstance, been made to their reserves. And in certain cases the office premiums (on non-participating assurances) are actually less than the net premiums according to the valuation basis in use! That this is not a mere exaggeration of language is seen from a table given below showing the average non-profit rates of 66 companies transacting business in the United Kingdom, and giving the net premiums by the H^M Table at $2\frac{1}{2}$ per-cent interest for comparison. Several companies now value at $2\frac{1}{2}$ per-cent, and a good many instances will be found of the net valuation premium exceeding the full office premium.

Comparison of average Non-profit Whole-Life Rates of 66 Companies transacting business in the United Kingdom and H^M $2\frac{1}{2}$ per-cent premiums.

Age	Average Premium per-cent	H ^M $2\frac{1}{2}$ per-cent net
21	1.683	1.576
25	1.829	1.742
30	2.079	2.003
35	2.387	2.322
40	2.796	2.723
45	3.329	3.253
50	4.029	3.943
55	5.012	4.870
60	6.333	6.134

In point of consistency and common sense, we think the Canadian course superior to the British; nor do we believe that it will be found practical for long to neglect the "cost of production", and pay a too

exclusive regard to considerations of competition in current premium tariffs. Many of the scales of premiums at present in use are based upon 4 per-cent interest, as will be evident from a comparison of any compilation of the premiums of the various companies with those specimen rates suggested by Dr. Sprague nearly eighteen years ago (see *J.I.A.*, xxii, 396). Now the employment of 4 per-cent for such a purpose is no longer a matter that would be seriously entertained; so that, unless the expenses of obtaining new assurances and managing the entire business of an office are less than they were eighteen years ago, rates such as these are based on false assumptions. But as to the expense of new business, it is notorious that it is rather higher than lower; and on this ground alone, rates ought probably to be put up for logical reasons. The question is one of the greatest importance to Life Offices, and delay in grappling with it is only justified by the fact that we are on the eve of the publication of the results of the new collective experience of assured lives, the joint work of the Institute and Faculty of Actuaries. If the net premiums of the new Table should not be materially less than those of the H¹ Table, it will be most desirable—we might almost say, imperative—in the common interest, that the premiums charged by Life Assurance Companies in Great Britain should be increased.

The same potent factor in life assurance enters just as largely into the question of the proper basis for reserves as it does into that of the proper basis for office premiums. We have seen that the net premium method was the embodiment of two great principles: the use of a "normal or true" mortality table, and the valuation as an asset of the mathematical net premium only, whereby the future loading was left intact as a provision for expenses, contingencies, and (in participating cases) profits. But the reduction in interest has naturally led to a lower rate being used in valuations, and therefore a larger net premium being valued. And as the office premium is a fixed quantity, each change in this direction has reduced the loading left over for future expenses and profits; until at the present moment there are offices whose non-profit premiums are practically equal to their valuation net premiums, so that no loading at all is reserved for future purposes. How does the net premium method justify itself in circumstances such as these? It is indeed difficult to say. The popular idea among business men, not so very long ago, was that if a company left untouched its future loadings, and they formed a larger percentage of the premium revenue than the total expenses, the financial position was satisfactory; and this was quite a common line of comment in the better informed insurance journals. But by largely increasing its reserves, and therefore improving its financial position, a company would invite criticism, according to this criterion, which would be less and less favourable, until the *reductio ad absurdum* would be reached at the point when, owing to an exceptionally strong basis, the net premiums valued (the same elements of interest and mortality being involved in the valuation factors) would be equivalent to the total office premiums receivable, and no provision at all would be made for future expenses and profits out of the premiums! Competition refuses to allow itself to be bound by any cast-iron theoretical system of valuation, or rates of premium would have increased as the

net premiums valued increased. But this would only have put the matter right as regards the newly-effected policies; and the whole anomaly would remain in respect of the older contracts put upon the books when the reserves were so much lighter and an adequate loading was shown at valuation. Look at it how we will, there is something in modern tendencies to weaken the foundations of the net premium system of valuation. We may either face the difficulty boldly, and say that future expenses are left to be discharged out of surplus interest and miscellaneous profits; or we may elect to make a suitable deduction from the gross premiums (or, what is the same thing in the case assumed, the net premiums) to form a provision for the working charges. But in either event, there is a departure from the principles so often expounded as being the essence of the method, and the ark is left with weak defences.

Nearly fifty years ago, the late John Finlaison—the first President of the Institute of Actuaries—deposed as follows: “I never saw nor can I conceive the case of an office accepting premiums that would be below the full arithmetical risk.” Again “If it were conceivable that any office would insure at premiums below the arithmetical premiums, that, I confess, is a course that would lead to insolvency, but I never saw it. The mercantile premium is, of course, always more than the arithmetical premium, so as to leave a margin of profit.” (*See Evidence before Select Committee on Assurance Associations*, 1853, Q. 694 and 698).

The circumstances of to-day are not those of 1853. Companies now accept premiums less than their valuation net premiums, and insolvency does not ensue, but, on the contrary, prosperity. And so it is with the net premium method. Devised more than half a century ago to correct grave faults in systems then in vogue, and to satisfy the scientific conscience; strongly supported as it has been down to the present times as being the safest and most trustworthy means of estimating assurance liabilities; it now has to stand the brunt of examination in conditions unforeseen and the slow growth of years; and it can cause no surprise that, its powers of adaption being limited, it ceases to have a general applicability.

For these reasons it seems quite within the bounds of possibility that the twentieth century will record the supersession of the net premium system by some other method, more elastic, flexible, and adaptable to changing conditions—one that will follow more closely the known facts and elements in the case of offices of varying status and characteristics, and not impose too inexorable a scale of measurement all round. But so complex is the problem, so manifold the factors, so various the conditions, that a long process of evolution will probably be gone through before a solution is reached. The new method will, we may conjecture, pay more regard to the office premium payable under a contract of assurance than does the net premium method. It may take account of the great discrepancy in the cost of obtaining new policies and of managing old business, though this is not so certain. But the position of the strongest companies of to-day in connection with their valuation reserves prepares us for a change when their number shall be reinforced, as it is bound to be sooner or later, by a large body of adherents. And

a new generation of actuaries will have this and—in spite of the absurd view often heard expressed that the actuarial field has been already exhaustively cultivated—many other equally important questions as the task which they must undertake to justify their existence.

Nor is the outlook much clearer as far as methods of distribution are concerned. At present, scientific opinion may be said to be chiefly taken up in discussing the rival merits of the uniform or compound reversionary bonus systems and the “contribution” systems which allot the surplus, in some measure, according to the ascertained sources of profit. The latter are the lineal successors of the older “contribution” method of Sheppard Homans, and aim at a literal fulfilment of the maxim that the surplus should go to those who have created it, in the proportion of their respective contributions. Two forms of this method are now practised. One, known as Dr. Sprague’s (or the Equity and Law) plan¹⁶, divides the surplus into two parts, the excess interest realized over the valuation rate forming one part, and the balance of surplus the other. The former is then distributed in proportion to the reserves on which the excess interest has been earned, and the latter in proportion to the premiums paid in the valuation period. The second method, known as Mr. T. G. C. Browne’s (or the Guardian) method¹⁶, distributes the surplus into three groups by special analysis, the first being the profit from loadings (after deduction of all expenses); the second, the profit from excess interest; and the third, that arising from favourable mortality. The last-named result is then allotted to the first and second groups in rateable proportion; and the two quantities thus deduced are divided in the ratio of the loadings and the reserves (at the *end* of the period) respectively.

Thus both of these contribution methods are elaborate, if not indeed cumbrous in application. But if this were the only objection, the ingenuity of actuaries might be depended upon to simplify the process or arrive at a rough approximation to its results by easy machinery. Unfortunately other defects come to light in examining the practical effects of these methods. Dr. Sprague’s method is found to cast the mortality fluctuations very largely upon recently entered policies, though they are more probably attributable to the working of the older contracts; and thus to benefit the younger members, or the reverse, according as the mortality result is favourable or otherwise. Mr. Browne’s method, by its rateable apportionment of mortality profit or loss between excess interest earnings and loadings, avoids to a partial extent this drawback; but it loses something of its scientific exactness by reason of the fact that any alteration of the valuation basis (and therefore of the loadings) affects the shares which the new and old members respectively take in the mortality profit, which does not practically depend on the valuation basis at all. So that if the valuation basis of a company using this method were so greatly strengthened that the loading profit disappeared, the mortality profits would all go to swell the interest profits divisible according to the reserves, and therefore apportionable almost exclusively among old contracts. Now, whether mortality profit may be reasonably so divided is a question for discussion and

investigation, though unless a select table is used in determining the result much of the so-called mortality profit is merely "suspended mortality" in respect of new policies entirely, which cannot by any stretch of actuarial conscience be deemed to be the contribution of older policies to surplus. However, our present contention is not that any given result of the working of the scheme at any point of time cannot be justified, but that a system which deals with mortality profit without any regard to its sources and origin loses claim to scientific exactitude, and can only be supported on other and practical grounds.

These points have been ably investigated by writers to whom references are subjoined; and we shall not do more than say that to advise a company to adopt either of the "contribution methods" above discussed is not to open up for it a haven of refuge from the perplexing requirements of the problem. Rather would it be to launch it upon a troubled sea where the soundings would need to be taken at frequent intervals, and the course perhaps varied from time to time.

But if, for a moment, we recur to Dr. Sprague's method, and imagine a case of a company valuing at a full rate of interest and consequently making little or no excess interest, the application of this plan becomes identical with that of the old "premium (or loading) system", (*see* p. 79) which was adopted for so long a period by many companies. This gave a constant cash ratio at all divisions of profit after the first, and, on the assumption of a uniform rateable surplus, reversionary bonuses diminishing at each successive distribution resulted. Each time the basis of valuation is strengthened by the use of a lower rate of interest, a certain amount of surplus is withdrawn mainly from the older members. This requires at once an adjustment to be made in the system of distribution, as henceforth interest profit will accrue which will benefit policies of long standing, and in the meantime they are bearing by far the larger share of the burden of increasing the reserves. Eventually, when the valuation standard has been placed upon the highest level, no further drafts upon surplus will probably be required, and the interest profit will be at a maximum. Again, this condition necessitates, in fairness, re-adjustment of the profit-rights of various classes of members in favour of those of long duration.

Hence, after considerable experience of such matters, we have come to the conclusion that a fair method of distribution for a company, having average rates of premium and no very marked characteristics, whose reserves are in process of being brought to the high level of strength, is the uniform reversionary bonus method. And further that, for companies whose reserves stand in little or no need of further augmentation, the compound reversionary bonus method has much to recommend it. And thus, if there is one method more than another among the several existing systems which is likely to gain favour in the 20th century, we may hazard the opinion—in spite of the dangers of prophecy—that it is the compound reversionary bonus plan. Once let the idea be admitted that it is appropriate to declare "reversionary dividends" on the capital assured, and that the oldest policies that more largely bear the cost

of the increases in reserves should be preferentially treated, and this method for its simplicity and convenience will be found worthy of much consideration. Its inelasticity can be surmounted, for new policies, by an adjustment of rates whenever any great or permanent change takes place in the conditions under which Life Assurance is carried on; and for existing policies, by augmenting the reserves so as to leave an appropriate margin of interest available for bonus purposes. Many authorities have expressed the view that a margin of 1 per-cent in the interest is sufficient to justify the adoption of this method, in which we concur. More minute investigations will probably be made in future with the object of showing the real incidence of profit from mortality, surrenders, lapses and other sources; and there is little reason to fear that they will prove detrimental to the continuance of the compound reversionary bonus method.

This, however, is for the future to disclose. A retrospect over the long years covered—however inadequately—by the present essay teaches us a lesson of humility. It gives us a story of constant ebb and flow, of change of valuation methods, change of distribution methods, change of practice and theory, change of financial and commercial conditions above all. It is then simply to court rebuke for us to usurp authority or dogmatize with an air of decision on the problems of to-day or the solutions of to-morrow. Let us refrain from indulging in hard and fast opinions on matters always liable to change. And let us remember that science never recedes; that the march is ever forward; that just as we recall, with something akin to amazement, the heresies of the past, so a future generation, with extended view and enlarged information, will deal with our short-sightedness and imperfections in much the same spirit. The lesson of life in its individual and corporate aspects is “change with continuity to combine”; and the maxim applies to actuarial work as much as to any other specialized form of human endeavour.

APPENDIX.

CONTAINING LIST OF AUTHORITIES REFERRED TO IN THE PRECEDING PAPER.

1. Address to the Actuarial Society of Edinburgh, by the President, James Sorley, F.R.S.E. (*Transactions of the Society*, vol. iii, No. 14, see page 420-1).

2. *Report on the Valuations of the Equitable Society.* By the Actuary, H. W. Manly (1896).

3. *On the Valuation of Annuities and Assurances.* By Joshua Milne (London, 1815).

4. Paper by Benjamin Gompertz, F.R.S., read before the Royal Society on 29 June 1820. See also *Memoir of the late Benjamin Gompertz.* By M. N. Adler, M.A. (*J.I.A.*, xiii, 1).

5. Augustus De Morgan. For a list of his chief contributions to actuarial science see *J.I.A.*, xxv, 140.

6. *Comparative View of the Various Institutions for the Assurance of Lives.* By Charles Babbage, F.R.S. (1826).

7. *Tables of Life Contingencies.* By Griffith Davies, F.R.S. (London, 1825).

8. See a paper *On the Methods pursued in Valuing the Risks of Life Assurance Companies, and on the Distribution of Surplus.* By Charles Jellicoe, 25 February 1850 (reprinted *J.I.A.*, x. 325).

9-10. *On the Proper Mode of estimating the Liabilities of Life Insurance Companies.* R. Tucker (*J.I.A.*, x. 312). T. B. Sprague (*J.I.A.*, xi. 90).

11. W. P. Pattison (*J.I.A.*, ix. 350-1).

12. *On the Equitable Distribution of Surplus.* By Sheppard Homans (*J.I.A.*, xi. 121).

13. See papers *On the effect of different standards of valuation as illustrated by the use of "hypothetical" or "model" offices.* By H. W. Manly (*J.I.A.*, xiv. 249), J. Valentine (*J.I.A.*, xviii. 229), and G. King (*J.I.A.*, xx. 233).

14. *On Selection.* See J. A. Higham (*J.I.A.*, i. 179 : xx. 1) : and T. B. Sprague (*J.I.A.*, xv. 328 : xxi. 229, 391, 407).

15. See J. Sorley's Presidential Address to the Actuarial Society of Edinburgh (referred to in (1) above Table E. p. 433).

16. See G. J. Lidstone (*J.I.A.*, xxxii. 73) on Dr. Sprague's Method. T. G. C. Browne on his own method (*J.I.A.*, xxxii. 194).

BRITISH OFFICES LIFE TABLES, 1893.

JOINT MORTALITY INVESTIGATION OF THE INSTITUTE OF ACTUARIES AND THE FACULTY OF ACTUARIES IN SCOTLAND.

Official Symbols adopted by Joint Committee, 12 June 1903.

A DESIRE having been expressed that, in order to assist in securing uniformity on the part of future writers making use of the Experience, an authorized set of symbols should be adopted to indicate the different sections of the Select and Aggregate Experience dealt with in the Investigation, the following symbols were approved by the Joint Committee at their meeting on 12 June 1903. These will appear on page v of the Select Assurance Tables now about to be published.

THE BRITISH OFFICES LIFE TABLES, 1893.

FULL AGGREGATE TABLES—Males :—

Whole-Life Participating Assurances	OM
Whole-Life Non-Participating Assurances	ONM
Endowment Assurances	OEM
Assurances with Limited Payments	OLM
Assurances with Increasing Scale of Premiums	OIM
Temporary Assurances	OTM
Contingent Assurances	OCM
Joint Assurances	OJM

“TRUNCATED” AGGREGATE TABLES—Males:—

Whole-Life Participating Assurances (excluding the first five years of Assurance)	} $O^{M(5)}$
&c., &c., &c., &c.	

SELECT TABLES—Males:—

Whole-Life Participating Assurances . . .	$O^{[M]}$
&c., &c., &c.	&c.

THE BRITISH OFFICES LIFE ANNUITY TABLES, 1893.

Male Annuitants' Tables (Aggregate) . . .	O^{am}
Male Annuitants' Tables (Select) . . .	$O^{[am]}$

Female Lives.

F instead of M, and *f* instead of *m* throughout.

ERRATA.

British Offices (O^M) Life Tables, 1893. Page 63:—

Temporary Annuities, 3 per-cent $a_{41:\overline{36}|}$. For 19·652, read 16·652.

Unadjusted Data:—

Endowment Assurances and Minor Classes of Assurance.
Introduction, page ix. Joint Life Assurances—
Female lives. Years of Risk “After Ten Years.”
For 41,090, read 41,093.

CORRESPONDENCE.

INSTITUTE OF ACTUARIES' TEXT-BOOK, Part I (New Edition).

To the Editor of the Journal of the Institute of Actuaries.

SIR,—My attention has been drawn from time to time to sundry mistakes and misprints in the New Edition of the *Text-Book*, Part I. I append a list of these in the belief that it will be a convenience to future students of the book to be able to refer to them in the *Journal*, and in the hope that few (if any) errors still remain undetected. It has not been thought necessary to include obvious typographical errors—such as defectively printed brackets—a few of which (varying, probably, from copy to copy) may occur in the book.

I am indebted to Mr. D. J. McG. McKenzie for the information (which may be of service to other members of the profession) that the Fourth Edition of *Pereire's Tables* (published in 1896), gives supplementary tables of the values of $(1+i)^n$, v^n , a_n , and $\frac{1}{a_n}$ at rates of interest proceeding by $\frac{1}{16}$ from $\frac{1}{2}$ to $\frac{3}{8}$ per-cent ($n=1$ to $n=300$)

and by $\frac{1}{5}$ from $1\frac{1}{2}$ to 2 or $2\frac{5}{5}$ per-cent ($n=1$ or 101 to $n=200$). The results are given to ten places of decimals in the case of $(1+i)^n$ for rates of interest from $\frac{1}{5}$ to $1\frac{3}{5}$ per-cent, and to seven or eight places in the remaining cases.

I am, Sir,

Your obedient Servant,

R. TODHUNTER.

ERRATA.

Page	Line*	Error	Correction
9	3 <i>b</i>	$e^{-n\delta}$	$e^{-n\delta'}$
13	6 <i>t</i>	$\overline{m-2}$	$\overline{2m-1}$
13	7 <i>t</i>	$\overline{m+2}$	$\overline{2m+1}$
13	11 <i>t</i>	$\overline{m+2}$	$\overline{2m+1}$
13	12 <i>t</i>	$\overline{m-2}$	$\overline{2m-1}$
13	4 <i>b</i>	$-\dots$	$+\dots$
13	2 <i>b</i>	$\frac{f^3}{3m^3} - \dots$	$\frac{f^3}{3m^2} + \dots$
14	1 <i>t</i>	$1-i+\frac{i^2}{2!}-\frac{i^3}{3!}-\dots$	$1-i+i^2-i^3+\dots$
14	4 <i>t</i>	$-\dots$	$+\dots$
16	1 <i>t</i>	$\left(1+\frac{j}{m}\right)^{mn}$	$P\left(1+\frac{j}{m}\right)^{mn}$
26	10 <i>t</i>	S_k	S_r
26	Note	$\frac{k^3}{2!}$	$\frac{k^3}{3!}$
41	9 <i>b</i>	powers of n	powers of x
50	5 <i>t</i>	$a_{n-p}^{(1)}$	$a_{n-p}^{(p)}$
54	3 <i>t</i>	$a_{n-p}^{(m)}$	$a_{n-p}^{(p)}$
62	1 <i>t</i>	r^{2m-n+1}	r^{2n-m+1}
67	2 <i>t</i>	t_1^-	t_1^{-p}
95	2 <i>b</i>	$\sum_{r=1}^{r=n-1}$	$\sum_{r=1}^{r=n}$

* The letters t and b denote that the lines are to be counted from the top and bottom respectively.

100		Correspondence.	[JULY 1903.]
Page	Line*	Error	Correction
100	15 <i>t</i>	$4\frac{1}{2}$	$4\frac{1}{2}$ per-cent
121	Formula 21	$\frac{i'I'' - I'i'}{I'' - I' - i'' - i'}$	$\frac{i'I'' - I'i''}{I'' - I' - i'' + i'}$
163	7 <i>b</i>	a	x
168	18 <i>t</i>	NMQ	NMR
169	9 <i>b</i>	when $x=0$	when $h=0$
171	13 <i>t</i>	denominator, and	denominator and
175	8 <i>b</i>	x^{n-}	x^{n-2}
181	11 <i>t</i>	$s_{n }$	$\bar{s}_{n }$
184	2 <i>t</i>	$\int_c \frac{du}{dx}$	$\int_v \frac{du}{dx} dx$
185	9 <i>t</i>	$\int_a^b \frac{d\phi}{dc} dx$	$\int_a^b \frac{d\phi(x_1c)}{dc} dx$
188	8 <i>b</i>	$\int_b^a \phi(x) dx$	$\int_a^b \phi(x) dx$
189	9 <i>t</i>	Taylor's Theroem	Taylor's Theorem
196	6 <i>b</i>	$\int_0^m e^{-\delta t} dt$	$\int_0^m e^{-\delta t} dt$
198	3 <i>t</i>	$\int_0^m e^{-\delta_0 t + \frac{rt^2}{2}} dt$	$\int_0^m e^{-\delta_0 t + \frac{rt^2}{2}} dt$

* The letters *t* and *b* denote that the lines are to be counted from the top and bottom respectively.

VOL. XXXVIII.

PART II.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

No. CCXIV.—OCTOBER 1903.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: 8, RUE LAMARTINE, 8.
BERLIN: CARLSTRASSE 11. MELBOURNE: MCCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

PRINTED BY C. & E. LAYTON, FARRINGDON STREET. E.C.
TO WHOM ALL COMMUNICATIONS FOR THE EDITOR SHOULD BE SENT, POST PAID.

Price 2s. 6d.

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JOURNAL

OF THE

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"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—Bacon.

VOL. XXXVIII.—PART II.

OCTOBER 1903.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: S, RUE LAMARTINE, S.

BERLIN: CARLSTRASSE 11.

MELBOURNE: McCARRON, BIRD & CO.

NEW YORK: THE SPECTATOR COMPANY.

[The Council of the Institute of Actuaries wish it to be understood, that while they consider it their duty to give, from time to time, publicity to certain of the papers presented to the Institute, they do not hold themselves responsible for the opinions put forward therein.]

LONDON:
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGTON STREET.

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JOURNAL

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INSTITUTE OF ACTUARIES.

On the Valuation of Staff Pension Funds. Part 2.—Widows' and Children's Pensions. By HENRY WILLIAM MANLY, Actuary of The Equitable Life Assurance Society, and Past-President of the Institute of Actuaries. With Tables by HERBERT FOOT, B.A., of The Northern Assurance Company, Fellow of the Institute of Actuaries.

[Read before the Institute, 27 April 1903.]

I HAVE been asked to explain how the benefits of a pension to widows and an allowance to children can be valued when they are included in a pension scheme; and as I have had no experience whatever of a fund providing such benefits, I am, of course, eminently qualified to lecture on the subject. At least, I approach the subject with an unprejudiced mind, and am not affected by anything I may have said or done.

The problem I am about to investigate is different from the ordinary Widows' Funds; because I am assuming that it is compulsory on every member of the staff, whether bachelor, husband, or widower, to contribute towards the benefits, either by a fixed annual sum for the whole of life or by a percentage of his salary, or of his salary and pension.

I have made several enquiries as to the nature of the benefits offered, and I find that the pension generally takes the form of a fixed annuity to the widow, and, on the death of the widow, a continuation of the annuity until the youngest surviving child reaches a certain age, say 14, 16, or 21. There is generally a provision that the annuity shall cease on the

re-marriage of the widow, but even in that case it is sometimes continued for the benefit of the children. Sometimes an allowance is made for every child left by a married man until the attainment of a certain age, say 14 or 16, in addition to the pension to the widow. This additional benefit is of the form of Mr. G. King's "Family Annuities", see *J.I.A.*, vol. xxx, p. 291. I have come across one fund which provides a pension to the widow based in some way on the retiring allowance which the husband would have received if he had retired at the date of his death.

It not infrequently happens that a consolation is given to the persistent bachelor, and the widower leaving no children, in the shape of the payment at death of a fixed sum or the return of contributions without interest.

As I propose to use, when suitable, Mr. A. Hewat's Tables in his "Widows' and Pension Funds", based on the experience of the Widows' Funds of the Scottish Banks, for which he has kindly given his permission, I may at once state that no provision will be made for forfeiture of the widow's pension in the event of re-marriage, and that second marriages are not provided for. (See Hewat, p. 80, "Widows' and Pension Funds.")

We are now entering on a new and wide field of investigation, and shall require a new notation, in which the symbols shall be so distinctive that they cannot be mistaken for others we are in the habit of using. In this I have found it necessary to break away from the signs hitherto adopted for reasons which will appear obvious. For instance, w which has hitherto been used to represent a widow, I have already appropriated for withdrawals, and $wrmq_x$, which has been used to represent the probability of a widower marrying between the ages x and $x+1$, is much too complex when a simpler symbol can be found.

PRIMARY SYMBOLS.

As p_x is used to represent the probability of a person aged x living from age x to age $x+1$, and q_x to represent the probability of a person aged x dying between the ages x and $x+1$, I propose that m_x shall represent the probability of a person aged x marrying between the ages of x and $x+1$. This has a certain advantage, because m without the suffix will represent the probability of marrying. The following is the scheme of notation I propose to use:—

B = bachelor.

H = husband.

K = widower.

M = married man.

W = widow.

O = orphans, that is, all the children left by a husband or widower.

YC = youngest child.

$B l_x$ = number of bachelors living at the age x .

$H l_x$ = „ husbands „ „ x .

$K l_x$ = „ widowers „ „ x .

$M l_x$ = „ married men „ „ $x = H l_x + K l_x$.

$B d_x$ = number of bachelors dying between the ages of x and $x + 1$.

$H d_x$ = number of husbands dying between the ages of x and $x + 1$.

$K d_x$ = number of widowers dying between the ages of x and $x + 1$.

$M d_x$ = number of married men dying between the ages of x and $x + 1 = H d_x + K d_x$.

$B p_x$ = the probability of a bachelor aged x living from age x to age $x + 1$.

$H p_x$ = the probability of a husband aged x living from age x to age $x + 1$.

$K p_x$ = the probability of a widower aged x living from age x to age $x + 1$.

$M p_x$ = the probability of a married man aged x living from age x to age $x + 1$.

$B q_x$ = the probability of a bachelor aged x dying between the ages of x and $x + 1$.

$H q_x$ = the probability of a husband aged x dying between the ages of x and $x + 1$.

$K q_x$ = the probability of a widower aged x dying between the ages of x and $x + 1$.

$M q_x$ = the probability of a married man aged x dying between the ages of x and $x + 1$.

m_x , or $B m_x$ = the probability of a bachelor aged x marrying between the ages x and $x + 1$.

m_x^2 , or Km_x = the probability of a widower aged x marrying between the ages of x and $x+1$.

B_xm = the probability of a bachelor aged x marrying.

K_xm = „ „ widower „ „

x is used to denote the age of a male.

y „ „ age of a female, but generally the age of the widow at the death of her husband.

Wa = the present value of an annuity of 1 to a widow.

W_ya = the present value of an annuity of 1 to a widow aged y .

$Oa(n)$ = present value, at the death of a husband or widower, of an annuity of 1 to each of the children, until they reach the age of n .

$YCa(n)$ = present value, at the death of a married man, of an annuity of 1 until the youngest child attains the age of n .

$K.YCa(n)$ = present value, at the death of a widower, of an annuity of 1 until the youngest child attains the age of n .

$E(u)$, or $Ea(n)$ = the addition, at the death of a husband, to the present value of an annuity of 1 to a widow to provide for the continuance of the annuity until the youngest child attains the age of n .

$K-C$ = a childless widower.

$K-C(n)$ = a widower without children under the age of n .

$\overline{K-C}(n)d_x$ = the number of widowers without children under the age of (n) dying between the ages x and $x+1$.

If it be desired to represent the probability of a person dying a bachelor, or a husband, or a widower, we can use b , h , and k respectively for such probabilities, so that

B_xb = probability of a bachelor aged x dying a bachelor.

B_xh = „ „ „ „ husband.

B_xk = „ „ „ „ widower.

H_xh = „ „ husband aged x dying a husband.

H_xk = „ „ „ „ widower.

K_xh = „ „ widower aged x dying a husband.

K_xk = „ „ „ „ widower.

It will sometimes be necessary to use a symbol to represent the number of bachelors marrying in a year, or marrying and dying the same year—similarly with husbands becoming widowers and widowers marrying. These will be represented thus:—

Bhl_x = the number of bachelors becoming husbands between the ages of x and $x+1$.

Hkl_x = the number of husbands becoming widowers between the ages of x and $x+1$.

Khl_x = the number of widowers becoming husbands between the ages of x and $x+1$.

Bhd_x = number of bachelors marrying and dying as husbands between the ages x and $x+1$.

Hkd_x = number of husbands becoming widowers and dying as widowers between the ages x and $x+1$.

Khd_x = number of widowers marrying and dying as husbands between the ages x and $x+1$.

Other symbols will be developed as the work proceeds.

CONSTRUCTION OF TABLE SHOWING THE NUMBER OF BACHELORS, HUSBANDS AND WIDOWERS DYING AT EACH AGE.

We have now to construct a working Table. It must in the first place be a Table showing the numbers remaining, withdrawing and dying each year out of a certain number entering at a given age. That we have in my Table No. 3 (*J.I.A.*, vol. xxxvi, p. 261). It is not necessary to separate those who retire, because the liability we have to deal with now, arises only on death, whether before or after retirement. In the second place we have to carry the deaths to the extreme limit of life; and for this purpose I have adopted the English Life No. 3 Experience. In the third place, we have to find, out of all those who die at each age, how many are bachelors, how many are husbands, and how many widowers. Fourthly, we have to determine the average age of the widow at the death of the husband; and, finally, we require to know how many of the husbands and widowers will leave children, and the average age of the youngest child at the death of the father.

Statistics on these points are very limited, and are not always given in a form convenient for use.

MARRIAGE RATES.

The number of husbands dying at each age must of necessity depend largely upon the marriage rate. Now the marriage rate differs according to occupation ; and this has been clearly shown by Messrs. Deuchar, Hewat, Huie, Meikle, and Sprague, in the experiences of the different Widows' Funds in Scotland, and the Peerage Families. The Ministers and Schoolmasters, who attain their maximum income early in life, marry earlier than the bankers and advocates who wait until their incomes will enable them to marry comfortably. I imagine that the marriage rate in the class for whom these Funds are created will correspond more closely to that prevailing in a bank staff than in any other ; and, consequently, I decided to adopt Mr. Archibald Hewat's experience of Scottish Bankers.

On page 63 of his work on "Widows' and Pension Funds" he gives a Marriage and Mortality Table, based on the experience of the Widows' Funds of the Scottish Banks, showing the numbers living separated into bachelors and married, and the numbers dying separated into bachelors and married, as well as the number of bachelors marrying. The numbers dying in my extended Table were divided into bachelors dying and married dying, in the same proportion as in Mr. Hewat's Table. Having thus obtained the numbers dying married, it became necessary to sub-divide them into husbands and widowers. This was done, up to age 65, in the proportion of husbands dying to widowers dying by means of Mr. G. King's Table C in his valuable paper "On Family Annuities." The number of bachelors, husbands and widowers living at the age of 65 and over, are not sub-divided according to ages in the Census Returns, and, in consequence, we had to have recourse to another method of separating our married men at death, after that age, into husbands and widowers. This was done by means of Dr. Farr's English Life Tables. It was assumed that all the married men of the age of 65 would have wives of the average age of 10 years younger than themselves, and we ascertained how many couples, out of a given number of couples of husband and wife living at age 65, would survive every year to the end of the Table. By this means we were able to extend Mr. King's curve for husbands to the end of life, and it was particularly gratifying to find that the total bachelors, husbands and widowers, which we thus obtained, corresponded very closely

with the Census Returns. The question of marriage or re-marriage after 65 was excluded, because in these Funds marriage after 65 is either not recognized, or, if it is, a large fine is imposed.

The result of this distribution of the deaths is shown in Table 45 on pp. 138-9; and, until more trustworthy statistics are forthcoming, I venture to think that this Table is the best that can be produced. The mortality amongst husbands is greater after age 47 than in Mr. Hewat's experience of married men in the class of Scottish Bankers, so that the liability, according to my Table, will be on the safe side.

WIDOWS' AGE.

Having found the number of husbands who die at each age we had next to find the ages of the widows which they leave on the Fund. On page 81 of his work Mr. Hewat gives a Table of the difference of age between husband and wife at marriage, based on the actual experience of the Scottish Bankers, of which the following is an extract :

TABLE J.

Husband's Age at Marriage	Average Age of Wife	Difference
20	22	- 2
25	24	+ 1
30	26	4
35	27	8
40	29	11
45	29	16
50	33	17
55	37	18
60	40	20

There will not, of course, be so much difference of age between the wife and husband at the death of the husband, especially as the majority of marriages takes place between the ages of 27 and 40. After due consideration of the probable survivors, I fixed upon the following scale :

TABLE K.

Average Age of Husband at Death	Average Age of Widow at Death of Husband	Difference
25 $\frac{1}{2}$	24	1 $\frac{1}{2}$
30 $\frac{1}{2}$	28	2 $\frac{1}{2}$
35 $\frac{1}{2}$	31 $\frac{1}{2}$	4
40 $\frac{1}{2}$	34 $\frac{1}{2}$	6
45 $\frac{1}{2}$	38	7 $\frac{1}{2}$
50 $\frac{1}{2}$	42	8 $\frac{1}{2}$
55 $\frac{1}{2}$	46 $\frac{1}{2}$	9
60 $\frac{1}{2}$	51	9 $\frac{1}{2}$
65 $\frac{1}{2}$	55 $\frac{1}{2}$	10

The difference between the ages of husband at death and widow for the intermediate ages of husband at death was determined by first differences (see Table 46).

If these figures err at all, they err upon the side of making the difference of age, especially after age 40, too large; but if that is so, then the error is upon the safe side, because in my Tables I shall be making a larger provision for the annuity to the widow than is necessary.

The present value of an annuity of 1 payable to the widow during the remainder of her life was taken from Mr. Hewat's Table II (page 87).

CHILDREN'S BENEFITS.

In order to ascertain the ages and the number of children left by a married man at his death, I have had to go to the same source as Mr. King did when he constructed his Table of "Family Annuities", namely, the "Statistics of the Colony of New Zealand." In vol. xxx of the *Journal*, on page 300, Mr. King gave a copy of the "Table showing, for the year 1890, "the total number of males who died at each year of age from "20 to 65; the number of married males who died (a) leaving "no children, and (b) leaving children; and the number and "ages of the children living at the time of the father's death." In the statistics of the Colony for the year 1899 a similar Table is given for the five years 1895 to 1899 inclusive, for all ages of the father up to 65, and also a Table for the year 1899, for all ages of males at death from 20 to 99. As I shall make very considerable use of this Table, I will give the figures, grouping five ages together.

TABLE I.

ORPHANHOOD OF CHILDREN.

Table showing the Total number of Married Men stated in the Registers of the Colony of New Zealand, as having died (from age 20 to 65, during the five years 1895 to 1899, and from age 65 to the end of life, for the year 1899) (a) Childless; (b) leaving Children; and the number and ages of the Children left.

Ages of Married Men at Death	Central Age	Childless	Leaving Children	AGES OF CHILDREN													Age not specified										
				0	1	2	3	4	5	6	7	8	9	10	11	12		13	14	15	16	17	18	19	20	21 and over	
Under 23	—	4	5	4	2	
23-27	25	33	71	40	19	58	51	13	9	5	23	19	11	8	0	1	
28-32	30	53	170	46	53	87	84	51	45	38	89	65	65	51	39	24	21	14	9	1	4	
33-37	35	54	300	81	54	87	91	85	87	73	98	101	91	100	93	95	77	74	59	48	38	26	16	7	4	62	
38-42	40	70	360	59	47	84	84	80	91	98	98	113	119	107	106	125	101	132	78	125	82	82	67	59	159	69	
43-47	45	73	452	46	49	60	65	92	71	107	96	102	122	129	130	150	126	161	150	149	153	124	143	123	591	87	
48-52	50	85	518	32	31	57	62	59	85	98	96	98	101	108	115	129	152	164	187	173	183	214	183	211	204	1482	143
53-57	55	140	755	25	30	39	49	56	71	82	98	101	59	79	71	101	99	134	114	166	152	181	179	203	203	2736	127
58-62	60	130	810	9	18	19	36	24	35	40	55	59	6	6	5	12	9	12	17	20	18	29	19	43	30	957	33
63-67	65	37	223	2	..	3	2	2	3	2	5	2	2	2	1	3	2	6	8	6	10	15	17	15	895	49	
68-72	70	23	196	1	3	..	1	1	1	1	1	1	1	2	2	4	2	2	2	8	2	8	..	720	29
73-77	75	25	155	1	1	1	..	1	3	1	498	35
78-82	80	13	107	1	1	1	1	259	19
83-87	85	7	58	1	107	2
88-92	90	3	19	1

No attempt has been made to distribute the numbers in the column headed "Age not specified", consequently that column has been ignored.

The marriage rate in the Colony is no doubt different from that prevailing amongst the class we are dealing with, but I agree with Mr. King that no great error will be committed if it be assumed that the number of children to each deceased married man is much the same in New Zealand as in Great Britain.

There are two ways in practice of providing for orphans; one by continuing the widow's annuity until the youngest child reaches a certain age, the other by granting an annuity to each child until the attainment of a certain age.

CONTINUATION OF WIDOW'S ANNUITY TO YOUNGEST CHILD.

This plan of providing for the children presents two distinct problems: (1) The value, on the death of a husband, of a deferred annuity to the youngest child to commence at the death of the widow, and to continue until the youngest surviving child attains a certain age; (2) the value on the death of a widower of an annuity until the youngest child reaches the assigned age. The former is a term survivorship-annuity, the latter is, practically, an immediate term-annuity-certain.

Our statistics do not distinguish between husbands' and widowers' children, nor do they give any information as to the youngest child in the family. These defects prevent us from arriving at anything like extreme accuracy in our estimates, but we shall produce results which, if not exact, will at least be on the safe side. The better to explain the methods pursued, I here give a Table showing the different stages by which the values were produced.

Now there can only be one youngest child for each deceased husband leaving a family; consequently, the 170 married men (which for the present we will assume to be husbands) leaving children, who died at ages within the group of which 30 is the centre, would leave 170 youngest children. If then we assume that the youngest 170 children are the youngest children of the 170 different families, we shall certainly be placing our estimate on the safe side, because some of these will undoubtedly be first and second from the youngest in the same family, and the youngest of some families will be older. On the other hand the fact that widowers' children are included would have the tendency, when widowers are treated as husbands, to make the youngest child somewhat older than the husbands' youngest children alone; but the proportion of widowers who die to husbands who die is very small, so that our assumption will still keep us on the side of safety.

The youngest 170 children opposite central age 30 are, 46 aged 0, 53 aged 1, 58 aged 2, and to make up the 170 we take 13 aged 3. On the principle that the ages are scattered equally throughout the year, we shall assume that all those entered under age 0 are, on the average, $\frac{1}{2}$ a year old; all those entered under age 1 are, on the average, $1\frac{1}{2}$ years' old, and so on. The average age of the youngest child will, therefore, be $\{(46 \times \frac{1}{2}) + (53 \times 1\frac{1}{2}) + (58 \times 2\frac{1}{2}) + (13 \times 3\frac{1}{2})\} \div 170 = \frac{293}{170} = 1.7$.

Having ascertained the average age of the youngest child, which we will call x' , our first object will be to find the addition to be made to the value of the widow's annuity at the death of the husband to provide for the continuance of the annuity to the youngest child until the age of, say 14, in the event of the widow dying before such time. We have not to provide for the youngest child alone, for if the youngest dies the next youngest in the family takes his place, and so on, until the youngest surviving reaches the age of 14. The value of the annuity for the youngest child, therefore, is not much short of an annuity-certain; so that by assuming, as I have done, that the value of the annuity to the youngest child is the value of an annuity-certain for the term between the age of the youngest child at the death of a husband, and the age when the annuity is to cease, we are arranging that the estimated value shall be on the safe side. Huie assumes that every husband leaves a child just born, and that such child will certainly live 21 years, which is an exaggerated view to take of the case.

The term during which a provision has to be made for the youngest child, until the age of 14, is $14-x'$, and the value of a continuous annuity-certain for that period is approximately $\frac{1}{2} + a_{\overline{14-x'-\frac{1}{2}}}$. The value of a continuous annuity to the widow on the death of the husband for the same term is approximately $\frac{1}{2} + \frac{1}{2}a_{\overline{14-x'-\frac{1}{2}}}$. The addition, therefore, to be made to the value of the widow's annuity to provide for the youngest child until 14, is

$$a_{\overline{14-x'-\frac{1}{2}}} - \frac{1}{2}a_{\overline{14-x'-\frac{1}{2}}},$$

consequently, we only tabulate $a_{\overline{14-x'-\frac{1}{2}}}$.

The values of the term annuities on the lives of widows were based on the H^F Table, because the rate of mortality in that Experience corresponded fairly well with Mr. Hewat's rate of mortality amongst widows, and the values could be easily obtained from the Institute H^F Tables. As Mr. Hewat does not publish the D and N columns in his Tables we had no ready means of calculating the term values on his experience.

The difference between the annuity-certain for the youngest child and the term annuity on the life of the widow, will give the addition to be made to the widow's annuity *per husband leaving children under 14 years of age*. It will be seen, on reference to the Table, that there are sometimes more married men leaving children than there are children under 14, and this is the case at central age 60 at death and over. In order to find the addition to the widow's annuity per husband leaving children, we require to multiply the value already obtained by the total number of children under 14, and divide by the number of married men leaving children; that is to say, for central age 60, the value in col. (10), which is .137, must be multiplied by $\frac{782}{840}$, which produces .128; for central age 65, the value in col. (10) must be multiplied by $\frac{69}{223}$, and so on. In this way col. (11) was produced, showing the addition to the widow's annuity *per husband leaving children*. But then it is not every husband who leaves a child; and on comparing cols. (2) and (3) it will be seen that 15 per-cent of the total married men die childless, and that the ratio is nearly the same at all ages. By multiplying, then, the values in col. (11) by .85, we obtain the addition to the widow's annuity *per husband dying*, which values are inserted in col. (12). These values form a fairly regular series, and the values for each age (see Table 48) were obtained by means of a curve drawn graphically through the values at the central ages.

PROVISION FOR YOUNGEST CHILD ON DEATH OF A WIDOWER.

I have already explained that, owing to the New Zealand statistics not distinguishing between husbands and widowers in the married men who die, it is quite impossible to separate their children; consequently, we have no choice between assuming that the ages of the children left by widowers are the same as those left by husbands and widowers together, or, making an empirical assumption as to the possible increase in the ages of the children on the death of a widower compared with the ages on the death of a husband. I prefer to assume that the ages of the children are exactly the same, for the reason that, by doing so, the provision I make for the children of husbands and widowers taken together will correspond exactly with the provision for the children left by married men at death according to the statistics. The number of widowers dying at ages when the children are young is very small, even when only one marriage is allowed for; but, if second and third marriages are taken into consideration, it will be subsequently shown that the number of widowers dying leaving young children is very small indeed.

At present, we do not take second marriages into consideration, and we accordingly assume that the values in col. (7) in the above Table, with the addition of $\cdot 5$, will represent the value of the continuous annuity to the youngest child of a widower leaving children under 14. These values reduced, first to the values per widower leaving children, and then to the values per widower dying, in the same way as the additions to the widow's annuity were treated, will produce the values in col. (15), which were also graduated graphically in order to obtain the values for each age of widower at death.

ANNUITY TO EACH OF THE CHILDREN OF A HUSBAND OR WIDOWER ON THE DEATH OF THE FATHER; OTHERWISE CALLED ORPHANS' ANNUITIES.

I have adopted the term "orphans' annuities" in preference to "family annuities" used by Mr. King, because although, in a narrow sense, the children of the same parents considered collectively apart from their parents may be called the family, it in no way implies that the liability arises only on the death of the father. On the other hand, although, I suppose, in its strict sense, an orphan means a child bereft of both parents, it is now most commonly used to signify a child whose father is dead, and therefore more clearly expresses the status of the child who is to receive the annuity.

We have not now to distinguish between husbands' children and widowers' children, consequently we have to calculate the value of the annuities as at the moment of death of a married man.

Mr. King based his annuities on the English Life No. 3 Experience, and I think wisely so, because he was dealing with a population problem, but that Table does not represent the mortality experience of the class with which we are dealing. Mr. Hewat in his work on "An Investigation of the Marriage and Mortality Experience of a Scottish Ministers' Widows' and Orphans' Fund, &c.", gives a Table of the mortality experience amongst children, taken out to four places of decimals. I am content to use only three places of decimals to represent the probability of dying in a year. The following Tables show the rate of mortality I have used, and the values of the temporary annuities deduced therefrom. I have not used the elaborate formula of Mr. King for a continuous term annuity, being satisfied with the approximate formula, $\frac{1}{2} + n - x' - \frac{1}{2}a_{x'}$, as a slight difference in the rate of mortality would neutralize any difference produced by extreme accuracy in calculating benefits.

The rate of children's mortality which I have assumed is lighter than that used by Mr. King, and, consequently, my temporary annuities are larger than his.

TABLE N.

Table of the Mortality Experience amongst Children.

Age x	q_x	l_x	d_x
0	·062	10,000	620
1	·016	9,380	150
2	·010	9,230	92
3	·009	9,138	83
4	·008	9,055	72
5	·007	8,983	63
6	·006	8,920	53
7	·005	8,867	45
8	·004	8,822	35
9	·004	8,787	35
10	·004	8,752	35
11	·004	8,717	35
12	·003	8,682	26
13	·002	8,656	17
14	·002	8,639	18
15	·003	8,621	25
16	·004	8,596	34
17	·004	8,562	35
18	·005	8,527	43
19	·006	8,484	50
20	·006	8,434	51
21	...	8,383	...

TABLE O.

Value of Continuous Temporary Annuity.

Year of Age	To AGE 14		To AGE 16		To AGE 21	
	3 per-cent	4 per-cent	3 per-cent	4 per-cent	3 per-cent	4 per-cent
0- 1	10·281	9·690	11·440	10·697	14·018	12·865
1- 2	9·995	9·457	11·237	10·518	14·003	12·898
2- 3	9·409	8·939	10·706	10·089	13·593	12·564
3- 4	8·764	8·361	10·114	9·568	13·116	12·167
4- 5	8·085	7·746	9·487	9·012	12·605	11·738
5- 6	7·372	7·093	8·826	8·420	12·063	11·276
6- 7	6·624	6·401	8·132	7·791	11·488	10·781
7- 8	5·843	5·671	7·405	7·124	10·880	10·251
8- 9	5·029	4·902	6·644	6·420	10·240	9·687
9-10	4·183	4·097	5·854	5·681	9·572	9·092
10-11	3·309	3·256	5·037	4·910	8·882	8·472
11-12	2·405	2·377	4·192	4·105	8·168	7·824
12-13	1·468	1·459	3·316	3·262	7·426	7·144
13-14	·500	·500	2·407	2·380	6·652	6·427
14-15	1·469	1·459	5·849	5·676
15-16	·500	·500	5·023	4·897
16-17	4·175	4·089
17-18	3·300	3·248
18-19	2·397	2·370
19-20	1·465	1·456
20-21	·500	·500

My Tables of orphans' annuities were constructed in the way described by Mr. King in his paper on "Family Annuities" (*J.I.A.*, xxx, p. 303), but using only the numbers for the groups of five ages of married men at death (Table L). The curve drawn through the values for central ages was almost exactly of the same form as Mr. King's. These Tables will be found on pages 146-7.

If my values "to age 14" at 3 per-cent be compared with Mr. King's values for the same limiting age and rate of interest, it will be found that my values are less than his, particularly from ages 30 to 40, notwithstanding that my annuities are higher, which shows that the number of children in a family in New Zealand had decreased in the ten years from 1890 to 1900.

Having given the particulars of the Tables upon which all our calculations are to be based, I think the best way of showing the construction of the Valuation Tables will be to follow the same course as before; that is, to give solutions of the values of the different benefits in the form of problems. By this means every investigation is kept clear and distinct.

Problem Ic.—Assuming that the members of a staff consist of bachelors, husbands, and widowers, distributed in the pro-

portions according to the Tables, what would be the present value in respect of a person aged x of an annuity of 1 to a widow for the remainder of her life to commence on the death of a husband? No marriage after age 65 and no second marriage to be recognized. What annual premium payable for the whole of life by each member of the staff, entering at age x , would be required to provide such benefit?

Out of l_x persons living at age x , Hd_x husbands will die between the ages x and $x+1$; say, on the average, at age $x+\frac{1}{2}$. Hd_{x+1} husbands will die between the ages of $x+1$ and $x+2$; say, on the average, at age $x+1\frac{1}{2}$; and so on. If the average age of the widow at the death of a husband dying at the average age of $x+\frac{1}{2}$ is y , and if we assume that $\bar{a}_y = \frac{1}{2} + a_y$ approximately, then we shall have to provide for $Hd_x \times (\frac{1}{2} + a_y)$ at age $x+\frac{1}{2}$, $Hd_{x+1} \times (\frac{1}{2} + a_{y+1})^*$ at age $x+1\frac{1}{2}$, &c., and the present value of the annuity in respect of a person living at age x will be

$$\frac{Hd_x(\frac{1}{2} + a_y)v^{x+\frac{1}{2}} + Hd_{x+1}(\frac{1}{2} + a_{y+1})v^{x+1\frac{1}{2}} + \dots}{l_x v^x}$$

If now we call $Hd_x(\frac{1}{2} + a_y)v^{x+\frac{1}{2}} = {}^wC_x$, and sum the values of wC_x so that ${}^wM_x = {}^wC_x + {}^wC_{x+1} + {}^wC_{x+2} + \dots$, then the present value of the widows' annuity will be represented by

$$\frac{{}^wM_x}{D_x^{(3)}}$$

The annual premium, in the ordinary way, would be found by substituting $N_{x-1}^{(3)}$ for D_x ; but if the premium is to be payable monthly, and the funds invested quarterly, then we must apply similar adjustments as in our other problems, and use

$$(vN_{x-1}^{(3)} - \frac{1}{2}M_x^{(3)} - \frac{1}{2}{}^wM_x^{(4)})(1 + \frac{3}{5}i)$$

for the denominator.

Problem IIc.—What is the present value, in respect of a person aged x , of the additional payment required to provide for the continuance of the widow's annuity of 1 from her death until the youngest surviving child reaches the age of (n) , and the corresponding annual premium payable for the whole of life?

We have already ascertained (page 113) the value of the addition at the death of the husband, and agreed to call such value $E(n)_x$. Out of l_x persons, therefore, living at age x , we

* a_{y+1} does not mean that the annuity is for the age 1 year older than y , but $y+1$ is the age of the widow on the death of a husband at the age of $x+1\frac{1}{2}$.

shall have to provide in respect of Hd_x husbands dying between the ages x and $x+1$, $Hd_x \cdot E(n)_x$; and in respect of Hd_{x+1} husbands dying between the ages $x+1$ and $x+2$, $Hd_{x+1} \cdot E(n)_{x+1}$; and so on. The present value of the addition in respect of a person aged x will therefore be

$$\frac{Hd_x \cdot E(n)_x \cdot v^{x+\frac{1}{2}} + Hd_{x+1} \cdot E(n)_{x+1} \cdot v^{x+1\frac{1}{2}} + \dots}{l_x v^x}$$

If then we call $Hd_x \cdot E(n)_x \cdot v^{x+\frac{1}{2}} = {}^{E(n)}C_x$, and sum the values of ${}^{E(n)}C_x$ so that ${}^{E(n)}M_x = {}^{E(n)}C_x + {}^{E(n)}C_{x+1} + {}^{E(n)}C_{x+2} + \dots$, the present value of the addition to provide for the continuance of the widow's annuity until the youngest surviving child reaches the age of n , will be represented by

$$\frac{{}^{E(n)}M_x}{D_x^{(3)}},$$

and the annual premium will be either

$$\frac{{}^{E(n)}M_x}{N_{x-1}^{(3)}} \text{ or } \frac{{}^{E(n)}M_x}{(vN_{x-1}^{(3)} - \frac{1}{2}M_x^{(3)} - \frac{1}{2}M_x^{(4)})(1 + \frac{3}{8}i)},$$

according as the premium is to be paid yearly in advance, or deducted from salary.

Problem IIIc.—What is the present value, in respect of a person aged x , of an annuity of 1 to the youngest surviving child of a widower until the age of (n) , from the moment of death of the widower; and what the corresponding annual premium?

We have already ascertained (page 113) the present value, at the death of a married man, of an annuity of 1 until the youngest surviving child reaches the age of n , and agreed to call the value $YCa(n)_x$.

Out of l_x persons living at the age of x , Kd_x will die as widowers between the ages of x and $x+1$, and the annuities to be provided will be $Kd_x \cdot YCa(n)_x$. Kd_{x+1} will die as widowers between the ages $x+1$ and $x+2$, and the annuities to be provided for them will be $Kd_{x+1} \cdot YCa(n)_{x+1}$; and so on. So that the present value in respect of a person aged x will be

$$\frac{Kd_x \cdot YCa(n)_x \cdot v^{x+\frac{1}{2}} + Kd_{x+1} \cdot YCa(n)_{x+1} \cdot v^{x+1\frac{1}{2}} + \dots}{l_x v^x}.$$

If now we call $Kd_x \cdot YCa(n)_x \cdot v^{x+\frac{1}{2}} = {}^{K \cdot YCa(n)}C_x$ and sum the values so that ${}^{K \cdot YCa(n)}M_x = {}^{K \cdot YCa(n)}C_x + {}^{K \cdot YCa(n)}C_{x+1} + \dots$, the

present value of an annuity of 1 to the youngest surviving child of a widower until the age of n will be

$$\frac{{}^{\kappa} \cdot \text{VCO}(n) M_x}{D_x^{(3)}}$$

and to find the annual premium we use the same denominator as in the two previous Problems.

Problem IVc.—What portion of salary payable during active service, equated to 1 of salary at age x , would be required to provide for an annuity of 1 to a widow and an annuity of 1 from the death of a widow or widower until the youngest child reaches the age of n ?

The value of the benefits have been ascertained in Problems Ic, IIc, and IIIc, and, equating the payments to the benefits, we have

$$\begin{aligned} P \{ \frac{\infty}{s} - \frac{1}{2} ({}^d M_x^{ls} + {}^w M_x^{ls} + {}^r M_x^{ls}) \} (1 + \frac{s}{s} i) &\div D_x^s \\ &= ({}^w M_x + {}^E(n) M_x + {}^{\kappa} \cdot \text{VCO}(n) M_x) \div D_x^{(3)} \end{aligned}$$

$$\text{and } P = \frac{{}^w M_x + {}^E(n) M_x + {}^{\kappa} \cdot \text{VCO}(n) M_x}{(1 + \frac{s}{s} i) \{ \frac{\infty}{s} - \frac{1}{2} ({}^d M_x^{ls} + {}^w M_x^{ls} + {}^r M_x^{ls}) \}} \times \frac{D_x^s}{D_x^{(3)}}$$

Problem Vc.—What is the present value, in respect of a person aged x , of an annuity of 1 to each of the children of a family, from the moment of death of the father, until they respectively attain the age of (n) ?

Here we must remember that both husbands and widowers leave children which are treated alike, and consequently we have to base our calculations on the number of children left by married men.

Out of l_x persons living at age x , Md_x married men will die between the ages x and $x+1$, say, on the average, at age $x+\frac{1}{2}$. Md_{x+1} married men will die between the ages $x+1$ and $x+2$, say, on the average at age $x+1\frac{1}{2}$, and so on. The present value of an annuity to the orphans left by a married man dying between the ages x and $x+1$, until they respectively reach the age of (n) is $Oa(n)_x$; consequently the present value of the orphans' annuities in respect of a person living at age x will be

$$\frac{Md_x \cdot Oa(n)_x \cdot v^{x+\frac{1}{2}} + Md_{x+1} \cdot Oa(n)_{x+1} \cdot v^{x+1\frac{1}{2}} + \dots}{l_x v^x}.$$

If now we call $Md_x \cdot Oa(n)_x \cdot v^{x+\frac{1}{2}} = {}^{Oa(n)}C_x$, and sum the values of ${}^{Oa(n)}C_x$, so that ${}^{Oa(n)}M_x = {}^{Oa(n)}C_x + {}^{Oa(n)}C_{x+1} + {}^{Oa(n)}C_{x+2} + \dots$

then the present value of the orphans' annuities of 1 each until they respectively reach the age of (n) will be represented by

$$\frac{{}^{oa(n)}M_x}{D_x^{(3)}}.$$

Problem VIc.—What is the present value, in respect of each person living at age x , whether bachelor, husband or widower, of 1 payable on the death of a bachelor?

Here out of l_x persons living at age x , Bd_x bachelors will die between the ages of x and $x+1$; Bd_{x+1} will die between the ages $x+1$ and $x+2$, and so on: so that the present value of 1 on the death of a bachelor will be

$$\frac{Bd_x \cdot v^{x+\frac{1}{2}} + Bd_{x+1} \cdot v^{x+1\frac{1}{2}} + Bd_{x+2} \cdot v^{x+2\frac{1}{2}} + \dots}{l_x v^x}$$

If then we call $Bd_x \cdot v^{x+\frac{1}{2}} = {}^{Bd}C_x$, and sum these values so that ${}^{Bd}C_x + {}^{Bd}C_{x+1} + {}^{Bd}C_{x+2} + \dots = {}^{Bd}M_x$, the present value required will be represented by

$$\frac{{}^{Bd}M_x}{D_x^{(3)}}.$$

It is not, I believe, usual in these funds to provide for a return of contributions on withdrawal from the staff; but if such a rule is introduced it is only necessary to add to the benefit side of the equation of payments and benefits the value $P \cdot {}^wR_x$, or $P \cdot {}^wR_x^s$ according as P is a fixed annual payment, or a proportion of salary; or, to be more exact,

$$P({}^wR_x - \frac{1}{2} {}^wM_x)(1 + \frac{1}{2}i), \text{ or } P({}^wR_x^s - \frac{1}{2} {}^wM_x^{ts})(1 + \frac{1}{2}i),$$

using, of course, the proper denominator of D_x or D_x^s . (See Problems IA and VB.)

Problem VIIc.—What is the present value, in respect of a person aged x , of the assurance of 1 on the death of a widower leaving no children under the age of (n)?

This seems to be a benefit which is rapidly coming to the front; and in the absence of statistics, anything like an exact calculation is out of the question. It will be seen from Table L that the proportion of married men dying childless is 33 per-cent at age 25, and 25 per-cent at age 30, and after that about 15 per-cent. If we assume that, as a rule, the last child we should have to deal with would be born when the husband's age was 55, we might with safety assume that the number of widowers who would become claims, if the children's benefits ceased at

age 16, would be 20 per-cent of Kd_x up to age 30, the percentage increasing by 1 each year up to age 45, when the number would be 25 per-cent of Kd_x ; then increasing by 2 each year to age 55, then increasing by 3 each year up to age 65, when the number would be 75 per-cent of Kd_x ; and then increasing by 5 each year up to age 70, when the claims for that age and over would amount to 100 per-cent of Kd_x .

If the children's benefits are continued till age 21, then I think the percentage might be deferred 5 years; that is to say, take 20 per-cent of Kd_x up to age 35, increase the percentage by 1 each year to age 45, and so on.

The symbol for a childless widower is $(\kappa - c)$, and for a widower without children under the age of (n) , $\kappa - c(n)$. If, now, the column we are supposed to have just constructed be called $\kappa - c(n)d_x$, then out of l_x persons living at age x , $\kappa - c(n)d_x$ will die between the ages x and $x+1$ as widowers leaving no children under the age of (n) . Then, following the same reasoning as in Problem VIc; if $\kappa - c(n)d_x \times v^{x+1} = \kappa - c(n)^l C_x$, and $\kappa - c(n)^l M_x = \kappa - c(n)^l C_x + \kappa - c(n)^l C_{x+1} + \dots$, the present value of the assurance of 1 on the death of a widower leaving no children under age (n) will be represented by

$$\frac{\kappa - c(n)^l M_x}{D_x^{(3)}}.$$

If the contributions are to be returned on the death of a widower leaving no child on the Funds, then Tables would have to be formed of $\kappa - c(n)^l R_x$, or $\kappa - c(n)^l M_x^s$, and $\kappa - c(n)^l R_x^s$. (See Problems IA and VB.)

Sometimes the contribution is a fixed percentage of the salary and pension, in which case it is not quite so easy to find the value of the contributions.

Problem VIIIc.—What is the present value, at age x , of future salary and pension to the end of life, equated to 1 of salary at age x ?

We have to deal with this problem in three sections, namely:

- (i) The present value of the salary during active service, equated to 1 of salary at age x ;
- (ii) The present value of the pension in respect of those who retire at the pension age, say 65, from the date of their retirement to the end of life, equated to 1 of salary at age x ;

- (iii) The present value of the pension in respect of those who retire before the pension age, from the date of their retirement to the end of life, equated to one of salary at age x ;

and for these we must use the values based upon my Table 4. The value of (i) we already know, namely,

$$\frac{\{N_x^s - \frac{1}{2}(dM_x^{ls} + wM_x^{ls} + rM_x^{ls})\}(1 + \frac{3}{8}i)}{D_x^s}$$

but (ii) and (iii) must depend upon the mode in which the pension is calculated.

Let us suppose, first, that the retiring allowance is a fixed percentage of average salary for every year of service, say π per cent.; then the value of (ii) will be

$$\frac{\pi}{100} \cdot \frac{\Sigma s_x(N_{65} + \frac{1}{2}D_{65})}{D_x^s}$$

But this is not now a liability payable at the moment of the occurrence of the event, but an asset dependent upon the accumulation of capital; and so, according to the rule upon which I have based the adjustments, namely, that the investments are not made immediately, but, on the average, every quarter, the above value should be multiplied by $v^{\frac{3}{4}}$.

The value of (iii) will be (see Problem XB.)

$$\frac{\pi}{100} \frac{(r^a R_x^s - \frac{1}{2} r^a M_x^{ls})(1 + \frac{3}{8}i)}{D_x^s}$$

and the total value of the future salary and pension to the end of life, equated to one of salary at age x , will be

$$\left[\{N_x^s - \frac{1}{2}(dM_x^{ls} + wM_x^{ls} + rM_x^{ls})\}(1 + \frac{3}{8}i) + \frac{\pi}{100} \cdot \Sigma s_x(N_{65} + \frac{1}{2}D_{65}) \cdot v^{\frac{3}{4}} \right. \\ \left. + \frac{\pi}{100} (r^a R_x^s - \frac{1}{2} r^a M_x^{ls})(1 + \frac{3}{8}i) \right] \div D_x^s.$$

If past service has to be taken into account in calculating the pension on retirement, then the complete value of (ii) based on past and present salary, — (not equated to 1 of salary) — will be

$$\frac{\pi}{100} \left\{ \frac{(\text{total past salary}) \times (N_{65} + \frac{1}{2}D_{65})}{D_x} \right. \\ \left. + \frac{(\text{present salary}) \times \Sigma s_x(N_{65} + \frac{1}{2}D_{65})}{D_x^s} \right\} v^{\frac{3}{4}}$$

and the complete value of (iii) will be

$$\frac{\pi}{100} \left\{ (\text{total past salary}) \times \frac{{}^{ra}M_x}{D_x} \right. \\ \left. + (\text{present salary}) \times \frac{{}^{ra}R_x^s - \frac{1}{2} {}^{ra}M_x^{ls}}{D_x^s} \cdot (1 + \frac{3}{8}i) \right\}$$

so that the complete total value is

$$\left\{ (\text{present salary}) \times \frac{{}^{ra}R_x^s - \frac{1}{2} ({}^dM_x^{ls} + {}^wM_x^{ls} + {}^rM_x^{ls})}{D_x^s} \cdot (1 + \frac{3}{8}i) \right\} \\ + \frac{\pi}{100} \left\{ (\text{total past salary}) \times \frac{(N_{65} + \frac{1}{2}D_{65}) \cdot v^{\frac{5}{8}} + {}^{ra}M_x}{D_x} \right\} \\ + \frac{\pi}{100} \left[(\text{present salary}) \left\{ \frac{\Sigma s_x}{s_x} \cdot \frac{(N_{65} + \frac{1}{2}D_{65}) \cdot v^{\frac{5}{8}}}{D_x} \right. \right. \\ \left. \left. + \left(\frac{{}^{ra}R_x^s}{D_x^s} - \frac{1}{2} \frac{{}^{ra}M_x^{ls}}{D_x^s} \right) (1 + \frac{3}{8}i) \right\} \right]$$

which looks more formidable than it really is.

In the second place, let us suppose that the retiring allowance is a fixed percentage of the last salary for every year of service, says π per-cent: then the value of (ii) will be

$$\frac{\pi}{100} \cdot \frac{(65-x)s_{64} \cdot (N_{65} - \frac{1}{2}D_{65})}{D_x^s} \cdot v^{\frac{5}{8}},$$

and the value of (iii) will be

$$\frac{\pi}{100} \cdot \frac{({}^{ra}R_x^{ls} - \frac{1}{2} {}^{ra}M_x^{ls})(1 + \frac{3}{8}i)}{D_x^s},$$

and the total value of the future salary and pension to the end of life, equated to 1 of salary at age x , will be

$$\left[\left\{ {}^{ra}R_x^s - \frac{1}{2} ({}^dM_x^{ls} + {}^wM_x^{ls} + {}^rM_x^{ls}) \right\} (1 + \frac{3}{8}i) \right. \\ \left. + \frac{\pi}{100} (65-x)s_{64} \cdot (N_{65} - \frac{1}{2}D_{65}) \cdot v^{\frac{5}{8}} \right. \\ \left. + \frac{\pi}{100} ({}^{ra}R_x^{ls} - \frac{1}{2} {}^{ra}M_x^{ls})(1 + \frac{3}{8}i) \right] \div D_x^s.$$

If past service has to be taken into account in calculating the pension on retirement, we have to consider the number of years already served. Say the number is n , then the complete value of (ii), based on past and future service, will be

$$(\text{present salary}) \times \frac{\pi}{100} \cdot \frac{(65+n-x)s_{64} \cdot (N_{65} - \frac{1}{2}D_{65})}{D_x^s} \cdot v^{\frac{5}{8}},$$

and the value of (iii) will be

$$(\text{present salary}) \times \frac{\pi}{100} \cdot \frac{n \cdot {}^{ra}M_x^{ls} + {}^{ra}R_x^{ls} - \frac{1}{2} {}^{ra}M_x^{ls}}{D_x^s} \cdot (1 + \frac{3}{8}i),$$

and the complete total value of the future salary and pension to the end of life will be

$$\begin{aligned} & (\text{present salary}) \times \left[\left\{ \mathbb{N}_x^s - \frac{1}{2} ({}^d\mathbf{M}_x^{ls} + {}^w\mathbf{M}_x^{ls} + {}^r\mathbf{M}_x^{ls}) \right\} \left(1 + \frac{3}{8}i \right) \right. \\ & \quad + \frac{\pi}{100} (65 + n - x) s_{64} (N_{65} - \frac{1}{2} D_{65}) \cdot v^{\frac{3}{8}} \\ & \quad \left. + \frac{\pi}{100} (n {}^ra\mathbf{M}_x^{ls} + {}^ra\mathbf{R}_x^{ls} - \frac{1}{2} {}^ra\mathbf{M}_x^{ls}) \left(1 + \frac{3}{8}i \right) \right] \div D_x^s. \end{aligned}$$

The usual scales of retiring allowances can be introduced by appropriately applying the solutions given in Problems XIII_B, XIV_B, and XV_B.

Problem IXc.—To find the present value, in respect of a person of the age x at entrance, of the return of total salary and pension, without interest, on the death of a bachelor, equated to 1 of salary at age x .

This would be a very simple matter if it were not for the early retirements, because we could assume that all the bachelors in our Table No. 45 received full salaries up to the pension age, and then retired on full pension.

If the pension is π per-cent of average salary for the total number of years' service, then s for age 65 and over would be, taking 17 as the average age at entrance, $\frac{\pi}{100} \cdot \Sigma_{17}^{64} s$; or, if the pension at the pension age is $\frac{2}{3}$ rds of last salary, then s for 65 and over would be $\frac{2}{3} s_{64}$.

We should then proceed in the same way as in Problem V_B, and construct a Table of $v^{x+\frac{1}{2}} \mathbf{B}d_x$ for the whole of life, which we have called ${}^{bd}\mathbf{C}_x$; sum these values like the \mathbf{M} column, and call the new values ${}^{bd}\mathbf{M}_x$, a column which we have already obtained in Problem VI_c; multiply each value of ${}^{bd}\mathbf{M}_x$ by s , calling the result ${}^{bd}\mathbf{M}_x^s$, and then sum these values again, thus producing a column which we will call ${}^{bd}\mathbf{R}_x^s$, and the value of the return of total salary and pension on the death of a bachelor, out of l_x persons (including pensioners) living at the age x would be represented by

$$\frac{{}^{bd}\mathbf{R}_x^s}{D_x^s}$$

where D_x^s is calculated on the l_x 's in Table 3.

The work could be shortened by adding all the $\mathbf{B}d_x$'s from age 65 inclusive to the end of life, thus producing $\mathbf{B}l_{65}$, and multiplying $\mathbf{B}l_{65}$ by $\frac{v^{65+\frac{1}{2}} \mathbf{R}_{65}^{(3)}}{D_x^{(3)}} \times (\text{average pension at age 65})$,

because no marriages are allowed after the pension age, and, consequently, the bachelors dying after that age are the numbers dying in the ordinary mortality Tables.

Even assuming that no one retires before the pension age, the values we have obtained would be in excess by half the last year's salary, namely, $\frac{1}{2} {}^{vl}M_x^s$, a value which has not been calculated.

If past service is to be taken into consideration, then the value will be—

$$\frac{(\text{total past salary}) {}^{vl}M_x + {}^{vl}R_x^s}{D_x^s}.$$

The rule that no marriage after retirement on pension is recognized would enable us to make an exact valuation of the benefit, if anyone thought it worth while to take the trouble. We could find out how many bachelors retire at each age, and how many remain in the fund until the pension age. We could then deal with those remaining on the fund in the same way as above; and for those who retire early we should have to find an average pension at each age of retirement, and make the liability from the age of retirement the value of an increasing assurance of the retiring allowance according to Table 8; that is,

$$v^{x+\frac{1}{2}}(r_x \times \frac{R_x^{(r)}}{D_x^{(r)}} \times \text{average pension age at } x).$$

Another and equally good way would be to find an average age at retirement, by multiplying the number in the column "Retiring" in Table 4 by the age, including those who retire at the pension age, and dividing the sum of the results by the total number retiring.

The last plan would be quite safe, and as near accuracy as we might ever wish to get.

Problem Xc.—To find the present values, in respect of a person aged x at entrance, of the return of total salary and pension, without interest, on the death of a widower leaving no children under the age of (n) , equated to 1 of salary at age x .

In the first place we have to find the number of widowers dying at each age leaving no children under the age of (n) , as described in Problem VIIc; and then proceed as in Problem IXc, forming columns of $v^{x+\frac{1}{2}}(\kappa - C)d_x$, $(\kappa - c)^l M_x$, $(\kappa - c)^l M_x^s$, and $(\kappa - c)^l R_x^s$. The required value would then be represented by

$$\frac{(\kappa - c)^l R_x^s}{D_x^s},$$

the D_x^s being calculated on Table 3.

There is no way of shortening this work as in the previous Problem, because widowers are constantly emerging from the husbands as they lose their wives. We, however, need not trouble about the early retirements, as the number of widowers dying do not become large until the pension age is reached.

Problem XIc.—A firm whose staff is represented by the particulars given on page 242, vol. xxxvi, is anxious to start a widows' pension fund, which all those now on the staff and of the age of 30 and under, and all future entrants, must join. The benefits are to be £20 a year to the widow, commencing from the moment of death of her husband, and, in the event of the early death of the widow, a continuation of the annuity until the youngest child reaches the age of 16. In the event of a widower dying leaving children, an annuity of £20 a year is to be paid until the youngest reaches the age of 16. On the death of a bachelor, the sum of £25 is to be paid to his relations. Each member of the fund is to contribute a percentage of his salary and pension until his death. The scale of pension is the Government scale, namely, $\frac{1}{60}$ th of last salary for every year of service not exceeding 40, and retirement at 65 is compulsory. The firm will pay a sum down to start the fund, and will guarantee 4 per-cent interest on the investments, free of income tax. There are to be no expenses connected with the management of the fund. No one over the age of 30 is to be allowed to join except on payment of a fine commensurate with the risk. No second marriage is to be allowed except on payment of a fine commensurate with the risk, and no marriage after 65 is to be recognized. A widow on re-marriage is to forfeit her annuity.

What percentage of salary and pension should each member contribute to the fund, and what sum should the firm pay down to start the fund?

As our Tables do not provide for second marriages, nor for the forfeiture of the widows' pension on re-marriage, we must leave them out of our present calculations.

To find the proper contribution, it is necessary to determine the average age on entrance to the staff. This will vary according to the nature of the business, but in our imaginary staff I think 18 would be the proper age to take. It is not desirable to take too young an age, because you may be fixing the scale too low and endangering the Fund, while if you take an age higher than the average, you will have a margin to keep the Fund in safety.

On the benefit side we have

$$\begin{aligned}
 \left. \begin{array}{l} \text{Present value of widow's annuity} \\ \text{of £20} \end{array} \right\} &= \frac{{}^w a M_{1s}}{D_{1s}^{(3)}} \times 20 \\
 \left. \begin{array}{l} \text{Present value of continuation of} \\ \text{annuity till youngest child is 16} \end{array} \right\} &= \frac{{}^E(16) M_{1s}}{D_{1s}^{(3)}} \times 20 \\
 \left. \begin{array}{l} \text{Present value of annuity of £20 to} \\ \text{children of widower until} \\ \text{youngest child is 16} \end{array} \right\} &= \frac{{}^K.vca(16) M_{1s}}{D_{1s}^{(3)}} \times 20 \\
 \left. \begin{array}{l} \text{Present value of £25 on death of} \\ \text{bachelor} \end{array} \right\} &= \frac{{}^E d M_{1s}}{D_{1s}^{(3)}} \times 25
 \end{aligned}$$

On the payment side we have

Present value of contribution during whole period of active service

$$= \frac{P \left\{ {}^N s_{1s} - \frac{1}{2} ({}^d M_{1s} + {}^w M_{1s} + {}^r M_{1s}) \right\} (1 + \frac{2}{3} i)}{D_{1s}^{(4)}}$$

Present value of contribution from those who retire on pension at age 65 (see Problem XIV B, *J.I.A.*, vol. xxxvii, p. 204)

$$= P \cdot \frac{{}^2 s_{64} (N_{65} + \frac{1}{2} D_{65}) v^{\frac{5}{3}}}{D_{1s}^{(4)}}.$$

Present value of contributions from those who retire on pension before the age of 65

$$= P \cdot \frac{\frac{1}{60} ({}^r a R_{1s} - {}^r a R_{5s} - \frac{1}{2} {}^r a M_{1s}) (1 + \frac{2}{3} i)}{D_{1s}^{(4)}}.$$

As $D_{1s}^{(3)}$ is the same as $D_{1s}^{(4)}$, we shall find, by collecting all the benefits together, and all the payments together, that

$$\begin{aligned}
 P = & \frac{({}^w a M_{1s} \times 20) + ({}^E(16) M_{1s} \times 20) + ({}^K.vca(16) M_{1s} \times 20) + ({}^E d M_{1s} \times 25)}{\left\{ {}^N s_{1s} - \frac{1}{2} ({}^d M_{1s} + {}^w M_{1s} + {}^r M_{1s}) \right\} (1 + \frac{2}{3} i)} \\
 & + \frac{{}^2 s_{64} (N_{65} + \frac{1}{2} D_{65}) v^{\frac{5}{3}} + \frac{1}{60} ({}^r a R_{1s} - {}^r a R_{5s} - \frac{1}{2} {}^r a M_{1s}) (1 + \frac{2}{3} i)}{D_{1s}^{(4)}},
 \end{aligned}$$

which produces .0150, or $1\frac{1}{2}$ per-cent.

To make a valuation in order to find what the reserve should be, or in other words to find the amount which the firm should pay down to start the fund, we must have a valuation schedule.

The most difficult part of this Problem is to find the present value of the future salaries and pensions on which we have to calculate the value of the contributions, so we will consider the contributions first.

From Problem VIIIc we find the value of future salary and pension (allowing for no more than 40 years' service to count for pension) to be :

Present salary multiplied by

$$\begin{aligned} & \{ \mathbb{N}_x^s - \frac{1}{2} ({}^d\mathbb{M}_x^{ls} + {}^w\mathbb{M}_x^{ls} + {}^r\mathbb{M}_x^{ls}) \} (1 + \frac{3}{8}i) \\ & + \frac{1}{6v} \left(\begin{array}{c} 65+n-x \\ \text{not exceeding } 40 \end{array} \right) {}^{s_{64}}(\mathbb{N}_{65} + \frac{1}{2}\mathbb{D}_{65}) v^{\frac{5}{8}} \\ & + \frac{1}{6v} (n {}^ra\mathbb{M}_x^{ls} + {}^ra\mathbb{R}_x^{ls} - {}^ra\mathbb{R}_{x+40-n}^{ls} - \frac{1}{2} {}^ra\mathbb{M}_x^{ls}) (1 + \frac{3}{8}i) \end{aligned}$$

and all divided by \mathbb{D}_x^s .

The last two terms contain values based on past service ; but the second term will not trouble us now, because in all cases in our supposed Fund, $65+n-x$ will exceed 40.

We will proceed then, first, to obtain the details for valuing the pensions on early retirement.

PARTICULARS OF FUND			PART OF VALUATION SCHEDULE		
Number of Members	Number of years service = n	Annual salary receivable by all the members in col. (1)	Salary \times past years of service (2) \times (3)	$\frac{{}^ra\mathbb{R}_{x+40-n}^{ls}}{\mathbb{D}_x^s}$	Col. (3) \times Col. (5)
(1)	(2)	(3)	(4)	(5)	(6)
Present Age 20					
10	5	450	2,250	15.974	7,188.3
20	4	900	3,600	14.305	12,874.5
10	3	450	1,350	12.549	5,647.1
5	2	200	400	10.776	2,155.2
5	1	250	250	8.984	2,246.0
50	...	2,250	7,850	...	30,111.1
Present Age 30					
5	15	375	5,625	15.974	5,990.3
5	14	375	5,250	14.305	5,364.4
15	13	1,310	17,030	12.549	16,439.2
15	12	1,275	15,300	10.776	13,739.4
5	11	460	5,060	8.984	4,132.6
4	10	600	6,000	7.178	4,306.8
1	8	105	840	3.677	386.1
50	...	4,500	55,105	...	50,358.8

All the rest of the work, except when $(65 + n - x)$ is less than 40, is based upon totals for each present age.

VALUATION SCHEDULE—(Continued)

Present age	Total annual salary	Total salary × past years of service	$\left\{ \frac{N_x^s}{D_x^s} - \frac{1}{2} \frac{M_x^{ls}}{D_x^s} + \frac{wM_x^{ls}}{D_x^s} + \frac{rM_x^{ls}}{D_x^s} \right\} \div D_s$	$(8) \times (10)$	$\frac{S_{64}}{S_x}$	$\frac{N_{65} + \frac{1}{2} D_{65}}{D_x}$	$(8) \times (12) \times (13)$
(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
20	2,250	7,850	24.035	54,078.8	5.02	.193	2179.9
30	4,500	55,105	19.427	87,421.5	2.51	.487	5500.7
Totals	6,750	141,500.3	7680.6

VALUATION SCHEDULE—(Continued)

Present age	$\frac{raM_x^{ls}}{D_x^s}$	$(9) \times (15)$	$\frac{raR_x^{ls}}{D_x^s} - \frac{1}{2} \frac{raM_x^{ls}}{D_x^s}$	$(8) \times (17)$	Number of Members on Fund	$\frac{waM_x}{D_x}$
	(15)	(16)	(17)	(18)	(19)	(20)
20	1.136	8,917.6	42.039	94,587.8	50	.809
30	1.430	78,800.1	38.711	174,199.5	50	1.940
Totals	...	87,717.7	...	268,787.3	100	...

VALUATION SCHEDULE—(Continued)

$(19) \times (20)$	$\frac{E^{(16)} M_x}{D_x}$	$(19) \times (22)$	$\frac{\times VCa^{(16)} M_x}{D_x}$	$(19) \times (24)$	$\frac{B^d M_x}{D_x}$	$(19) \times (26)$
(21)	(22)	(23)	(24)	(25)	(26)	(27)
40.45	.021	1.05	.027	1.350	.053	2.65
97.00	.048	2.40	.067	3.350	.066	3.30
137.45	...	3.45	...	4.700	...	5.95

Valuation Balance Sheet.

LIABILITIES.

By present value of annuity to widows of £20, col. (21) \times 20 . . .	=	2,749.0
„ „ „ continuation of annuity after death of widow until youngest child is 16, col. (23) \times 20 . . .	=	69.0
„ „ „ £20 to children of widowers until youngest is 16, col. (25) \times 20 . . .	=	94.0
„ „ „ £25 on death of bachelors, col. (27) \times 25 . . .	=	148.8
		<u>3,060.8</u>

ASSETS.

To present value of salary during active service, col. (11)	141,500.3	
Increase by $\frac{3}{8}i = 1\frac{1}{2}\%$	2,122.5	
	<u>143,622.8</u>	
To present value of pension after 65, col. (14) $\times \frac{2}{3} =$	5120.4	
multiplied by $v^{\frac{5}{6}} =$ decreased by $2\frac{1}{2}\%$ nearly	128.0	
	<u>4,992.4</u>	
To present value of pension on early retirement:		
col. (16) =	87,717.7	
col. (18) =	268,787.3	
	<u>356,505.0</u>	
deduct, col. (6) =	30,111.1	
	<u>50,358.8</u>	
	80,469.9	
	<u>276,035.1</u>	
increased by $1\frac{1}{2}\%$	4,140.5	
	<u>280,175.6</u>	
	$\div 60 = 4,669.6$	
	<u>153,284.8</u>	
$1\frac{1}{2}\%$ of £153,284.8 =		2,299.3
Deficit		761.5
		<u>£3,060.8</u>

The sum which the firm should pay down to start the Fund is £761. 10s.

Problem XIIc.—What is the present value, in respect of a person aged x at entrance, of an annuity to a widow based on the husband's pension, or the pension he would have been entitled to at the date of his death? And what would be the value of a continuation of the same annuity to the children of a widow or widower until the youngest reaches the age of (n)?

Here the question of retirement, and the mortality amongst those who retire, form important elements in the Problem; consequently we require to use Table 4, where those who retire early are separated from the others.

It will be necessary to divide the Problem into three parts, and investigate separately the values of the benefits in respect of

- (1) Those who die while in the service.
- (2) Those who retire before the pension age.
- (3) Those who retire at the pension age.

For 1, the d 's in Table 4 must be divided into Bd_x , Hd_x and Kd_x ; and for 2, the d 's in Table 8 must be divided into $Bd_x^{(r)}$, $Hd_x^{(r)}$ and $Kd_x^{(r)}$, and extended to the end of life. This can stop at age 64 if the values of ${}^{wa}M_x$, &c., by Table 3 have already been calculated.

Now we know the value of the widow's annuity on the death of her husband, namely, $a_y + \frac{1}{2}$, but we want also to know the value of the widow's annuity per retired person *at the date of retirement*. It is necessary, therefore, to ascertain the values of $\frac{{}^{wa}C_x}{D_x}$ and $\frac{{}^{wa}M_x}{D_x}$ according to Table 8; and, for the children's annuities, to find the values of $\frac{{}^{E(n)}M_x}{D_x}$ and $\frac{{}^{K.YC(n)}M_x}{D_x}$ also by the same Table.

Using the index (r) to distinguish the invalid pensioners Mortality Table, we may call

$\frac{{}^{wa}M_x^{(r)}}{D_x^{(r)}} = {}^{wa}a_x^{(r)}$ = the value, per retired person aged x , of a widow's annuity of 1.

$\frac{{}^{E(n)}M_x^{(r)}}{D_x^{(r)}} = {}^{E(n)}a_x^{(r)}$ = the value, per retired person aged x , of the addition to provide for continuation of the widow's annuity until the youngest child is (n) years of age.

$\frac{{}^{K.YC(n)}M_x^{(r)}}{D_x^{(r)}} = {}^{K.YC(n)}a_x^{(r)}$ = the value, per retired person aged x , of an annuity of 1 to the children of a widower until the youngest reaches the age of (n) .

When these Tables are constructed we shall be in a position to proceed with the investigation of the Problem; but we must first know the principle on which the pensions are calculated. Let us, in the first place, take the case where the pension is based on the number of years' service of the husband, and the average salary he received.

Following the same reasoning as in my Problem XB (*J.I.A.*, vol. xxxvi, p. 238) we proceed thus:

Out of l_x persons living at the age x , in receipt of an average

salary of s each, the liability for widows' annuities, equal to the total amount of the salaries received by their husbands, would be—

In respect of Hd_x persons dying— $Hd_x \times s_x \times (a_y + \frac{1}{2})$	And in respect of r_x persons retiring— $r_x \times s_x \times Wa_x^{(r)}$
In respect of Hd_{x+1} persons dying— $Hd_{x+1}(s_x + s_{x+1})(a_{y+1} + \frac{1}{2})$	And in respect of r_{x+1} persons retiring— $r_{x+1}(s_x + s_{x+1})Wa_{x+1}^{(r)}$
In respect of Hd_{x+2} persons dying— $Hd_{x+2}(s_x + s_{x+1} + s_{x+2})(a_{y+2} + \frac{1}{2})$	And in respect of r_{x+2} persons retiring— $r_{x+2}(s_x + s_{x+1} + s_{x+2})Wa_{x+2}^{(r)}$
\vdots	\vdots
In respect of Hd_{64} persons dying— $Hd_{64} \cdot \sum_x^{64} s \cdot (a_{y+64-x} + \frac{1}{2})$	And in respect of r_{64} persons retiring— $r_{64} \cdot \sum_x^{64} s \cdot Wa_{64}^{(r)}$

In respect of l_{65} persons retiring at the pension age $l_{65} \sum_x^{64} s \cdot Wa_{65}^{(r)}$.

The present value of all the annuities equated to 1 of salary at age x will be

$$\begin{aligned} & \left[\{ Hd_x \times (a_y + \frac{1}{2}) + r_x \cdot Wa_x^{(r)} \} s_x \cdot v^{x+1} + \{ Hd_{x+1} \times (a_{y+1} + \frac{1}{2}) \right. \\ & \quad \left. + r_{x+1} \cdot Wa_{x+1}^{(r)} \} (s_x + s_{x+1}) v^{x+2} + \dots + \{ Hd_{64} \times (a_{y+64-x} + \frac{1}{2}) \right. \\ & \quad \left. + r_{64} \cdot Wa_{64}^{(r)} \} \sum_x^{64} s \cdot v^{65} \right] + l_{65} \cdot \sum_x^{64} s \cdot Wa_{65}^{(r)} \cdot v^{65} \end{aligned}$$

divided by $l_x s_x v^x$.

Separating the symbols inside the large bracket, we have

$$\begin{aligned} (Hd_x(a_y + \frac{1}{2}) + r_x \cdot Wa_x^{(r)}) s_x \cdot v^{x+1} &= (Hd_x \cdot (a_y + \frac{1}{2}) + r_x Wa_x^{(r)}) s_x \cdot v^{x+1} \\ (Hd_{x+1} \cdot (a_{y+1} + \frac{1}{2}) + r_{x+1} \cdot Wa_{x+1}^{(r)}) (s_x + s_{x+1}) v^{x+2} \\ &= (Hd_{x+1} \cdot (a_{y+1} + \frac{1}{2}) + r_{x+1} \cdot Wa_{x+1}^{(r)}) s_x \cdot v^{x+2} \\ & \quad + (Hd_{x+1} \cdot (a_{y+1} + \frac{1}{2}) + r_{x+1} \cdot Wa_{x+1}^{(r)}) s_{x+1} v^{x+2} \\ & \quad \vdots \qquad \qquad \qquad \vdots \qquad \qquad \qquad \vdots \end{aligned}$$

If now we first form Tables of $Hd_x(a_y + \frac{1}{2})$ and $r_x \cdot Wa_x^{(r)}$, then add the two values together, forming a Table of $\{ Hd_x(a_y + \frac{1}{2}) + r_x \cdot Wa_x^{(r)} \}$, and multiply the value by v^{x+1} , thus forming a Table of $\{ Hd_x(a_y + \frac{1}{2}) + r_x \cdot Wa_x^{(r)} \} v^{x+1}$, which we will call ${}^{w.ra}C_x$; and sum these values, so that

$${}^{w.ra}C_x + {}^{w.ra}C_{x+1} + {}^{w.ra}C_{x+2} + \dots + {}^{w.ra}C_{64} = {}^{w.ra}M_x;$$

then multiply ${}^{w.ra}M_x$ by s_x making ${}^{w.ra}M_x^s$, and then sum these values so that

$${}^{w.ra}R_x^s = {}^{w.ra}M_x^s + {}^{w.ra}M_{x+1}^s + \dots$$

we can represent the present value of an annuity to the

widow, of her husband's pension based on average salary and number of years' service equated to 1 of salary, in respect of a person aged x at entrance, and dying or retiring before the pension age, by the convenient symbol

$$\frac{{}^{w.r.a}R_x^s}{D_x^s}.$$

Adding to this the value of the widow's pension in respect of the l_{65} persons who retire at the pension age,

$$= \frac{l_{65} v^{65} \cdot \sum_x {}^{64}s \cdot {}^{w.a}_{65}}{l_x s_x v^x} = \frac{D_{65}^s \cdot \sum_x {}^{64}s \cdot {}^{w.a}_{65}}{D_x^s}$$

we have for the complete value of the widow's annuity, per person aged x at entrance,

$$\frac{{}^{w.r.a}R_x^s + D_{65}^s \cdot \sum_x {}^{64}s \cdot {}^{w.a}_{65}}{D_x^s}.$$

The value ${}^{w.r.a}R_x^s$, however, requires adjustment, because it has been assumed that the full year's salary in the year of retirement was paid, and the whole has been over-discounted by half a year. The proper adjustment is to deduct $\frac{1}{2} {}^{w.r.a}M_x^{ls}$ (which will be found later), and increase the difference by half a year's interest. The corrected value will therefore be

$$\frac{(1 + \frac{1}{2}i)({}^{w.r.a}R_x^s - \frac{1}{2} {}^{w.r.a}M_x^{ls}) + D_{65}^s \cdot \sum_x {}^{64}s \cdot {}^{w.a}_{65}}{D_x^s}.$$

For the continuation of the annuity for the benefit of the children of a widow or widower until the youngest child attains the age of n ; we shall have (equating our values to a denominator of $l_x s_x v^x$)

In respect of Hd_x persons dying as husbands at age x ,

$$Hd_x \times E(n) \times s_x \times v^{x+1}$$

In respect of Kd_x persons dying as widowers at age x ,

$$Kd_x \times YCa(n) \times s_x \times v^{x+1}$$

In respect of r_x persons who retire at age x

$$(r_x \times E(n)_x^{(r)} + r_x \times K.YCa(n)_x^{(r)})(s_x \times v^{x+1}).$$

By separating the symbols as before, it will be found that a Table would have first to be constructed of

$$\left\{ (Hd_x \times E(n) + (Kd \times YCa(n))) + (r_x \times E(n)_x^{(r)} + (r_x \times K.YCa(n)_x^{(r)})) \right\} v^{x+1}$$

which could be called ${}^{vc(n).ra}C_x$.

Then these values of ${}^{vc(n).ra}C_x$ would have to be summed to form a Table which might be called ${}^{vc(n).ra}M_x$. The values in this Table would have to be multiplied by s_x , forming a Table of ${}^{vc(n).ra}M_x^s$, and these new values summed again producing a Table of ${}^{vc(n).ra}R_x^s$.

In respect of those who retire at the pension age 65, the value equated to a denominator of $l_x s_x v^x$ will be $l_{65} \times \sum_x^{64} s(E(n)_{65} + K.YCa(n)_{65}) \cdot v^{65}$, and the complete value for the continuation of the annuity will be

$$\frac{{}^{vc(n).ra}R_x^s + D_{65} \cdot \sum_x^{64} s(E(n)_{65} + K.YCa(n)_{65})}{D_x^s}.$$

The value ${}^{vc(n).ra}R_x^s$ is, however, too large by reason of our assuming that the last year's salary is paid in full, and by the whole being over-discounted by half a year. The correct value would therefore be

$$\frac{(1 + \frac{1}{2}i)({}^{vc(n).ra}R_x^s - \frac{1}{2} \cdot {}^{vc(n).ra}M_x^{ls}) + D_{65} \cdot \sum_x^{64} s(E(n)_{65} + K.YCa(n)_{65})}{D_x^s}$$

Secondly, let us assume that the pension is based on last salary and number of years' service. The same process as before will have to be followed, up to the construction of the Table of values of ${}^{w.ra}C_x$, when each value would have to be multiplied by s_x , and the resulting values then summed, so that

$${}^{w.ra}C_x s_x + {}^{w.ra}C_{x+1} s_{x+1} + {}^{w.ra}C_{x+2} s_{x+2} + \dots = {}^{w.ra}M_x^{ls}.$$

These values would have to be summed again, so that

$${}^{w.ra}M_x^{ls} + {}^{w.ra}M_{x+1}^{ls} + {}^{w.ra}M_{x+2}^{ls} + \dots = {}^{w.ra}R_x^{ls}.$$

The value of the widow's pension, in respect of the l_{65} persons who retire at the pension age 65, will be

$$\frac{1}{l_x s_x v^x} \cdot l_{65} v^{65} \cdot (65 - x) s_{64} \times W a_{65} \},$$

and the value of the widow's annuity per person aged x at entrance will be

$$\frac{{}^{w.ra}R_x^{ls} + D_{65}(65 - x) s_{64} \times W a_{65}}{D_x^s},$$

or, using the proper adjustments,

$$\frac{(1 + \frac{1}{2}i)({}^{w.ra}R_x^{ls} - \frac{1}{2} {}^{w.ra}M_x^{ls}) + D_{65}(65 - x) s_{64} \times W a_{65}}{D_x^s}.$$

For the continuation of the annuity for the benefit of the children of a widow or widower until the youngest arrived at the age of (n) we follow the same process as above up to the construction of the Table of ${}^{vc(n).ra}C_x$. These values would have to be multiplied by s_x , and then summed, so that

$${}^{vc(n).ra}C_x s_x + {}^{vc(n).ra}C_{x+1} s_{x+1} + \dots = {}^{vc(n).ra}M_x^{ls}.$$

Summing these values in like manner, we shall have

$${}^{vc(n).ra}M_x^{ls} + {}^{vc(n).ra}M_{x+1}^{ls} + \dots = {}^{vc(n).ra}R_x^{ls},$$

and the value, in respect of the l_{65} persons who retire at the pension age 65, will be

$$\frac{1}{l_x s_x v^x} \{ l_{65} v^{65} \cdot (65 - x) s_{64} \cdot (E(n)_{65} + K \cdot YCa(n)_{65}) \},$$

so that the value of the addition to the value of the widow's annuity (making the usual correction) will be

$$\frac{(1 + \frac{1}{2}i)({}^{vc(n).ra}R_x^{ls} - \frac{1}{2}{}^{vc(n).ra}M_x^{ls}) + D_{65}(65 - x) s_{64} (E(n)_{65} + K \cdot YCa(n)_{65})}{D_x^s}$$

By means of Problems XIII_B, XIV_B and XV_B, it would be a simple matter to apply the above formulas to the usual modes of calculating pensions.

I have not attempted to make any Tables for this kind of pension to widows and children; because I have not found the plan to be in practical use with either of these modes of calculating pensions; but, from signs which I have observed, I should not be surprised to hear of an attempt to put this plan into operation. The younger members might do worse than exercise their powers on the construction of Tables for this kind of benefit.

Problem XIIIc.—To ascertain the fine payable on second marriage.

One of the difficult problems connected with these Funds is to determine what fine should be paid on re-marriage. All the Tables are based on first marriage only; and if the contributions are based on such Tables, without any addition for the contingencies of second and third and subsequent marriages, then the widower who marries again is bringing on to the Fund a liability which he is not paying for. He undertook to make a contribution for the whole of life, and the Fund undertook the risk of making a payment if he died a bachelor, the risk of paying

an annuity to his widow by the first marriage, the risk of paying an annuity to his children, and the risk of paying a sum on his death as a widower. The risk of his dying a bachelor ceased when he married, the risk of paying an annuity to his widow by the first marriage ceased when his wife died, the risk of paying an annuity to his children may or may not have ceased, and the risk of paying a sum at his death is still in existence. He now wants to bring on to the Fund the risks attaching to a second marriage, that is, the risk of paying an annuity to his widow by the second marriage, and the risk of paying an annuity to his children by the second marriage; but by doing so he is reducing the risks left by his original contract, for the value of the annuity to his children by the first wife will be smaller, and the risk of his dying a widower without children under a certain age will be less.

If we assume that the value of the reduction in the old risks will cover the value of the annuity to the children of the new marriage, there remains the additional risk of the annuity to the widow, which is $a_y - a_{xy}$, and that is clearly the fine which should be paid.

The cost, however, is prohibitive; and you will never persuade a man that it is right and just that he should go on paying all his life for other people's widows, and not be allowed to leave his own widow on the Fund, even if it is by a second marriage, without paying an additional sum equal to the full value of the risk.

In all the Funds I have examined fine on second marriage is quite nominal compared with the correct amount; but then the contributions and the values of the risks have generally been calculated with an excess of caution which has justified the imposition of only a small fine.

If, however, we are going to calculate the values of the contributions more closely, we shall have to calculate the fines more closely, or find some way of making them merely nominal, or doing without them altogether. We can only do this by including the risk of second and subsequent marriages in the original calculations. But how?

The difficulty is considerably increased when the contribution is a fixed percentage of salary and pension.

Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.

TABLE 44.

Simple Commutation Columns (according to Tables 3 and 45).

4 PER-CENT				3 PER-CENT			
Age x	$D_x^{(3)}$	Age x	$D_x^{(3)}$	Age x	$D_x^{(3)}$	Age x	$D_x^{(3)}$
15	11105	60	343.93	15	12837	60	614.10
16	9839.0	61	319.28	16	11484	61	575.61
17	8581.4	62	295.57	17	10113	62	538.05
18	7469.1	63	272.79	18	8887.9	63	501.40
19	6552.1	64	251.01	19	7872.3	64	465.84
20	5793.8	65	230.41	20	7028.9	65	431.77
21	5158.9	66	211.11	21	6319.5	66	399.44
22	4627.2	67	192.95	22	5723.1	67	368.62
23	4176.1	68	175.63	23	5215.3	68	338.79
24	3789.7	69	159.14	24	4778.7	69	309.97
25	3456.0	70	143.49	25	4400.2	70	282.20
26	3164.7	71	128.69	26	4068.5	71	255.55
27	2908.0	72	114.74	27	3774.8	72	230.06
28	2680.8	73	101.66	28	3513.7	73	205.81
29	2478.6	74	89.462	29	3280.2	74	182.88
30	2297.3	75	78.141	30	3069.7	75	161.28
31	2134.2	76	67.716	31	2879.5	76	141.12
32	1986.3	77	58.186	32	2706.0	77	122.44
33	1852.3	78	49.538	33	2547.9	78	105.25
34	1730.0	79	41.768	34	2402.7	79	89.605
35	1618.0	80	34.851	35	2269.1	80	75.492
36	1515.1	81	28.755	36	2145.4	81	62.891
37	1420.6	82	23.449	37	2031.0	82	51.786
38	1333.2	83	18.883	38	1924.7	83	42.107
39	1252.5	84	15.005	39	1825.7	84	33.783
40	1177.5	85	11.757	40	1733.0	85	26.728
41	1107.5	86	9.076	41	1645.9	86	20.833
42	1042.6	87	6.897	42	1564.4	87	15.985
43	981.59	88	5.155	43	1487.2	88	12.063
44	924.76	89	3.789	44	1414.7	89	8.953
45	871.40	90	2.732	45	1346.0	90	6.517
46	821.26	91	1.933	46	1280.9	91	4.657
47	774.16	92	1.341	47	1219.1	92	3.263
48	729.93	93	.909	48	1160.6	93	2.233
49	688.10	94	.604	49	1104.7	94	1.497
50	648.54	95	.393	50	1051.4	95	.983
51	611.29	96	.248	51	1000.6	96	.627
52	576.21	97	.154	52	952.29	97	.392
53	542.90	98	.092	53	905.98	98	.237
54	511.32	99	.054	54	861.55	99	.139
55	481.02	100	.030	55	818.35	100	.078
56	451.72	56	776.00
57	423.02	57	733.74
58	395.95	58	693.44
59	369.45	59	653.31

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 45.

Showing the number living and remaining on the staff at each age according to Table 3, and the numbers who die at each age as Bachelors, Husbands, and Widowers; no second marriage being allowed for.

Age	Living and remaining on Staff	Dying	Dying Bachelors	Dying Married	Dying Husbands	Dying Widowers	Age
x	$l_x^{(3)}$	$d_x^{(3)}$	Bd_x	Md_x	Hd_x	κd_x	x
15	20,000	72	72	15
16	18,428	68	68	16
17	16,716	64	64	17
18	15,131	59	59	18
19	13,804	55	55	19
20	12,695	52	51.9	.1	.1	...	20
21	11,756	49	48.7	.3	.3	...	21
22	10,966	47	46.2	.8	.8	...	22
23	10,293	45	43.4	1.6	1.6	...	23
24	9,714	43	40.5	2.5	2.5	...	24
25	9,213	42	38.3	3.7	3.7	...	25
26	8,774	42	37.0	5.0	4.9	.1	26
27	8,385	41	34.8	6.2	6.1	.1	27
28	8,039	40	32.5	7.5	7.4	.1	28
29	7,730	40	30.0	10.0	9.8	.2	29
30	7,451	40	28.0	12.0	11.8	.2	30
31	7,199	41	26.6	14.4	14.1	.3	31
32	6,968	41	24.6	16.4	16.0	.4	32
33	6,758	41	22.7	18.3	17.8	.5	33
34	6,564	42	21.2	20.8	20.2	.6	34
35	6,385	43	19.7	23.3	22.6	.7	35
36	6,218	43	18.0	25.0	24.2	.8	36
37	6,063	45	17.4	27.6	26.7	.9	37
38	5,918	46	16.5	29.5	28.4	1.1	38
39	5,782	47	16.1	30.9	29.7	1.2	39
40	5,653	49	16.1	32.9	31.5	1.4	40
41	5,530	50	16.0	34.0	32.4	1.6	41
42	5,414	53	16.4	36.6	34.8	1.8	42
43	5,301	54	16.2	37.8	35.9	1.9	43
44	5,194	57	16.5	40.5	38.3	2.2	44
45	5,090	60	16.9	43.1	40.6	2.5	45
46	4,989	63	17.4	45.6	42.8	2.8	46
47	4,891	65	17.7	47.3	44.2	3.1	47
48	4,796	69	18.6	50.4	46.8	3.6	48
49	4,702	73	19.4	53.6	49.5	4.1	49
50	4,609	76	19.8	56.2	51.6	4.6	50
51	4,518	79	20.2	58.8	53.7	5.1	51
52	4,429	83	20.8	62.2	56.5	5.7	52
53	4,340	87	21.4	65.6	59.2	6.4	53
54	4,251	92	22.4	69.6	62.4	7.2	54
55	4,159	97	23.2	73.8	65.6	8.2	55
56	4,062	103	24.3	78.7	69.3	9.4	56
57	3,956	108	25.1	82.9	72.4	10.5	57
58	3,851	114	25.8	88.2	76.4	11.8	58
59	3,737	119	26.4	92.6	79.5	13.1	59

Hypothetical Experience of Staff Pension Fund for Widows and Orphans.

TABLE 45—(continued).

Showing the number living and remaining on the staff at each age according to Table 3, and the numbers who die at each age as Bachelors, Husbands, and Widowers; no second marriage being allowed for.

Age	Living and remaining on Staff	Dying	Dying Bachelors	Dying Married	Dying Husbands	Dying Widowers	Age
x	$l_x^{(3)}$	$d_x^{(3)}$	Bd_x	Md_x	Hd_x	Kd_x	x
60	3618.0	125.0	26.8	98.2	83.3	14.9	60
61	3493.0	130.0	26.9	103.1	86.4	16.7	61
62	3363.0	135.0	26.9	108.1	89.3	18.8	62
63	3228.0	139.0	26.8	112.2	91.2	21.0	63
64	3089.0	140.0	26.2	113.8	91.0	22.8	64
65	2949.0	139.0	25.5	113.5	89.0	24.5	65
66	2810.0	139.0	25.1	113.9	87.4	26.5	66
67	2671.0	142.5	25.4	117.1	87.8	29.3	67
68	2528.5	145.7	25.9	119.8	87.6	32.2	68
69	2382.8	148.4	26.4	122.0	86.9	35.1	69
70	2234.4	150.3	26.8	123.5	85.5	38.0	70
71	2084.1	151.6	27.1	124.5	83.6	40.9	71
72	1932.5	151.8	27.2	124.6	81.0	43.6	72
73	1780.7	151.0	27.1	123.9	77.8	46.1	73
74	1629.7	149.3	26.8	122.5	74.1	48.4	74
75	1480.4	146.2	26.4	119.8	69.6	50.2	75
76	1334.2	141.9	25.9	116.0	64.5	51.5	76
77	1192.3	136.6	25.4	111.2	59.0	52.2	77
78	1055.7	130.0	24.8	105.2	53.0	52.2	78
79	925.7	122.4	23.7	98.7	47.1	51.6	79
80	803.3	114.0	22.2	91.8	41.2	50.6	80
81	689.3	104.7	21.4	83.3	35.0	48.3	81
82	584.6	95.0	18.8	76.2	29.9	46.3	82
83	489.6	85.0	16.4	68.6	24.9	43.7	83
84	404.6	74.9	14.2	60.7	20.3	40.4	84
85	329.7	65.0	12.0	53.0	16.2	36.8	85
86	264.7	55.5	10.1	45.4	12.6	32.8	86
87	209.2	46.6	7.8	38.8	9.7	29.1	87
88	162.6	38.3	6.4	31.9	7.1	24.8	88
89	124.3	31.1	5.2	25.9	5.1	20.8	89
90	93.2	24.6	4.1	20.5	3.5	17.0	90
91	68.6	19.1	3.2	15.9	2.4	13.5	91
92	49.5	14.6	2.4	12.2	1.5	10.7	92
93	34.9	10.8	1.8	9.0	1.0	8.0	93
94	24.1	7.8	1.3	6.5	.6	5.9	94
95	16.3	5.6	.9	4.7	.4	4.3	95
96	10.7	3.8	.6	3.2	.2	3.0	96
97	6.9	2.6	.4	2.2	.1	2.1	97
98	4.3	1.7	.3	1.4	...	1.4	98
99	2.6	1.1	.2	.99	99
100	1.5	.6	.1	.55	100
101	.9	.4	.1	.33	101
102	.5	.222	102
103	.3	.222	103
104	.1	.111	104
105	105

Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.

TABLE 46.

*Value, on the Death of a Husband, of a continuous Annuity of 1 to
the Widow for the remainder of her life.*

Age of Husband at Death	Age of Widow at Death of Husband	VALUE OF $a_y + \frac{1}{2}$		Age of Husband at Death
x	y	4 per-cent	3 per-cent	x
20	22.0	18.311	21.547	20
21	22.4	18.278	21.496	21
22	22.8	18.245	21.445	22
23	23.2	18.211	21.394	23
24	23.6	18.177	21.341	24
25	24.0	18.143	21.288	25
26	24.8	18.072	21.178	26
27	25.6	17.998	21.065	27
28	26.4	17.922	20.950	28
29	27.2	17.844	20.832	29
30	28.0	17.764	20.712	30
31	28.7	17.690	20.602	31
32	29.4	17.614	20.490	32
33	30.1	17.537	20.376	33
34	30.8	17.457	20.259	34
35	31.5	17.374	20.139	35
36	32.1	17.302	20.034	36
37	32.7	17.227	19.925	37
38	33.3	17.151	19.815	38
39	33.9	17.073	19.704	39
40	34.5	16.992	19.589	40
41	35.2	16.897	19.453	41
42	35.9	16.799	19.315	42
43	36.6	16.697	19.173	43
44	37.3	16.593	19.028	44
45	38.0	16.487	18.880	45
46	38.8	16.361	18.706	46
47	39.6	16.231	18.527	47
48	40.4	16.097	18.344	48
49	41.2	15.959	18.157	49
50	42.0	15.818	17.967	50
51	42.9	15.655	17.747	51
52	43.8	15.486	17.522	52
53	44.7	15.311	17.291	53
54	45.6	15.131	17.053	54
55	46.5	14.944	16.810	55
56	47.4	14.751	16.559	56
57	48.3	14.552	16.301	57
58	49.2	14.345	16.036	58
59	50.1	14.131	15.765	59

Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.

TABLE 46—(continued).

*Value, on the Death of a Husband, of a continuous Annuity of 1 to
the Widow for the remainder of her life.*

Age of Husband at Death	Age of Widow at Death of Husband	VALUE OF $a_y + \frac{1}{2}$		Age of Husband at Death
x	y	4 per-cent	3 per-cent	x
60	51·0	13·911	15·488	60
61	51·9	13·687	15·205	61
62	52·8	13·457	14·919	62
63	53·7	13·224	14·628	63
64	54·6	12·986	14·334	64
65	55·5	12·745	14·037	65
66	56·5	12·471	13·702	66
67	57·5	12·191	13·362	67
68	58·5	11·904	13·015	68
69	59·5	11·609	12·662	69
70	60·5	11·305	12·300	70
71	61·5	10·993	11·929	71
72	62·5	10·671	11·551	72
73	63·5	10·339	11·164	73
74	64·5	9·998	10·769	74
75	65·5	9·648	10·366	75
76	66·5	9·290	9·957	76
77	67·5	8·928	9·545	77
78	68·5	8·572	9·143	78
79	69·5	8·233	8·760	79
80	70·5	7·908	8·395	80
81	71·5	7·594	8·044	81
82	72·5	7·291	7·705	82
83	73·5	6·996	7·376	83
84	74·5	6·705	7·054	84
85	75·5	6·418	6·736	85
86	76·5	6·130	6·419	86
87	77·5	5·843	6·104	87
88	78·5	5·556	5·791	88
89	79·5	5·267	5·478	89
90	80·5	4·977	5·163	90
91	81·5	4·679	4·842	91
92	82·5	4·370	4·513	92
93	83·5	4·049	4·171	93
94	84·5	3·718	3·821	94
95	85·5	3·388	3·473	95
96	86·5	3·060	3·129	96
97	87·5	2·734	2·790	97

**Hypothetical Experience of Staff Pension Fund
for Widowers and Orphans.**

TABLE 47.

Present value, on the death of a Married Man, of a continuous Annuity of 1 until the youngest surviving child attains the age of 21, 16, or 14.

Age of Widower at Death	4 PER-CENT			3 PER-CENT			Age of Widower at Death
	Annuity till Age of			Annuity till Age of			
	21	16	14	21	16	14	
<i>x</i>	YCa(21)	YCa(16)	YCa(14)	YCa(21)	YCa(16)	YCa(14)	<i>x</i>
25	11·74	9·59	8·61	12·79	10·25	9·12	25
26	11·71	9·56	8·57	12·75	10·21	9·08	26
27	11·68	9·52	8·53	12·71	10·17	9·03	27
28	11·65	9·48	8·48	12·67	10·12	8·98	28
29	11·62	9·43	8·43	12·63	10·07	8·93	29
30	11·58	9·38	8·38	12·59	10·02	8·87	30
31	11·54	9·33	8·33	12·54	9·96	8·81	31
32	11·50	9·28	8·28	12·49	9·90	8·75	32
33	11·46	9·23	8·22	12·44	9·84	8·69	33
34	11·41	9·18	8·16	12·38	9·78	8·63	34
35	11·36	9·13	8·10	12·32	9·71	8·56	35
36	11·31	9·07	8·04	12·26	9·64	8·48	36
37	11·26	9·00	7·97	12·20	9·56	8·39	37
38	11·20	8·93	7·89	12·13	9·48	8·30	38
39	11·13	8·85	7·80	12·05	9·39	8·21	39
40	11·06	8·76	7·71	11·97	9·29	8·11	40
41	10·98	8·67	7·60	11·88	9·19	8·00	41
42	10·90	8·57	7·50	11·78	9·08	7·88	42
43	10·82	8·46	7·39	11·68	8·96	7·76	43
44	10·74	8·35	7·27	11·57	8·84	7·63	44
45	10·64	8·24	7·15	11·46	8·71	7·49	45
46	10·54	8·12	7·02	11·35	8·58	7·35	46
47	10·44	8·00	6·89	11·23	8·44	7·20	47
48	10·33	7·88	6·75	11·11	8·30	7·05	48
49	10·22	7·75	6·60	10·98	8·15	6·89	49
50	10·10	7·60	6·45	10·84	7·98	6·72	50
51	9·97	7·43	6·27	10·69	7·80	6·52	51
52	9·82	7·24	6·08	10·52	7·60	6·30	52
53	9·65	7·04	5·86	10·32	7·37	6·06	53
54	9·57	6·82	5·61	10·10	7·12	5·80	54
55	9·26	6·56	5·33	9·86	6·85	5·51	55
56	9·02	6·28	5·00	9·60	6·54	5·17	56
57	8·75	5·90	4·62	9·29	6·19	4·77	57
58	8·44	5·58	4·21	8·94	5·78	4·34	58
59	8·08	5·13	3·76	8·54	5·30	3·86	59

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 47—(continued).

Present value, on the death of a Married Man, of a continuous Annuity of 1 until the youngest surviving child attains the age of 21, 16, or 14.

Age of Widower at Death	4 PER-CENT			3 PER-CENT			Age of Widower at Death
	Annuity till Age of			Annuity till Age of			
	21	16	14	21	16	14	
<i>x</i>	YC _a (21)	YC _a (16)	YC _a (14)	YC _a (21)	YC _a (16)	YC _a (14)	<i>x</i>
60	7.65	4.61	3.27	8.04	4.75	3.34	60
61	6.97	4.08	2.76	7.34	4.20	2.82	61
62	6.32	3.48	2.26	6.64	3.58	2.32	62
63	5.68	2.87	1.80	5.94	2.95	1.86	63
64	5.04	2.26	1.37	5.23	2.32	1.42	64
65	4.41	1.67	1.06	4.53	1.70	1.09	65
66	3.78	1.29	.86	3.88	1.32	.88	66
67	3.22	1.03	.70	3.30	1.05	.72	67
68	2.74	.85	.58	2.81	.87	.59	68
69	2.32	.72	.48	2.38	.73	.49	69
70	1.94	.60	.39	1.99	.62	.41	70
71	1.60	.51	.32	1.64	.52	.33	71
72	1.30	.43	.26	1.33	.44	.27	72
73	1.10	.37	.21	1.13	.38	.22	73
74	.94	.31	.17	.96	.32	.17	74
75	.79	.26	.14	.81	.27	.14	75
76	.66	.22	.11	.68	.22	.11	76
77	.56	.18	.09	.57	.18	.09	77
78	.46	.15	.08	.48	.15	.08	78
79	.39	.12	.06	.40	.12	.06	79
80	.32	.10	.05	.33	.11	.05	80
81	.27	.09	.04	.28	.09	.04	81
82	.23	.07	.03	.23	.07	.03	82
83	.19	.06	.02	.20	.06	.02	83
84	.16	.04	.01	.16	.05	.01	84
85	.13	.0313	.03	...	85
86	.11	.0211	.02	...	86
87	.09	.0109	.01	...	87
88	.0707	88
89	.0505	89
90	.0404	90
91	.0202	91
92	.0101	92

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 48.

Value, on the death of a Husband, of a continuous Reversionary Annuity of 1 to commence on the death of the Widow, and continue until the youngest surviving child reaches the age of 21, 16, or 14.

Age of Husband at Death	4 PER-CENT			3 PER-CENT			Age of Husband at Death
	Annuity till age of			Annuity till age of			
	21	16	14	21	16	14	
<i>x</i>	E(21)	E(16)	E(14)	E(21)	E(16)	E(14)	<i>x</i>
25	1.096	.698	.551	1.235	.765	.597	25
26	1.096	.696	.548	1.234	.762	.593	26
27	1.095	.694	.545	1.232	.758	.589	27
28	1.094	.691	.541	1.230	.754	.585	28
29	1.092	.687	.537	1.228	.750	.580	29
30	1.090	.682	.533	1.225	.745	.575	30
31	1.087	.677	.528	1.222	.740	.569	31
32	1.084	.672	.522	1.218	.735	.563	32
33	1.080	.667	.516	1.213	.729	.556	33
34	1.075	.661	.509	1.207	.722	.549	34
35	1.070	.655	.502	1.201	.715	.542	35
36	1.064	.648	.494	1.194	.706	.534	36
37	1.058	.640	.486	1.187	.697	.526	37
38	1.052	.632	.478	1.179	.688	.517	38
39	1.046	.623	.470	1.171	.678	.507	39
40	1.040	.614	.462	1.162	.668	.497	40
41	1.033	.605	.454	1.153	.658	.487	41
42	1.025	.596	.445	1.144	.648	.477	42
43	1.017	.587	.436	1.134	.637	.467	43
44	1.009	.578	.427	1.123	.625	.456	44
45	1.000	.568	.417	1.112	.613	.445	45
46	.991	.557	.406	1.101	.600	.433	46
47	.982	.546	.395	1.090	.587	.420	47
48	.972	.533	.383	1.078	.573	.406	48
49	.962	.519	.369	1.066	.557	.391	49
50	.951	.504	.354	1.051	.539	.375	50
51	.938	.486	.336	1.035	.519	.356	51
52	.922	.464	.316	1.016	.495	.334	52
53	.903	.440	.295	.993	.469	.311	53
54	.880	.415	.272	.966	.442	.286	54
55	.853	.388	.246	.934	.413	.258	55
56	.822	.356	.217	.898	.377	.228	56
57	.787	.320	.188	.857	.337	.196	57
58	.748	.284	.159	.811	.297	.164	58
59	.703	.248	.131	.759	.258	.134	59

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 48—(continued).

Value, on the death of a Husband, of a continuous Reversionary Annuity of 1 to commence on the death of the Widow, and continue until the youngest surviving child reaches the age of 21, 16, or 14.

Age of Husband at Death	4 PER-CENT			3 PER-CENT			Age of Husband at Death
	Annuity till age of			Annuity till age of			
	21	16	14	21	16	14	
<i>x</i>	E(21)	E(16)	E(14)	E(21)	E(16)	E(14)	<i>x</i>
60	·650	·212	·109	·699	·220	·111	60
61	·574	·182	·091	·615	·189	·093	61
62	·498	·154	·077	·531	·160	·079	62
63	·422	·128	·066	·447	·133	·068	63
64	·352	·106	·057	·370	·110	·058	64
65	·282	·088	·049	·296	·092	·050	65
66	·230	·076	·042	·241	·080	·043	66
67	·192	·065	·036	·200	·068	·037	67
68	·161	·054	·031	·168	·057	·032	68
69	·139	·046	·026	·145	·048	·027	69
70	·123	·040	·021	·128	·042	·022	70
71	·111	·034	·017	·115	·036	·018	71
72	·101	·029	·014	·105	·030	·014	72
73	·092	·024	·011	·096	·025	·011	73
74	·083	·020	·009	·086	·021	·009	74
75	·075	·016	·007	·078	·017	·007	75
76	·068	·013	·005	·071	·014	·005	76
77	·061	·011	·004	·064	·011	·004	77
78	·055	·009	·003	·058	·009	·003	78
79	·050	·007	·003	·052	·007	·003	79
80	·045	·006	·002	·047	·006	·002	80
81	·040	·005	·002	·042	·005	·002	81
82	·035	·004	·001	·036	·004	·001	82
83	·030	·003	·001	·031	·003	·001	83
84	·025	·002	...	·026	·002	...	84
85	·020	·001	...	·021	·001	...	85
86	·016	·017	86
87	·012	·013	87
88	·009	·009	88
89	·006	·006	89
90	·004	·004	90
91	·002	·002	91
92	92

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 49.

Value, on the death of a Married man, of continuous Annuities of 1 to each of the children until they respectively attain the age of 21, 16, or 14.

Age of Married man at Death	4 PER-CENT			3 PER-CENT			Age of Married man at Death
	Orphans' Annuities till age of			Orphans' Annuities till age of			
	21	16	14	21	16	14	
<i>x</i>	O <i>a</i> (21)	O <i>a</i> (16)	O <i>a</i> (14)	O <i>a</i> (21)	O <i>a</i> (16)	O <i>a</i> (14)	<i>x</i>
20	0	0	0	0	0	0	20
21	5.20	4.33	3.92	5.67	4.63	4.16	21
22	8.10	6.70	6.05	8.80	7.15	6.40	22
23	10.22	8.64	7.60	11.10	9.21	8.04	23
24	12.08	10.07	8.82	13.10	10.71	9.30	24
25	13.74	11.12	9.87	14.88	11.81	10.40	25
26	15.37	12.17	10.83	16.58	12.91	11.42	26
27	16.85	13.22	11.67	18.20	14.01	12.30	27
28	18.36	14.27	12.50	19.80	15.11	13.12	28
29	19.86	15.33	13.31	21.40	16.21	13.96	29
30	21.37	16.38	14.17	23.00	17.31	14.84	30
31	22.89	17.44	14.94	24.60	18.41	15.64	31
32	24.41	18.42	15.65	26.20	19.41	16.38	32
33	25.92	19.32	16.43	27.80	20.37	17.16	33
34	27.45	20.12	17.01	29.38	21.16	17.74	34
35	28.64	20.85	17.35	30.62	21.87	18.10	35
36	29.73	21.21	17.14	31.70	22.24	17.88	36
37	30.53	21.17	16.80	32.50	22.20	17.52	37
38	31.15	20.86	16.27	33.10	21.80	16.96	38
39	31.20	20.31	15.67	33.15	21.25	16.36	39
40	30.87	19.66	15.06	32.75	20.54	15.73	40
41	30.37	18.96	14.45	32.22	19.80	15.09	41
42	29.77	18.25	13.84	31.60	19.05	14.45	42
43	29.15	17.53	13.23	30.90	18.29	13.80	43
44	28.52	16.81	12.62	30.18	17.54	13.15	44
45	27.89	16.09	12.01	29.45	16.78	12.50	45
46	27.18	15.37	11.39	28.70	16.03	11.86	46
47	26.46	14.65	10.77	27.90	15.27	11.21	47
48	25.73	13.93	10.15	27.10	14.52	10.56	48
49	24.95	13.21	9.53	26.26	13.76	9.92	49
50	24.06	12.49	8.92	25.30	13.01	9.27	50
51	22.93	11.77	8.31	24.10	12.25	8.62	51
52	21.60	11.04	7.70	22.70	11.50	7.98	52
53	20.27	10.32	7.09	21.30	10.74	7.33	53
54	18.94	9.60	6.47	19.90	9.99	6.68	54
55	17.61	8.88	5.85	18.50	9.23	6.04	55
56	16.29	8.16	5.23	17.10	8.48	5.40	56
57	14.97	7.44	4.70	15.70	7.72	4.85	57
58	13.65	6.72	4.21	14.30	6.97	4.34	58
59	12.33	6.00	3.76	12.90	6.21	3.86	59

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 49—(continued).

Value, on the death of a Married man, of continuous Annuities of 1 to each of the children until they respectively attain the age of 21, 16, or 14.

Age of Married man at Death	4 PER-CENT			3 PER-CENT			Age of Married man at Death
	Orphans' Annuities till age of			Orphans' Annuities till age of			
	21	16	14	21	16	14	
<i>x</i>	<i>Oa</i> (21)	<i>Oa</i> (16)	<i>Oa</i> (14)	<i>Oa</i> (21)	<i>Oa</i> (16)	<i>Oa</i> (14)	<i>x</i>
60	11·01	5·28	3·27	11·50	5·46	3·34	60
61	9·69	4·56	2·76	10·10	4·70	2·82	61
62	8·37	3·84	2·26	8·70	3·95	2·32	62
63	7·05	3·11	1·80	7·30	3·20	1·86	63
64	5·76	2·39	1·37	5·90	2·45	1·42	64
65	4·52	1·71	1·06	4·65	1·75	1·09	65
66	3·78	1·29	·86	3·88	1·32	·88	66
67	3·22	1·03	·70	3·30	1·05	·72	67
68	2·74	·85	·58	2·81	·87	·59	68
69	2·32	·72	·48	2·38	·73	·49	69
70	1·94	·60	·39	1·99	·62	·41	70
71	1·60	·51	·32	1·64	·52	·33	71
72	1·30	·43	·26	1·33	·44	·27	72
73	1·10	·37	·21	1·13	·38	·22	73
74	·94	·31	·17	·96	·32	·17	74
75	·79	·26	·14	·81	·27	·14	75
76	·66	·22	·11	·68	·22	·11	76
77	·56	·18	·09	·57	·18	·09	77
78	·46	·15	·08	·48	·15	·08	78
79	·39	·12	·06	·40	·12	·06	79
80	·32	·10	·05	·33	·11	·05	80
81	·27	·09	·04	·28	·09	·04	81
82	·23	·07	·03	·23	·07	·03	82
83	·19	·06	·02	·20	·06	·02	83
84	·16	·04	·01	·16	·05	·01	84
85	·13	·03	...	·13	·03	...	85
86	·11	·02	...	·11	·02	...	86
87	·09	·01	...	·09	·01	...	87
88	·07	·07	88
89	·05	·05	89
90	·04	·04	90
91	·02	·02	91
92	·01	·01	92

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 50.

*Commutation Columns for finding the Present Value per Member
(whether Bachelor, Husband, or Widower) of an Annuity of 1
to a Widow to commence at the moment of death of a Husband.*

Age	4 PER-CENT		3 PER-CENT		Age
x	${}^{wa}C_x^*$	${}^{wa}M_x^\dagger$	${}^{wa}C_x$	${}^{wa}M_x$	x
20	·82	4686·13	1·18	8731·16	20
21	2·36	4685·31	3·42	8729·98	21
22	6·04	4682·95	8·82	8726·56	22
23	11·59	4676·91	17·09	8717·74	23
24	17·38	4665·32	25·86	8700·65	24
25	24·69	4647·94	37·07	8674·79	25
26	31·32	4623·25	47·41	8637·72	26
27	37·34	4591·93	57·00	8590·31	27
28	43·37	4554·59	66·77	8533·31	28
29	54·98	4511·22	85·36	8466·54	29
30	63·37	4456·24	99·21	8381·18	30
31	72·51	4392·87	114·49	8281·97	31
32	78·78	4320·36	125·44	8167·48	32
33	83·90	4241·58	134·74	8042·04	33
34	91·13	4157·68	147·60	7907·30	34
35	97·57	4066·55	159·38	7759·70	35
36	100·05	3968·98	164·83	7600·32	36
37	105·67	3868·93	175·59	7435·49	37
38	107·60	3763·26	180·34	7259·90	38
39	107·71	3655·66	182·07	7079·56	39
40	109·32	3547·95	186·38	6897·49	40
41	107·52	3438·63	184·84	6711·11	41
42	110·39	3331·11	191·38	6526·27	42
43	108·84	3220·72	190·27	6334·89	43
44	110·96	3111·88	195·58	6144·62	44
45	112·37	3000·92	199·73	5949·04	45
46	113·03	2888·55	202·53	5749·31	46
47	111·35	2775·52	201·12	5546·78	47
48	112·43	2664·17	204·71	5345·66	48
49	113·36	2551·74	208·07	5140·95	49
50	112·62	2438·38	208·38	4932·88	50
51	111·53	2325·76	207·96	4724·50	51
52	111·62	2214·23	209·74	4516·54	52
53	111·18	2102·61	210·55	4306·80	53
54	111·36	1991·43	212·50	4096·25	54
55	111·18	1880·07	213·80	3883·75	55
56	111·47	1768·89	216·00	3669·95	56
57	110·47	1657·42	215·68	3453·95	57
58	110·49	1546·95	217·38	3238·27	58
59	108·91	1436·46	215·90	3020·89	59

$$* {}^{wa}C_x = v^{x+\frac{1}{2}} H d_x(a_y + \frac{1}{2}).$$

$$\dagger {}^{wa}M_x = \sum_x {}^{wa}C_x.$$

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 50—(continued).

*Commutation Columns for finding the Present Value per Member
(whether Bachelor, Husband, or Widower) of an Annuity of 1
to a Widow to commence at the moment of death of a Husband.*

Age	4 PER-CENT		3 PER-CENT		Age
x	${}^{wa}C_x^*$	${}^{wa}M_x^\dagger$	${}^{wa}C_x$	${}^{wa}M_x$	x
60	108.02	1327.55	215.77	2804.99	60
61	105.99	1219.53	213.31	2589.22	61
62	103.57	1113.54	210.02	2375.91	62
63	99.94	1009.97	204.18	2165.89	63
64	94.16	910.03	193.83	1961.71	64
65	86.91	815.87	180.23	1767.88	65
66	80.30	728.96	167.73	1587.65	66
67	75.82	648.66	159.53	1419.92	67
68	71.03	572.84	150.52	1260.39	68
69	66.07	501.81	141.04	1109.87	69
70	60.87	435.74	130.88	968.83	70
71	55.65	374.87	120.49	837.95	71
72	50.33	319.22	109.75	717.46	72
73	45.03	268.89	98.92	607.71	73
74	39.88	223.86	88.23	508.79	74
75	34.76	183.98	77.45	420.56	75
76	29.82	149.22	66.93	343.11	76
77	25.21	119.40	56.98	276.18	77
78	20.91	94.19	47.61	219.20	78
79	17.16	73.28	39.35	171.59	79
80	13.86	56.12	32.03	132.24	80
81	10.87	42.26	25.31	100.21	81
82	8.57	31.39	20.11	74.90	82
83	6.59	22.82	15.56	54.79	83
84	4.95	16.23	11.78	39.23	84
85	3.64	11.28	8.72	27.45	85
86	2.60	7.64	6.27	18.73	86
87	1.83	5.04	4.46	12.46	87
88	1.23	3.21	3.01	8.00	88
89	.80	1.98	1.98	4.99	89
90	.50	1.18	1.25	3.01	90
91	.31	.68	.78	1.76	91
92	.17	.37	.44	.98	92
93	.10	.20	.26	.54	93
94	.05	.10	.14	.28	94
95	.03	.05	.08	.14	95
96	.01	.02	.04	.06	96
97	.01	.01	.02	.02	97

$$* {}^{wa}C_x = v^{x+\frac{1}{2}} H d_x(a_j + \frac{1}{2}).$$

$$\dagger {}^{wa}M_x = \sum_x {}^{wa}C_x.$$

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 51.

Commutation Columns for finding the present value per Member (whether Bachelor, Husband, or Widower) of an Annuity of 1 to commence at the death of a Widow and continue until the youngest surviving child reaches the age of 21, 16, or 14.

Age	4 PER-CENT			3 PER-CENT			Age
	Annuity till Age of			Annuity till Age of			
	21	16	14	21	16	14	
x	$E^{(21)}M_x^*$	$E^{(16)}M_x$	$E^{(14)}M_x$	$E^{(21)}M_x$	$E^{(16)}M_x$	$E^{(14)}M_x$	x
20	229.04	118.96	84.32	398.95	197.91	137.50	20
21	228.99	118.93	84.30	398.88	197.87	137.47	21
22	228.85	118.84	84.23	398.68	197.75	137.38	22
23	228.49	118.61	84.05	398.17	197.44	137.13	23
24	227.79	118.17	83.70	397.18	196.83	136.65	24
25	226.74	117.50	83.17	395.68	195.90	135.93	25
26	225.25	116.55	82.42	393.53	194.57	134.89	26
27	223.35	115.34	81.47	390.77	192.86	133.56	27
28	221.08	113.90	80.34	387.44	190.81	131.97	28
29	218.43	112.23	79.03	383.52	188.41	130.11	29
30	215.07	110.11	77.38	378.49	185.34	127.73	30
31	211.18	107.68	75.48	372.62	181.77	124.98	31
32	206.72	104.90	73.32	365.83	177.66	121.82	32
33	201.87	101.89	70.99	358.37	173.16	118.37	33
34	196.70	98.70	68.52	350.35	168.34	114.69	34
35	191.09	95.25	65.86	341.56	163.08	110.69	35
36	185.08	91.57	63.04	332.06	157.42	106.40	36
37	178.93	87.82	60.18	322.24	151.61	102.01	37
38	172.44	83.89	57.20	311.78	145.47	97.37	38
39	165.84	79.92	54.20	301.05	139.21	92.66	39
40	159.24	75.99	51.23	290.23	132.95	87.98	40
41	152.55	72.04	48.26	279.17	126.59	83.25	41
42	145.98	68.19	45.37	268.21	120.34	78.62	42
43	139.24	64.27	42.45	256.87	113.92	73.89	43
44	132.61	60.44	39.61	245.62	107.60	69.26	44
45	125.86	56.58	36.75	234.08	101.18	64.57	45
46	119.04	52.71	33.91	222.32	94.70	59.86	46
47	112.19	48.86	31.11	210.40	88.20	55.17	47
48	105.45	45.11	28.40	198.57	81.83	50.61	48
49	98.66	41.39	25.72	186.54	75.44	46.08	49
50	91.83	37.70	23.10	174.32	69.06	41.60	50
51	85.06	34.11	20.58	162.13	62.81	37.25	51
52	78.38	30.65	18.19	150.00	56.73	33.08	52
53	71.73	27.31	15.91	137.84	50.80	29.08	53
54	65.17	24.11	13.77	125.75	45.09	25.29	54
55	58.69	21.06	11.77	113.71	39.58	21.73	55
56	52.34	18.17	9.94	101.83	34.33	18.45	56
57	46.13	15.48	8.30	90.12	29.41	15.48	57
58	40.16	13.05	6.87	78.78	24.95	12.89	58
59	34.40	10.86	5.65	67.79	20.92	10.67	59

* $E^{(n)}M_x = \sum v^{x+1} H d_x \cdot E(n)_x$.

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 51—(continued).

*Commutation Columns for finding the present value per Member
(whether Bachelor, Husband, or Widower) of an Annuity of
1 to commence at the death of a Widow and continue until
the youngest surviving child reaches the age of 21, 16, or 14.*

Age	4 PER-CENT			3 PER-CENT			Age
	Annuity till Age of			Annuity till Age of			
	21	16	14	21	16	14	
x	$E^{(21)}M_x^*$	$E^{(16)}M_x$	$E^{(14)}M_x$	$E^{(21)}M_x$	$E^{(16)}M_x$	$E^{(14)}M_x$	x
60	28.98	8.95	4.64	57.40	17.39	8.83	60
61	23.93	7.30	3.79	47.66	14.33	7.28	61
62	19.48	5.89	3.09	39.03	11.68	5.98	62
63	15.65	4.70	2.50	31.55	9.43	4.87	63
64	12.46	3.73	2.00	25.31	7.57	3.92	64
65	9.91	2.96	1.59	20.31	6.08	3.14	65
66	7.99	2.36	1.26	16.51	4.90	2.50	66
67	6.51	1.87	.99	13.56	3.92	1.97	67
68	5.32	1.47	.77	11.17	3.11	1.53	68
69	4.36	1.15	.59	9.23	2.45	1.16	69
70	3.57	.89	.44	7.61	1.92	.86	70
71	2.91	.67	.33	6.25	1.47	.63	71
72	2.35	.50	.24	5.09	1.11	.45	72
73	1.87	.36	.17	4.09	.82	.32	73
74	1.47	.26	.12	3.24	.60	.22	74
75	1.14	.18	.08	2.54	.43	.15	75
76	.87	.12	.05	1.96	.30	.10	76
77	.65	.08	.03	1.48	.21	.07	77
78	.48	.05	.02	1.10	.14	.05	78
79	.35	.03	.01	.80	.09	.03	79
80	.25	.02	.00	.57	.06	.02	80
81	.17	.0139	.04	.01	81
82	.11	.0026	.02	.00	82
83	.0717	.01	...	83
84	.0410	.00	...	84
85	.0206	85
86	.0103	86
87	.0001	87

$$* E^{(n)}M_x = \sum v^{x+1} H d_x \cdot E(n)_x.$$

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 52.

Commutation Columns for finding the present value per Member (whether Bachelor, Husband, or Widower) of an Annuity of 1 to commence on the death of a Widower and continue until the youngest surviving child reaches the age of 21, 16 or 14.

$${}^{K.YCa(n)}M_x = \Sigma v^x + \frac{1}{2} K d_x . YCa(n).$$

Age	4 PER-CENT			3 PER-CENT			Age
	Annuity till age of			Annuity till age of			
	21	16	14	21	16	14	
x	$K.YCa(21)M_x$	$K.YCa(16)M_x$	$K.YCa(14)M_x$	$K.YCa(21)M_x$	$K.YCa(16)M_x$	$K.YCa(14)M_x$	x
25	259.00	156.10	121.33	476.46	276.03	210.96	25
26	259.00	156.10	121.33	476.46	276.03	210.96	26
27	258.59	155.76	121.03	475.88	275.56	210.55	27
28	258.19	155.44	120.74	475.32	275.11	210.15	28
29	257.81	155.13	120.46	474.77	274.67	209.76	29
30	257.08	154.54	119.93	473.71	273.83	209.01	30
31	256.38	153.97	119.42	472.69	273.02	208.29	31
32	255.37	153.16	118.69	471.21	271.84	207.25	32
33	254.08	152.12	117.76	469.30	270.32	205.91	33
34	252.54	150.88	116.66	466.99	268.49	204.30	34
35	250.77	149.46	115.39	464.31	266.37	202.43	35
36	248.79	147.87	113.98	461.29	263.99	200.33	36
37	246.63	146.14	112.44	457.95	261.37	198.02	37
38	244.30	144.28	110.79	454.33	258.53	195.53	38
39	241.58	142.11	108.87	450.05	255.19	192.60	39
40	238.74	139.85	106.88	445.55	251.68	189.54	40
41	235.58	137.34	104.67	440.49	247.75	186.11	41
42	232.13	134.62	102.28	434.92	243.44	182.36	42
43	228.42	131.71	99.73	428.88	238.79	178.32	43
44	224.69	128.79	97.18	422.75	234.08	174.24	44
45	220.57	125.58	94.39	415.92	228.86	169.73	45
46	216.11	122.12	91.39	408.46	223.19	164.85	46
47	211.35	118.45	88.22	400.42	217.11	159.64	47
48	206.33	114.60	84.90	391.87	210.68	154.16	48
49	200.78	110.37	81.27	382.33	203.55	148.11	49
50	194.77	105.81	77.39	371.91	195.81	141.57	50
51	188.36	100.99	73.30	360.70	187.56	134.62	51
52	181.61	95.96	69.06	348.80	178.88	127.36	52
53	174.47	90.69	64.64	336.09	169.70	119.75	53
54	166.89	85.16	60.04	322.50	160.00	111.77	54
55	158.76	79.37	55.28	307.98	149.76	103.43	55
56	150.15	73.27	50.32	292.30	138.87	94.67	56
57	140.91	66.83	45.19	275.31	127.30	85.52	57
58	131.28	60.33	40.10	257.48	115.42	76.37	58
59	121.24	53.69	35.09	238.76	103.32	67.28	59

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 52—(continued).

Commutation Columns for finding the present value per Member (whether Bachelor, Husband, or Widower) of an Annuity of 1 to commence on the death of a Widower and continue until the youngest surviving child reaches the age of 21, 16 or 14.

$$K.YCa(n) M_x = \sum v^{x+\frac{1}{2}} K d_x.YCa(n).$$

Age	4 PER-CENT			3 PER-CENT			Age
	Annuity till age of			Annuity till age of			
	21	16	14	21	16	14	
x	K.YCa 21 M_x	K.YCa 16 M_x	K.YCa 14 M_x	K.YCa 21 M_x	K.YCa 16 M_x	K.YCa 14 M_x	x
60	110.98	47.18	30.31	219.48	91.36	58.57	60
61	100.35	40.78	25.77	199.44	79.52	50.24	61
62	89.92	34.67	21.64	179.54	68.13	42.59	62
63	79.68	29.03	17.98	159.86	57.52	35.71	63
64	69.80	24.04	14.85	140.77	48.04	29.73	64
65	60.64	19.93	12.36	123.05	40.18	24.92	65
66	52.36	16.80	10.37	107.04	34.17	21.07	66
67	44.98	14.28	8.69	92.64	29.27	17.80	67
68	38.29	12.14	7.24	79.49	25.09	14.94	68
69	32.28	10.27	5.98	67.54	21.38	12.43	69
70	26.95	8.62	4.88	56.83	18.08	10.22	70
71	22.31	7.18	3.94	47.42	15.17	8.30	71
72	18.35	5.92	3.14	39.32	12.59	6.66	72
73	15.05	4.82	2.47	32.52	10.32	5.28	73
74	12.21	3.87	1.92	26.59	8.34	4.14	74
75	9.77	3.05	1.47	21.45	6.63	3.21	75
76	7.72	2.37	1.12	17.07	5.20	2.47	76
77	6.03	1.81	.84	13.44	4.01	1.88	77
78	4.64	1.37	.62	10.43	3.05	1.40	78
79	3.53	1.02	.44	7.98	2.29	1.00	79
80	2.65	.74	.30	6.04	1.68	.68	80
81	1.96	.52	.19	4.51	1.19	.44	81
82	1.43	.35	.11	3.32	.81	.26	82
83	1.02	.22	.06	2.39	.52	.13	83
84	.71	.13	.03	1.67	.31	.05	84
85	.48	.07	.01	1.13	.16	.01	85
86	.31	.0374	.07	...	86
87	.19	.0146	.02	...	87
88	.1127	88
89	.0615	89
90	.0307	90
91	.0103	91
				.01	92

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 53.

*Commutation Columns for finding the present value per Member
(whether Bachelor, Husband, or Widower) of an Annuity of
1 to each of the children until age 16 on the death of a married
man (whether Husband or Widower).*

$${}^{0a(16)}M_x = \Sigma .v^{x+\frac{1}{2}}Md_x . {}^{0a(16)}x.$$

4 PER-CENT				3 PER-CENT			
Age x	${}^{0a(16)}M_x$	Age x	${}^{0a(16)}M_x$	Age x	${}^{0a(16)}M_x$	Age x	${}^{0a(16)}M_x$
20	3411.5	60	246.42	20	5506.0	60	470.89
21	3411.5	61	198.09	21	5506.0	61	381.21
22	3410.9	62	155.94	22	5505.3	62	302.52
23	3408.7	63	120.17	23	5502.4	63	235.21
24	3403.2	64	91.25	24	5495.0	64	180.24
25	3393.6	65	69.58	25	5482.0	65	138.81
26	3378.5	66	54.71	26	5461.4	66	110.15
27	3357.0	67	43.89	27	5431.9	67	89.09
28	3329.1	68	35.35	28	5393.4	68	72.37
29	3294.1	69	28.38	29	5344.6	69	58.58
30	3245.9	70	22.63	30	5276.8	70	47.12
31	3186.5	71	17.94	31	5192.5	71	37.65
32	3113.5	72	14.09	32	5088.0	72	29.80
33	3029.1	73	10.94	33	4966.2	73	23.31
34	2934.1	74	8.37	34	4827.7	74	17.98
35	2825.9	75	6.31	35	4668.9	75	13.66
36	2705.2	76	4.69	36	4490.4	76	10.24
37	2578.5	77	3.44	37	4301.4	77	7.57
38	2444.2	78	2.49	38	4099.1	78	5.52
39	2308.3	79	1.78	39	3893.0	79	3.98
40	2175.0	80	1.25	40	3688.7	80	2.81
41	2042.9	81	.85	41	3484.6	81	1.92
42	1916.3	82	.56	42	3287.1	82	1.27
43	1790.1	83	.35	43	3088.6	83	.80
44	1669.8	84	.20	44	2897.5	84	.46
45	1550.9	85	.10	45	2706.8	85	.24
46	1434.5	86	.04	46	2518.3	86	.10
47	1321.3	87	.01	47	2333.4	87	.03
48	1213.7			48	2156.0		
49	1108.9			49	1981.5		
50	1007.3			50	1810.7		
51	910.40			51	1646.4		
52	818.57			52	1489.2		
53	730.95			53	1337.6		
54	647.91			54	1192.7		
55	569.10			55	1053.8		
56	494.76			56	921.7		
57	424.73			57	796.1		
58	360.05			58	679.1		
59	300.28			59	569.95		

Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.

TABLE 54.

*Commutation Columns for finding the present value per Member
(whether Bachelor, Husband, or Widower) of 1 payable
immediately on the death of a Bachelor.*

$${}^{\text{Ed}}M_x = \Sigma .v^{x+\frac{1}{2}} .Bd_x.$$

4 PER-CENT				3 PER-CENT			
Age x	${}^{\text{Ed}}M_x$	Age x	${}^{\text{Ed}}M_x$	Age x	${}^{\text{Ed}}M_x$	Age x	${}^{\text{Ed}}M_x$
15	467.00	60	40.04	15	632.62	60	79.51
16	427.80	61	37.54	16	587.00	61	75.03
17	392.20	62	35.13	17	545.34	62	70.66
18	359.98	63	32.81	18	507.18	63	66.42
19	331.42	64	30.59	19	473.03	64	62.32
20	305.82	65	28.50	20	442.13	65	58.43
21	282.59	66	26.55	21	413.81	66	54.75
22	261.63	67	24.70	22	388.02	67	51.23
23	242.52	68	22.90	23	364.26	68	47.78
24	225.25	69	21.14	24	342.59	69	44.36
25	209.75	70	19.41	25	322.96	70	40.98
26	195.66	71	17.72	26	304.93	71	37.64
27	182.57	72	16.08	27	288.03	72	34.37
28	170.73	73	14.50	28	272.59	73	31.18
29	160.10	74	12.98	29	258.59	74	28.09
30	150.67	75	11.54	30	246.05	75	25.13
31	142.20	76	10.17	31	234.68	76	22.30
32	134.47	77	8.88	32	224.20	77	19.60
33	127.59	78	7.66	33	214.79	78	17.03
34	121.49	79	6.52	34	206.36	79	14.59
35	116.01	80	5.47	35	198.71	80	12.33
36	111.11	81	4.53	36	191.81	81	10.27
37	106.81	82	3.65	37	185.69	82	8.35
38	102.81	83	2.91	38	179.95	83	6.71
39	99.16	84	2.29	39	174.66	84	5.32
40	95.74	85	1.77	40	169.65	85	4.15
41	92.45	86	1.35	41	164.79	86	3.19
42	89.31	87	1.01	42	160.10	87	2.41
43	86.21	88	.76	43	155.43	88	1.82
44	83.27	89	.56	44	150.95	89	1.35
45	80.39	90	.40	45	146.52	90	.98
46	77.55	91	.28	46	142.12	91	.70
47	74.74	92	.19	47	137.72	92	.49
48	71.99	93	.13	48	133.37	93	.33
49	69.21	94	.08	49	128.93	94	.22
50	66.43	95	.05	50	124.44	95	.14
51	63.70	96	.03	51	119.99	96	.09
52	61.02	97	.02	52	115.58	97	.06
53	58.37	98	.01	53	111.17	98	.04
54	55.75			54	106.77	99	.02
55	53.11			55	102.30	100	.01
56	50.48			56	97.80		
57	47.83			57	93.23		
58	45.20			58	88.64		
59	42.60			59	84.06		

**Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.**

TABLE 55.

Multipliers for use in a Valuation.

INTEREST 4 PER-CENT.

x	$\frac{w_a M_x}{D_x^{(3)}}$	$\frac{E(21) M_x}{D_x^{(3)}}$	$\frac{E(16) M_x}{D_x^{(3)}}$	$\frac{E(14) M_x}{D_x^{(3)}}$	$\frac{K, YCa(21) M_x}{D_x^{(3)}}$	$\frac{K, YCa(16) M_x}{D_x^{(3)}}$	$\frac{K, YCa(14) M_x}{D_x^{(3)}}$	$\frac{Oa(16) M_x}{D_x^{(3)}}$	$\frac{Ed M_x}{D_x^{(3)}}$	x
20	·809	·040	·021	·015	·045	·027	·021	·589	·053	20
21	·908	·044	·023	·016	·050	·030	·024	·661	·055	21
22	1·012	·049	·026	·018	·056	·034	·026	·737	·057	22
23	1·120	·055	·028	·020	·062	·037	·029	·816	·058	23
24	1·231	·060	·031	·022	·068	·041	·032	·898	·059	24
25	1·345	·066	·034	·024	·075	·045	·035	·982	·061	25
26	1·461	·071	·037	·026	·082	·049	·038	1·068	·062	26
27	1·579	·077	·040	·028	·089	·054	·042	1·154	·063	27
28	1·699	·082	·042	·030	·096	·058	·045	1·242	·064	28
29	1·820	·088	·045	·032	·104	·063	·049	1·329	·065	29
30	1·940	·094	·048	·034	·112	·067	·052	1·413	·066	30
31	2·058	·099	·050	·035	·120	·072	·056	1·493	·067	31
32	2·175	·104	·053	·037	·129	·077	·060	1·568	·068	32
33	2·290	·109	·055	·038	·137	·082	·064	1·635	·069	33
34	2·403	·114	·057	·040	·146	·087	·067	1·696	·070	34
35	2·513	·118	·059	·041	·155	·092	·071	1·747	·072	35
36	2·620	·122	·060	·042	·164	·098	·075	1·785	·073	36
37	2·724	·126	·062	·042	·174	·103	·079	1·815	·075	37
38	2·823	·129	·063	·043	·183	·108	·083	1·833	·077	38
39	2·919	·132	·064	·043	·193	·113	·087	1·843	·079	39
40	3·013	·135	·065	·044	·203	·119	·091	1·847	·081	40
41	3·105	·138	·065	·044	·213	·124	·094	1·845	·083	41
42	3·195	·140	·065	·044	·223	·129	·098	1·838	·086	42
43	3·281	·142	·065	·043	·233	·134	·102	1·824	·088	43
44	3·365	·143	·065	·043	·243	·139	·105	1·806	·090	44
45	3·444	·144	·065	·042	·253	·144	·108	1·780	·092	45
46	3·517	·145	·064	·041	·263	·149	·111	1·747	·094	46
47	3·585	·145	·063	·040	·273	·153	·114	1·707	·097	47
48	3·650	·144	·062	·039	·283	·157	·116	1·663	·099	48
49	3·708	·143	·060	·037	·292	·160	·118	1·612	·101	49
50	3·760	·142	·058	·036	·300	·163	·119	1·553	·102	50
51	3·805	·139	·056	·034	·308	·165	·120	1·489	·104	51
52	3·843	·136	·053	·032	·315	·167	·120	1·421	·106	52
53	3·873	·132	·050	·030	·321	·167	·119	1·346	·108	53
54	3·895	·127	·047	·027	·326	·167	·117	1·267	·109	54
55	3·909	·122	·044	·024	·330	·165	·115	1·183	·110	55
56	3·916	·116	·040	·022	·332	·162	·111	1·095	·112	56
57	3·918	·109	·037	·020	·333	·158	·107	1·004	·113	57
58	3·907	·101	·033	·017	·332	·152	·101	·909	·114	58
59	3·888	·093	·029	·015	·328	·145	·095	·813	·115	59

Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.

TABLE 56.

Multipliers for use in a Valuation.

INTEREST 3 PER-CENT.

x	$\frac{wa M_x}{D_x^{(3)}}$	$\frac{E(21) M_x}{D_x^{(3)}}$	$\frac{E(16) M_x}{D_x^{(3)}}$	$\frac{E(14) M_x}{D_x^{(3)}}$	$\frac{K.YCa(21) M_x}{D_x^{(3)}}$	$\frac{K.YCa(16) M_x}{D_x^{(3)}}$	$\frac{K.YCa(14) M_x}{D_x^{(3)}}$	$\frac{Oa(16) M_x}{D_x^{(3)}}$	$\frac{Ed M_x}{D_x^{(3)}}$	x
20	1.242	.057	.028	.020	.068	.039	.030	.783	.063	20
21	1.381	.063	.031	.022	.075	.044	.033	.871	.065	21
22	1.525	.070	.035	.024	.083	.048	.037	.962	.068	22
23	1.672	.076	.038	.026	.091	.053	.040	1.055	.070	23
24	1.821	.083	.041	.029	.100	.058	.044	1.150	.072	24
25	1.972	.090	.045	.031	.108	.063	.048	1.246	.073	25
26	2.123	.097	.048	.033	.117	.068	.052	1.342	.075	26
27	2.276	.104	.051	.035	.126	.073	.056	1.439	.076	27
28	2.429	.110	.054	.038	.135	.078	.060	1.535	.078	28
29	2.581	.117	.057	.040	.145	.084	.064	1.629	.079	29
30	2.730	.123	.060	.042	.154	.089	.068	1.719	.080	30
31	2.876	.129	.063	.043	.164	.095	.072	1.803	.082	31
32	3.018	.135	.066	.045	.174	.100	.077	1.880	.083	32
33	3.156	.141	.068	.046	.184	.106	.081	1.949	.084	33
34	3.291	.146	.070	.048	.194	.112	.085	2.009	.086	34
35	3.420	.151	.072	.049	.205	.117	.089	2.058	.088	35
36	3.543	.155	.073	.050	.215	.123	.093	2.093	.089	36
37	3.661	.159	.075	.050	.226	.129	.097	2.118	.091	37
38	3.772	.162	.076	.051	.236	.134	.102	2.130	.093	38
39	3.878	.165	.076	.051	.247	.140	.106	2.132	.096	39
40	3.980	.168	.077	.051	.257	.145	.109	2.129	.098	40
41	4.078	.170	.077	.051	.268	.151	.113	2.117	.100	41
42	4.172	.171	.077	.050	.278	.156	.117	2.101	.102	42
43	4.260	.173	.077	.050	.288	.161	.120	2.077	.105	43
44	4.343	.174	.076	.049	.299	.165	.123	2.048	.107	44
45	4.420	.174	.075	.048	.309	.170	.126	2.011	.109	45
46	4.489	.174	.074	.047	.319	.174	.129	1.967	.111	46
47	4.550	.173	.072	.045	.328	.178	.131	1.914	.113	47
48	4.606	.171	.071	.044	.338	.182	.133	1.858	.115	48
49	4.654	.169	.068	.042	.346	.184	.134	1.794	.117	49
50	4.692	.166	.066	.040	.354	.186	.135	1.722	.118	50
51	4.722	.162	.063	.037	.361	.187	.135	1.646	.120	51
52	4.743	.158	.060	.035	.366	.188	.134	1.564	.121	52
53	4.754	.152	.056	.032	.371	.187	.132	1.476	.123	53
54	4.755	.146	.052	.029	.374	.186	.130	1.384	.124	54
55	4.746	.139	.048	.027	.376	.183	.126	1.288	.125	55
56	4.729	.131	.044	.024	.377	.179	.122	1.188	.126	56
57	4.707	.123	.040	.021	.375	.174	.117	1.085	.127	57
58	4.670	.114	.036	.019	.371	.166	.110	.979	.128	58
59	4.624	.104	.032	.016	.366	.158	.103	.872	.129	59

Hypothetical Experience of Staff Pension Fund
for Widows and Orphans.

TABLE 56—(continued).

Multipliers for use in a Valuation.

INTEREST 3 PER-CENT.

x	$\frac{w_a M_x}{D_x^{(3)}}$	$\frac{E(21) M_x}{D_x^{(3)}}$	$\frac{E(16) M_x}{D_x^{(3)}}$	$\frac{E(14) M_x}{D_x^{(3)}}$	$\frac{K.YCa(21) M_x}{D_x^{(3)}}$	$\frac{K.YCa(16) M_x}{D_x^{(3)}}$	$\frac{K.YCa(14) M_x}{D_x^{(3)}}$	$\frac{Oa(16) M_x}{D_x^{(3)}}$	$\frac{Ed M_x}{D_x^{(3)}}$	x
60	4.568	.094	.028	.014	.357	.149	.095	.767	.129	60
61	4.498	.083	.025	.013	.346	.138	.087	.662	.130	61
62	4.416	.073	.022	.011	.334	.127	.079	.562	.131	62
63	4.320	.063	.019	.010	.319	.115	.071	.469	.132	63
64	4.211	.054	.016	.008	.302	.103	.064	.387	.134	64
65	4.095	.047	.014	.007	.285	.093	.058	.321	.135	65
66	3.975	.041	.012	.006	.268	.086	.053	.276	.137	66
67	3.852	.037	.011	.005	.251	.079	.048	.242	.139	67
68	3.720	.033	.010	.005	.235	.074	.044	.214	.141	68
69	3.581	.030	.009	.004	.218	.069	.040	.189	.143	69
70	3.433	.027	.007	.003	.201	.064	.036	.167	.145	70
71	3.279	.024	.006	.002	.186	.059	.032	.147	.147	71
72	3.119	.022	.005	.002	.171	.055	.029	.130	.149	72
73	2.953	.020	.004	.002	.158	.050	.026	.113	.152	73
74	2.782	.018	.003	.001	.145	.046	.023	.098	.154	74
75	2.608	.016	.003	.001	.133	.041	.020	.085	.156	75
76	2.431	.014	.002	.001	.121	.037	.018	.073	.158	76
77	2.256	.012	.002	.001	.110	.033	.015	.062	.160	77
78	2.083	.010	.001099	.029	.013	.052	.162	78
79	1.915	.009	.001089	.026	.011	.044	.163	79
80	1.752	.008	.001080	.022	.009	.037	.163	80
81	1.593	.006	.001072	.019	.007	.031	.163	81
82	1.446	.005064	.016	.005	.025	.161	82
83	1.301	.004057	.012	.003	.019	.159	83
84	1.161	.003049	.009	.001	.014	.157	84
85	1.027	.002042	.006009	.155	85
86	.899	.001035	.003005	.153	86
87	.779	.001029	.001002	.151	87
88	.663022151	88
89	.557017151	89
90	.462011150	90
91	.378006150	91
92	.300003150	92

DISCUSSION.

Mr. MANLY said that the Paper was not yet finished, and he hoped to be able to show how to ascertain the different benefits, not for individuals—bachelors, husbands, and widowers all put together—but for bachelors, husbands, and widowers separately; and to include the risks of second marriages. Such figures had not been produced before; and he never saw a way of arriving at them himself until he had made the very interesting table, which was marked No. 57*, tracing the husbands to their death or widowerhood. Table 58*, which had cost an enormous amount of labour, would also, he hoped, be considered an extremely interesting one, showing out of a given number of bachelors who started at the age of 15, namely, 200,000, how many died as bachelors, how many as husbands after the first marriage, and how many as widowers of the first marriage, husbands of the second marriage, widowers of the second marriage, and husbands of the third marriage, and widowers of the fourth marriage, and so on until, as would be noticed, they had really disappeared after the fourth marriage, for very few of them entered into that state. For this table he was indebted to Mr. Workman, who had shown the most exemplary patience in working the tables out for him. He thought that some very excellent results would be produced afterwards.

Mr. E. C. THOMAS said that Mr. Manly had set himself many distinct problems, the number of which seemed to be only limited by the possible combinations of widows, children, and other relations on the one hand, with husbands, widowers, and bachelors on the other. Some of these problems had been already investigated by previous writers, but Mr. Manly had introduced two fresh complications, namely, the elements of withdrawal and average salary. One of the greatest difficulties with which Mr. Manly, in common with every other investigator, had had to contend, was to find suitable material on which to base his necessary tables, and one was at once struck with the great variety of the sources to which he had been obliged to resort for this purpose. Those sources comprised his own tables, based on the experience of certain railway pension Funds, Mr. Hewat's tables based on the experience of Scottish banks, the census returns of New Zealand, and the H¹ Mortality Table. There might, therefore, be a possible objection raised to Mr. Manly's tables on account of the composite nature of their bases. Possibly a consideration of the various items involved might help to remove that objection. He thought that although the present paper was a continuation of Mr. Manly's previous paper, the kind of experience now in Mr. Manly's mind was more that of a bank than of a railway. He imagined that there would be no objection raised against the use of Mr. Hewat's tables drawn from the experience of banks. With regard to Mr. Manly's own tables, previously published, they had been criticized as comprising a very low mortality rate and a very high lapse rate. They might naturally expect that a low mortality rate would usually be experienced in these Funds, for not only were members generally selected at the outset, but selection operated at

* Tables 57 and 58 will be published later.

the other end by the retirement, compulsory or otherwise, of the weakly members. With regard to the withdrawal rate, experience had accumulated to show that for a hypothetical table, unaffected by any exceptional circumstances, Mr. Manly's table would probably be as suitable a basis as any other that could be devised. In one case—that of a bank's Fund—it had been possible to use Mr. Manly's withdrawal rate in its entirety. With regard to the figures based on the New Zealand census returns, it might be objected that New Zealand was a very much younger country than England, and subject to very different economic conditions; but this could be met by the unanswerable argument that the figures were apparently the only ones obtainable. One would naturally expect the number of children per marriage to be greater in New Zealand than in England, but in the discussion on Mr. King's paper on "Family Annuities", Mr. Morris Fox had stated that he considered that both the marriage rate and the birth rate were lower in New Zealand than in England. He took this to mean the birth rate per thousand of the population living, and this might follow upon a lower marriage rate, though not necessarily so. But what was required in the present instance was the number of children left per married man dying, and the New Zealand figures would probably err in this respect, if at all, on the side of safety. It was in accordance with reasonable expectation that the later figures obtainable by Mr. Manly showed a falling-off in this respect compared with the earlier statistics used by Mr. King. On the whole, he thought that they might conclude that Mr. Manly had been able to keep on the safe side in these figures as in the others. For lack of reliable information Mr. Manly had been forced in many cases to make assumptions. Mr. Manly seemed to have a remarkable gift for placing his hand upon the key of the situation, and this was exemplified in the short table on page 111. By calculating the extra liability involved in the continuance of the annuity to the youngest child after the death of the widow, the succeeding calculations for this benefit were greatly simplified. There were two assumptions made by Mr. Manly, both on the safe side, which would probably command general approval. The first was that the n youngest children tabulated for each age of the father at death, would represent the youngest children of the n fathers who died. This would certainly over-estimate the liability, particularly in respect of the fathers dying in middle life. The second assumption was that the annuity to the youngest child would be equivalent to the value of the annuity-certain for the same term, and would commend itself as an easy, effective, and sound solution of the difficulty. It was far less stringent than the assumption made by Mr. Huie that each husband dying would leave a child who would certainly live twenty-one years. It might be considered severe in the case of very young children, and it might be possible to obtain a little relief from the following consideration. The children that would take the place in the benefit of those who died, must necessarily be older than the originals, and the term would consequently be shortened. They might take it that on the average the difference would be about two years. They might, therefore, use the annuity-certain for two years less than the full

term, and add the value of the last two payments discounted with mortality as well as interest. Where the rate of child mortality was as low as in the present case, the difference would not be very large; but as they had so many assumptions on the safe side, he thought it would be desirable that they should obtain what relief was available. One of the greatest services which Mr. Manly had done was to provide them with a comprehensive system of notation, and it was a remarkable instance of his skill that he had been able to keep so successfully each distinct element apart, and provide them with a well-defined symbol for each, at the same time maintaining consistency with the scheme already propounded in his earlier paper. Although it might look complicated, it would be found to be in reality simple and straightforward. The paper was a further illustration, if such were needed, of the immense power of the commutation column. Mr. Manly extended into a much wider sphere than before the principle he developed so ably in his previous paper of providing a special commutation column for each kind of benefit, from which the value could be at once obtained. With reference to the benefit investigated in Problem VIIc, a payment on the death of a widower without children under a certain age, it was a little difficult to follow Mr. Manly's assumptions as to the number of widowers which would come on to the Fund for that particular benefit. From ages 25 to 40 it appeared to be merely a question of the number dying childless; and the proportion of these among married men was stated as 33 per-cent. at 25, decreasing to 15 per-cent. after age 30. Unless, therefore, there were fewer widowers than husbands dying childless, they might expect the series to begin at 33 per-cent. at 25, and run down to 15 per-cent. at, say, age 40, and thence steadily rise to 100 per-cent. The series actually adopted by Mr. Manly started with 20 per-cent. and increased steadily up to 100 per-cent. at age 70. Perhaps the most interesting part of the paper was the short valuation schedule given on pages 128-130. It would be seen that all unnecessary labour had been avoided, but the essential features were set out with such clearness that they could be readily grasped and understood.

Mr. G. J. LIDSTONE said that after a diligent study of Mr. Manly's formulas, he could not help thinking that if Mr. Manly had worked throughout on the numbers existing in the middle of the year of age, his formulas would have been considerably simplified, and he would have got rid of the complicated adjustments referred to, more especially in the earlier paper. Then Mr. Manly's notation was distinctly awe-inspiring, and one was tempted to ask whether a pictorial notation could not be purchased at almost too great a price—one, in fact, almost sighed for a more arbitrary notation, which would be a little more easily written. Mr. Manly had given, with extraordinary completeness, theoretically exact solutions of a number of problems which arose in connection with these Funds. It was probable that in practice they would, if they could, use slightly rougher methods with a view of minimizing labour; but it was quite evident that a full study of these theoretically exact solutions must be the necessary preliminary to the consideration of what those rougher methods might be. The kernel of the paper was Mr. Manly's

adoption of what had been called the "Collective" method, suggested first, he believed, by Mr. Ralph P. Hardy to Mr. Meikle, and used by the latter with great success in 1875 in connection with the Ministers' Widows' Fund. The same method and principle was also used very successfully by Mr. King in his classic paper on "Family Annuities." According to that method, instead of investigating and seeking to combine various component forces, the statistics of which were quite insufficient for proper treatment, they dealt with the resultant effect of those causes as focussed and exhibited in those cases, whether of members living or of members dying, which they were able to bring under observation. As Mr. G. F. Hardy had said (*J.I.A.* xxxi., 469): "Given certain rates of mortality, marriage, &c., operating fairly constantly, it should follow that among the numbers surviving to successive ages the proportion of married, the average age of their wives"—one might add, also, the number and average age of their children—"and the average liability that would be created by the death of each member would be practically a function of the age." Having brought the calculations into that form, the benefits were treated as varying assurances payable at the death of the members, the amount of the assurance being the average value of the benefits at the moment of death of the member. It was important to observe that that average value could be determined equally well by examining the cases of members living, as it could by examining the cases of members who had actually died, for there was no reason to think that the members dying were anything else than an average sample of those existing. For example, Mr. Manly had applied the proportions of husbands, widowers, and married men existing to obtain the relative numbers of each dying in the year. When he came to deal with the values of orphans' annuities at the date of the death of the parents, he found it more convenient to derive his information from statistics of persons dying, because those statistics happened to be ready at his hand. But it was obvious that equally good results could have been obtained by examining the same number of persons living and taken at random from the population. This was a very important point when they had to consider how far the statistics could be derived from the records of the Funds with which they were dealing. The numbers dying in these Funds during a reasonable period of observation would doubtless generally be too small for statistical treatment. But if the figures were taken from the numbers living over a period of years, they might frequently get quite sufficient material for their purpose. For example, Mr. King based his valuation of orphan annuities on about 1,500 observations; the same number, he thought, might be easily obtained by an examination of the numbers living in an average widows' Fund. This point was of great importance, because it was obviously desirable to get as much information as they could from the Fund itself, if only for purposes of test. However much they might admire the way in which Mr. Manly had pieced together statistics drawn from various sources, they could not but have a slight feeling of anxiety as to how far such a delicate mosaic formed a solid and substantial basis for the erection of tables of valuation. It was desirable to get back to bed-rock whenever they could, and to examine

the statistics of the Funds with which they were dealing. There were one or two practical problems with which he had met, and to which it might be worth while to refer. As Mr. Thomas had pointed out, Mr. Manly had assumed that the widows' Fund was to be worked in conjunction with a pension Fund, and therefore he naturally treated the contribution as being a percentage of the salary. Where, however, the widows' Fund existed independently, they sometimes found that instead of the contribution being a percentage of the salary, the contribution varied, not continuously, but by steps according as the salary passed a certain limit. For instance: In a Fund with which he (Mr. Lidstone) had had to deal, the annual contribution was £1 for salaries under £100, £2 for salaries between £100 and £200, and £3 where the salary was £200 or upwards. There were two ways in which they could deal with a case of that kind. The first, and perhaps the best way, was to form a table of the average contributions payable at each age in the same way as Mr. Manly had formed a table of average salary. Another way was to determine the average age at which the increase of salary, and the consequent increase of contribution, took place, and to value the contribution as being unchanged between those points, treating the increment of contribution as taking place at those average ages. Another point which had to be borne in mind when a widows' Fund existed independently of a pension Fund, was that there was usually a certain option exercisable against the Fund at the pension age. He believed that it was usual to give pensioners the option of continuing in the Fund (subject to their paying the maximum contribution) or of retiring—indeed, it was difficult to prevent their having the option of retiring if there were no pension from which the contributions could be deducted. The probability was that those who were bachelors or widowers with no likelihood of marrying would drop their contribution to the Fund, because the benefit was worth practically nothing, whereas married men and men with families would probably maintain their membership in force. This, of course, had to be allowed for, as the benefits which were dropped at the same time as the contributions were practically worth very little, if anything, at the date of the pension age, so that there was practically a loss, in such cases, of the value of the future contributions. Of course, the easy way to get over this was to assume that a certain proportion of members would cease membership at the pension age, and to reduce the value of the contribution after that age by the same proportion.

Mr. GEORGE KING said that the paper was an exceedingly heavy one, and he felt that he must repeat Mr. Manly's expression, and say that he rose to speak with an unprejudiced mind. The first thing that struck him was the immense magnitude and completeness of Mr. Manly's labours. When he (Mr. King) wrote the paper on "Family Annuities" he was quite unaware that anyone had done anything of the kind before, although he afterwards heard of what had been done by Mr. R. P. Hardy and Mr. Meikle, which had been mentioned by Mr. Lidstone. However, the work which he did on the subject had, he thought, put things in an entirely different light, and had shown how the benefits could best be valued.

When he (Mr. King) wrote the paper, he had specially in his mind what he called Family Annuities, and what Mr. Manly had now called Orphans' Annuities; but at the close of the paper he gave a formula for similar benefits for widows put in the same form, and just as he had transformed the integral for orphans' annuities into commutation columns, so he had had in his mind to transform the integral for widows' annuities also into commutation columns. But this Mr. Manly had now done. Ever since he (Mr. King) wrote the paper, he had been thinking of continuing it whenever opportunity offered, and whenever the right statistics came along, but so far his intention had not been fulfilled. However, only about six months ago he was collaborating in a Fund with Mr. H. J. Nahmer, a Fellow of the Faculty of Actuaries who resided in Dublin, and Mr. Nahmer and he made up their minds, from the statistics which they then had, to continue the paper. Mr. Manly had now relieved them of that labour, and he was sure that they were very much obliged to him. He thought, however, that some of the figures that were derived from their work might be useful, and he should refer to that subject later on. The difficulty which Mr. Manly had had, and the difficulty which stopped him (Mr. King) from continuing his paper, was the absence of suitable statistics. Mr. Manly had to a large extent overcome that difficulty by a most skilful use of the statistics of the Scottish banks, obtaining, however, facts from elsewhere to supplement them. He was not sure, however, that in preparing the tables the ideas had been fully carried out, although in using the tables they had been carried out. Those tables enabled them theoretically, as Mr. Lidstone had said, to solve exactly the most intricate problems which came before the actuary in this very special branch of consulting practice. In his paper on "Family Annuities", he said that they must not take the statistics from those living, but that, if they wanted to be exact, they must take them at the date of death. He did not remember exactly what was in his mind when he wrote that, but he would not now put the statement in the same form, because he entirely agreed that they might safely take the condition as to marriage and as to family, either at any point in life, or at the date of death, and if they used those statistics properly they got correct results. In fact, in the Fund which he had already referred to, he had, because of the very meagre number of deaths, made use of the statistics at a particular date of those living, and he believed that he got quite as good results that way as he would have had if he had had a large body of deaths. However, in the paper on "Family Annuities", in the same paragraph that he had been commenting upon, he found another statement: "To take the average difference between the ages of bridegrooms and brides would understate the values of annuities to widows, because the average difference between the ages of husbands who died and the ages of their wives is greater than in the case of bridegrooms and brides." He now found Mr. Manly, after having given the average difference of age of bridegrooms and brides, saying that there would not, of course, be so much difference of age between the wife and the

husband at the death of the husband, especially as the majority of marriages took place between the ages of 27 and 40. So here Mr. Manly apparently made a statement the exact opposite to his (Mr. King's), and one might think at the first blush that one or other statement or both of them must be wrong. But notwithstanding the extraordinary contradiction between their statements, he ventured to submit that they were both right. They could well imagine that of two people who had looked through a telescope, one might maintain that the telescope enlarged distant objects and brought them nearer, and that the other person might maintain that the telescope made distant objects smaller and sent them farther away. These statements would seem contradictory, but they were reconciled when it was found that the two persons had looked through different ends of the telescope. That was exactly what Mr. Manly and he had done. He took the average difference of age between bridegrooms and brides, and then he looked ahead, and he remembered that those who had wives older than themselves, or not much younger, were more likely to be left widowers than those who married young wives, and that, speaking of the death of husbands later on, they would be the survivors therefore of those who had comparatively young wives, because the others would be in the category of widowers and would not come under observation as husbands; so that from that point of view his statement was absolutely correct. Mr. Manly, however, went to the other end of the telescope, and looked at the ages of bridegrooms and brides, and then he took the deaths of husbands at that particular moment of age, and he remembered that those who were husbands then and dying, were the survivors of those who married earlier in life when the difference between the ages of bridegrooms and brides was not so great; and therefore Mr. Manly maintained, and perfectly correctly from that point of view, that the difference of age between husband and wife at the moment of the husband's death was not so great as the difference of age between brides and bridegrooms, the bridegroom being of the age of the husband of whom they were speaking. This reconciled the apparently absolutely contradictory statements; and it showed how statistics might bear various meanings, and give various lessons, according to the way in which they were regarded. There was one point in which he did not quite agree with Mr. Manly. Having got the average difference of age between husband and wife, he proceeded, in making his calculations, to take an annuity on the wife at that average age. A little investigation would show that, when the wives were young, that overstated the value of the annuity, and that when the wives were old it understated it. And they could not be sure which way the average error would go. In the case of the table that Mr. Manly had used, the turn he thought began somewhere about 70, but he believed that in the British Offices Annuity Tables it would be found to begin younger, somewhere about 60. He was not prepared to say exactly how much error there would be in this. In fact, the error would depend on the amount of the differences in the ages of the wives. There was a better plan, and a plan which he had himself followed. He multiplied the value of the annuity at each one of the

individual ages of the wives by the number of wives at that particular age, and then took the sum of all the annuities so found and divided by the total number of wives. That gave the average value of the annuity, as distinguished from the annuity at the average age. He thought that that gave more correct results, and it occasioned very little more trouble. He therefore would suggest that it was the better course to follow. He had spoken of the Fund which Mr. Nahmer and he had been dealing with. It was a Fund relating to the Irish clergy, and it was of rather a special kind. It was compulsory on all bachelor clergymen who entered the ministry of the church of Ireland, and it was voluntary on the part of all married clergymen who entered that ministry, so that every bachelor was compelled on entering the ministry to join the Fund, and if he remained a bachelor he never got any benefit from it at all. The Fund provided annuities to widows, and it provided annuities to children at the death of the father, whether the mother was alive or not, until those children reached the age of twenty-one. They did not value the Fund by means of its own experience by the new methods, but by means of adjustment of Mr. Hewat's tables on the old methods of Huie and Hewat. He had, however, worked out the statistics roughly, not with the same exactness as if he had had a larger body of facts, on the new method. He prepared the values of what Mr. Manly called the Orphans' Annuities, and he also got the value of Widows' Annuities. Unfortunately, on account of the smallness of the Fund, and it having been established barely more than thirty years, he could not carry the annuities for widows up to the necessary point. He could not carry them beyond age sixty-five, and, of course, they were wanted right on to the end of life. For the children he was able to get tables which were sufficiently complete for practical purposes. The fewness of the children per marriage was very remarkable. His table did not include the whole members of the Fund, whether husbands, or widowers, or bachelors, but only the husbands. That table he had the pleasure to submit. It showed the number of husbands, and the number of children of each age up to age twenty-one for each age of the husbands, and from that he had prepared a table of family annuities, and he might give a comparison with Mr. Manly's table. His were $3\frac{1}{2}$ per-cent., and he had taken the mean between Mr. Manly's 3 per-cent. and 4 per-cent. values. His figures did not extend below age thirty. He might remind the meeting that a man could not become a clergyman until he was twenty-four, and he probably would not enter the ministry of the church of Ireland until later than that, and in almost every case when he entered he was a bachelor.

As to the average ages of the wives compared with the husbands, he found that Mr. Manly gave a much greater difference in his tables than the Irish clergy showed. At age 30 the average difference according to the Irish clergy was 1·8, the wife being that much younger. Mr. Manly gave 2·5. So, at age 50, the Irish clergy showed 5·7, while Mr. Manly gave 8·5. Mr. Manly's tables would certainly err on the safe side, both as regarded the values of the children's benefits and the values of the widows' benefits; but he thought that that would be so seriously on the safe side as to upset

IRISH CLERGY.

Number of Children living at each Age, grouped according to Central Ages of Husbands, also value of Family Annuity and Average Age of Wife.

HUSBANDS		NUMBER OF CHILDREN AGED																				VALUE OF ORPHANS' ANNUITY TO AGE 21 3½ PER-CENT		Average Age of Wife
Central Age	Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Irish Clergy	Manly	
30	39	12	5	...	4	1	14.9	22.1	28.2
35	91	37	20	13	12	8	12	9	5	4	4	1	1	1	1	1	1	17.0	29.6	31.9
40	113	41	22	25	17	24	14	19	15	15	14	8	4	9	4	4	1	24.2	31.8	35.7
45	94	15	14	12	14	13	10	17	13	20	16	16	9	18	8	12	10	5	7	4	5	21.8	28.7	39.8
50	64	7	4	5	8	6	4	4	9	7	10	5	13	8	9	8	9	14	6	6	4	18.7	24.7	44.3
55	32	4	...	2	1	1	1	1	1	1	4	1	4	6	3	4	4	5	4	4	3	11.5	18.1	49.3
60	16	1	1	1	2	1	1	3	1	3	3	1	3	5	3	3	7.1	11.3	54.3

altogether such a Fund as that of the Irish clergy. This led him to say that he thought that Mr. Manly's industry and thoroughness might perhaps be a source of danger, because they would tempt those persons who were not so industrious, and who had Funds of this kind to value, to use Mr. Manly's tables. His own experience was most emphatically that, although for mortality purposes it might be safe to use one experience for another Fund, yet for pension purposes, and for widows' and orphans' annuities purposes, the Funds were so very different that he did not think that they could find two sufficiently near for them to apply the same experience to both. He had submitted one example in the case of the Irish clergy. They could not possibly use Mr. Manly's tables for them. To do so would give results which would be out of the question. He had had another case not long ago. He had had a great deal to do with certain large railway superannuation Funds, and a very large and a very old-established industrial corporation came to him not long ago for him to advise them as to the establishment of a Fund for themselves. They thought that he could take the railway statistics and apply them. He said, "No; I must have the statistics of your own Service before I can do anything at all." The result was that he found them so totally different that the railway statistics could not possibly be applied to the commercial undertaking. The rate of withdrawal in the commercial undertaking was very much lower than in the railway. The rate of early superannuation was very much higher. The rate of mortality was not so different as to be impossible of adjustment. But the rate of mortality after pension age in the industrial undertaking was lower, and the rate of salary in the industrial undertaking was much higher and rose much more rapidly from the minimum to the maximum, so that the two sets of statistics were not in any way comparable, and to establish a Fund for an industrial undertaking on railway experience would be to launch out into

inevitable ruin. He had a case in which an industrial undertaking some ten or eleven years ago adopted the exact ratios and rates of pension of a railway Fund without any actuarial advice and applied it to their staff, and they had come to him to value it. He was sure it would prove to be in a bad way.

The PRESIDENT (Mr. Hughes) said that he was very sorry that the paper had not attracted a larger number of speakers, but it was not altogether surprising that it had not done so. The subject had been described by Mr. King as a very heavy one, and he could quite understand that members would feel hesitation in speaking on a paper like this without giving it a great deal more consideration than they had had the opportunity of doing. They must thank Mr. Manly, not only for the paper and for the information that it contained, but for having put into the hands of the profession an exceedingly useful tool. He had supplied them from time to time with useful tools, and this last was not the least useful of them. The paper might, moreover, be put to a use which was not contemplated by the author. The mere exhibition of it, with its formidable looking formulas, might serve to impress with some sense of the seriousness of the subject those amateur philanthropists and benevolent employers who were apt to call upon actuaries in an airy sort of way to devise some little scheme for their work-people, comprising impossible benefits at merely nominal contributions, and, of course, at an equally nominal fee for settling the scheme.

Mr. MANLY, in reply, said that he had put in one or two kinds of benefits which had not come into actual use as yet, so far as he knew; but from observations which he had heard dropped from various sources he was inclined to think that they were soon coming to the front. It was marvellous how benefits of that sort were introduced from time to time, and how old Funds which had been conducted upon quite cautious and conservative principles, and which simply provided for widows' annuities, were asked to give more benefits. They wanted to know whether they could not give some benefit for the children, and then it was said, "Well, you know, bachelors are very discontented, they do not see why they should go on subscribing for other people's widows and families, when they get nothing out of it themselves." So an additional benefit providing some consolation to the persistent bachelors was now being introduced in the form of the return of their subscriptions at death. But widowers with no children also wanted some consolation; and so it was coming now to be quite fashionable to give something on the death of a widower leaving no children; and all these new benefits were supposed to be so small that the Funds could easily bear them. If this paper had no other object it would do a great service in showing really what the costs of these benefits would be to such a Fund; and they totalled up pretty costly. They meant, of course, that the widow must go with less annuity, or that the contributions must be increased, neither of which changes would be received with any great amount of delight. He had listened with very great interest to Mr. King's experience of the different Funds he had valued. He (Mr. Manly) was not putting forward these tables as standard tables for use. They were intended more to show the way in which the

experience must be worked up in order to arrive at the values of the different benefits. He quite agreed with Mr. King that the experience of one Fund with regard to its withdrawals, its mortality, its rate of retirement, and its marriage rate, would not necessarily apply to another Fund. One would never expect that the rate of withdrawal in a service like, for instance, the Government Civil Service of the higher class, would be the same as in a poorly paid staff like that of a railway company. Then with regard to the question of the effect of these pension Funds on the rate of withdrawal. There was a general idea—though, curiously, he had not found the idea really confirmed by statistics—that the establishment of such Funds induced the men to remain in the services. It might do so when they received such a scale of salary and pension as they got in the Government service, but he thought that present income had much more to do with the movement of the members of a staff than future benefits. There was a very common feeling on the part of young men of from twenty to thirty years of age that they would never reach the far-off pension age of sixty or sixty-five, and they would consequently very much rather have an increase of their salary at the present time, when, they considered, the money would be much more valuable to them, than take the deferred pension into account. Moreover, the rule that the contributions shall be returned on withdrawal removes any feeling of loss when they leave. As to taking the average age of widows: If they had the material—that is to say, if they could ascertain from all the husbands living at each age the ages of their wives, and if they could make their average pension depend on the sum of the pensions to each of the wives, it would be perfection; but he did not think that they often got it. At any rate, he had no material of the kind on which to base his annuities, so he had to be content to assume an average age, as he had explained. He understood that Mr. Lidstone rather objected to his notation. He was very sorry for that. The notation which had been used before seemed to be very confusing, and he wanted something that would be very distinct, and that could not be mistaken for any other values or functions. He tried several methods before he could reduce the notation to anything like a system. If Mr. Lidstone, or anybody else, could make an improvement upon that notation, he should be quite glad to adopt it. He did not think that Old English type would have helped him. He had taken the clearest and most distinctive type which the printer had. As to the option to withdraw, to which Mr. Lidstone had referred, he took it that it was one of those Funds in which the contribution was a fixed contribution to the end of life, or something of that kind.

MR. LIDSTONE said that his case was a widows' Fund without a pension Fund at all.

MR. MANLY said that the option allowed to the bachelor or widower to withdraw introduced another element which would have to be taken into consideration. It had been suggested by Mr. Lidstone that he (Mr. Manly) could have made his tables simpler and his work very much lighter by assuming that the payments took place in the middle of the year, and so avoiding the introduction of adjustments. In this paper it was assumed that they all took place in the middle

of the year, but he did not think that Mr. Lidstone had tried the problem of ascertaining the return of contributions based on salary, on withdrawal, or retirement. If they tried to value those benefits in the middle of a year they got mixed up with half-year's salaries, and it became a very awkward problem indeed. He found it work very much better to assume that the members went out exactly at the end of the year, and to make the adjustment afterwards. When they had simple benefits, such as in this paper, he agreed that it was easier to assume the payments to be made in the middle of the year. He would like to refer to his table number 45, that which he had called his working table, because he was not altogether satisfied with it. It did not altogether represent what it was intended to represent. He had said that it had been assumed that only first marriages had been allowed for; that no second marriages were allowed for, and that there were no marriages after sixty-five. But, having distributed the married men into husbands and widowers, according to the census returns, he felt that there certainly must be second marriages in them, but how many he could not tell. Further, he was unable to tell exactly whether the method he had adopted of extending the table from age sixty-five to the end of life had given him exactly what he wanted, namely, the numbers dying out of first marriages only. The supplementary tables had settled those points. The husbands dying in table 45 were more numerous than the husbands dying from first marriages in table 58, and took in, roughly, half the second marriages, and that applied up to the age of seventy-five. After seventy-five the husbands dying in table 45 were less than the deaths of husbands of the first marriage in table 58, which was calculated scientifically. He fancied that the increase in the husbands dying in the first part of the table and the decrease in the latter part arose entirely from their having taken the census returns. Taking the whole population, the marriages were earlier than in the class with which they were likely to deal, and were less in the later part of life. They knew that the working classes really attained to their maximum wage-earning power when they had reached maturity. From 22 to 24 or 25 was the time at which they earned best wages and the time when they married; but after 55 or 60 they were not able to earn wages enough to keep a wife. If they had children, the children generally kept them, but the children would not keep another mother if they could help it. He thought this explanation accounted for the difference in the two tables, coupled with the fact that the census returns included second and subsequent marriages.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1902, published in 1903.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
Balance at the beginning of the Year	£ 248,876,376	£ 21,751,565	£ 270,627,941
Adjustments for one return discontinued (—£35,877); and for funds carried to other accounts on transfer of business (—£145,035)	—180,912	...	—180,912
	248,695,464	21,751,565	270,447,029
Premiums	22,458,422	9,611,129	32,069,551
Consideration for Annuities	1,981,605	84,049	2,065,654
Interest and Dividends (less Tax)	9,120,587	713,148	9,833,735
Increase in value of Investments	215,717	165	215,882
Fines, Fees, &c.	13,943	1,051	14,994
Capital Paid-up	7,753	6,910	14,663
Customs Timber Measuring, &c.	4,247	...	4,247
Transfers from other Accounts	1,139,006	67,136	1,206,142
Miscellaneous	28,772	2,181	30,953
	283,665,516	32,237,334	315,902,850
OUTGO	Ordinary Companies	Industrial Companies	TOTAL
Claims	£ 16,580,897	£ 3,738,426	£ 20,319,323
Cash Bonuses and Reduction of Premiums	1,200,496	...	1,200,496
Surrenders	1,291,970	41,108	1,333,078
Annuities	1,862,835	76,364	1,939,199
Commission	1,186,379	2,409,366	3,595,745
Expenses of Management	1,897,197	1,725,505	3,622,702
Bad Debts	5,157	369	5,526
Decrease in value of Investments	285,362	1,464	286,826
Interest on Capital and Dividends and Bonuses to Shareholders	309,483	518,600	828,083
Transfers to other Accounts	1,061,154	420,553	1,481,707
Miscellaneous	184,261	2,587	186,848
Balance* at the end of the Year	257,800,325	23,302,992	281,103,317
	283,665,516	32,237,334	315,902,850

* This Balance includes the whole of the Life and Annuity Funds (£277,365,922), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1902).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . . .	11,312,110	1,777,619	13,089,729
Life and Amnity Funds . . .	255,315,445	22,050,477	277,365,922
Fire Funds of Companies trans- acting Life Business . . .	10,883,652	...	10,883,652
Marine Funds of Companies trans- acting Life Business . . .	736,050	...	736,050
Reserve Funds . . .	4,110,015	903,130	5,013,145
Other Funds . . .	2,090,539	227,637	2,318,176
Profit and Loss Balances . . .	3,897,381	8,335	3,905,716
Depreciation and Investment Bal- ances . . .	1,416,845	24,326	1,441,171
Globe Annuitants (Liverpool and London) . . .	1,102,800	...	1,102,800
Outstanding Claims . . .	4,255,447	66,689	4,322,136
Outstanding Accounts . . .	674,533	42,656	717,189
Temporary Loans . . .	385,000	...	385,000
	296,179,817	25,100,869	321,280,686

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages . . .	84,120,226	2,494,955	86,615,181
Loans on Policies . . .	14,003,827	45,365	14,049,192
„ Rates . . .	25,733,305	9,328,956	35,062,261
British Government Securities . .	7,067,715	2,051,732	9,119,447
Indian and Colonial Government Securities . . .	19,136,041	266,496	19,402,537
Foreign Government Securities . .	10,313,809	338,444	10,652,253
Debentures . . .	52,002,844	2,109,054	54,111,898
Shares and Stocks . . .	35,749,800	146,662	35,896,462
Companies' own Shares . . .	626,339	...	626,339
Land and House Property and Ground Rents . . .	23,690,477	6,550,882	30,241,359
Life Interests and Reversions . .	8,117,439	2,016	8,119,455
Loans on Personal Security . . .	1,426,719	9,398	1,436,117
Agents' Balances and Outstanding Premiums . . .	5,727,543	648,802	6,376,345
Outstanding Interest . . .	2,733,957	222,395	2,956,352
Cash, Deposits, Stamps, &c. . .	5,378,064	324,424	5,702,488
Customs Timber Measuring Ba- lances, &c. . .	2,245	...	2,245
Deficiencies, Preliminary Expenses, &c. . .	349,467	561,288	910,755
	296,179,817	25,100,869	321,280,686

INCREASE (+) or DECREASE (−) in the Chief Items of this Year's SUMMARY as compared with the corresponding Items for the previous Year.

	Ordinary Companies	Industrial Companies
INCOME.	£	£
Premiums	+ 662,803	+ 315,082
Consideration for Annuities	+ 265,578	+ *77,063
Interest and Dividends (less Tax)	+ 210,234	+ 36,968
Net Result of Realization and Re-valuation of Investments	− 82,964	+ 7,566
OUTGO.		
Claims	− 356,779	− 128,111
Annuities	+ 87,042	+ *68,506
Surrenders	+ 166,515	+ 3,049
Commission	+ 19,814	+ 103,524
Expenses of Management	+ 71,681	+ 85,454
LIABILITIES.		
Paid-up Capital (including sundry Share- holders' Balances)	− 417,224	+ *106,329
Life and Annuity Funds	+ 9,185,642	+ 1,571,974
ASSETS.		
Mortgages (including Loans on Rates)	+ 2,047,715	+ 1,370,774
Life Interests and Reversions	+ 657,113	+ 3
Loans on Policies	+ 815,447	− 732
British Government Securities	+ 1,254,946	+ 146,171
Indian and Colonial Government Securities	+ 167,294	− 95,577
Foreign Government Securities	− 135,317	− 77,545
Debentures	+ 2,090,844	+ 26,273
Shares and Stocks	+ 744,224	+ 41,029
Companies' own Shares	− 3,727	...
Land and House Property and Ground Rents	+ 847,613	+ 411,751
Loans on Personal Security	+ 146,576	+ 1,457

* Increase due to inclusion of figures of Nelson & Co.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 94, of which 76 have been classed as Ordinary, 11 as Industrial, and 7 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately.

During the year three names have been removed from the Official List of Companies, namely, Absolute, which is in liquidation; Lancashire; and United Kent Life; in which cases the business was transferred. And four names have been added, namely, Bells Stores, Limited; British Endowment Tea, Limited; British Widows' Assurance Company, Limited; and the Canada Life Assurance Company; in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

SUMMARY OF THE ASSURANCES IN FORCE, as shown by the last Returns of the Companies.
ORDINARY BUSINESS.

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur-	Net
	No.	Amount	No.	Amount	No.	Amount	ances Amount	Amount
ASSURANCES.		£		£		£	£	£
Whole Term of Life	800,158	352,478,330	153,902	72,416,069	954,060	454,894,399	28,403,918	426,490,481
Limited number of								
Premiums . . .	49,391	31,591,451	10,935	4,724,250	60,326	36,315,701	1,940,941	34,374,760
Endowments . . .	849,549	414,069,781	164,837	77,140,319	1,014,386	491,210,100	30,344,859	460,865,241
Endowment Assur-	1,473	386,190	18,932	5,110,868	20,405	5,497,058	12,000	5,485,058
ances . . .	913,625	152,444,398	59,903	18,925,043	973,528	171,369,441	2,654,532	168,714,909
Joint Lives . . .	16,453	3,344,528	2,825	1,052,191	19,278	4,396,719	250,116	4,146,603
Last Survivor . . .	557	708,190	1,005	1,479,489	1,952	2,187,679	393,447	1,794,232
Contingent . . .	37	70,363	4,247	6,363,981	4,284	6,434,344	1,524,425	4,909,919
Issue . . .	11	25,250	1,518	5,510,379	1,529	5,535,629	1,707,772	3,827,848
Miscellaneous . . .	2,525	1,340,238	8,777	12,128,815	11,302	13,469,053	1,851,412	11,617,641
	1,784,530	572,388,938	262,134	127,711,067	2,046,664	700,100,005	38,738,563	661,361,442
ANNUITIES.								
Immediate	35,007	1,831,589	51,707	1,779,882
Deferred	12,737	372,103	24,919	347,184
	47,744	2,203,692	76,626	2,127,066

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur-	Net
	No.	Amount	No.	Amount	No.	Amount	ances Amount	Amount
ASSURANCES.						£	£	£
Whole Term of Life	19,108,774	185,200,131	1,180	185,198,951
Limited number of								
Premiums	1,343	8,143	...	8,143
Endowments	19,110,117	185,208,274	1,180	185,207,094
Endowment Assur-	1,426,042	12,918,585	...	12,918,585
ances	305,521	3,416,468	88	3,416,380
Joint Lives	374,820	6,002,484	...	6,002,484
Contingent	4	1,910	400	1,510
Miscellaneous	2	2,100	1,000	1,100
	21,216,506	207,549,821	2,668	207,547,153
ANNUITIES.								
Immediate	65	2,034	...	2,034
Deferred	7	178	...	178
	72	2,212	...	2,212

The above figures are based on Returns deposited, for the most part, during the last five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of two Companies, namely, the Customs Fund and the Northern, the amount of business at a more recent date has been included. The figures of the Colonial and Foreign Companies have been excluded, as their Returns do not separately show the extent of business in the United Kingdom.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, APRIL 1903.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART I).*Examiner*—PROF. S. L. LONEY, M.A.*Supervisors*—MESSRS. S. G. WARNER, and L. F. HOVIL.*First Paper.*

1. A man who owns £20,000 L.N.W. 4 per-cent. Preference Stock sells at 126 (per £100 Stock), and invests the proceeds in Local Loans 3 per-cent. Stock at 101; allowing $\frac{1}{2}$ per-cent. brokerage on each transaction, find to the nearest penny the change in his income.

2. A and B start together from the same point upon a walking match round a circular course. After half an hour A has walked three complete circuits, and B four and a half. Assuming that each walks with uniform speed, find when B next overtakes A.

3. Solve the equations—

$$(1) \quad \frac{75-x}{3x+3} + \frac{80x+21}{5(3x+2)} = \frac{23}{x+1} + 5;$$

$$(2) \quad \begin{cases} x^2 - 2y^2 = 16, \\ \sqrt{x+y} + \sqrt{x-y} = 4 \end{cases}$$

4. Show that the arithmetic, geometric, and harmonic means between two positive quantities form a descending geometric progression.

If a^2 , b^2 , and c^2 are in arithmetic progression, show that $b+c$, $c+a$, and $a+b$ are in harmonic progression.

5. Find the number of permutations of n things taken all together, which are not all different.

Of n things p are alike and the rest all different; find the number of permutations that can be made of the n things taken $n-2$ at a time.

6. Obtain in their simplest forms the coefficients of x^n in the expansions of $(1-x)^{-5}$ and $(1-x)^{\frac{5}{3}}$.

Find the coefficient of x^{20} in the expansion of

$$(1+x+x^2+\dots+x^{10})^5.$$

7. State and prove the rule for writing down the characteristic of the logarithm of any number.

Making use of the tables, find the value of

$$\sqrt[3]{\frac{64.935 \div \sqrt[5]{.00037}}{(48.73)^{\frac{1}{4}} \times (.029)^3}}$$

8. Sum the series

$$(1) \quad \frac{1}{1.2.3} + \frac{5}{2.3.4} + \frac{9}{3.4.5} + \frac{13}{4.5.6} + \dots \text{ to } n \text{ terms ;}$$

$$(2) \quad \frac{x}{(1-x)(1-x^3)} + \frac{x^3}{(1-x^3)(1-x^5)} \\ + \frac{x^5}{(1-x^5)(1-x^7)} + \dots \text{ to infinity, where } x < 1.$$

9. If there are two independent events the respective probabilities of which are known, find the probability that one will happen and one will not.

A man has two sovereigns and four shillings in his pocket; from these he selects at random 3 coins, and again chooses from these three coins at random, the first coin for A, the second for B, and the third for C. Find the values of the expectations of A, B, and C.

10. The odds in a given race against three horses are 11 to 4, 13 to 3, and 7 to 2 respectively. Find the chance that one of them should win the race, a dead-heat being assumed to be impossible.

11. Prove the formula

$$\Delta^n u_x = u_{x+n} - nu_{x+n-1} + \frac{n(n-1)}{1.2} u_{x+n-2} + \dots + (-1)^n u_x.$$

Given that $u_0 = 66566$, $u_1 = 65152$, $u_2 = 63677$, and $u_3 = 62136$, find the value of u_9 .

12. Prove Lagrange's Interpolation formula, viz. :—

$$u_x = u_a \frac{(x-b)(x-c) \dots (x-k)}{(a-b)(a-c) \dots (a-k)} + u_b \frac{(x-a)(x-c) \dots (x-k)}{(b-a)(b-c) \dots (b-k)} \\ + \dots + u_k \frac{(x-a)(x-b) \dots}{(k-a)(k-b) \dots}.$$

Find the simplest algebraic expression in x which has the values 5, 3, 9, 47, and 165 when x has the values 0, 1, 2, 3, 4 respectively.

Second Paper.

13. By selling eggs at 15*d.* per dozen, a man gains 20 per-cent; if he sold them at 16*d.* per dozen, how much per-cent. would he gain?

14. Simplify the expressions—

$$(1) \quad \frac{x^3 + 2x^2 - 29x - 30}{x^3 - 3x^2 - 34x + 120} ;$$

$$(2) \quad \frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-a)(b-c)} + \frac{c^3}{(c-a)(c-b)}.$$

15. Explain how and why a meaning is given to x^n when n is a fraction or a negative quantity, and obtain the meaning of $x^{\frac{1}{2}}$ and $x^{-\frac{1}{2}}$.

Find a rationalizing factor for $x^{\frac{1}{2}} - y^{\frac{1}{2}}$.

16. Find the sum of the squares of the first n natural numbers.

Show that the sum of n terms of the series, whose r th term is $1^2 + 2^2 + \dots + r^2$, is

$$\frac{n(n+1)^2(n+2)}{12}.$$

17. Find the number of combinations of n things, taken r together, without assuming the formula for permutations.

Show that in the number of combinations of 10 things, taken 4 together, any particular thing occurs in two-fifths of the whole number.

18. Prove the truth of the Binomial Theorem for a positive integral exponent.

Show that the coefficient of x^r in $(1+x)^{n+1}$ is equal to the sum of those of x^r and x^{r-1} in $(1+x)^n$.

19. Given an algebraic fraction whose denominator is of the form $(x-a)(x-b) \dots (x-k)$, show how to resolve it into a sum of fractions whose denominators are $x-a$, $x-b$, \dots $x-k$, giving the simplest rule for the calculation of the corresponding numerators.

Find the general term in the expansion of $\frac{1}{(1-x)(1+x)^2}$ in powers of x .

20. State the Rule of Proportional Parts, giving the limits within which it is true.

Given $\log 52347 = 4.7188918$ and

$\log 52348 = 4.7189001$,

find $\log 5234729$, and the number whose logarithm is $\bar{1}.7188977$.

21. Prove the Exponential Theorem.

Show that the expansion of $\log \frac{1+x+x^2}{1-x+x^2}$ in powers of x is a series of odd powers of x in which the coefficient of x^n is $\frac{2}{n}$, unless n be a multiple of 3, when it is $-\frac{4}{n}$.

22. Find the general solution in positive integers of the equation $ax + by = c$.

A man spent £477 in buying horses at £29 each, and cows at £17 each; how many of each did he buy?

23. A die, with six faces, is thrown three times, and the sum of the throws is 12; find the chance (1) that the first throw was a four; (2) that four was thrown each time.

24. Prove that—

$$u_0 + u_1 + u_2 + \dots + u_n \\ = (n+1)u_0 + \frac{(n+1) \cdot n}{1 \cdot 2} \Delta u_0 + \frac{(n+1) \cdot n \cdot (n-1)}{1 \cdot 2 \cdot 3} \Delta^2 u_0 + \dots$$

Apply this theorem to find the sum of the series

$$1^2 + 2^2 + 3^2 + \dots + (n+1)^2.$$

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART II).

Examiners—MESSRS. H. J. BAKER, A. D. BESANT, B.A., J. SPENCER,
and A. W. TARN.

First Paper.

1. Assuming one rate of interest throughout, obtain prospectively and retrospectively the value of a Capital Redemption Policy of 1 taken out n years ago for a period of t years at an annual premium of P_t , and prove the identity of the two expressions.

art 33 page 24
see also
art 42 page 34

Calculate the net level annual premium for a capital redemption assurance of £100 payable at the expiration of 50 years assuming $3\frac{1}{2}$ per-cent. interest for the first 10 years, 3 per-cent. for the next 20 years, and $2\frac{1}{2}$ per-cent. thereafter.

2. A loan of £460 is repayable by 10 annual instalments of principal and interest, commencing at £100 and decreasing by £10 annually.

Assuming that the lender treats the matter as a 5 per-cent. investment for the first three years, find

- (1) The capital outstanding immediately after the third instalment has been paid.
- (2) The rate of interest yielded on the investment during the remainder of the term of repayment.

3. Define l_x , d_x , p_x , m_x , μ_x , and prove that

$$m_x = q_x + \frac{(q_x)^2}{2} + \frac{(q_x)^3}{4} + \dots$$

What are the approximate conditions under which

$$\mu_x > = < q_x ?$$

4. What are Select Mortality Tables?

Express in Commutation symbols

$$P_{[x]:\bar{n}}, \quad A_{[x]:[y]}, \quad t \cdot A_{[x]}, \quad nV_{[x]:t}.$$

5. C and D have the perpetual right of alternate presentation to a living of the annual value of £ k . Assuming that C has the next right of presentation, that the present incumbent is aged y , and that his successors will all be aged x on appointment, find the sum which C should pay D to purchase his interest.

6. Explain clearly what is meant by "Policy-Values", and describe the retrospective method of obtaining them.

If at each age in a given mortality table $A_x = \frac{x}{100}$, find the value of a whole-life policy issued at a yearly premium $15\frac{1}{2}$ years ago on a life then aged 35.

You may assume the rate of interest to be such that $d = .04$.

7. Prove mathematically that

$$a_{\overline{e}|x} > a_x,$$

and, hence, assuming uniform distribution of deaths, show that

$$\overline{A}_x > v^{e_x}.$$

8. Deduce the annual premium, at rate of interest i , for a deferred annuity-due payable during the life of x to commence n years hence, all premiums paid to be returned with compound interest at the rate j in the event of death occurring before the annuity commences.

9. Explain the principle of uniform seniority. To what mortality tables does it apply? - [22] 1244 and [32]

Having given $\log_{10} e$, how would you construct a table of the values of t so that $a_{x:x+h} = a_{x+t:x+t}$ at all rates of interest?

5.649 30 p 346 10. Show how you would construct from a table of l_x a table of annuity-values, and from these show how to deduce the corresponding continuous annuities.

§ 39 p 232 11. Show that, if Makeham's law holds,

$$\overline{A}_{xy} = \frac{1}{1 + c^{y-x}} \cdot [\overline{A}_{xy} + (1 - c^{y-x}) \cdot \log_{es} \cdot \overline{a}_{xy}].$$

12. Find the value of $P_{44:52}$ as accurately as possible from the following data:—

$$P_{40:50} = \text{£}5. 16s. 9d. \quad P_{45:50} = \text{£}6. 3s. 9d.$$

$$P_{40:55} = \text{£}6. 13s. 11d. \quad P_{50:55} = \text{£}6. 14s. 4d.$$

$$P_{40:60} = \text{£}7. 17s. 8d. \quad P_{45:55} = \text{£}7. 0s. 1d.$$

Second Paper.

5.649 p 11 - 13. Assuming that a purchaser desires to realize interest at the rate i , investigate a formula for determining the price to be paid for a bond of 1 securing an annual dividend of j and redeemable at a premium n years hence.

£100 irredeemable stock bearing an annual dividend of £4 was bought 20 years ago for £90. Find at what figure it must now be sold in order that the vendor may realise 5 per-cent. per annum on the transaction.

14. Find, in terms of the nominal yearly rate, the value of an annuity-certain for n years:

- (i) payable momentarily, and interest convertible half-yearly;
- (ii) payable m times a year, and interest convertible momentarily.

✓ Prove that $a_n^{(2)} = \left(1 + \frac{i}{2}\right) a_n$ approximately.

15. In a population of 1,000,000, hitherto stationary, the birth-rate begins to increase at the rate of 1 per-cent. per annum. What is the population at the end of three years, assuming a uniform distribution of births and deaths throughout the year?

16. Prove the following formulæ, and give a verbal interpretation in each case:

✓ (1) $A_{x+n} = {}_nV_x(1 - A_x) + A_x$

(2) $a_{x:n} = \frac{1 - A_{x:n+1}}{d} - 1.$

17. A select life x desires a contingent assurance against y , with the proviso that, if he be alive at the death of y , he shall have the option of converting his policy, as at the next renewal date, into a whole-life assurance at the ordinary annual premium applicable to his then age. Obtain an expression for the net annual charge required for this option.

18. Prove that

$${}_nV_{xt} = \frac{a_{xt}}{a_{xt}} \cdot a_{x+n:t-n} - a_{x+n:t-n}$$

and that ${}_nV_{xt} = v^{t-n} p_{x+n} \cdot \frac{a_{xn}}{a_{xt}}.$

Show that the sum of these expressions $= {}_nV_{xt}.$

✓ 19. In consideration of a yearly premium of $\frac{v^{20}}{a_{x:20}}$ an Assurance Company offers a life aged x a policy securing a sum of 1 payable at the expiration of 20 years, if x be then alive; and a sum of S payable 15 years after his death if this event take place during the 20 years. Find the value of S .

20. Prove that $\hat{a}_x = a_x + \bar{A}_x \frac{i - \delta}{i\delta}$ approximately, and write down an analogous equation giving the value of $\hat{a}_x^{(m)}.$

21. A life office desires to investigate its mortality experience since its commencement. Describe how you would deduce an aggregated unadjusted table of q_x .

22. Having given the values of $\log D_x$ at all ages, how would you obtain, by means of Gauss's Logarithms, a table of a_{xx} at the same rate of interest?

✓ Describe the method you would adopt for the construction and verification of a table of $a_{xn}.$

23. Obtain by means of the calculus of finite differences an accurate expression for the value of $\frac{1}{i} a^{(2)}_x$.

24. Write down the integrals of

$$\frac{1}{x^2} \log x, \quad \bar{A}_{xy}, \quad \bar{A}_{xyz}, \quad \bar{a}_{yz:x}^2.$$

Calculate by means of a formula of approximate summation the numerical value at 3 per-cent. interest of $\bar{A}_{62:70}^1$.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III).

Examiners—MESSRS. O. KENTISH, E. A. RUSHER, R. TODHUNTER, M.A.,
and J. D. WATSON.

First Paper.

1. Specify the various methods that have been employed for the determination of the ages at entry in the compilation of the mortality experience of assured lives, and state, with reasons, which method you consider best.

If, in a life office experience the total number of entrants at ages x and $x+1$ next birthday were equal, and if, on examination of the exact ages of the entrants, the number entering at each exact age were found to be proportional to the fraction of a year elapsed since the last integral age, show that the determination of the age at entry by the "nearest age" method would have the effect of overstating the average age of the entrants at nearest age x by one month.

2. Specify the various sections into which the British Offices Mortality Experience, 1893, is divided, and indicate the general character of the mortality shown in each section.

How were the Withdrawals scheduled in the compilation of the Experience?

3. What are the objects of graduation, and what tests would you apply in considering the merits of a given graduation of a table of the values of q based on a given experience?

Investigate the effect of graduating by Woolhouse's method a table of the values of $\log p_x$ which could be accurately represented by Makeham's formula.

4. Describe Makeham's method of graduation. What are the practical difficulties in its application?

Show how the method may be used for the purpose of graduating a small experience.

5. What table or tables of mortality would you use in making the periodic valuations of (a) An Ordinary Life Assurance Company; (b) An Industrial Life Assurance Company?

State, with reasons, in what circumstances you would consider it proper to value office premiums (less a percentage) instead of net premiums, and what precautions you would take to exclude negative values.

6. A Company grants with-profit whole-life policies at premiums reduced by anticipation of a simple reversionary bonus (quinquennial and interim) of 1 per-cent. per annum, the reductions being calculated by the $H^M 4\frac{1}{2}$ per-cent. Table. What valuation-formula would, in your opinion, most closely conform to the principles of an $H^M 3$ per-cent. valuation as applied to these policies, the average renewal date being eight months after the date of valuation? How would you calculate surrender values for policies of this description, and on what terms would you convert them into ordinary policies entitled to full profits from the date of conversion?

7. In making an $H^M 2\frac{1}{2}$ per-cent. net premium valuation how would you deal with whole-life policies subject to limited premiums—the office rates having been calculated by commutation of the ordinary whole-life rates by the H^M (Select) $3\frac{1}{2}$ per-cent. Table?

8. Given a summary of the valuation of a Company's whole-life assurances, showing for each valuation-age the totals of the sums assured, the bonuses and the $H^M 3$ per-cent. net premiums, and their respective values by the $H^M 3$ per-cent. Table, how would you estimate roughly the extra reserve required on an $H^M 2\frac{1}{2}$ per-cent. basis?

9. How would you deal in a valuation with assurances and re-assurances participating in the profits of other companies?

In what circumstances (if any) would you consider it inadvisable to guarantee the bonus of another company?

10. State how you would endeavour to determine in practice whether or not an extra should be charged on account of a given occupation, and what the extra (if any) should be, indicating any available sources of information known to you.

Would you take into account the possibility that a proposer engaged in a hazardous occupation, who had passed the necessary medical and other tests in connection with a proposal for assurance, might be a better life than the average of his class?

11. Upon what bases as to mortality, interest and loading would you calculate office non-profit premiums for:

(a) Joint whole-life assurances;

(b) Reversionary annuities at annual premiums?

12. On the death of the survivor of A, aged 70, and B, aged 75, C, aged 25, if then living, will become entitled, in the event of B (who is married but childless) leaving no issue, to an estate of the estimated value of £100,000. He wishes to raise £10,000 on security of his reversion without incurring any liability for payment of interest, &c. unless and until he comes into possession of the estate, and to retain the right to redeem at any time on payment of principal, premiums, and costs, with compound interest. Draft a letter indicating the lines on which you would be prepared to deal with the matter.

Second Paper.

13. Given (a) the numbers of widows entering on observation at various ages last birthday from 20 to 60, (b) the numbers thereof passing out of observation at mean ages by death or re-marriage, and the numbers remaining under observation on a given 31st of December, (c) that the possibility of re-marriage after age 60 may be neglected, and that the mortality from age 60 onwards may be taken to be in accordance with that shown by a standard table, explain fully how you would construct on the basis of the experience a table of annuity-values suitable for valuing annuities payable during widowhood to the widows remaining on the books at a given date.

14. Sketch a convenient form of register for keeping a continuous record of the mortality experience of a Life Assurance Company.

15. What arguments might be advanced for and against the adoption of the British Offices' Life Tables, 1893, or one of them, for the valuation of the liabilities of a Life Assurance Company whose recent valuations have been made by the H^M and $H^{M(5)}$ Tables at 3 per-cent.?

What special considerations (if any) would arise in the case of a Company whose new business had recently largely increased, and which had maintained for many years a fairly large compound reversionary bonus which it is not desired to increase temporarily?

16. Discuss generally the effect of Discontinuances upon the mortality experience of a Life Assurance Company.

17. What is the distinguishing characteristic of the Contribution Method of distributing surplus? Specify the points of difference between Dr. Sprague's and Mr. Browne's modifications of the method.

18. On the amalgamation of the business of two Companies having the same valuation basis and method of distribution, but different scales of with-profit premiums, it is decided to transact future business at the rate of one of the two Companies and to continue the method of distribution employed by both Companies before the amalgamation. How would you allow for the difference in the premium rates in distributing the surplus if the method of distribution were (1) the Contribution Method, (2) the Simple Reversionary Bonus Method?

19. A Company distributes its surplus quinquennially by the Simple Reversionary Bonus Method. What points should be considered in deciding whether the same or different rates of bonus should be declared in respect of whole-life assurances and endowment assurances, and in fixing the relative proportion of the rates, if different?

20. In distributing its surplus a Company gives its policyholders (whether participating for the first time or not) the option of taking (a) a reversionary bonus calculated on the sum assured and existing bonuses, (b) a cash payment, (c) a permanent reduction of premium, (d) a reduction of premium for five years. Assuming the valuation to have been made on an H^M and $H^{M.5}$ 3 per-cent. basis, and the rate of reversionary bonus to have been fixed, on what bases would you calculate the corresponding cash payment and reductions of premium?

21. What are the arguments for and against the adoption of a table of with-profit premiums reduced by anticipation of bonus? Specify and give the distinguishing features of the different tables of this description now in use, and state which of them (if any) appear to you to be least open to the objections commonly urged against such systems in general.

22. What points would you bear in mind in deciding upon the methods and bases to be employed in the calculation of surrender-values for whole-life and endowment assurance with-profit policies?

A whole-life policy for £1,000, with bonus additions of £300, effected 20 years ago on a life then aged 24, at an annual premium of £21, is to be converted into a paid-up non-profit policy. The life assured is about to incur an extra risk (of a permanent character), for which an extra premium of £1 per-cent. would ordinarily be charged. How would you fix the amount of the paid-up policy?

23. Give the formulæ and the rates of interest, mortality and loading, which you would adopt in the calculation of rates, in the case of a life aged 30, for—

(a) A 15-year term assurance.

(b) A 15-year term assurance, with the option during the whole period of converting the policy into a whole-life or endowment assurance policy, at the rate for the age attained.

24. Value for purchase the absolute reversion expectant on the death of a female aged 63, to one-third (less £300) of the following fund:—

A paid-up non-profit policy for £1,000 on a male life aged 60.
£1,000 Queensland 4 per-cent. bonds, 1915.

Freehold property in London, let to weekly tenants at gross rentals of £200 per annum.

£1,300 (of which the above-mentioned sum of £300 forms part) to be brought into hotchpot by beneficiaries.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART IV).

Examiners—MESSES. T. G. ACKLAND, J. BLAKEY, J. E. FAULKES, B.A.,
and W. O. NASH.

First Paper.

1. State what is meant by each of the expressions "sub-mortgage", "contributory mortgage", "statutory mortgage", "stock mortgage", and write a short note on the doctrine of clogging the equity of redemption, giving some instances of agreements constituting a clog.

2. State how the position of the law as to the assignment, at the present time, by a married woman of her reversionary interests in personal property, varies according to date of marriage and date of accrual of title.

3. State fully, giving reasons, how you would deal with the case of a lost policy (*a*) during the currency of the contract (*b*) where the loss is discovered only on a claim arising.

4. State concisely the law as to insurable interest in connection with life policies. Draft a question dealing with the point for insertion in a "life of another" proposal form.

Criticise, in the light of a recent decision, the following statement: The value of the interest of an employer in the life of his servant does not depend upon the amount paid as wages, but on the expectation of pecuniary advantage to the employer from the engagement.

5. An Employees' Pension Fund is to be established on the basis of a percentage contribution on the salary, to be paid by the members, varying according to age at entry, and a fixed percentage of the salary, to be paid by the employer. The benefits include a pension, according to term of service, vesting at a fixed age or on earlier disability, and a return of the employee's contributions only, without interest, in the event of death or resignation before the grant of pension. It is proposed that, on the establishment of the Fund, employees should be allowed the option of reckoning past years of service, upon payment, both by employer and employee, of the *past* contributions which would have been payable if the member had entered the Fund at the commencement of his term of service, with compound interest thereon. State the grounds for your objections, if any, to this proposed arrangement.

6. What data would you require for the purposes of the valuation of a Friendly Society granting

- (*a*) funeral benefit on death of member
- (*b*) funeral benefit on death of member's wife
- (*c*) full sick-pay
- (*d*) reduced sick-pay

where it is desired to base the valuation upon the actual experience of the Society as to mortality and sickness during an observed period of ten years? Draft a form of card for conveniently recording the required data.

7. Enumerate the various Statements and Returns to be made under—

(a) the Companies Acts 1862 to 1900

(b) the Life Assurance Companies Acts 1870 to 1872

by a Limited Liability Company fully established and transacting life assurance business only.

8. Give particulars of the agreement existing between any one of the Indian Railway Companies and the Indian Government.

9. In what order do the following British Railway Securities rank :—

Preference Stock

Preferred Ordinary Stock

Debenture Stock

Guaranteed Stock

Lloyd's Bonds

Consolidated Ordinary Stock ?

Give your opinion of each as an investment for Life Office funds, dealing particularly with the immediate yield, and the prospect of there being a profit or loss on realization.

10. Under what conditions may Colonial Government Stocks registered in the United Kingdom be constituted Trustee Investment Securities ?

11. State the points to be considered by a Municipal Corporation which has to decide whether it shall issue a loan in stock to the public at a fixed price, or by tender with a stated minimum price ; and if the issue shall be underwritten.

12. State the items of National Revenue derived from the currency system of the United Kingdom.

How may the Bank of England deal with its notes which have been issued for more than forty years and not presented for payment ?

Second Paper.

13. Describe the method or methods in which a mortgage of leasehold property may be carried out.

In the case of a mortgage of leaseholds under the Conveyancing Act 1881, what covenants are implied by a mortgagor conveying as beneficial owner ?

14. What is meant by (i) administration *durante absentia*, (ii) administration *cum testamento annexo*, (iii) administration *de bonis non* ?

A, who effected a policy on B's life, predeceased B. A's legal personal representative also predeceased B. State whose discharge you would require on a claim arising, assuming that the policy has never been assigned, and bearing in mind that the expression "legal personal representative" covers both an executor and an administrator.

15. A life policy which has become a claim is presented for payment, duly discharged in accordance with the company's requirements, by the payee's solicitor. The discharge does not contain any express authority for payment to the solicitor, who, however, asks for the cheque to be made payable to him. Discuss the position, assuming the payee to be (i) an executor, (ii) a trustee, (iii) a mortgagee, (iv) a holder under an absolute assignment.

16. Give the provisions of the Customs and Inland Revenue Act 1888, as repealed and re-enacted by the Stamp Act 1891, specially referring to the stamping of assignments of life policies, and illustrate them by reference to (i) absolute assignments, (ii) mortgages. What are the present rates of stamp duty on (a) legal mortgages, (b) transfers of mortgage, (c) reconveyances? How would you deal in this connection with a payment under a life policy which has been mortgaged to a bank to secure an account current, no amount being mentioned in the deed?

17. A Pension Society grants annuities to its members, to commence on attainment of a fixed age, in consideration of periodical contributions from the members. After the Society has been in existence for some years, a valuation discloses a deficiency, which is mainly attributable to the mortality experienced having been materially lighter than that upon which the scale of members' contributions was based. Upon what general lines would you advise the Society to proceed, in order that it may ultimately be placed in a solvent position? Give separate consideration to the cases of (a) existing pensioners; (b) members whose pensions have not yet vested; (c) future members of the Society.

18. Give a brief outline of the Annual and Valuation Returns prescribed by the Friendly Societies Acts; and state in what manner you consider that the requirements of the Acts could be modified for the better attainment of the objects sought.

19. Draft a form of Life Policy Register. What additional books (other than Books of Account and Valuation Books) are required by a Mutual Life Office? Give a short description of each.

20. Discuss the relative merits of Local Loans Stock, India 3 per-cent. Stock, and Bank of England Stock, for the investment at the present time of the £20,000 deposit required by the Life Assurance Companies Act 1870.

21. Discuss the merits of (so called) permanent investments as compared with temporary investments, for the funds of a Life Office.

State to what degree the following classes of investment possess the quality of permanence; and give briefly your opinion as to their suitability for Life Office funds—paying attention to the immediate yield and the probability of an ultimate profit or loss on realization:—

- (a) freehold ground rents
- (b) freehold chief (or head) rents
- (c) mortgages of freehold land
- (d) brewery debentures
- (e) irredeemable stocks of British Corporations.

22. Draft a report to the Directors of a Life Office discussing the advisability of their commencing to transact the business of granting Loans on Personal Security. Assume that the Directors have wide powers of investment, and large funds at their disposal.

23. In the event of the Government not having sufficient funds in hand to provide for the payment of the dividends on the National Debt when they become due, how, and at what cost, is the necessary amount obtained?

24. What restriction exists against selling the shares of a Joint Stock Bank with the object of depreciating their market value? Why should Banking Companies be protected differently from other Companies in this respect?

PROCEEDINGS OF THE INSTITUTE.—SESSION 1902-1903.

First Ordinary Meeting, 24 November 1902.

The first ordinary meeting of the Session 1902-1903 was held at the Hall of the Institute, on the 24th day of November 1902.

The President (Mr. W. HUGHES) in the Chair.

The President delivered an inaugural address.

Second Ordinary Meeting, 29 December 1902.

The President (Mr. W. HUGHES) in the Chair.

Mr. Benjamin Bell Maxwell, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On the Comparative Reserves of Life Assurance Companies according to Various Tables of Mortality at Various Rates of Interest", was read in abstract by the author, Mr. George King.

The following gentlemen took part in the discussion:—Messrs. G. Green, R. P. Hardy, T. G. Ackland, E. C. Thomas, H. P. Calderon, H. W. Manly, and the President.

Third Ordinary Meeting, 26 January 1903.

The President (Mr. W. HUGHES) in the Chair.

A paper entitled "Temporary Assurances", was read in abstract by the author, Mr. W. Palin Elderton.

The following gentlemen took part in the discussion:—Messrs. G. E. May, G. King, R. Todhunter, H. P. Calderon, G. J. Lidstone, T. G. Ackland, and the President.

Fourth Ordinary Meeting, 23 February 1903.

Mr. R. P. HARDY (Vice-President) in the Chair.

A paper entitled "Further Remarks on the Valuation of Endowment Assurances in Groups", was read in abstract by the author, Mr. G. J. Lidstone.

The following gentlemen took part in the discussion:—Messrs. H. A. Thomson, G. King, C. D. Higham, T. G. Ackland, W. P. Elderton, T. E. Young, H. W. Manly, S. G. Warner, and the Chairman.

Fifth Ordinary Meeting, 30 March 1903.

The President (Mr. W. HUGHES) in the Chair.

A paper entitled "The Mortality Experience of the Imperial Forces during the War in South Africa, 11 October 1899 to 31 May 1902", by Messrs. F. Schooling and E. A. Rusher, was read in abstract by Mr. Schooling.

The following gentlemen took part in the discussion:—Messrs. A. G. Mackenzie, W. A. Workman, S. G. Warner, A. F. Burridge, H. W. Manly, A. Hewat, G. Todd, T. G. Ackland, and the President; also Surgeon-General W. F. Stevenson, M.B., C.B., Lieutenant-Colonel E. M. Wilson, C.B., C.M.G., D.S.O., and Major T. McCulloch, M.B. (visitors).

Sixth Ordinary Meeting, 27 April 1903.

The President (Mr. W. HUGHES) in the Chair.

A paper entitled "On the Valuation of Staff Pension Funds, Part 2, Widows' and Children's Pensions", by Mr. H. W. Manly, with Tables by Mr. H. Foot, was read in abstract by Mr. Foot.

The following gentlemen took part in the discussion:—Messrs. E. C. Thomas, G. J. Lidstone, G. King, and the President.

The Fifty-sixth Annual General Meeting, 8 June 1903.

The President (Mr. W. HUGHES) in the Chair.

The proceedings at the Annual General Meeting will be found on page 197.

REPORT, 1902-1903.

The Council have the pleasure to report to the members upon the progress of the Institute during the session of 1902-1903, the fifty-fifth year of its existence.

There has been a *decrease* of 14 in the number of members, as compared with the previous year. 115 candidates have been admitted as Probationers, and 51 as Students conditionally on their passing Part I of the Examination. At the end of the official year in which the Institute was incorporated by the Royal Charter the number of members was 434, while ten years later, at 31 March 1895, it was 775. Since that time the numbers have been as follows:

On 31 March 1896,	788,
„ 1897,	826,
„ 1898,	860,
„ 1899,	834,
„ 1900,	822,
„ 1901,	818,
„ 1902,	842,
„ 1903,	828.

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

Schedule of Membership, 31 March 1903.

	Honorary Members	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1902 .	1	216	268	332	25	842
ii. Withdrawals by						
(1) Death	1	2	1	1	86
(2) Resignation	2	6	11	...	
(3) Default in pay- ment of Sub- scriptions	8	54	...	
	1	213	252	266	24	756
iii. Additions to Membership						
(1) By Election	1	72
(2) By Order of Council	71	...	
(3) By Re-instatement	
	1	213	253	337	24	828
iv. Transfers						
(1) By Examination:						
<i>from Associates</i>	11
<i>to Fellows</i>	11
	1	224	242	337	24	828
(2) By Examination:						
<i>from Students</i>	1
<i>to Fellows</i>	1
	1	225	242	336	24	828
(3) By Examination:						
<i>from Students</i>	20
<i>to Associates</i>	20
v. Number of Members in each class on 31 March 1903 .	1	225	262	316	24	828

The Council have, with great regret, to report the loss by death of three Fellows, Mr. F. Laing, Mr. W. McCabe, and Mr. T. Y. Strachan; two Associates, Mr. C. Povah and Mr. Reuben Watson; one Student, Mr. H. G. S. Lawson; and one Corresponding Member, Mons. H. Pimentel.

The Annual Subscriptions, together with admission and other fees, amounted to £1,897. 17s. 6d., showing an increase of £116. 0s. 6d. as compared with those of the previous year. The total Income for the year was £2,803. 15s. 5d., and the total Expenditure £2,530. 15s. 5d. The Revenue Account and Balance Sheet are given herewith (p. 194).

The stock in hand of the Institute publications on 31 March was as follows :

No. of Copies	Description of Work
12,689	Parts of <i>Journal</i> .
509	Index to Vols. 1 to 10.
993	„ to Vols. 21 to 30.
1,278	<i>Text-Book</i> , Part I (New Edition).
1,597	„ Part II (Second Edition).
707	Government Joint-Life Annuity Tables.
783	Select Life Tables.
549	A Short Collection of Actuarial Tables.
263	Messenger Prize Essay (Friendly Societies).
49 <i>in cloth</i> } . . . {	Lectures on Finance and Law (Clare and
2,931 <i>in paper</i> } . . . {	Wood Hill).
1,694	Lectures on the Companies Acts (A. C. Clauson).
1,673	Lectures on the Law of Mortgage (W. G. Hayter).
873	Lectures on the Measurement of Groups and Series (A. L. Bowley).
721	Transactions of the Second International Congress of Actuaries.

The following papers were submitted at the sessional meetings of the Institute, namely :

- 24 *November* 1902.—An Inaugural Address by the President, Mr. W. Hughes.
- 29 *December* 1902.—“On the Comparative Reserves of Life Assurance Companies according to various Tables of Mortality at various Rates of Interest.”—Mr. George King.
- 26 *January* 1903.—“Temporary Assurances.”—Mr. W. Palin Elderton.
- 23 *February* 1903.—“Further Remarks on the Valuation of Endowment Assurances in Groups.”—Mr. G. J. Lidstone.
- 30 *March* 1903.—“The Mortality Experience of the Imperial Forces during the War in South Africa, 11 October 1899 to 31 May 1902.”—Messrs. F. Schooling and E. A. Rusher.
- 27 *April* 1903.—“Valuation of Staff Pension Funds: Part 2. Widows' and Children's Pensions.”—Mr. H. W. Manly. With Tables by Mr. H. Foot.

For the Examinations held in the United Kingdom and the Colonies on 17, 18, 20, and 21 April last, 266 entries were received, namely :

106	for Part	I.
88	„	II.
53	„	III.
19	„	IV.

The results of the Examinations will be duly announced.* The Council warmly acknowledge the valuable services of the Honorary Examiners and Supervisors.

* These results for the United Kingdom and the Colonies, combined, are given on page 195.

A series of six lectures on the Measurement of Groups and Series have been delivered by Mr. A. L. Bowley, Teacher of Statistics at the School of Economics in connection with the University of London. It was perhaps inevitable, from the nature of the subject, that these lectures should prove somewhat less attractive than some of the earlier series, which dealt with subjects of more general interest, and the attendance was consequently comparatively small. In their printed form, however, they will prove of the highest interest and value to students, more especially as the existing literature on the subject is not voluminous, nor easily accessible.

His Royal Highness the President of the Royal Commissioners of the Patriotic Fund has again asked for a valuation of certain of their Funds, including the Transvaal War Fund and the Indian Mutiny Relief Fund, which latter is in process of transfer to the Royal Commissioners of the Patriotic Fund. A Report on the Indian Mutiny Relief Fund has been presented, and the valuations of the other Funds are in progress.

The Fourth International Congress of Actuaries is to be held in New York in September next; Delegates have been nominated to attend on behalf of the Institute.

The labours of the Joint Committee of the Institute of Actuaries and the Faculty of Actuaries on Mortality Investigation are now almost concluded. During the year, the Committee have published two volumes of Mortality and Monetary Tables deduced from the graduated data; the one, comprising Aggregate Tables, based upon the experience of Whole-Life Participating Assurances Male Lives (O^M and $O^{M(5)}$); the other, based upon the experience of life annuitants, comprising Tables for single and joint lives, with discrimination of sex, set out in the form of Select or Extended Tables. The Mortality and Monetary Tables, based upon Mr. G. F. Hardy's Graduation of the Experience of Whole-Life Participating Assurances on Male Lives, in the form of SELECT Tables, (O^{MJ}), have been completed, and are in the printers' hands for early publication. This will form the seventh and final volume of the Tables published under the authority of the Joint Committee; and it is proposed to include in this closing volume some account of the processes and methods followed throughout the investigation, in the collation and graduation of the data, and in the computation of the deduced Tables.

The Council are pleased to announce that the Institute has entered on possession of the new Hall in Staple Inn Buildings. Accommodation is thus afforded for conducting the whole of the examinations on the Institute's own premises, and for lectures, classes and study generally, under more comfortable conditions than have been hitherto available.

EXAMINATIONS, 1903.

Examinations were held on the 17th, 18th, 20th, and 21st of April, 1903, in the United Kingdom and the Colonies, at London, Edinburgh, Dublin, Adelaide, Melbourne, Sydney, Wellington, Montreal, Toronto, and Ottawa, with the following results, the names in each class being arranged in alphabetical order:—

PART I.

One hundred and six candidates sent in their names, of whom ninety-nine presented themselves, and fifty-nine passed, namely:—

Class I:

Bradbury, A. C.	Macneill, M.
Cooper, J. J.	Myers, H. D.
Ebihara, K.	Smith, W.
Hancock, E. J.	Strong, A. W.
Harris, E. A.	Warren, L. A. H.
Kirsopp, F.	Wolfenden, E. S.

Class II:

Bain, W. A.	Neill, W. A. H.
Bennett, R.	Northcott, J. A.
Carpenter, T. B. B.	Reynolds, W. D.
Farmer, E. C.	Sneddon, A. W.
Gopp, J. I.	Stebbing, G. W.
Jones, E. S.	Story, C.
Macaulay, F. R.	Townley, E. W.
Macdonald, C. J. A.	Vance, J. G.
Maltby, C. H.	Wilson, A. B.

Young, H. J.

Class III:

Acum, W. H.	Jefferson, J. A.
Baggs, H. E.	Macdonald, C. S.
Binney, C. E.	Macdougall, G.
Bodley, R. F.	Manly, G. W.
Capon, F. C.	Martin, F. C.
Cheshire, H. F.	Oates, T. P.
Child, R. H.	Paterson, E. R.
Cotton, A. S.	Raynes, H. E.
Downes, E. G.	Ridgway, W.
Ellis, T. B.	Robertson, B.
Fulford, W. J.	Robinson, E. W.
Green, W.	Stanford, H. W.
Harley, B.	Suddaby, W. A.
Hill, F. W.	Watson, A. R. D.

PART II.

Eighty-eight candidates sent in their names, of whom eighty-four presented themselves, and sixteen passed, namely:—

Class I:

None.

Class II:

Baxter, E. H.	Humphreys, H. T.
Gemmill, W.	Shovelton, S. T.
Hall, J. B.	Wilkinson, W. M.

Class III:

^ Ferguson, C. C.	Nicholls, A. W.
Jones, L. A. Monat	Watherston, C. F.
Jones, W. Monat	Wilton, H. G.
May, W. T.	^ Wood, W. A. P.
^ Moore, G. C.	Worth, B. O.

PART III.

Fifty-three candidates sent in their names, of whom forty-seven presented themselves, and twelve passed, namely:—

Class I:

None.

Class II:

Brown, Henry		Curjel, H. W.
^ Papps, P. C. H.		

Class III:

Chandler, T. R.		Diver, O. F.
† Collins, F. L.		†* Jarman, W. R.
†* Cross, H. T.		† Rae, J.
†* Denham, W.		Rietschel, H. J.
Wilson, J. S.		

PART IV.

Nineteen candidates sent in their names, all of whom presented themselves, and seven passed, namely:—

Class I:

None.

Class II:

† Bacon, J.

Class III:

†* Cross, H. T.	† Norris, C. A.
†* Denham, W.	Symmons, F. P.
†* Jarman, W. R.	Wandless, J. R.

Those marked (†) have now completed the examination for the Class of Fellow, and (*) passed in both parts of the Final Examination.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Monday, June 8, the President, Mr. William Hughes, in the chair.

The Report of the Council (given on p. 190) having been read,

The PRESIDENT, in moving the adoption of the report and accounts, remarked that the number of members was nearly double that at the date of the charter; but there had been very little variation for the last seven years, and he supposed it might now be considered to have arrived at something like a stationary condition, and that it was not likely to vary very much from these numbers, unless the new movement which had been taking place in the University of London, and still later in the University of Cambridge (in the one case establishing a school, and in the other a Tripos for Economics and Social Science), might have the effect of directing further attention to those particular matters with which they were concerned, and might bring in a new class of students. That might have some effect, either in one direction or the other, upon their numbers. He was sure they would all join in the expression of regret with which the Council had to report the death of some of its members. Mr. Laing had not been an active member of the Institute for some time. Mr. McCabe's influence upon their profession had been very great, especially with reference to the educational part of it. He had taken a great deal of interest in promoting the study of actuarial matters among the young members of the staff of insurance offices in Canada. Mr. Strachan was better known to them, although perhaps not to the younger members of the Institute. His energies were mostly directed to matters connected with friendly societies and similar bodies in Newcastle and the North of England. It was not too much to say that Mr. Reuben Watson's influence upon the finances of friendly societies had been greater than that of almost any other man that could be mentioned. The next matter with which the report dealt was finance. Their funds still increased, notwithstanding some rather exceptional expenses that had been incurred during the year, and the total now stood at a rather considerable sum. Although some doubt had been expressed as to whether the Institute really needed such a considerable sum of money, he was sure the Council would not find it too much for what they had to meet in times to come. Among other things, the provision of the new room would necessitate a little further expense. It was not too much to say that the papers which were read last Session had been of more than usual interest. Three of them had been of special interest to the profession only, dealing as they did with very technical matters. He referred to Mr. King's paper, Mr. Elderton's, and Mr. Lidstone's. The other two papers were of a kind to interest people beyond the immediate circle of their profession. The paper by Messrs. Schooling and Rusher on the mortality during the late war, and Mr. Manly's paper on Pension Funds, had attracted attention at a considerable distance beyond their limits. With regard to the examinations, which was the next point touched upon in the report, although the results were not announced at the time the report was drawn up, they had since appeared, and they had certainly created a feeling of great disappointment, not only amongst the

candidates themselves, but in other quarters, at the rather meagre results, the poor proportion between the successful and the unsuccessful candidates. And again, they had heard at least a whisper of suspicion that the papers were really too difficult for the purposes for which they were intended. But the test based upon the mere proportion between the successful and the unsuccessful was not a correct one for the purpose. Of course they were apt to compare similar numbers at University and School examinations; but they must not forget that the conditions here were entirely different. The students in the Schools and Universities had nothing else to do but to prepare for their examinations, and they were specially prepared for them by their tutors and lecturers. But in their own case the students were, for the most part, almost exclusively engaged in other business during the day. They had a good deal to occupy their attention and exhaust their energies during office hours, and it was only after office hours that they could turn their attention to the work of preparation for their examinations, with the result that they came to it with much less vigour than school-boys or University students. Therefore they must not conclude that because the proportions were what they were, that the fault was with the examiners. It was not with the examinees either, for the reason which was stated. It was, in fact, that the conditions were altogether different, and the results were not so deplorable as they appeared to be. The lectures that had been given during the past year by Mr. Bowley were on a subject of a peculiarly technical and abstruse character. They did not appeal to quite so many students as those on legal, financial and statistical matters, which preceded them, and hence the attendance was not quite so large as it was on former occasions. But this did not detract from the value of the lectures, which in their printed form he was sure would be of the highest interest and value to students. He thought that other subjects, of a still different nature, might be found for lectures in the future. They had had statistics, they had had law, and they had had finance; and he hoped they might find room, at some time, for lectures upon those branches of medicine with which they had to deal as insurance officials. They had had the satisfaction of doing some work again for the Commissioners of the Patriotic Fund. It had been a labour of considerable intricacy, involving a good deal of work, and those members of the Institute who had done the work were entitled to take to themselves a good part of the commendation which the Institute had received from the Commission. They had hoped that it would have been possible to have announced the definite actual completion of the mortality investigation. Although they could not quite do that, very little now remained. It had been suggested, and he thought the suggestion was a very good one, that anyone who was preparing any tables based on the new experience would do well to give notice of the fact to one of the Secretaries at the Institute, or to the Assistant Secretary, so that it might be on record, and so that before a man began to work on a heavy table, involving a great deal of labour, he might be able to ascertain whether his work had been done, or was being done, by somebody else. There was only one other matter to which he would draw their attention, and that was the new hall which had been added to their premises. It would be of considerable use to them, not only for the examinations, but for reading and study and other purposes.

Mr. R. P. HARDY, in seconding the motion, said that the Institute showed no signs of impairment by age; its energies still ranged with unabated vigour, and its productiveness was still of satisfying fulness and utility. He recorded his legitimate pride at belonging to a fellowship that, in the maturity of its age, could put forward a report exhibiting such excellent work. He could not but regret the slight diminution in the numbers of their members, more especially so when some of those losses recorded names only too familiar to them. Mr. Strachan had applied his knowledge to the marked benefit of the many local associations he had advised. Mr. McCabe, of Toronto, was a variously accomplished man, not merely in actuarial matters, but in others. Mr. Reuben Watson was in many ways a very remarkable man. Commencing life under disadvantages that only those that have resolutely fought their way upwards from the ranks of labour know, by dint of a perseverance honourable in its aims and unflinchingly directed in its course, he mastered their subjects and earned for himself that recognition accorded to him by his election as an Associate of the Institute. It was well known that the friendly societies would barely listen to the recommendations of the professional actuary. But Mr. Watson's intimate acquaintance with the classes that composed these societies had taught him how to approach them, and how to shape his explanations and his advice. Once again had the Institute been requisitioned in the public service, and the appeal had been promptly responded to. These reports set at rest many disturbing questions. Although these valuations were under the charge of a committee selected for their familiarity with the subject, by far the greater part of the labour and administration devolved upon Mr. Schooling, and he desired, on behalf of his colleagues, to offer this sincere tribute to Mr. Schooling's self-denying labours. They had noted with satisfaction the issue of the two volumes of the Monetary Tables, based upon the recently-investigated experience, and there now remained, and would shortly be published, the Select Tables, and the account of the technical processes employed. This would complete the work on which so many minds and so much devoted attention had been expended, both north and south. In view of the spread of Service Funds, an adequate treatment of the technique was necessary. Mr. Manly's paper was a distinct advance upon what had been previously done, and should be mastered by all students. They had, further, a paper of very great social importance on war mortality. One would have thought an investigation into the mortality of the recent war was a matter which the War Office would have undertaken. But it had been left to two of their members to perform this duty. In Mr. King's paper they had a bold and successful application of the old philosophical doctrine of form, showing how life assurance business throws itself into one of those eternal forms of grouping that all social events tend to assume. Mr. Lidstone's paper was of the analytical rather than of the constructive order. With the skill of a practical anatomist, he laid bare the centre of action, and showed how the complicated contributing machinery actually worked, and substituted a simplification that was a triumph of art.

The motion was unanimously agreed to.

Mr. GEO. R. JELICOE proposed the re-election of Messrs. H. E. Wilson and Bernard Woods as Auditors; and also the election of Mr. W. M. Monilaws

as their colleague for the ensuing year. In doing so he thanked Messrs. Leveaux, Wilson and Woods for their services during the past year. The audit could not have been more efficiently done than it had been by those gentlemen.

The resolution, seconded by Mr. H. WEATHERILL, was carried unanimously.

The PRESIDENT announced that the Officers recommended by the Council had been unanimously elected, as follows:—

President.

WILLIAM HUGHES.

Vice-Presidents.

ARTHUR FRANCIS BURRIDGE.

THOMAS G. C. BROWNE.

GEORGE KING.

ERNEST WOODS.

Council.

THOMAS GANS ACKLAND.

HENRY WALSINGHAM ANDRAS.

ARTHUR DIGBY BESANT, B.A.

JAMES BLAKEY.

THOMAS G. C. BROWNE.

ARTHUR FRANCIS BURRIDGE.

HENRY COCKBURN.

GEORGE STEPHEN CRISFORD.

*GORDON DOUGLAS.

*JOSEPH ERNEST FAULKS, B.A.

GEORGE FRANCIS HARDY.

CHARLES DANIEL HIGHAM.

LOUIS FREDERICK HOVIL.

WILLIAM HUGHES.

GEORGE KING.

GEORGE MACRITCHIE LOW.

HENRY WILLIAM MANLY.

GEOFFREY MARKS.

*ARTHUR ERNEST MOLYNEUX.

*GERALD HEMINGTON RYAN.

FREDERICK SCHOOLING.

JOHN BELL TENNANT.

GEORGE TODD, M.A.

RALPH TODHUNTER, M.A.

SAMUEL GEORGE WARNER.

ALFRED WILLIAM WATSON.

*JAMES DOUGLAS WATSON.

ERNEST WOODS.

*FRANK BERTRAND WYATT.

THOMAS EMLEY YOUNG, B.A.

Treasurer.

HENRY COCKBURN.

Honorary Secretaries.

FREDERICK SCHOOLING.

| GEORGE TODD, M.A.

* Not Members of the last Council.

Mr. F. B. WYATT proposed a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners, and Supervisors for the past year. He need hardly say that the dignity of the Institute had been thoroughly upheld and its progress maintained. With regard to the Examiners, he would like to make a special remark, having been an examiner himself. All knew what a difficult task they had. It was also a thankless task.

Mr. W. J. H. WHITTALL, in seconding the motion, said that all who had attended the meetings during the past Session, and especially the members of the Council, who had had more intimate experience of the President's conduct of the affairs of the Institute, were aware of the great qualifications which he possessed for that position, and of the services which he had

rendered. And the same applied to every member of the Council. He thought there was sometimes a tendency to forget the obligation which they owed to the Examiners for the trouble of going through the papers of those numerous candidates of whom they had heard from the President. The number of candidates who presented themselves for examination more than ever emphasized the question as one for discussion, whether the Examiners should not be remunerated in some way for their very heavy labour. Some years ago, when he (Mr. Whittall) was a member of the Examination Committee, he made something of a special endeavour to get the payment of Examiners seriously considered. He carried with him the Vice-President, who then presided over the Examination Committee, to the extent of considering the Part I examination, and the discussions which then took place led to the institution of the present system, which had worked well ever since. But he was not successful in getting the Chairman of the Committee to consider Parts II and III of the examination. From that time to this, so far as his knowledge went, nothing more had been done in that direction. But he could not help thinking that this was a matter which now called for consideration in view of the very great increase in the number of candidates who presented themselves. As they had only thanks to offer, they were under an exceptional debt of gratitude to the Examiners for their labours; but personally he was one of those who hoped to see the time when their gratitude could take a more tangible and practical shape.

The motion was carried.

The PRESIDENT said that whilst he thought all concurred in many of the remarks which Mr. Whittall had made on the subject of the onerous work performed by the Examiners, and some of his remarks as to their reward—tangible or intangible—there were two sides to every question; and although one side presented itself very strongly to Mr. Whittall's mind, there was another side which had, perhaps, to be regarded also. With respect to the duties of the Officers of the Institute, they owed a good deal to the Assistant Secretary, Mr. Jarvis. Mr. Jarvis had been in office one year, since the retirement of his predecessor, and all who came in contact with him highly appreciated the work which he had done for the Institute, and the assistance which he had rendered to themselves.

Mr. H. M. TROUNCER moved a vote of thanks to Messrs. Leveaux, Wilson, and Bernard Woods for their services during the past year as Auditors.

The resolution, seconded by Mr. E. A. RUSHER, was carried unanimously.

Mr. BERNARD WOODS briefly responded on behalf of the Auditors.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1902:

*By whom presented
(when not purchased).*

Accountants and Auditors, Society of

List of Members, &c., 1902-3, 1903-4.

The Society.

Accountants, Institute of Chartered, in England and Wales.

List of Members, 1903.

The Institute.

Actuaries, Faculty of

List of Members, 1903.

Transactions, 1902-3.

} *The Faculty.*

Actuarial Society of America.

Transactions, 1902-3.

The Society.

American Mathematical Society.

Transactions, 1902-3.

The Society.

American Statistical Association.

Transactions, 1902-3.

The Association.

Australian Mutual Provident Society

Fifty-fourth Annual Report, 1903.

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das Königreich Böhmen in Prag. 1901.

Die privaten Versicherungsunternehmungen in den im
Reichsrathe vertretenen Königreichen und Ländern
im Jahre, 1900.

Mitteilungen des Verbandes der österr. und ungar.
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Bankers.

Journal of the Institute of Bankers, 1902-3.

The Institute.

*By whom presented
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Belgium.

Bulletin du Comité permanent des Congrès Internationaux d'Actuaires.	}	<i>The Permanent Committee of Actuarial Congresses.</i>
Bulletin de l'Association des Actuaires Belges.		<i>The Association.</i>
Compte Rendu des Opérations et de la Situation de la Caisse Générale d'Épargne et de Retraite. 1902.	}	<i>The Belgian Government.</i>

"Biometrika."

Vol. I, and Vol. II, parts I, II, and III.

Purchased.

Containing *inter alia* :—

- "Data for the Problem of Evolution in Man", by A. O. Powys. (Vol. I, part I.)
- "Inheritance of the Duration of Life and the Intensity of Natural Selection in Man", by Miss M. Beeton and Prof. Karl Pearson. (Vol. I, part I.)
- "Tables for Testing the Goodness of Fit of Theory to Observation", by W. Palin Elderton. (Vol. I, part II.)
- "On the change in expectation of Life in Man during a period of circa 2,000 years", by Prof. Karl Pearson. (Vol. I, part II.)
- "On the Systematic Fitting of Curves to Observations and Measurements", by Prof. Karl Pearson. (Vol. I, part III, & Vol. II, part I.)
- "On the Influence of Previous Vaccination in cases of Smallpox", by W. R. Macdonell, LL.D. (Vol. I, part III.)
- "Interpolation by Finite Differences. (Two Independent Variables)", by W. Palin Elderton. (Vol. II, part I.)
- "A Further Study of Statistics relating to Vaccination and Smallpox", by W. R. Macdonell, LL.D. (Vol. II, part II.)
- "New Tables of the Probability Integral", by W. F. Sheppard. (Vol. II, part II.)
- "Graduation and Analysis of a Sickness Table", by W. Palin Elderton. (Vol. II, part III.)
- "On the Probable Errors of Frequency Constants." Editorial. (Vol. II, part III.)

British Offices Life Annuity Tables, 1893.

Tables deduced from the Graduated Experience of Life Annuitants, Male and Female. Select Tables.

The Joint Mortality Experience Committee.

*By whom presented
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Select Tables.

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Complete Digest of Interest, Surplus Earnings, and
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Questions set for Part I of the Examinations of
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Statistical Account of the Seven Colonies of Australasia,
1901-1902.

Statistics of the Six States of Australia and New
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The Decline in the Birth-rate of New South Wales,
and other phenomena of Child-birth.

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Elements of Life Insurance. 2nd edition, New York,
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Assecuranz Almanach, 1903.

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Bulletin de l'Institut des Actuaire Français.

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Tables de Mortalité, 1900. Des Rentiers et Assurés en cas de vie établies par le Comité des trois Compagnies "Générales," "Union," et "Nationale."

*Le Comité des
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Germany.

Deutscher Verein für Versicherungs-Wissenschaft.

List of Members, 1903.

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The Editor.

Zeitschrift des Königlich Preussischen Statistischen Bureau. 1902-1903.

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Zeitschrift für die gesamte Versicherungs-Wissenschaft. 1902-1903.

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Zustand und Fortschritte der deutschen Lebensversicherungs-Anstalten im Jahre 1902.

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Hobson (E. W.), Sc.D., F.R.S.

On the Infinite and the Infinitesimal in Mathematical Analysis.

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Archief voor de Verzekerings-Wetenschap, 1902-1903.

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Mededeelingen der Vereeniging voor Levensverzekering. 1902-1903.

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Nationale Levensverzekering-Bank. Verslag over het jaar 1902.

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Verplichte verzekering van werklieden tegen de geldelijke gevolgen van invaliditeit en ouderdom. Rapport by MM. E. Fokker, W. L. P. A. Molengraaff, Dr. W. A. Poort, and J. W. C. Tellegen.

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Net Premiums and Reserves on Joint Life Policies. Based on the American Table of Mortality graduated by the Makeham Formula at 3 per cent. and $3\frac{1}{2}$ per cent. interest.

*The New York
Life Insurance
Company.*

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Independent Order of Oddfellows, Manchester Unity
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Sickness and Mortality Experience, 1893-97, with }
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Directors.*

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Bollettino della Associazione Italiana per l'incremento
della Scienza degli Attuari. Decr. 1902. }
Casa Nazionale di Assicurazione per gli infortuni degli
operai sul lavoro. Reports 1899, 1900, 1901. }

G. Toja.

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An Elementary Treatise on the Calculus of Variations.

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Purchased.

Vol. V, containing *inter alia*:—

"An Endeavour to Estimate the Value of Certain
Doubtful Lives for Life Assurance", by
F. Le M. Grasett, M.B.

"Diseases of Occupation", by Prof. Oliver, M.D.

"Heart Diseases in their Relation to Life Assur-
ance", by Henry Handford, M.D.

"Industrial Life Assurance", by Joseph Burn.

"Life Assurance Investments", by N. B. Gunn.

"Mortgages", by J. R. Hart.

"Personal Accident Insurance", by Alfred Foot,
F.S.S.

"The New Mortality Tables of the Institute of
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Petit Traité d'Economie politique mathématique.

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Life Assurance Medical Officers' Association.

Transactions, 1900 and 1901.

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Annuities-Certain, Logarithmic Elements, Factors,
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The London Life Table. Based on the Mortality in the }
Decennium 1891-1900.

*The London
County Council.*

Another Copy.

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Palgrave (R. H. Inglis).

Bank Rate and the Money Market in England, France,
Germany, Holland and Belgium, 1844-1900.

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Parliamentary Papers.

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General Report, with Tables.

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Population Tables, with Reports. Vols. I and II.

Colonial Acts and Bills.

Life Assurance Policies (N.S.W.), 1895.

Life Insurance Companies (N.S.W.), 1862.

Old Age Pensions (N.S.W.), 1900.

Companies (Victoria), 1890.

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Old Age Pensions (Victoria), 1900, 1901.

Life Assurance Companies (S. Australia), 1882,
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Foreign Insurance Companies' Deposits (New
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Colonies.

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Public Service Board. Supplement to Sixth Annual Report, with an Appendix relating to the Superannuation Account, 1902.

Results of Census, taken 31 March 1901 :

Part II. Education of the People.

Part III. Religions of the People.

Part IV. Birthplaces of the People.

Part V. Conjugal Condition and Families.

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Statistical Register, 1901.

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Annual Reports of the Government Insurance Commissioner, 1901, 1902.

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Friendly Societies. Twenty-fifth Annual Report of the Registrar, 1902.

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Report for the quinquennium 1895-1899, together with the Mortality and Sickness Experience and Statistics. Being the Third Report of the Public Actuary, 1902.

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Emigration and Immigration.

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Seventh Annual Report, Parts I and II, and Eight Annual Report, Part I.

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Parliamentary Papers—(continued).

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Guide Book of the Registry Office, 1902 and 1903.

Government Department Securities.

Return of the amounts of British Government Securities held by the several Government Departments and other Public Offices on 31 March 1902.

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Life Assurance Companies.

Returns, 1902.

The Board of Trade.

National Debt.

Annuities. Account to 5 Jan. 1903.

Military Savings Banks. Account to 5 January 1903.

Return showing Liabilities and Assets at the close of each financial year, from 1835-6 to 1902-3.

Savings Banks and Friendly Societies. Account to 20 Nov. 1902.

Patriotic Fund.

Fortieth and Forty-first Annual Reports of the Royal Commissioners. 1902-3.

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Thirty-eighth detailed Annual Report on Births, Deaths, and Marriages for 1901, and Supplement.—Special Report on Cancer.

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Forty-sixth detailed Annual Report on Births, Deaths, and Marriages. (Abstracts of 1900.)

Forty-eighth Annual Report on Births, Deaths and Marriages during the year 1902; and Thirty-eight Annual Report on Vaccination.

Workmen's Compensation.

Statistics of Proceedings under the Acts of 1897 and 1900, and the Employers' Liability Act, 1880, during the year 1901.

Pearson (Karl), F.R.S.

On Certain Properties of the Hypergeometrical Series, and on the Fitting of such Series to Observation Polygons in the Theory of Chance.

On Lines and Planes of Closest Fit to Systems of Points in Space.

On some Applications of the Theory of Chance to Racial Differentiation.

The Author.

*By whom presented
(when not purchased).*

Pearson (Karl), F.R.S.—(continued).

On the Criterion that a given System of Deviations
from the Probable in the case of a Correlated
System of Variables is such that it can be reason-
ably supposed to have arisen from Random
Sampling.

The Author.

On the Mathematical Theory of Errors of Judgment,
with special reference to the Personal Equation.

Pearson (Karl), F.R.S., and Lee (Alice), B.Sc.

On the Distribution of Frequency (Variation and
Correlation) of the Barometric Height at Divers
Stations.

Prof. Pearson.

Tables of $G(r, v)$ —Integrals. Report to the British
Association for the Advancement of Science.
With Appendix.—Tables of $F(r, v)$ and $H(r, v)$
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Periodicals.

Accountants' Magazine.

Purchased.

Insurance Record.

The Editor.

Post Magazine.

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Ratcliffe (Henry).

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The Trade Relations of the British Empire.

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Philosophical Transactions, 1894–1902.

Various Contributed Papers on the Mathematical
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Thiele (T. N.).

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The Workmen's Compensation Act, 1897. A plea for
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Toja (Guido).

Nota Attuariale sulla casse-pensioni basate su sistemi
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Twelfth Census, 1900:—

Vols. I and II. Population.

Vols. III and IV. Vital Statistics.

Vols. V. and VI. Agriculture.

Vols. VII. to X. Manufactures.

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Staple Inn and its Story.

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Yorkshire, Insurance Institute of
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{ *The Insurance
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Yule (G. Undy).

On the Theory of Correlation.

Prof. Pearson.

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Friendly Societies' Valuation and other Tables, 1896.

Purchased.

Coghlan (T. A.).

Childbirth in New South Wales; a Study in
Statistics, 1900.

R. P. Hardy.

Journal of the Institute of Actuaries.

1850-1903, 37 Vols.

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Meech (Levi W.).

System and Tables of Life Insurance; (Thirty American
Offices' Experience); N.D.

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The Causes of Death among the Assured in the
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Dr. T. B. Sprague.

Neison (F. G. P.), F.L.S.

Contributions to Vital Statistics; 3rd edition, 1857.

Purchased.

Poisson (S. D.).

Recherches sur la Probabilité des Jugements en Matière
Criminelle et en Matière Civile; Paris 1837.

Dr. T. B. Sprague.

VOL. XXXVIII.

PART III.

JOURNAL
OF THE
INSTITUTE OF ACTUARIES.

No. CCXV.—JANUARY 1904.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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CHARLES AND EDWIN LAYTON,
FARRINGTON STREET.

PARIS: 8, RUE LAMARTINE, S.
BERLIN: CARLSTRASSE 11. MELBOURNE: M. CARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

PRINTED BY C. & E. LAYTON, FARRINGTON STREET, E.C.
TO WHOM ALL COMMUNICATIONS FOR THE EDITOR SHOULD BE SENT, POST PAID.

Price 2s. 6d.

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JANUARY 1904.

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PARIS: 5, RUE LAMARTINE, 5.
BERLIN: CARLSTRASSE 11. MELBOURNE: MCCARRON, BIRD & CO.
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[The Council of the Institute of Actuaries wish it to be understood, that while they consider it their duty to give, from time to time, publicity to certain of the papers presented to the Institute, and to abstracts of the speeches delivered at the Sessional Meetings, they do not hold themselves responsible for the opinions put forward therein.]

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PRINTED BY CHARLES AND EDWIN LAYTON,
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NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Comparative Mortality among Assured Lives of Abstainers and Non-Abstainers from Alcoholic Beverages. By RODERICK MACKENZIE MOORE, Actuary to the United Kingdom Temperance and General Provident Institution.

[Read before the Institute, 30 November 1903.]

INTRODUCTION.

(1.) THE effect of alcohol on the human economy and on the health and prosperity of the community has occupied the close attention of statesmen, physiologists and philanthropists for many generations past, and the question of total abstinence has been probably the most widely discussed social problem for more than half a century. That excess, as compared with moderation in the use of alcohol, is prejudicial in every way is a proposition hardly to be disputed. And many authorities who have given the matter careful consideration contend that even as moderation is superior to excess, so is total abstinence from alcohol superior to moderation. It is the purpose of this paper to examine the truth of this contention in regard to Mortality; with which must be closely associated the questions of health and general well being.

(2.) In our daily experience we know that of all the points to be considered in connection with a proposal for Life Assurance, by far the most important, in the majority of cases, is the question of "habits", by which general term "habits" in this connection we nearly always refer to the use, moderately or otherwise, of alcoholic beverages. This fact gives importance to the subject from a technical standpoint.

(3.) But if any further apology be necessary for bringing this question forward, I may be permitted to refer to the remarks of our President in his opening address last session in which (pp. 369-370 of the *Journal* for January 1903) he points out that no "Actuary of these Societies" (*i.e.*, Assurance Societies which distinguish between Abstainers and Non-Abstainers) "has ever made any contribution to our proceedings giving particulars of the experience of so-called Temperance Sections. It is much to be desired that we should have detailed information on a matter so important . . . proving with minute and scientific accuracy the truth of the oft repeated assertions as to the effect of total abstinence upon the duration of life." I trust the figures I have now the privilege of submitting to you, will help to fill the void pointed out by our President.

(4.) Some eighteen months back the writer was requested by the Directors of the United Kingdom Temperance and General Provident Institution to continue, and complete to the end of the year 1901, the investigation made nearly twenty years previously by Mr. R. P. Hardy, into the mortality arising among the Members of that Institution to the end of the year 1882, with special reference to the relative mortality of Total Abstainers from alcoholic beverages and Non-Abstainers. This work has now been completed, and, by permission of the Institution's Directors, the results are embodied herein.

HISTORY AND PRACTICE OF THE INSTITUTION.

(5.) It is necessary to give some account of the Office whose statistics we are dealing with. The United Kingdom Temperance and General Provident Institution was founded in 1840, under the name of the United Kingdom Total Abstinence Life Association (which title was altered about a year later to Temperance Provident Institution) for the Mutual Assurance of the lives of Total Abstainers only. The premiums were somewhat lower than those now charged, and were not intended to yield any substantial profit. Its first Policy was dated 14 January 1841, and was granted on the life of one of its founders, who died so recently as 1896, in his 82nd year.

(6.) In 1849, Non-Abstainers were admitted, and the present title and scale of premium were adopted about 1851. The same premiums have since been charged for both Abstainers and Non-Abstainers, the former receiving any advantage arising from their superior vitality, in the form of additional bonus.

(7.) Persons are eligible for the Temperance Section who do not take alcohol as a beverage in any form. The continued adherence of the assured to the principles of Abstinence is checked by an annual declaration to that effect. Such persons are described throughout this paper as "Abstainers", "Temperance Lives", or "Teetotalers." Persons who do not come within these conditions are eligible for the "General Section" only, and are herein described as "Non-Abstainers." If an Abstainer cease to abstain he is transferred to the General Section, and on the other hand, policyholders in the General Section who become Abstainers are generally eligible for transfer to the Temperance Section. The effect of these "Transfers" on the Mortality of the two sections respectively will be dealt with later on in this paper, but it may be here stated that such transfers are not made if the assured be known to be in bad health or of intemperate habits, and transfers from the Temperance Section are not now made in the cases of persons over 70 years of age. Moreover, the "Transfers" are comparatively very few in number. A complete record has been kept throughout the career of the Institution of the section (Temperance or General) in which every policy has been included year by year, and of any transfers between those sections. Some 125,000 Policies have been issued to the end of 1901, including Endowment Assurances, Joint Life Assurances, Annuities, and other special Policies.

(8.) The same care is exercised in admitting a new applicant into either section as regards family history and personal condition. But, possibly if there were any unfavourable information on either of these points the case may have a better chance of acceptance if the life be an Abstainer. It should thus be made clear that the Temperance Section has never been favoured or nursed, with the view of securing more favourable mortality results, either by the admission of unexceptionable lives only in the first instance or by the removal or transfer of inferior or doubtful lives.

(9.) As far as it is possible to generalize over so long a period as 61 years, it may be said that the practice of the Institution has been throughout as described in paragraphs 7 and 8. The writer can speak from personal knowledge in reference to the last 25 years, and from information derived from the knowledge of past and present officials extending over nearly the whole of the period under observation.

(10.) A question of importance is the relative pecuniary positions of the persons assured in the two sections of the Institution. Table I shows the average sum assured of the whole-life participating policies in force at each quinquennial valuation from

TABLE I.

Showing the average Sum Assured in the Institution in force at successive Valuations. Whole-Life Participating Policies.

Year	Abstainers	Non-Abstainers
	£	£
1865	187	198
1870	191	205
1875	208	220
1880	230	232
1885	252	246
1890	265	264
1895	285	289
1900	299	321

AVERAGE SUM ASSURED under Whole-Life Participating Policies as per last Valuation Returns of seven leading Ordinary Life Offices, transacting mainly a provident class of business £575

1865 onwards, in the Abstainers' and Non-Abstainers' Sections respectively. It will be seen that they are approximately equal throughout, although the Non-Abstainers show a small superiority. On the other hand, it will be seen hereafter in Tables III and IV that the Abstainers enter at a rather younger average age than the Non-Abstainers. It will also be seen that the average sum assured has steadily increased in each section.

From these figures it may reasonably be assumed that the positions in life of the assured in the two sections are much the same, and this view is confirmed by my knowledge of the working of the Institution's business.

In the same table I have shown the average sum assured in force in seven selected Offices transacting a large business, mainly of what may be called a provident character as distinguished from that connected with Mortgages, &c. I have not included any of the "Law" Offices or any Offices effecting Industrial business. It will be seen that the average is nearly double that of the Institution in 1900.

METHOD OF DEALING WITH THE DATA.

(11.) Particulars of each Policy issued by the Institution were taken on a card of similar form to that used by the English

Offices in the first (20 Offices) experience of the Institute of Actuaries. The card is as follows : the letters in brackets being added here for reference.

Pol. No. (a)	Dept. (b)	£ (c)
With Profits. (d)	Nature of } Assurance { (e)	
Life (f)	Sex (g)	
Rated up (h)	years	
How rated up (i)		
Year of Entry (j)	18	(l)
Year of Exit (k)	18	
True age at Entry (m)		
True age at Exit (n)	n	
Mode of Exit (o)		
Cause of Death (p)		
Remarks		

(a) Policy Number. Policies for every number from 1 to 125,007 have been issued, with the exception of 330 numbers omitted for some unknown reason in the year 1850. The cards were therefore mechanically numbered by the printer, and every Policy thus accounted for. The final disposal by lapse, surrender, death or otherwise was ascertained from the Policy registers. As a further check, particulars of all cancellments were taken on separate small slips, which were sorted into numerical order and collated with the cards. The cards which then remained as not

cancelled were compared with the Registers of Policies existing at 31 December 1901. By these means it was guaranteed (1) that every Policy was included in the observations; (2) that no cancelled Policy could be treated as still in force; and (3) that all the deaths were included.

(b) Department. This is for office purposes, and shows the nature of the Policy, and whether in the Temperance or General Section.

(c) The sum assured.

(d) The syllable "out" was added for non-participating Policies. These were, however, very few in number, being less than 2 per-cent of the whole. (No use has been made of (c) and (d) in the present investigation).

(e), (f), and (g) These items do not call for any explanation.

(h) Rated up—years. The majority of "Rated-up" Policies issued since 1866 are charged with a decreasing debt, which extinguishes itself on completion of the expectation of life at the real entry age, and in some cases earlier. In the case of Endowment Assurances the debt is usually extinguished on the completion of the term of the Policy, that term being generally shorter than the expectation. The number of years to be added to the age, of which the debt is assumed to be the equivalent, is inserted in the space. In some few cases the premium corresponding to the advanced age has been paid. Where small extras for Gout, Rheumatism, Rupture, &c., have been added to and paid with the premium, the equivalent number of years' addition to the age has been inserted in the space. In the case of females, the extra for sex has in all cases been ignored in this investigation.

(i) How rated up; *i.e.*, Whether by decreasing debt, or by addition to the premium. But no distinction has been made between these cases.

(j) and (k) Years of entry and exit. These are calendar years. In the cases of Policies in force at the close, (k) was of course left blank.

(l) The difference between (j) and (k), namely—the duration of the Policy, was inserted at this point. If the Policy were in force the difference between the year of entry and 1901 was inserted in pencil at the point *l^a*.

(m) This is, in the cases of lives accepted at ordinary rates, or rated up on the debt plan, the Office age next birthday corresponding to the premium. In cases where an extra is payable with the premium, it is the true age next birthday.

(n) The age of exit is obtained by adding the "duration" to the age at entry. If the Policy were in force this age was inserted in pencil at the point n^2 .

(o) Mode of exit. Void (or lapse), Surrender or otherwise cancelled, Matured Endowment (officially called Claim), Transfer from the Temperance to the General Section or *vice versa*, or Death were marked by the initial letters V, S, C, T and D respectively. Voids were dated as becoming so in the year in which the premium fell in default.

(p) Cause of death. Not inserted.

(12.) Ages and period at which the Policy was "Exposed to Risk." For Office entry age x , and duration n years, the exposure was half-a-year at age $(x-1)$, and in the case of Surrenders, Transfers and Matured Endowments, half-a-year at age $(x+n-1)$. In the case of Deaths and existing, a full year at age $(x+n-1)$. Under Voids, $\frac{2}{3}$ th of a year at age $(x+n-1)$. Policies being renewable on the first days of January, April, July and October, they must go void on one of these dates, and the average exposure will therefore be $\frac{2}{3}$ th of a year. For each of the intermediate ages between $(x-1)$ and $(x+n-1)$ of course a full year's exposure. It will be noted that when a Policy is transferred from the Temperance to the General Section it passes out of the observations. It is separately dealt with as a "Transferred" Policy from the date of such transfer, as will be explained later on. It will also be borne in mind that the general effect of these adjustments is to very slightly under-estimate the exposures, and correspondingly over-estimate the rates of mortality. The effect of this being almost infinitesimal, it is of no practical consequence.

It will be seen that the Institute of Actuaries' (1869, 20 Offices' Experience) method has been adopted in dealing with the ages and duration of the Policies. A portion of this investigation was commenced by Mr. R. P. Hardy (with the assistance of the writer) so far back as 1883, before much of the criticism to which that method had been subjected had appeared, and the work has naturally been continued on the same lines. But apart from this, the writer ventures to express his opinion that the original Institute method has the merit of simplicity, and is very suitable for old experience, where exact data, date of birth, death, &c., cannot be obtained. Given the Office age at entry (x) equivalent to the premium, the calendar year of entry and the calendar year and mode of exit, and your material is complete.

The life enters at age $(x-1)$. If he die or retire the following year, it is at age (x) , and so on. Again, an important point in this investigation is that the method may more easily be understood by persons who are interested in the question but have no knowledge of actuarial technicalities.

(13.) The cards being completed they were sorted into firstly

(a) Males and Females. Then each sex into

(b) Abstainers and Non-Abstainers. Then

(c) Whole-Life Policies; Endowment Assurances and others; and

(d) Healthy and rated-up lives.

(14.) Of this material by far the largest and most important two divisions are those of Whole-Life Policies on Healthy Male Lives of Abstainers and Non-Abstainers respectively, and it is on their experience that our main conclusions must be based. The Female experience is, naturally, proportionately small. The experience of Endowment Assurances consists very largely of recent business, and of a large proportion of short term Policies, and as, moreover, the bulk of such business, if matured, passes out of observation at ages 55 and 60, the results based thereon cannot be accepted with any degree of confidence. However, the experience of these minor classes is given for what it is worth.

(15.) The cards of each final sub-division to be investigated were then sorted into entry ages (x) and the number at each entry age was entered in Column (2) at age $(x-1)$ on the following form:

Age (x)	Entered at Age (x)	PASSED FROM OBSERVATION AT AGE (x) AS					
		Transfd.	Void	Surr'd.	Endow't. Matured	Died	Existing 31 Dec. 1901
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

Sum of Columns (3) to (8)	Brought forward from Age ($x-1$) $\left\{ \begin{array}{l} 10+(2) \\ - (9) \end{array} \right\} x-1$	$\begin{array}{l} (2) \\ + (3) + (5) \\ + (6) \end{array}$	$\frac{1}{2}$ (11)	$\frac{5}{8}$ (4)	Exposed to Risk (2)+(10) -(12)-(13)	Died (7)	Rate of Mortality (15) \div (14)	Age (x)
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)

The same cards were then sorted into "Mode of Exit", *i.e.*, "Transfer", "Void", "Surrender", "Matured", "Death", or "Existing", and each mode of exit was sorted into exit ages. The number at each exit age ($x+n$) was entered in its proper column against age ($x+n-1$) and the numbers exposed to Risk at each age were obtained by the method indicated in the headings of the remaining columns of the form above.

(16.) The results put forward in the paper, except where otherwise stated, are based upon the entire experience of the Institution from 1841 to 1901 inclusive, a period of 61 years. It may be suggested that it would have been better to have taken the period 1863-1892, or the last 30 years' experience 1872-1901, as agreeing more closely with the O^M data. I venture to think, however, that the Institution's more extended experience is better. If the last 30 years only had been dealt with I believe the rates of mortality deduced therefrom would have been lighter than those based upon the total experience, because the new business would have formed a larger proportion of the whole. Later on is given the experience of the last 13 years' new business, showing a very low death rate. It is obvious that the smaller the proportion of old business combined with this, the lower would be the rate of mortality. But the most important point is that the same method and period be used for both sections.

(17.) The Total Experience of the Institution's working at various dates is also given later on.

(18.) The experience given throughout is that of "Policies" and not "Lives"; that is to say, every separate Policy on one life is regarded as a separate risk. The writer believes that the profession does not now insist on the superiority of the "Lives" method, some eminent authorities holding that if duplicate Policies issued simultaneously be eliminated, the "Policies" method is the preferable test. An experiment was made at an early stage of this investigation in reference to this question. In a large body of "Policies" the proper corrections for duplicates were made, and the results of each method "Policies" and "Lives" are given in Table I.A. For practical purposes these results may be considered identical throughout the important periods of life; and bearing in mind the great labour involved in adjusting the cards for duplicate Policies, it was decided to adhere to the "Policy" method.

TABLE IA.

Showing the results of dealing with "Policies" or "Lives" in a Mortality Investigation.

Ages	28,383 POLICIES			AFTER ELIMINATING DUPLICATES THERE REMAINED 26,858 LIVES			Ratio of (4) to (7) × 100	Ages
	Exposed to Risk	Died	Rate of Mortality per 100	Exposed to Risk	Died	Rate of Mortality per 100		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
-19	2,441	10	·410	2,331	10	·429	96·6	-19
20-24	8,306	60	·722	8,134	59	·725	99·6	20-24
25-29	23,384	153	·654	22,671	143	·631	104·	25-29
30-34	38,867	312	·803	36,984	298	·806	99·6	30-34
35-39	47,309	409	·865	44,270	383	·865	100·	35-39
40-44	47,471	509	1·072	44,146	472	1·069	100·	40-44
45-49	41,636	579	1·391	38,651	531	1·374	101·	45-49
50-54	32,126	569	1·771	29,835	514	1·723	103·	50-54
55-59	21,912	563	2·569	20,456	512	2·503	103·	55-59
60-64	13,200	448	3·394	12,305	416	3·381	100·	60-64
65-69	6,848	341	4·980	6,393	320	5·005	99·5	65-69
70-74	3,040	226	7·433	2,801	206	7·356	101·	70-74
75-79	1,027	120	11·688	933	112	12·000	97·4	75-79
80 & up.	253	56	22·140	224	50	22·330	99·2	80 & up.
All Ages	287,820	4,355	...	270,134	4,026	All Ages

THE COMBINED (ABSTAINERS AND NON-ABSTAINERS) EXPERIENCE.

(19.) Dealing first with the combined mortality of Abstainers and Non-Abstainers under Whole-Life Policies on Healthy Males we have the results exhibited in Table II. Col. 4 gives the actual annual Rate of Mortality per-cent, and Col. 6 gives the ratio of that rate (multiplied by 100) to the O^M standard. It will be seen that up to age 24 the experience has been unfavourable; but onwards to age 74 it has been well below the standard. For the group of ages 75-79 there is a noticeable increase—the rate just exceeding the standard; but beyond that age, with an experience of some 700 deaths, there is a decided recovery. This point should be kept in view in connection with the Abstainers' experience as shown later.

TABLE II.

*Healthy Males. ABSTAINERS and NON-ABSTAINERS combined.
Whole-Life Policies. "Transfers to" excluded.*

Entrants 1841-1901.

Experience 1841-1901.

Ages	Exposed to Risk	Died	Rate of Mortality per-cent per annum unadjusted	Adjusted O^M rate of Mortality central ages of groups	Taking O^M as 100, the Institution's combined rate of Mortality is	Ages
(1)	(2)	(3)	(4)	(5)	(6)	(7)
- 9	151	1	·662	- 9
10-14	1,641	3	·183	·345	53·1	10-14
15-19	6,595	40	·607	·375	162·	15-19
20-24	25,276	136	·538	·431	125·	20-24
25-29	59,839	290	·485	·523	92·7	25-29
30-34	93,520	529	·566	·648	87·3	30-34
35-39	115,203	735	·636	·804	79·3	35-39
40-44	123,027	949	·772	1·001	77·1	40-44
45-49	117,308	1,231	1·050	1·277	82·2	45-49
50-54	103,079	1,455	1·411	1·693	83·4	50-54
55-59	82,853	1,721	2·077	2·338	88·9	55-59
60-64	60,424	1,796	2·972	3·344	88·9	60-64
65-69	39,698	1,878	4·730	4·900	96·5	65-69
70-74	22,182	1,500	6·762	7·281	92·9	70-74
75-79	10,131	1,119	11·050	10·882	102·	75-79
80-84	3,236	512	15·820	16·240	97·4	80-84
85-89	680	145	21·339	24·001	88·9	85-89
90-94	104	30	28·850	31·788	82·2	90-94
95-99	6	1	16·660	48·276	34·5	95-99
All Ages	864,953	14,071	All Ages

MORTALITY OF THE NON-ABSTAINERS.

(WHOLE-LIFE HEALTHY MALES.)

(20.) Coming now to the classification of the Institution's members into "Abstainers" and "Non-Abstainers" we shall deal first with the Mortality among the Non-Abstainers Healthy Males, Whole-Life Policies. There were 31,776 of these passing through 466,942 years of life. Of these 8,947 died. Their complete experience is shown in Table III and the summary thereof. It will be seen that the Mortality among the Non-Abstainers in the Institution during the younger ages (say to age 34) was in excess of the O^M , but for all the higher ages, and throughout the more important part of the Mortality Table, it shows a remarkable agreement with the O^M rate. Taking the Expected Deaths according to the O^M Table (obtained by multiplying the actual exposures by the O^M rate of Mortality—a method which, if not theoretically correct, will serve our purpose) we find the total from age 35 upwards to be 8,414, the

TABLE III.—*Summary.*
Healthy Males. NON-ABSTAINERS.
Whole-Life Policies. "Transfers to" excluded.
Entrants 1841-1901.
Experience 1841-1901.

Ages	Exposed to Risk	Died	Rate of Mortality per-cent per annum unadjusted	Adjusted On rate of Mortality % Central Ages of Groups	Expected Deaths by On	Taking On as 100, the Non-Abstainers' Experience is	Ages
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
- 9	74	- 9
10-14	590	1	.170	.345	2	49.3	10-14
15-19	2,104	10	.475	.375	8	127.	15-19
20-24	9,516	63	.662	.431	41	154.	20-24
25-29	27,009	157	.579	.523	142	111.	25-29
30-34	46,965	339	.722	.648	304	111.	30-34
35-39	61,106	495	.810	.804	491	101.	35-39
40-44	67,423	645	.957	1.001	675	95.6	40-44
45-49	65,931	846	1.283	1.277	842	100.	45-49
50-54	58,941	992	1.683	1.693	998	99.4	50-54
55-59	47,879	1,136	2.373	2.338	1,119	101.	55-59
60-64	35,161	1,148	3.265	3.344	1,176	97.6	60-64
65-69	23,219	1,176	5.065	4.900	1,138	103.	65-69
70-74	12,857	922	7.171	7.281	936	98.5	70-74
75-79	5,780	614	10.623	10.882	629	97.6	75-79
80-84	1,890	307	16.252	16.240	307	100.	80-84
85-89	358	79	22.607	24.001	86	94.2	85-89
90-94	49	16	32.653	34.788	17	93.9	90-94
95-99	1	1	100.000	48.276	...	207.	95-99
All Ages	466,943	8,947	8,911	...	All Ages

actual deaths being 8,377, a difference of less than one-half per-cent, while for some of the individual quinary groups of ages the numbers are identicleal. It will be fair, therefore, to assume the O^M as a standard representing the Mortality in the Institution of Healthy Males who are not Abstainers.

MORTALITY OF THE ABSTAINERS.

(WHOLE-LIFE HEALTHY MALES.)

(21.) We now arrive at the main subject of this paper, the Mortality of Abstainers. We have 29,094 Whole-Life Policies passing through 398,010 years of life. Of these 5,124 died. Their complete experience is shown in Table IV and the summary thereof. We see that for ages below 20 the mortality is considerably above the O^M standard, and for the quinary groups of ages 20-24 it is also distinctly higher than the O^M . But for ages 25 and onwards up to age 74 the results are much in favour of the Abstainers. This advantage increases up to the group of ages 40-44, when it is at its maximum, being under 55 per-cent of the O^M rate. After that age the rate very slowly approximates to the standard, being $71\frac{1}{2}$ per-cent at ages 55-59 and 85 per-cent at the ages 70-74. For the group 75-79 the Abstainers' rate is in excess of the O^M , but for the remainder of life it is distinctly below. In general terms we may say that the feature of the Abstainers' Mortality is a great saving after early manhood throughout all the working years of life; which saving increases up to middle age, and gradually rises to the normal rate of healthy life as old age is reached at say ages 70 or 75.

For the group of ages 74-79 the rate, as already pointed out is well above the standard; and this peculiarity is exhibited, to a greater or less degree, in all classes of Policies, as will be shown hereafter. This would be consistent with a general deferment of Mortality consequent on a prolongation of the working years of life. The additional deaths that would have taken place between the ages of, say, 25 and 75, if the normal rate of Mortality had prevailed between those years, are postponed to appear after the latter age. And it is evident that in the five years immediately following that age, namely, during ages 75-79, an undue proportion of these lives have fallen in. But under normal conditions, *i.e.*, if the O^M rate had prevailed throughout, they would have died earlier. It will be seen that the expected deaths by the O^M Table at ages 75-79 were 474, the actual being 505. But if we take all ages over 74 to the end of life, the expected deaths were 792, the actual being 790.

(22.) In regard to the younger ages below 25, we have already seen in paragraph (20) that the experience of the Non-Abstainers has also been unfavourable. So far we have compared the Abstainers' experience with the O^M standard. Column (9) of

Table IV (summary) gives a comparison of the Abstainers' experience with that of the Non-Abstainers actually; and although for ages below 19 the Non-Abstainers show a superiority, yet for the group of

TABLE IV—Summary.
Healthy Males. ABSTAINERS.
Whole-Life Policies. "Transfers to" excluded.
Entrants 1841-1901.
Experience 1841-1901.

Ages	Exposed to Risk	Died	Rate of Mortality per-cent per annum Unadjusted	Adjusted Rate of Mortality per-cent per annum of Groups	Expected Deaths by (a)	Taking (a) as 100, the Abstainers' Experience is	Rate of Mortality per-cent per annum, Non-Abstainers' Experience Table III	Taking Non-Abstainers' Experience as 100, Abstainers' Experience is	Ages
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
- 9	77	1	1.299	- 9
10-14	1,051	2	.190	.345	4	55.1	.170	112.	10-14
15-19	4,491	30	.668	.375	17	177.	.475	141.	15-19
20-24	15,760	73	.463	.431	68	108.	.662	69.9	20-24
25-29	32,740	133	.406	.523	171	77.8	.579	70.1	25-29
30-34	46,555	190	.408	.648	302	62.9	.722	56.5	30-34
35-39	54,097	240	.444	.801	435	55.2	.810	54.8	35-39
40-44	55,604	304	.547	1.001	557	54.6	.957	57.2	40-44
45-49	51,377	385	.749	1.277	656	58.7	1.283	58.5	45-49
50-54	44,138	463	1.049	1.633	747	62.0	1.633	62.4	50-54
55-59	34,974	585	1.673	2.338	818	71.5	2.373	70.6	55-59
60-64	25,263	648	2.565	3.344	845	76.7	3.265	78.5	60-64
65-69	16,479	702	4.260	4.900	808	86.9	5.045	84.0	65-69
70-74	9,325	578	6.199	7.281	679	85.1	7.171	86.5	70-74
75-79	4,351	505	11.607	10.882	474	107.	10.623	110.	75-79
80-84	1,346	205	15.230	16.240	219	93.6	16.252	93.7	80-84
85-89	322	66	20.497	21.001	77	85.8	22.607	90.7	85-89
90-94	55	14	25.455	34.788	19	73.7	32.653	77.9	90-94
95-99	5	48.276	5	...	100.000	...	95-99
All Ages	398,010	5,124	6,839	All Ages

ages 20-24 the Abstainers have the advantage. Briefly, we may say that although the Abstainers' results below age 25 are inferior to

TABLE V.
Healthy Males. Excluding first five years of Assurance. Entrants 1841-1896.
Whole-Life Policies. "Transfers to" excluded. Experience 1846-1901.

Ages	ABSTAINERS				NON-ABSTAINERS				TAKING (OM) AS 100				TAKING HM) AS 100				Ages
	Exposed to Risk	Died	Rate of Mortality per cent per annum	Rate of Mortality per cent per annum	Exposed to Risk	Died	Rate of Mortality per cent per annum	Taking Non-Abstainers as 100, Experience is	Rate of Mortality per cent Central Ages	Abstainers' Experience is	Non-Abstainers' Experience is	Rate of Mortality per cent Central Ages	Abstainers' Experience is	Non-Abstainers' Experience is	Rate of Mortality per cent Central Ages	Abstainers' Experience is	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
-9	1
10-14	79	147.4
15-19	936	6	6.41	459	2	4.35	147.4	619	100.5	68.2	68.2	3.02	145.7	99.1	10-14
20-24	3,496	16	4.58	1,499	15	1.001	45.8	638	68.9	150.5	150.5	4.10	44.6	97.4	15-19
25-29	12,138	66	5.44	6,418	60	9.35	58.2	7.09	76.7	131.9	131.9	1.028	54.7	94.1	20-24
30-34	25,756	118	4.58	20,210	179	8.86	51.7	7.77	59.0	114.0	114.0	9.91	49.5	95.7	25-29
35-39	37,153	178	4.79	36,526	332	9.09	52.7	8.86	54.1	102.6	102.6	9.26	44.8	85.0	30-34
40-44	43,061	250	5.81	48,679	507	1.012	55.8	1.056	55.0	98.7	98.7	1.070	44.8	85.0	35-39
45-49	42,731	335	7.84	53,489	750	1.402	55.9	1.320	59.4	106.2	106.2	1.158	50.2	90.0	40-44
50-54	38,728	412	1.064	51,426	902	1.754	60.7	1.731	61.5	101.3	101.3	1.410	54.4	97.3	45-49
55-59	32,234	512	1.682	43,918	1,065	2.425	69.4	2.375	70.8	102.1	102.1	1.879	56.6	93.4	50-54
60-64	24,002	617	2.571	33,390	1,128	3.378	76.1	3.373	76.2	100.2	100.2	2.508	67.1	96.7	55-59
65-69	16,049	684	4.262	22,475	1,148	5.108	83.4	1.918	86.7	103.9	103.9	3.558	72.3	95.0	60-64
70-74	9,234	578	6.260	12,607	914	7.250	86.1	7.290	85.9	99.5	99.5	5.036	83.6	100.3	65-69
75-79	4,317	503	11.652	5,745	611	10.635	109.5	10.885	107.0	97.7	97.7	7.555	82.9	96.0	70-74
80-84	1,324	203	15.327	1,880	307	16.334	93.7	16.240	94.4	100.6	100.6	11.549	100.9	92.1	75-79
85-89	322	66	20.497	356	78	21.910	93.6	21.001	85.4	91.3	91.3	17.230	89.0	94.8	80-84
90-94	55	14	25.455	49	16	32.653	78.0	34.788	73.2	93.8	93.8	23.279	88.1	94.3	85-89
95-99	5	1	1	100.000	...	48.276	35.938	70.7	90.8	90-94
All Ages	291,624	4,588	...	339,182	8,015	100.000	95-99

the O^M standard, they are, however, better than those of the Non-Abstainers. For the remaining years of life the ratios are much the same as in the O^M comparison.

(23.) In Table V we have the rates of Mortality, Abstainers

TABLE VI.
Healthy Males. Entrants previous to 1889.
Whole-Life Policies. "Transfers to" excluded.
Entrants 1841-1888.
Experience 1841-1901.

Ages	ABSTAINERS			NON-ABSTAINERS			Taking Non-Abstainers' as 100, Experience Is	Rate of Mortality per centum Central Ages	Rate of Mortality per centum Central Ages	Ages
	Exposed to Risk	Died	Rate of Mortality per centum	Exposed to Risk	Died	Rate of Mortality per centum				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
- 9	76	1	1.311	73	- 9
10-14	874	2	.229	558	1	.180	127.70	.345	.619	10-14
15-19	3,442	28	.813	1,966	10	.508	159.90	.375	.638	15-19
20-24	12,278	68	.553	8,666	58	.669	82.66	.431	.665	20-24
25-29	27,007	114	.422	24,897	150	.602	70.06	.523	.709	25-29
30-34	40,112	179	.446	43,669	318	.728	61.29	.618	.777	30-34
35-39	48,352	220	.455	57,534	466	.810	56.17	.804	.886	35-39
40-44	50,818	292	.575	61,142	625	.975	58.97	1.001	1.056	40-44
45-49	47,681	365	.766	63,339	825	1.303	58.78	1.277	1.320	45-49
50-54	41,548	440	1.059	56,958	967	1.697	62.39	1.693	1.731	50-54
55-59	33,299	566	1.700	46,579	1,114	2.392	71.07	2.338	2.375	55-59
60-64	24,370	631	2.589	34,462	1,434	3.291	78.67	3.344	3.373	60-64
65-69	16,075	687	4.273	22,934	1,163	5.072	84.24	4.900	4.918	65-69
70-74	9,184	571	6.217	12,742	919	7.213	86.20	7.281	7.290	70-74
75-79	4,314	501	11.610	5,745	611	10.630	109.20	10.882	10.885	75-79
80-84	1,326	202	15.220	1,879	305	16.240	93.73	16.210	16.240	80-84
85-89	318	63	19.810	358	79	22.060	89.78	24.001	24.001	85-89
90-94	55	14	25.450	49	16	32.650	77.95	34.788	34.788	90-94
95-99	5	1	1	100.000	...	48.276	48.276	95-99
All Ages	361,137	4,944	...	446,551	8,762	All Ages

and Non-Abstainers, Whole-Life, Healthy Males, *excluding the first five years of assurance* from the experience; i.e., corresponding to the $H^{M(5)}$ and $O^{M(5)}$ Tables. The Abstainers and Non-Abstainers

TABLE VII.
Healthy Males. Entrants previous to 1889.
Whole-Life Policies. "Transfers to" excluded.
Entrants 1841-1888.
Experience 1889-1901.

Ages	ABSTAINERS				NON-ABSTAINERS				Taking Non-Abstainers' as 100, Experience is	Central Ages		Ages
	(1)	Exposed to Risk	Died	Rate of Mortality per cent per annum	(2)	Exposed to Risk	Died	Rate of Mortality per cent per annum		(9)	(10)	
- 9									(8)			(1)
10-14	30	18	10-14
15-19	268	746	71	345	619	15-19
20-24	1,090	4	...	367	235	375	638	20-24
25-29	3,465	9	...	260	1,058	431	665	25-29
30-34	7,674	30	...	391	3,318	12	...	1134	22-93	523	709	30-34
35-39	12,738	45	...	353	6,958	25	...	753	51-92	648	777	35-39
40-44	16,070	90	...	560	11,302	88	...	862	40-97	804	886	40-44
45-49	17,101	118	...	690	15,297	196	...	778	71-98	1001	1056	45-49
50-54	16,580	179	...	1080	17,350	295	...	1281	53-88	1277	1320	50-54
55-59	14,682	257	...	1749	17,764	397	...	1700	63-52	1693	1731	55-59
60-64	11,645	274	...	2352	15,833	509	...	2235	78-27	2338	2375	60-64
65-69	8,618	357	...	4143	12,312	618	...	3215	73-16	3344	3373	65-69
70-74	5,695	324	...	5689	7,881	559	...	5020	82-53	4900	4918	70-74
75-79	2,992	358	...	11960	3,953	658	...	7093	80-20	7281	7290	75-79
80-84	962	144	...	14980	1,409	233	...	10321	115-90	10882	10885	80-84
85-89	235	47	...	20000	294	60	...	16539	90-57	16240	16240	85-89
90-94	39	9	...	23070	47	15	...	20408	98-20	21001	24001	90-94
95-99	5	1	1	...	31915	72-28	34788	34788	95-99
All Ages	119,899	2,247	115,101	3,476	...	100000	...	48276	48276	All Ages

are compared with one another, and each with the two standard Tables. The Abstainers here show as marked a superiority as in the complete experience.

TABLE VIII.
Healthy Males. Entrants 1889-1901.
Whole-Life Policies. "Transfers to" excluded.
Entrants 1889-1901.
Experience 1889-1901.

Ages	ABSTAINERS			NON-ABSTAINERS			Taking Non-Abstainers' as 100 Abstainers' Experience is	Ages
	Exposed to Risk	Died	Rate of Mortality per cent per annum	Exposed to Risk	Died	Rate of Mortality per cent per annum		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
- 9	1	1	- 9
10-14	176	32	10-14
15-19	1,050	138	15-19
20-24	3,481	2	.191	850	5	.589	24.40	20-24
25-29	5,733	5	.144	2,201	7	.318	104.09	25-29
30-34	6,443	19	.331	3,296	21	.637	26.80	30-34
35-39	5,746	11	.171	3,572	29	.812	42.86	35-39
40-44	4,787	20	.348	3,281	20	.609	41.22	40-44
45-49	3,693	12	.251	2,592	21	.810	66.83	45-49
50-54	2,590	20	.542	1,983	25	1.260	70.47	50-54
55-59	1,675	23	.888	1,301	22	1.691	67.10	55-59
60-64	893	17	1.903	698	14	2.004	94.97	60-64
65-69	404	15	3.713	286	13	4.553	81.55	65-69
70-74	140	7	5.000	115	3	2.609	192.50	70-74
75-79	37	4	10.810	35	3	8.570	126.20	75-79
80-84	20	3	15.000	11	2	18.182	84.65	80-84
85-89	4	3	75.000	85-89
90-94	90-94
95-99	95-99
All Ages	36,873	180	...	20,392	185	All Ages

In Table VI are the rates of Mortality during the complete period 1841-1901 of the Healthy Male Assurers *who entered before 1889, i.e.,* excluding the new business of the last 13 years

TABLE IX.
Rated-up Males. True Ages.
Whole-Life Policies. "Transfers to" excluded.
Entrants 1841-1901.
Experience 1841-1901.

Ages	ABSTAINERS			NON-ABSTAINERS			Ages
	Exposed to Risk	Died	Rate of Mortality per cent per annum	Exposed to Risk	Died	Rate of Mortality per cent per annum	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
- 9	17	1	2.268	2
10-14	44	15
15-19	262	116
20-24	1,712	13	.760	1,008	7	.862	...
25-29	4,418	20	.450	3,443	32	.929	109.30
30-34	6,442	36	.559	6,136	57	.929	48.44
35-39	7,316	32	.437	7,745	88	1.136	60.17
40-44	7,318	40	.547	8,409	97	1.154	38.47
45-49	6,563	61	.975	8,153	152	1.864	47.40
50-54	5,253	72	1.371	7,079	133	1.879	52.31
55-59	3,871	74	1.912	5,594	145	2.592	72.96
60-64	2,585	90	3.481	3,964	149	3.759	73.77
65-69	1,587	55	3.466	2,427	110	4.531	92.60
70-74	890	73	8.198	1,390	129	9.282	76.49
75-79	379	51	13.456	579	62	10.717	88.33
80-84	123	17	13.821	201	31	15.423	125.56
85-89	43	5	11.628	53	12	22.641	89.62
90-94	6	5	83.334	7	2	28.571	51.36
95-99	291.67
All Ages	48,859	618	...	56,321	1,207

1889-1901. The O^M and $O^{M(5)}$ rates are shown for convenience, but they do not admit of a direct comparison. The ratios of the Abstainers to the Non-Abstainers are given in Column (8).

TABLE X.
Rated-up Males. Rated-up Ages.
Whole-Life Policies. "Transfers to" excluded.
Entrants 1841-1901.
Experience 1841-1901.

Ages	ABSTAINERS			NON-ABSTAINERS			Taking Non-Abstainers' as 100, Experience is	Rate of Mortality per annum (Central Ages)		TAKING O^M AS 100		Ages
	Exposed to Risk	Died	Rate of Mortality per-cent per annum	Exposed to Risk	Died	Rate of Mortality per-cent per annum		(8)	(9)	Abstainers' Experience is	Non-Abstainers' Experience is	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
-9	5	-9
10-14	24	10-14
15-19	50	1	2.000	17	15-19
20-24	247	1	.405	165	20-24
25-29	1,329	9	.677	1,087	11	.608	66.62	.375	532.40	25-29
30-34	3,964	16	.404	3,330	29	1.012	66.90	.431	93.97	141.00	...	30-34
35-39	6,218	27	.434	5,945	55	.870	46.44	.523	129.40	193.50	...	35-39
40-44	7,525	46	.611	7,936	104	.925	46.92	.618	62.34	134.20	...	40-44
45-49	7,848	43	.548	8,775	93	1.311	46.60	.804	53.98	115.00	...	45-49
50-54	7,077	74	1.016	8,629	152	1.060	51.70	1.001	61.04	131.00	...	50-54
55-59	5,561	75	1.349	7,230	151	1.762	59.36	1.277	42.91	83.00	...	55-59
60-64	4,011	77	1.920	5,622	144	2.089	64.58	1.633	61.77	104.10	...	60-64
65-69	2,561	94	3.670	3,857	155	2.562	74.94	2.338	57.70	89.35	...	65-69
70-74	1,475	70	4.745	2,245	150	4.019	91.33	3.344	57.41	76.61	...	70-74
75-79	650	70	10.778	1,002	94	6.683	70.99	4.900	74.90	82.02	...	75-79
80-84	228	30	13.187	353	45	9.381	114.90	7.281	65.16	91.79	...	80-84
85-89	69	8	11.594	102	17	12.766	103.30	10.882	99.08	86.22	...	85-89
90-94	17	7	41.177	18	6	16.586	69.87	16.240	81.21	78.63	...	90-94
95-99	33.333	123.60	24.001	48.29	69.12	...	95-99
All Ages	48,859	648	...	56,321	1,207	48.276	All Ages

Table VII shows the experience of the same body during the years 1889-1901 only, *i.e.*, after the exclusion of new business.

Table VIII gives the experience of *new entrants only* during the years 1889-1901.

Tables V, VI, VII and VIII are "Select" experience of Whole-Life Policies on Healthy Males, and are each parts of the general experience given in Tables III and IV. In each of these divisions the superior vitality of the Abstainers is conspicuous.

RATED-UP LIVES.

(24.) Table IX gives the experience of "Rated-up" Lives Whole-Life Policies at their true ages.

Table X of the same lives at the ages corresponding to the ratings.

These are, of course, a separate body of persons from those in the preceding Tables; but the superiority of the Abstainer is still evident in both Tables.

ENDOWMENT ASSURANCES.

(25.) Table XI gives the rates of Mortality under Endowment Assurances Healthy Males over the entire period 1841-1901. Here, again, the Abstainers show the best results.

FEMALES.

(26.) Table XII, Whole-Life Policies, Healthy Females: The H^F Table being taken as a standard. The difference between the Abstainers and Non-Abstainers is not so marked as among the Males. The Abstainers are generally as much superior to the Standard H^F as are the Male Abstainers to the O^M , but the Female Non-Abstainers are also superior to the Standard, though to a less extent than the Abstainers. It is not unreasonable to assume that the Female Non-Abstainer of the assuring class is more temperate and careful in her habits, *i.e.*, more closely approaches the Abstainer, than does the corresponding Male, and this, I suggest, as the reason of her greater comparative vitality.

(27.) Table XIII shows in a form convenient for comparison the ratios (multiplied by 100) of the Abstainers' rate of Mortality to that of the Non-Abstainers in the various classes given in Tables IV, V, VI, VIII, IX, X, XI, and XII. It may be read thus in any class, taking Column (2) as an example. Of 100 persons, Non-Abstainers, dying, of the ages 50-54, according to the experience only 62 would die among a similar body of Abstainers.

TABLE XI.

Healthy Males. Endowment Assurances.
"Transfers to" excluded.
Entrants 1841-1901.
Experience 1841-1901.

Ages	ABSTAINERS.				NON-ABSTAINERS.			Taking Non-Abstainers' as 100, Abstainers' Experience is	Ages
	Exposed to Risk	Died	Rate of Mortality per-cent per annum		Exposed to Risk	Died	Rate of Mortality per-cent per annum		
(1)	(2)	(3)	(4)		(5)	(6)	(7)	(8)	(9)
- 9	48		31	- 9
10-14	1,052	1	.095		266	1	.377	25.20	10-14
15-19	5,368	15	.279		1,466	3	.205	136.10	15-19
20-24	21,539	70	.325		8,804	36	.405	80.24	20-24
25-29	37,243	159	.427		21,329	97	.455	93.84	25-29
30-34	40,858	165	.404		29,037	124	.427	94.62	30-34
35-39	36,960	155	.419		29,858	191	.640	65.46	35-39
40-44	20,606	164	.554		25,963	190	.732	75.68	40-44
45-49	21,287	139	.653		19,585	186	.950	68.74	45-49
50-54	12,870	127	.987		12,145	152	1.252	78.93	50-54
55-59	7,018	112	1.596		6,753	121	1.792	89.06	55-59
60-64	2,063	66	3.199		1,762	50	2.838	112.70	60-64
65-69	122	4	3.265		94	2	2.128	153.40	65-69
70-	5		9	70-
All Ages	216,039	1,177	...		157,192	1,153	All Ages

TABLE XII.

Healthy Females. Whole-Life Policies.

"Transfers to" excluded.

Entrants 1841-1901.

Experience 1841-1901.

Ages	ABSTAINERS			NON-ABSTAINERS			Taking Non-Abstainers' as 100, Experiences	Rate of Mortality per cent. per annum Central Ages	TAKING 11F AS 100		Ages
	Exposed to Risk	Died	Rate of Mortality per cent. per annum	Exposed to Risk	Died	Rate of Mortality per cent. per annum			Abstainers' Experiences	Non-Abstainers' Experiences	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
9	75	54	1	1.863	-9
10-14	344	5	1.456	228	71.75	.514	283.27	...	10-14
15-19	637	2	.287	500	2	.400	106.00	.707	40.60	56.58	15-19
20-24	1,135	8	.705	1,203	8	.665	71.06	.792	89.02	83.97	20-24
25-29	1,897	13	.685	2,904	28	.964	136.60	1.182	57.96	81.56	25-29
30-34	2,707	27	.997	4,333	36	.730	87.86	1.157	86.18	63.10	30-34
35-39	3,454	31	.897	6,756	69	1.021	87.36	1.204	74.51	84.80	35-39
40-44	3,927	33	.843	7,983	77	.965	87.06	1.294	65.15	74.58	40-44
45-49	4,212	47	1.116	8,657	111	1.282	87.68	1.388	80.41	92.36	45-49
50-54	4,161	51	1.225	8,588	120	1.397	85.15	1.562	78.42	89.43	50-54
55-59	3,719	64	1.721	7,971	148	1.857	92.68	1.989	86.58	93.41	55-59
60-64	3,139	83	2.614	6,339	208	3.105	85.15	2.816	92.92	109.10	60-64
65-69	2,280	94	4.122	5,065	225	4.442	92.77	4.340	94.95	102.40	65-69
70-74	1,412	109	7.720	3,239	222	6.854	112.60	6.917	111.60	99.08	70-74
75-79	629	72	11.449	1,607	166	10.328	110.80	10.976	104.30	94.08	75-79
80-84	183	35	19.126	534	101	18.917	138.90	13.170	145.20	143.60	80-84
85-89	36	10	27.778	105	21	20.000	300.00	23.696	117.20	84.39	85-89
90-94	3	2	66.667	27	6	22.222	...	23.200	287.30	95.76	90-94
95-99	3	2	66.667	...	33.775	...	197.30	95-99
All Ages	31,013	686	...	67,056	1,551	66.667	All Ages

TABLE XIII.

Summary of the Ratios of Mortality Rates of Abstainers (per-cent per annum) to those of Non-Abstainers.

TAKING NON-ABSTAINERS AS 100, THE ABSTAINERS' RATES UNDER THE VARIOUS GROUPS ARE

Ages	Males				Females Whole-Life	Males (Whole-Life)				Ages
	Whole Life	Endowment Assurance	Rated-up Lives			Excluding 1st Five Years of Assurance	Entrants 1841-1888 during 1841-1901	New Entrants 1889-1901 only		
			True Ages	Rated-up Ages						
									(3)	
- 9	- 9	
10-14	112.0	25.2	71.8	147	128	...	10-14	
15-19	141.0	136	106	45.8	160	...	15-19	
20-24	69.9	80.2	109	66.6	71.1	58.2	82.7	24.4	20-24	
25-29	70.1	93.8	48.4	66.9	137	51.7	70.1	101	25-29	
30-34	56.5	94.6	60.2	46.4	87.9	52.7	61.3	26.8	30-34	
35-39	54.8	65.5	38.5	46.9	87.4	55.8	59.0	42.9	35-39	
40-44	57.2	75.7	47.4	46.6	87.1	55.9	58.8	41.2	40-44	
45-49	58.5	68.7	52.3	51.7	87.7	60.7	62.4	66.8	45-49	
50-54	62.4	78.8	73.0	59.4	92.7	69.4	71.1	70.5	50-54	
55-59	70.6	89.1	73.8	64.6	92.7	76.1	78.7	67.1	55-59	
60-64	78.5	113	92.6	74.9	85.2	83.4	84.2	95.0	60-64	
65-69	84.0	153	76.5	91.3	92.8	86.4	86.2	81.6	65-69	
70-74	86.5	...	88.3	71.0	113	110	109	193	70-74	
75-79	110.0	...	126	115	111	93.7	93.7	126	75-79	
80-84	93.7	...	89.6	103	101	93.6	93.7	84.7	80-84	
85-89	90.7	...	51.4	69.9	139	78.0	89.8	...	85-89	
90-94	77.9	...	292	124	300	78.0	78.0	...	90-94	
95-99	95-99	

(28.) It will be noticed how closely the ratios resemble one another in their general results. The curious temporary increase in the Abstainers' rate, as compared with the standard in the group of ages 74-79, and the recovery at the later ages is clearly shown in every class of Policy.

TABLE XIV.
Healthy Males. Whole-Life Policies. Rates of Mortality in various periods compared with 1841-1901.

Ages	ABSTAINERS. RATES OF MORTALITY PER-CENT PER ANNUM					NON-ABSTAINERS. RATES OF MORTALITY PER-CENT PER ANNUM					Ages
	1841-82 42 Years	1841-88 48 Years	1841-1901 61 Years	Taking 1841-1901 as 100		1841-82 42 Years	1841-88 48 Years	1841-1901 61 Years	Taking 1841-1901 as 100		
				1841-82	1841-88				1841-82	1841-88	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
- 9	1.167	1.320	1.299	89.6	101.	- 9
10-14	.282	.237	.190	148.	125.	.197	.185	.170	116.	109.	10-14
15-19	.849	.820	.668	127.	123.	.471	.529	.475	99.8	111.	15-19
20-24	.668	.571	.463	144.	123.	.717	.688	.662	113.	104.	20-24
25-29	.500	.447	.406	123.	110.	.663	.580	.579	115.	100.	25-29
30-34	.475	.460	.408	116.	113.	.799	.726	.722	111.	101.	30-34
35-39	.498	.492	.444	112.	111.	.858	.802	.810	106.	99.	35-39
40-44	.635	.582	.547	116.	106.	1.082	1.016	.957	113.	106.	40-44
45-49	.901	.807	.749	121.	108.	1.409	1.309	1.283	110.	102.	45-49
50-54	1.077	1.045	1.019	103.	99.6	1.752	1.696	1.683	104.	101.	50-54
55-59	1.680	1.661	1.673	100.	99.3	2.581	2.488	2.373	109.	105.	55-59
60-64	2.814	2.806	2.565	110.	109.	3.436	3.355	3.265	105.	103.	60-64
65-69	4.121	4.425	4.260	96.7	104.	5.019	5.131	5.065	99.2	101.	65-69
70-74	6.363	7.079	6.199	103.	114.	7.418	7.406	7.171	103.	103.	70-74
75-79	10.178	10.823	11.607	87.7	93.2	11.485	11.331	10.623	108.	107.	75-79
80-84	14.634	15.891	15.230	96.1	104.	19.512	15.335	16.252	120.	94.4	80-84
85-89	18.557	19.277	20.497	90.5	94.	47.826	29.687	22.607	212.	131.	85-89
90-94	38.462	31.250	25.455	151.	123.	...	50.000	32.653	...	153.	90-94
95-99	100.000	95-99

The main results of the principal divisions, namely :

Healthy Males—Whole-Life Policies,
Do. Endowment Assurances,
Rated-up Males,
Healthy Females,

as compared with the Non-Abstainers' standard of each division, are exhibited in pictorial form in Diagram 1. It will be seen that 40 degrees of the ordinates have been omitted to save space. The curves indicated by the several lines all take the same form. Approaching, and sometimes exceeding, the standard of the Non-Abstainers at the younger ages, they show a very great superiority throughout middle life, and approximate to the standard again in old age. It should be noted that the curves cannot be compared with one another.

(29.) Table XIV gives the rates of Mortality of Healthy Males, Whole-Life Policies, from 1841 to the close of the years 1882 and 1888, in both the Abstainers' and Non-Abstainers' Sections respectively, and compares these rates with those of the full period 1841-1901. The general improvement in both sections is very obvious, thus confirming my belief, expressed in paragraph 16, that, had a more recent section of the Institution's history been dealt with, the results deduced would have been even more favourable than those now shown.

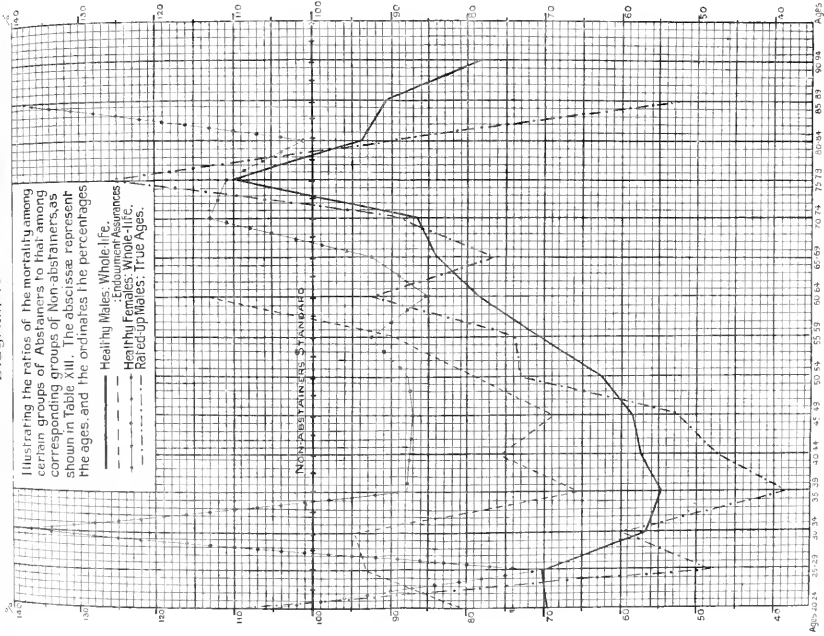
ON THE EFFECT OF TRANSFERS BETWEEN THE ABSTAINERS' AND NON-ABSTAINERS' SECTIONS.

(30.) In Tables III and IV only original entrants are included, as stated in the headings thereof. "Transfers to" either section are excluded altogether, and "Transfers from" are treated as having passed from observation at the time of transfer.

It has been suggested by persons not conversant with the working of the Institution that a large proportion of the lives transferred from the Abstainers' to the Non-Abstainers' Section may be in poor health or of doubtful habits; and that the Mortality of the Abstainers' Section is thus lightened, and that of the Non-Abstainers' Section correspondingly increased by such transfers. In other words, it is supposed that when an Abstainer's health deteriorates, if he take alcohol he is transferred to the Non-Abstainers' Section to die, and his death is included in the latter section. Now (A) an Abstainer is allowed to take alcohol temporarily as a medicine *bona fide*, and (B) he is not transferred if he be known to be in bad health or of intemperate habits. (In the latter case (B) a record is made in the Institution's

Illustrating the ratios of the mortality among certain groups of Abstainers to that among corresponding groups of Non-abstainers, as shown in Table XIII. The abscissæ represent the ages, and the ordinates the percentages.

— Healthy Males: Whole-life.
 - - - Endowment-Assurances
 - · - Healthy Females: Whole-life.
 - · - Raced-up Males: True Ages.



Registers that he is to receive Non-Abstainer's rate of Bonus only.) But apart from these facts the transfers have been so few that their effect can be but insignificant. Table XV shows the numbers transferred year by year between the two

TABLE XV.—Showing the Number of Transfers between Abstainers' and Non-Abstainers' Sections in each Year. Whole-Life Policies.

Year	TRANSFERS			Year	TRANSFERS			Year	TRANSFERS		
	To Non-Abstainers	To Abstainers	(3)		To Non-Abstainers	To Abstainers	(3)		To Non-Abstainers	To Abstainers	(3)
(1)	(2)	(3)		(1)	(2)	(3)		(1)	(2)	(3)	
1847	2	1865	49	26	...	1884	25	60	...
1848	5	1866	44	18	...	1885	60	98	...
1849	2	1867	31	7	...	1886	28	41	...
1850	6	1868	29	15	...	1887	32	29	...
1851	8	1869	28	10	...	1888	33	33	...
1852	19	1	1	1870	57	24	...	1889	29	24	...
1853	11	1	1	1871	35	40	...	1890	94	46	...
1854	11	3	3	1872	36	34	...	1891	33	41	...
1855	14	1873	26	30	...	1892	37	21	...
1856	11	8	...	1874	31	25	...	1893	25	26	...
1857	23	10	10	1875	68	44	...	1894	31	22	...
1858	13	8	8	1876	32	35	...	1895	73	40	...
1859	31	17	17	1877	33	45	...	1896	42	20	...
1860	32	10	10	1878	19	32	...	1897	34	4	...
1861	28	22	22	1879	21	47	...	1898	32	22	...
1862	35	23	23	1880	78	67	...	1899	29	16	...
1863	33	1881	26	65	...	1900	84	47	...
1864	42	24	24	1882	19	84	...	1901	182	14	...
				1883	34	85	...	Total	2,685	1,465	

Corresponding Policies in force—

31 December 1882	Abstainers	Non-Abstainers
31 December 1888	9,808	12,233
31 December 1901	11,245	11,338
	12,645	9,382

sections under Whole-Life Policies, and also the number of such Policies in force at certain dates. The increase in the transfers from the Temperance to the General Section in the years 1900

TABLE XVI—Summary.
Healthy Males. Whole-Life Policies.
Showing effect of TRANSFERS TO NON-ABSTAINERS.
Transfers 1847-1901.
Experience 1847-1901.

Ages	TRANSFERS TO NON-ABSTAINERS			TRANSFERS TO NON-ABSTAINERS COMBINED WITH NON-ABSTAINERS			TRANSFERS TO NON-ABSTAINERS COMBINED WITH ABSTAINERS			Ages
	Exposed to Risk	Died	Rate of Mortality per-cent per annum	Exposed to Risk	Died	Rate of Mortality per-cent per annum	Exposed to Risk	Died	Rate of Mortality per-cent per annum	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
- 9	74	77	1	1.299	- 9
10-14	5	595	1	.168	1,056	2	.189	10-14
15-19	37	2,141	10	.467	4,528	30	.663	15-19
20-24	332	4	1.205	9,848	67	.680	16,092	77	.479	20-24
25-29	1,179	6	.509	28,278	163	.576	33,919	139	.410	25-29
30-34	2,431	16	.658	49,396	355	.719	48,986	206	.421	30-34
35-39	3,296	31	.941	64,402	526	.817	57,393	271	.472	35-39
40-44	3,579	38	1.062	71,002	683	.962	59,183	342	.578	40-44
45-49	3,482	57	1.637	69,413	903	1.301	54,859	442	.806	45-49
50-54	3,050	46	1.508	61,991	1,038	1.674	47,188	509	1.079	50-54
55-59	2,566	86	3.352	50,145	1,222	2.423	37,510	671	1.787	55-59
60-64	1,879	59	3.141	37,040	1,207	3.259	27,142	707	2.605	60-64
65-69	1,182	80	6.771	24,401	1,256	5.147	17,661	782	4.428	65-69
70-74	627	47	7.496	13,484	969	7.186	9,952	625	6.280	70-74
75-79	271	23	8.487	6,051	637	10.527	4,622	528	11.424	75-79
80-84	87	21	24.138	1,977	328	16.599	1,433	226	15.771	80-84
85-89	16	2	12.500	374	81	21.658	338	68	20.116	85-89
90-94	4	1	25.000	53	17	32.075	59	15	25.423	90-94
95-99	1	1	100.000	5	95-99
All Ages	24,023	517	...	490,966	9,464	...	422,033	5,641	...	All Ages

TABLE XVII.—Summary.

Healthy Males. Whole-Life Policies.

Showing effect of TRANSFERS TO ABSTAINERS.

Transfers 1851-1901.

Experience 1851-1901.

Ages	TRANSFERS TO ABSTAINERS				TRANSFERS TO ABSTAINERS COMBINED WITH ABSTAINERS				TRANSFERS TO ABSTAINERS COMBINED WITH NON-ABSTAINERS				Ages
	Exposed to Risk	Died	Rate of Mortality per annum	(2)	(3)	(4)	(5)	(6)	(7)	Exposed to Risk	Died	Rate of Mortality per annum	
(1)										(8)	(9)	(10)	(11)
- 9	77	1	1-289	1	1-289	74	- 9
10-14	22	1,053	2	1-90	2	1-90	592	1	1-69	10-14
15-19	123	4,513	30	1-665	30	1-665	2,126	10	1-470	15-19
20-24	433	1	1-813	...	15,883	74	1-466	74	1-466	9,639	64	1-664	20-24
25-29	951	3	1-694	...	33,173	136	1-410	136	1-410	27,532	160	1-581	25-29
30-34	1,597	2	1-210	...	47,506	192	1-404	192	1-404	47,916	341	1-712	30-34
35-39	2,363	5	1-313	...	55,694	245	1-440	245	1-440	62,703	500	1-797	35-39
40-44	2,849	10	1-423	...	57,967	314	1-542	314	1-542	69,786	655	1-939	40-44
45-49	3,118	27	1-948	...	51,226	412	1-760	412	1-760	68,780	873	1-269	45-49
50-54	2,971	29	1-930	...	47,256	492	1-041	492	1-041	62,059	1,021	1-645	50-54
55-59	2,504	47	1-582	...	37,945	632	1-666	632	1-666	50,850	1,183	2-326	55-59
60-64	1,707	65	2-596	...	27,767	713	2-568	713	2-568	37,665	1,213	3-220	60-64
65-69	914	74	4-334	...	18,186	776	4-267	776	4-267	21,926	1,250	5-015	65-69
70-74	470	56	6-130	...	10,239	634	6-193	634	6-193	13,771	978	7-102	70-74
75-79	154	42	8-936	...	4,821	547	11-347	547	11-347	6,250	656	10-476	75-79
80-84	23	21	13-592	...	1,500	226	15-062	226	15-062	2,044	328	16-050	80-84
85-89	2	3	13-333	...	345	60	20-029	60	20-029	381	82	21-550	85-89
90-94	...	2	100-000	...	57	16	28-070	16	28-070	51	18	35-294	90-94
95-99	5	1	1	100-000	95-99
All Ages	20,203	387	418,213	5,511	...	5,511	...	187,146	9,334	...	All Ages

and 1901, was due to the issue of more definite and detailed instructions on some points on which it was found there had been some doubt and variation in practice in different districts. It is practically certain that many of those who declined to sign the declaration were really Abstainers, and they may apply for re-transfer later on. Be this as it may, it is obvious that the operations of these particular years, 1900 and 1901, cannot have any material effect on the aggregate experience 1841-1901.

A second set of cards was prepared for these transferred cases on which the year of entry and age at entry were respectively the year of transfer and the corresponding age. They were then dealt with independently in their new section in the form given in par. 15. The full statistics relating to them are given in Tables XVI and XVII. It will be seen that the number of entrants in Tables XVI and XVII differs from the exits by transfer in Tables IV and III the reason being that some of the former are re-transfers and a few of the latter are transfers to a different class of Policy; for example, from Whole-Life to Endowment Assurance.

(31.) In Table XVIII are shown, in respect of lives transferred from the Abstainers' to the Non-Abstainers' Section, the following rates of Mortality,

- (a) Among the transferred cases only.
- (b) Combining these cases with the section to which they are transferred. In practice they are so combined.
- (c) Combining the same cases with their original section; *i.e.*, treating them as if they had not been transferred at all.

Table XIX gives similar information in regard to transfers from the Non-Abstainers' to the Abstainers' Section.

(32.) Now what has been the effect of these transfers on the Non-Abstainers' Mortality? If we refer to Columns 2 and 6 of Table XVIII we see that the general rate of Mortality of these transferred lives has exceeded what may be termed the normal rate of Mortality of the Non-Abstainers. But on referring to Columns 3 and 7 of the same Table, we see that when the exposures and deaths of the transfers are combined with those of the general body of Non-Abstainers, the numbers of the former being small, no appreciable effect is made on the latter. This combination is what actually takes place in practice, and it is important that its operation should be fully understood.

(33.) I venture to state that the ratios in Column 7 of Table XVIII are a *complete refutation of the suggestion that the*

TABLE XVIII.
Healthy Males. Whole-Life Policies.
Rates of Mortality of Transfers to Non-Abstainers.
Experience 1847-1901.

Ages	RATES OF MORTALITY PER CENT PER ANNUM				TAKING NORMAL RATE AS 100				Ages
	Transfers to Non-Abstainers alone	Transfers to Non-Abstainers combined with Non-Abstainers	Transfers to Abstainers combined with Non-Abstainers	Normal rate of Non-Abstainers without transfers	Transfers to Non-Abstainers alone are	Transfers to Non-Abstainers combined with Non-Abstainers	Transfers to Abstainers combined with Non-Abstainers		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
10-14	...	168	169	170	...	98.83	99.43	10-14	
15-19	...	467	470	475	...	98.31	98.72	15-19	
20-24	1205	680	664	662	18200	102.70	100.30	20-24	
25-29	509	576	581	579	8730	99.47	100.30	25-29	
30-34	658	719	712	722	9114	99.59	98.63	30-34	
35-39	941	817	797	810	11620	100.90	98.10	35-39	
40-44	1062	962	939	957	11100	100.50	98.13	40-44	
45-49	1637	1301	1269	1283	12760	101.40	98.92	45-49	
50-54	1508	1674	1645	1683	8960	99.47	97.72	50-54	
55-59	3352	2423	2326	2373	11150	102.10	98.03	55-59	
60-64	3141	3259	3220	3265	9621	99.82	98.63	60-64	
65-69	6771	5117	5015	5065	13470	101.60	99.01	65-69	
70-74	7496	7186	7102	7171	10150	100.20	99.01	70-74	
75-79	8487	10527	10476	10623	7993	98.92	98.70	75-79	
80-84	24138	16599	16050	16252	14850	102.10	98.76	80-84	
85-89	12500	21658	21550	22607	5528	95.81	95.30	85-89	
90-94	25000	32075	35294	32653	7656	98.24	108.10	90-94	

TABLE XIX.
Healthy Males. Whole-Life Policies.
Rates of Mortality of TRANSFERS TO ABSTAINERS.
Experience 1851-1901.

Ages	RATES OF MORTALITY PER-CENT PER ANNUM					TAKING NORMAL RATE AS 100			Ages
	Transfers to Abstainers alone	Transfers to Abstainers combined with Abstainers	Transfers to Non-Abstainers combined with Abstainers	Normal rate of Abstainers without transfers	(5)	Transfers to Abstainers alone are	Transfers to Abstainers combined with Abstainers	Transfers to Non-Abstainers combined with Abstainers	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(9)
- 9	...	1.299	1.299	1.299	...	100.00	100.00	100.00	- 9
10-14190	.189	.190	...	100.00	99.47	99.47	10-14
15-19665	.663	.668	...	99.54	99.24	99.24	15-19
20-24	.813	.466	.479	.463	175.60	100.60	103.40	103.40	20-24
25-29	.694	.410	.410	.406	171.00	101.00	101.00	101.00	25-29
30-34	.210	.404	.421	.408	51.46	99.01	103.20	103.20	30-34
35-39	.313	.440	.472	.444	70.49	99.11	106.30	106.30	35-39
40-44	.423	.542	.578	.547	77.32	99.08	105.70	105.70	40-44
45-49	.918	.760	.806	.749	126.60	101.50	107.60	107.60	45-49
50-54	.930	1.041	1.079	1.049	88.65	99.24	102.80	102.80	50-54
55-59	1.582	1.666	1.787	1.673	94.56	99.59	106.80	106.80	55-59
60-64	2.596	2.568	2.605	2.565	101.20	100.10	101.60	101.60	60-64
65-69	4.334	4.267	4.428	4.260	101.70	100.20	103.90	103.90	65-69
70-74	6.130	6.193	6.280	6.199	98.90	99.91	101.30	101.30	70-74
75-79	8.936	11.347	11.424	11.607	76.97	97.77	98.38	98.38	75-79
80-84	13.592	15.062	15.771	15.230	89.23	98.88	103.50	103.50	80-84
85-89	13.333	20.029	20.116	20.497	65.01	97.70	98.13	98.13	85-89
90-94	100.000	28.070	25.423	25.455	392.70	110.20	99.84	99.84	90-94

Mortality of the Non-Abstainers has been increased by the importation into their ranks of unhealthy withdrawals from the Abstainers' Section.

(34.) A minor question is—Are the Non-Abstainers injured by the withdrawal of their select lives who may become Abstainers? Passing the indirect suggestion by this question that Abstinence is beneficial, we refer to Columns 4 and 8 of Table XVIII, which show what the Mortality of the Non-Abstainers would have been had the lives transferred not been withdrawn. Again we see that the results are not materially influenced.

(35.) Conversely, what has been the effect of the transfers on the Abstainers' Section? The chief question in this connection is—"Has the Abstainers' Section been relieved by the withdrawal of the lives transferred to the Non-Abstainers' Section?" We have already seen (paragraph 32) that the mortality among these has been rather in excess of that of the Non-Abstainers. Still more must it be in excess of the Abstainers' normal Mortality. Columns 4 and 8 of Table XIX show to what extent the Abstainers' Mortality would have been increased had these lives not been withdrawn. We see (col. 8) that there has been an increase, and had the numbers so withdrawn been large, the relief enjoyed by the Abstainers' Section must have been considerable. Columns 2 and 6, and Columns 3 and 7 of Table XIX show respectively the Mortality of transfers to the Abstainers' Section, and of the combination of these transfers with the Abstainers generally. It is interesting to compare Column 2 of Table XVIII with the corresponding Column 2 of Table XIX, showing the relative mortality of those transferred from the Abstainers' Section, and those transferred thereto.

MONETARY TABLES BASED UPON ABSTAINERS' MORTALITY.

(36.) From the data given in Table IV (Healthy Male Abstainer's Whole-Life Policies, Aggregate Experience 1841-1901) a Mortality Table (which I call T^M) has been prepared. It was assumed for ages below 23 that q_x (T^M) was equal to q_{x+2} (O^M), and for ages 75 and upwards that the T^M rate was the same as the O^M . For ages 23-74 the Abstainers' Mortality (Healthy Males Table IV) was followed, and graduated by a simple "graphic" method, for which no mathematical accuracy is claimed. The functions l_x , d_x , p_x , q_x , and the Commutation Columns (with 3 per-cent interest) D_x and N_x are given on Table XXI. In Table XXII are given for quinary ages the values of

TABLE XX.

Mortality of Healthy Males, Abstainers, Whole-Life Policies, 1841-1901 [T^M]. Showing out of 100,000 living at ages 10, 20, 30, &c., the numbers surviving to the succeeding decennial ages according to T^M and O^M Tables respectively. Also the additional survivors by the T^M Table.

Age attained	COMMENCING AGE 10				COMMENCING AGE 20				COMMENCING AGE 30			
	By T ^M	By O ^M	Additional Survivors by T ^M		By T ^M	By O ^M	Additional Survivors by T ^M		By T ^M	By O ^M	Additional Survivors by T ^M	
10	(1)	(2)	(3)		(4)	(5)	(6)		(7)	(8)	(9)	
20	100,000	100,000	
30	96,314	96,453	- 139		100,000	100,000	
40	92,249	91,942	+ 307		95,779	95,323	456		
50	88,373	85,467	2,906		91,755	88,609	3,146		100,000	100,000	...	
60	82,853	76,185	6,668		86,024	78,986	7,038		95,799	92,959	2,840	
70	72,036	62,073	9,963		74,793	64,355	10,438		89,815	82,863	6,952	
80	51,089	40,615	10,474		53,044	42,109	10,935		78,089	67,513	10,576	
90	20,372	15,530	4,842		21,152	16,101	5,051		55,382	44,175	11,207	
	2,094	1,596	498		2,174	1,655	519		22,084	16,891	5,193	
									2,269	1,736	533	
Age attained	COMMENCING AGE 40				COMMENCING AGE 50				COMMENCING AGE 60			
	(10)	(11)	(12)		(13)	(14)	(15)		(16)	(17)	(18)	
40	100,000	100,000	
50	93,754	89,139	4,615		100,000	100,000	
60	81,514	72,628	8,886		86,914	81,476	5,438		
70	57,811	47,521	10,290		61,662	53,314	8,351		100,000	100,000	...	
80	23,053	18,171	4,882		24,588	20,384	4,204		70,922	65,432	5,490	
90	2,369	1,867	502		2,527	2,095	432		28,281	25,019	3,262	
									2,906	2,571	335	

TABLE XXII.
Comparison of certain functions based upon ABSTAINERS' MORTALITY T^M.

Ages	e_x			a_x 3 PER-CENT			125 P _x 3 PER-CENT			Ages	
	T ^M	H ^M	O ^M	T ^M	H ^M	O ^M	T ^M		Carlisle, Being Institution's Whole-Life Rate		
							(8)	(9)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	£	s. d.	£	s. d.	(10)
10	55·021	49·791	51·159	25·471	24·148	24·669	1	1 7	1	9 2	10
15	50·973	45·661	47·323	24·687	23·158	23·751	1	4 6	1	13 3	15
20	46·949	41·561	43·182	23·863	22·043	22·705	1	7 9	1	17 4	20
25	42·967	37·905	39·083	22·930	21·038	21·539	1	11 8	2	2 7	25
30	38·827	34·181	35·067	21·786	19·867	20·264	1	16 11	2	8 10	30
35	34·595	30·516	31·159	20·428	18·587	18·885	2	3 10	2	15 7	35
40	30·328	26·899	27·360	18·856	17·176	17·394	2	13 1	3	4 11	40
45	26·100	23·292	23·668	17·084	15·594	15·782	3	5 5	3	15 5	45
50	22·016	19·806	20·107	15·165	13·896	14·054	4	1 10	4	10 6	50
55	18·130	16·462	16·722	13·125	12·094	12·239	5	4 2	5	13 8	55
60	14·554	13·330	13·571	11·054	10·236	10·384	6	14 7	7	4 9	60
65	11·338	10·512	10·716	9·011	8·418	8·551	8	16 11	8	19 3	65
70	8·491	7·995	8·208	7·037	6·657	6·810	11	18 3	11	15 1	70

e_x , a_x , and P_x , also $125 \cdot P_x$ for comparison with the Office Whole-Life Premiums of the Institution, which are Carlisle 3 per-cent with 25 per-cent loading. It will be observed that the difference between these Office premiums and the loaded T^M premium is, roughly, constant.

(37.) In Diagram II are shown the approximate curves of the function l_x according to the T^M and O^M Tables, the abscissæ being the ages, and the ordinates the numbers living. In Table XX are shown out of 100,000 living at the ages, 10, 20, 30, &c., the numbers surviving to the succeeding decennial ages according to the T^M and O^M Tables, also the additional survivors by the former Table at those succeeding decennial ages. This Table should be read thus, of 100,000 living at age 10, 82,853 would survive to age 50, according to the Institution's Temperance Mortality Experience. But according to the experience of ordinary Healthy Assured Lives (O^M) only 76,185 would survive, showing 6,668 additional survivors on the Temperance Experience. Or, out of 100,000 persons living at age 30, 78,089 would survive to age 60 on the Temperance Experience, and 67,513 on the Ordinary Healthy Lives' Experience, showing 10,576 additional survivors on the Temperance Experience.

(38.) The general increase in length of life which may be claimed as following on the practice of abstinence may be best presented in a popular form by a consideration of the function e_x , Table XXII. At age 30, when a man's constitution and habits of life may be considered as settled, we find the average number of years enjoyed thereafter by Non-Abstainers, according to the O^M Table, to be 35.1. But according to the Abstainers' Table it is 38.8 years, an increase of nearly 11 per-cent. At age 40 these figures are respectively 27.4 years and 30.3 years, also an increase of nearly 11 per-cent.

If an increased collective vitality (and what may reasonably be associated with it, namely, increased working capacity) to the extent of over 10 per-cent be an advantage to the community, these figures should be a striking testimony of the benefits that would arise from a general adoption of total abstinence at an early age, and the retention of the practice through life.

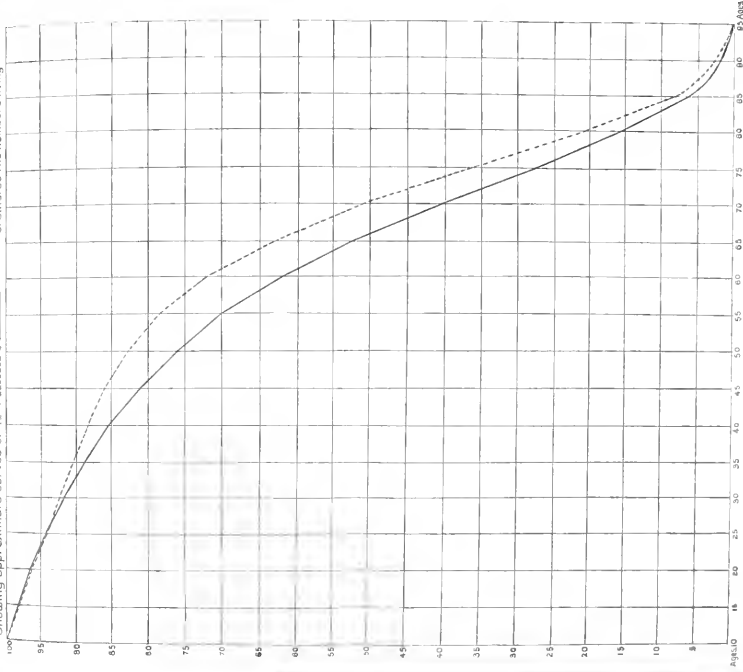
(39.) In conclusion, if it be granted—

- (a) That the average pecuniary positions of the Abstainers and Non-Abstainers assured in the Institution are approximately equal; or, in other words, that they belong to the same social class;

Diagram II

The abscissæ represent the ages and the ordinates the numbers living

Showing approximate curves of l_x T_x ----- 0



- (b) That the Members of the two sections are admitted, and afterwards dealt with on equal terms ;
- (c) That the methods used by me in this investigation are correct ;

then, bearing in mind that the point chiefly in view in this paper has been to compare the Mortality of a body of Abstainers with that of a similarly circumstanced body of Non-Abstainers, I submit that the data now put forward, and the results deduced therefrom, prove—

Firstly : That the Non-Abstainers assured in the Institution are good average lives, generally equal to the best accepted standard of assured life, namely, the O^M Table.

Secondly : That the Abstainers show a marked superiority to the Non-Abstainers throughout the entire working years of life, for every class of Policy, and for both sexes, however tested.

Thirdly : That this superiority has not been brought about by the operation of the “ Transfers ” between the two sections.

Fourthly : That the financial working of the Institution in the allotment of Bonuses as between the Abstainers’ and Non-Abstainers’ Sections has not been influenced by such transfers.

TABLE III.

Healthy Males. NON-ABSTAINERS.

Whole-Life Policies. "Transfers to" excluded.

Entrants 1841-1901.

Experience 1841-1901.

Age (x)	Entered at Age (x)	PASSED FROM OBSERVATION AT AGE (x) AS					Exposed to Risk	Died	Rate of Mortality	Age (x)
		Trans- ferred	Void	Surren- dered	Died	Existing 31 Dec. 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	0
1	1
2	2
3	3
4	4
5	3	2	5
6	6	...	1	6	6
7	8	12	7
8	10	21	8
9	17	...	1	33	9
10	37	...	4	...	1	...	59	1	·01709	10
11	29	...	4	85	11
12	35	...	8	1	112	12
13	49	1	6	2	144	13
14	63	...	9	2	...	1	190	14
15	64	1	7	1	243	15
16	100	1	14	3	2	...	310	2	·00645	16
17	117	2	21	3	1	3	394	1	·00254	17
18	173	1	31	1	4	10	505	4	·00793	18
19	241	3	43	5	3	6	652	3	·00460	19
20	379	4	61	8	6	8	891	6	·00674	20
21	570	8	99	9	10	16	1,251	10	·00799	21
22	827	10	130	12	13	17	1,787	13	·00728	22
23	943	10	181	24	18	30	2,451	18	·00734	23
24	1,042	14	256	16	16	46	3,136	16	·00510	24
25	1,107	22	257	32	22	43	3,850	22	·00571	25
26	1,181	16	287	27	18	60	4,605	18	·00391	26
27	1,264	15	324	35	35	59	5,393	35	·00649	27
28	1,285	25	311	49	33	71	6,195	33	·00533	28
29	1,419	31	306	53	49	80	7,056	49	·00695	29
30	1,367	41	343	52	51	83	7,903	51	·00645	30
31	1,306	30	311	59	55	89	8,691	55	·00633	31
32	1,389	29	332	71	89	92	9,476	89	·00939	32
33	1,238	41	341	93	78	100	10,154	78	·00766	33
34	1,267	35	359	100	66	119	10,741	66	·00615	34
35	1,218	34	335	100	85	125	11,321	85	·00751	35
36	1,147	47	331	74	103	104	11,833	103	·00870	36
37	1,039	33	314	100	97	129	12,272	97	·00790	37
38	1,032	47	268	87	97	127	12,662	97	·00766	38
39	976	41	301	96	113	144	13,018	113	·00868	39
40	860	58	273	97	150	137	13,250	150	·01132	40
41	810	34	255	92	114	152	13,396	114	·00851	41
42	797	46	259	108	135	164	13,535	135	·00997	42
43	687	39	220	122	107	163	13,586	107	·00788	43
44	690	37	190	100	139	217	13,656	139	·01018	44
45	570	42	205	111	165	228	13,584	165	·01215	45
46	525	40	172	115	148	216	13,401	148	·01105	46
47	461	36	171	89	160	214	13,218	160	·01211	47
48	392	30	146	94	190	232	12,991	190	·01463	48
49	447	38	119	82	183	216	12,737	183	·01438	49
Carried forward	29,187	942	7,606	2,124	2,556	3,502	280,808	2,556	...	

TABLE III (continued).

Age (x)	Entered at Age (x)	PASSED FROM OBSERVATION AT AGE (x) AS					Exposed to Risk	Died	Rate of Mortality	Age (x)
		Trans- ferred	Void	Surren- dered	Died	Existing 31 Dec. 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Brought forward	29,187	942	7,606	2,124	2,556	3,502	280,808	2,556	...	
50	354	38	117	87	194	234	12,498	194	01552	50
51	304	36	104	91	193	227	12,165	193	01588	51
52	292	34	100	93	190	230	11,813	190	01600	52
53	245	44	93	87	202	234	11,438	202	01766	53
54	226	26	89	84	213	245	11,027	213	01932	54
55	203	26	66	90	243	293	10,596	243	02293	55
56	145	32	67	73	230	264	10,056	230	02287	56
57	123	22	41	58	216	255	9,552	216	02261	57
58	112	21	48	43	219	277	9,082	219	02411	58
59	127	22	37	57	228	279	8,593	228	02653	59
60	80	25	31	62	235	236	8,074	235	02911	60
61	56	22	39	63	251	264	7,547	251	03326	61
62	58	10	23	71	248	207	6,979	248	03554	62
63	57	16	21	43	191	204	6,480	191	02943	63
64	44	6	11	49	223	234	6,072	223	03672	64
65	31	13	17	40	231	229	5,584	231	04137	65
66	30	2	10	49	256	176	5,091	256	05029	66
67	26	8	11	29	239	192	4,631	239	05160	67
68	20	5	4	29	233	204	4,182	233	05572	68
69	23	5	9	16	217	164	3,731	217	05816	69
70	9	5	6	23	180	203	3,335	180	05668	70
71	7	6	9	19	197	157	2,916	197	06756	71
72	6	7	7	25	194	119	2,533	194	07659	72
73	3	6	1	16	187	111	2,193	187	08525	73
74	2	3	4	7	155	109	1,880	155	08245	74
75	1	2	4	12	135	101	1,601	135	08435	75
76	1	1	3	9	143	75	1,350	143	10589	76
77	1	...	1	5	123	60	1,124	123	10913	77
78	1	1	1	5	95	68	937	95	10144	78
79	2	5	118	44	768	118	15355	79
80	6	86	40	602	86	14286	80
81	1	2	65	45	471	65	13800	81
82	3	63	24	359	63	17573	82
83	1	2	50	28	269	50	18557	83
84	43	14	189	43	22752	84
85	1	25	14	131	25	19011	85
86	1	22	8	92	22	24044	86
87	11	7	61	11	18033	87
88	12	...	43	12	27907	88
89	9	2	31	9	29032	89
90	6	...	20	6	35000	90
91	6	...	14	6	42857	91
92	3	8	...	00000	92
93	3	...	5	3	60000	93
94	1	...	2	1	50000	94
95	1	...	1	1	100000	95
96	96
97	97
98	98
99	99
All Ages	31,776	1,386	8,582	3,479	8,947	9,382	166,913	8,947	...	All Ages

TABLE IV.

*Healthy Males. ABSTAINERS.**Whole-Life Policies. "Transfers to" excluded.**Entrants 1841-1901.**Experience 1841-1901.*

Age (x)	Entered at Age (y)	PASSED FROM OBSERVATION AT AGE (x) AS					Exposed to Risk	Died	Rate of Mortality	Age (x)
		Trans- ferred	Void	Surren- dered	Died	Existing 31 Dec. 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	0
1	1
2	2
3	3
4	1	1	4
5	1	5
6	4	...	1	3	6
7	9	...	1	7	7
8	21	...	1	22	8
9	27	...	3	...	1	...	43	1	.022988	9
10	45	1	5	...	1	1	74	1	.013514	10
11	79	...	6	1	128	11
12	78	...	7	1	...	8	198	12
13	105	1	7	9	273	13
14	154	1	22	...	1	6	378	1	.002649	14
15	191	...	10	2	2	15	526	2	.003799	15
16	197	...	36	4	3	21	674	3	.004454	16
17	271	5	35	2	9	31	843	9	.010676	17
18	360	7	44	1	5	48	1,070	5	.004673	18
19	474	9	46	6	11	49	1,378	11	.007980	19
20	731	10	60	10	8	77	1,849	8	.004328	20
21	919	19	94	19	14	81	2,477	14	.005651	21
22	984	37	153	21	19	109	3,155	19	.006022	22
23	1,018	31	175	31	18	140	3,802	18	.004734	23
24	1,161	46	192	34	14	121	4,477	14	.003127	24
25	1,186	45	219	49	23	189	5,219	23	.004407	25
26	1,204	60	219	41	21	163	5,885	21	.003568	26
27	1,183	60	224	56	28	163	6,565	28	.004265	27
28	1,183	72	198	63	26	193	7,223	26	.003600	28
29	1,179	71	200	72	35	197	7,848	35	.004460	29
30	1,149	80	234	66	28	204	8,413	28	.003328	30
31	1,076	75	243	80	40	212	8,904	40	.004493	31
32	1,074	85	219	88	42	192	9,334	42	.004500	32
33	1,037	98	203	93	35	220	9,765	35	.003584	33
34	978	84	194	89	45	237	10,139	45	.004438	34
35	920	69	224	91	42	232	10,426	42	.004028	35
36	863	74	199	91	52	248	10,672	52	.004873	36
37	804	73	185	64	35	248	10,865	35	.003221	37
38	742	79	177	80	63	275	11,027	63	.005713	38
39	735	72	164	72	48	272	11,107	48	.004322	39
40	697	68	167	93	58	271	11,185	58	.005186	40
41	624	61	161	87	55	272	11,198	55	.004912	41
42	580	81	141	79	66	292	11,171	66	.005908	42
43	561	66	172	76	55	293	11,072	55	.004967	43
44	518	58	130	79	70	323	10,978	70	.006376	44
45	487	52	125	108	73	349	10,813	73	.006752	45
46	379	64	115	88	75	365	10,548	75	.007100	46
47	373	43	94	64	78	316	10,253	78	.007608	47
48	349	57	103	58	75	341	10,010	75	.007493	48
49	366	50	78	57	84	304	9,753	84	.008613	49
Carried forward	27,076	1,864	5,286	2,015	1,358	7,088	261,752	1,358	...	

TABLE IV—(continued).

Age (x)	Entered at Age (x)	PASSED FROM OBSERVATION AT AGE (x) AS					Exposed to Risk	Died	Rate of Mortality	Age (x)
		Trans- ferred	Void	Surren- dered	Died	Existing 31 Dec. 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Brought forward	27,076	1,864	5,286	2,015	1,358	7,088	261,752	1,358	...	
50	288	42	99	59	86	315	9,496	86	·009056	50
51	251	46	65	62	66	369	9,183	66	·007188	51
52	227	39	67	54	105	267	8,820	105	·011905	52
53	190	47	62	48	99	290	8,498	99	·011649	53
54	175	35	52	63	107	285	8,141	107	·013141	54
55	140	41	38	38	114	289	7,773	114	·014665	55
56	138	28	49	58	110	271	7,382	110	·014901	56
57	93	31	45	42	110	258	6,991	110	·015734	57
58	99	29	37	48	124	261	6,604	124	·018776	58
59	100	20	24	35	127	263	6,224	127	·020407	59
60	63	29	30	36	119	245	5,827	119	·020422	60
61	55	27	20	37	130	238	5,434	130	·023921	61
62	44	30	16	42	126	229	5,030	126	·025050	62
63	31	13	10	34	128	164	4,641	128	·027580	63
64	31	13	7	21	145	191	4,331	145	·033483	64
65	16	19	10	18	157	194	3,974	157	·039502	65
66	15	9	9	18	142	167	3,597	142	·039478	66
67	19	14	4	20	126	136	3,270	126	·038538	67
68	12	11	5	23	145	161	2,984	145	·048593	68
69	15	13	1	21	132	118	2,654	132	·049727	69
70	2	7	2	17	114	128	2,383	114	·047839	70
71	4	8	2	19	133	126	2,117	133	·062839	71
72	1	12	3	17	133	75	1,829	133	·072718	72
73	2	11	...	12	104	73	1,595	104	·065182	73
74	2	3	1	12	94	71	1,401	94	·067120	74
75	...	5	...	7	105	68	1,223	105	·085854	75
76	...	5	1	5	112	57	1,038	112	·107900	76
77	1	2	1	8	108	46	858	108	·125800	77
78	2	1	2	5	108	48	697	108	·154950	78
79	1	3	1	5	72	36	535	72	·134700	79
80	1	2	1	3	54	30	419	54	·128880	80
81	...	1	...	1	51	25	332	51	·153610	81
82	...	1	1	2	50	9	253	50	·197240	82
83	1	30	10	192	30	·156660	83
84	...	1	...	1	20	10	150	20	·133330	84
85	22	13	119	22	·184870	85
86	...	1	...	1	21	5	83	21	·253010	86
87	2	12	5	55	12	·218180	87
88	7	2	37	7	·189190	88
89	4	3	28	4	·142860	89
90	3	3	21	3	·142860	90
91	4	...	15	4	·266670	91
92	5	1	11	5	·454545	92
93	2	...	5	2	·400000	93
94	1	3	94
95	2	95
96	1	1	96
97	1	97
98	1	1	98
99	99
All Ages	29,094	2,463	5,951	2,911	5,124	12,645	398,010	5,124	...	All Ages

TABLE XVI.—*Healthy Males. Whole-Life Policies.*
Showing effect of TRANSFERS TO NON-ABSTAINERS.

Transfers 1847-1901.

Experience 1847-1901.

Age (x)	Trans- ferred to at Age (x)	PASSED FROM OBSERVATION AT AGE (x) AS					Exposed to Risk	Died	Rate of Mortality	Age (x)
		Trans- ferred from	Void	Surr'd	Died	Existing 31 Dec, 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
10	1				10
11				11
12				12
13				13
14	1				14
	2	5	
15				15
16				16
17	5				17
18	8	2				18
19	9	1	...	1				19
	22	1	...	3	37	
20	16	5				20
21	15	...	2	2				21
22	38	1	5				22
23	34	...	3	...	2	7				23
24	49	1	5	1	1	4				24
	152	1	10	1	4	23	332	4	·01205	
25	43	1	5	12				25
26	57	1	2	3	...	4				26
27	64	2	5	2	3	12				27
28	76	1	5	3	...	15				28
29	88	...	4	6	3	17				29
	328	5	21	14	6	60	1,179	6	·00509	
30	81	3	4	7	2	19				30
31	79	2	8	4	5	17				31
32	93	1	6	7	2	28				32
33	105	3	3	8	4	30				33
34	90	2	2	8	3	46				34
	448	11	23	34	16	140	2,431	16	·00658	
35	72	1	13	8	6	25				35
36	85	4	7	4	4	29				36
37	83	3	5	8	6	30				37
38	78	1	4	12	8	49				38
39	77	4	7	8	7	40				39
	395	13	36	40	31	173	3,296	31	·00941	
40	65	5	11	2	3	45				40
41	67	6	4	6	7	45				41
42	88	1	6	9	11	41				42
43	72	2	2	6	7	49				43
44	63	8	4	6	10	43				44
	355	22	27	29	38	223	3,579	38	·01062	
45	61	9	3	6	6	41				45
46	67	5	5	4	17	50				46
47	46	4	2	7	9	43				47
48	56	4	6	7	10	37				48
49	53	3	4	5	15	48				49
	283	25	20	29	57	219	3,482	57	·01637	
Carried forward	1,985	77	137	148	152	841	14,341	152	...	

TABLE XVI—continued.

Age (<i>x</i>)	Trans- ferred to at Age (<i>x</i>)	PASSED FROM OBSERVATION AT AGE (<i>x</i>) AS					Exposed to Risk	Died	Rate of Mortality	Age (<i>x</i>)
		Trans- ferred from	Void	Surr'd	Died	Existing 31 Dec. 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Brought forward	1,985	77	137	148	152	841	14,341	152	...	
50	47	5	1	11	12	52				50
51	49	3	3	6	11	29				51
52	47	7	3	4	7	36				52
53	52	4	2	5	7	49				53
54	38	5	2	8	9	43				54
	233	24	11	34	46	209	3,050	46	·01508	
55	45	...	1	4	12	31				55
56	37	4	3	5	18	50				56
57	43	3	1	4	17	35				57
58	35	3	1	2	15	40				58
59	25	3	...	5	24	30				59
	185	13	6	20	86	186	2,566	86	·03352	
60	32	1	2	4	9	34				60
61	36	2	1	2	15	41				61
62	32	1	...	6	9	39				62
63	19	3	...	3	17	40				63
64	15	4	1	3	9	23				64
	134	11	4	18	59	177	1,879	59	·03141	
65	24	1	...	2	14	22				65
66	10	2	...	4	18	27				66
67	17	1	16	19				67
68	13	4	13	15				68
69	15	2	19	21				69
	79	4	...	12	80	104	1,182	80	·06771	
70	8	5	11	15				70
71	9	8	12				71
72	12	2	16	11				72
73	13	1	5	25				73
74	4	7	13				74
	46	8	47	76	627	47	·07496	
75	6	1	8	5				75
76	5	1	3	11				76
77	2	1	5	9				77
78	1	4	5				78
79	3	3	2				79
	17	3	23	32	271	23	·08487	
80	2	6	5				80
81	1	8	3				81
82	1	...				82
83	1	...	1	...				83
84	1	5	...				84
	4	...	1	...	21	8	87	21	·24138	
85	1	1				85
86	1	2	1				86
87	2				87
88				88
89				89
	2	2	4	16	2	·12500	
90				90
91				91
92				92
93	1	...				93
94				94
	1	...	4	1	·25000	
All Ages	2,685	129	159	243	517	1,637	24,023	517	...	All Ages

TABLE XVII—*Healthy Males. Whole-Life Policies.*

Showing effect of TRANSFERS TO ABSTAINERS.

Transfers 1851-1901.

Experience 1851-1901.

Age (<i>v</i>)	Trans- ferred to at Age (<i>v</i>)	PASSED FROM OBSERVATION AT AGE (<i>v</i>) AS					Exposed to Risk	Died	Rate of Mortality	Age (<i>x</i>)
		Trans- ferred from	Void	Surr'd	Died	Existing 31 Dec. 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
10				10
11				11
12	1				12
13	1	...	1				13
14				14
	2	...	1	2	
15	2				15
16				16
17	2				17
18	1				18
19	3				19
	8	22	
20	4				20
21	7	1				21
22	11	1				22
23	9	...	2	...	1	1				23
24	13	...	2				24
	44	...	4	2	1	1	123	1	·00813	
25	22	...	1	2				25
26	20	1	2				26
27	17	1	2	...	1	1				27
28	26	4	2	2	2	...				28
29	26	6	3	2	...	2				29
	111	12	10	6	3	3	433	3	·00694	
30	41	2	1	2	...	1				30
31	29	...	1	3	1	3				31
32	29	5	3	1	...	1				32
33	42	4	...	4	...	3				33
34	37	5	5	1	1	5				34
	178	16	10	11	2	13	951	2	·00210	
35	33	1	4	4	...	4				35
36	52	5	5	1	...	4				36
37	35	5	3	2	...	4				37
38	49	1	1	3	3	1				38
39	42	3	1	...	2	1				39
	211	15	14	10	5	14	1,597	5	·00313	
40	58	2	7	2	2	1				40
41	38	6	5	6	1	3				41
42	46	6	3	7	2	8				42
43	38	5	1	7	3	7				43
44	43	5	1	4	2	6				44
	223	24	17	26	10	25	2,363	10	·00423	
45	47	6	3	4	5	13				45
46	46	2	3	7	3	7				46
47	37	2	4	1	8	11				47
48	33	1	1	6	4	15				48
49	38	2	1	1	7	13				49
	201	13	12	19	27	59	2,849	27	·00948	
Carried forward	978	80	68	74	48	115	8,340	48	...	

TABLE XVII—continued.

Age (x)	Trans- ferred to at age (x)	PASSED FROM OBSERVATION AT AGE (x) AS					Exposed to Risk	Died	Rate of Mortality	Age (x)
		Trans- ferred from	Void	Surr'd	Died	Existing 31 Dec. 1901				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Brought forward	978	80	68	74	48	115	8,340	48	...	
50	41	3	1	5	4	17				50
51	36	3	2	...	4	15				51
52	41	7	5	4	11	15				52
53	46	3	1	6	7	26				53
54	29	2	2	2	3	21				54
	193	18	11	17	29	94	3,118	29	·00930	
55	26	4	...	1	11	22				55
56	36	6	2	...	9	28				56
57	24	11	2	4	9	28				57
58	25	6	...	7	6	28				58
59	25	5	...	1	12	19				59
	136	32	4	13	47	125	2,971	47	·01582	
60	27	5	1	4	12	15				60
61	24	5	1	1	15	19				61
62	11	2	...	5	15	19				62
63	17	5	...	7	11	24				63
64	9	2	12	25				64
	88	17	2	19	65	102	2,504	65	·02506	
65	14	5		1	23	33				65
66	4	1		1	16	21				66
67	8	3		1	11	18				67
68	6	2		2	13	17				68
69	5	2		4	11	21				69
	37	13		9	74	110	1,707	74	·04334	
70	2	1	2	1	11	32				70
71	7	...		1	14	18				71
72	7	...		1	19	13				72
73	7	2		...	6	15				73
74	6	2		...	6	6				74
	29	5	2	3	56	84	914	56	·06130	
75	1	...			10	6				75
76	1	...			7	11				76
77			10	7				77
78	1	...	1		7	5				78
79	1		8	5				79
	4	...	1	...	42	34	470	42	·08936	
80		1	6	4				80
81			6	5				81
82			1	8				82
83			7	3				83
84			1	2				84
				1	21	22	154	21	·13592	
85	...	1		1				85
86	1	...				86
87	2	1				87
88				88
89				89
		1			3	2	23	3	·13333	
90	2	...				90
91				91
92				92
93				93
94	2	...	2	2	1·00000	94
All Ages	1,465	166	88	136	387	688	20,203	387	...	All Ages

TABLE XXI.

*Mortality Table and auxiliary columns based upon
ABSTAINERS' MORTALITY (T^M) and 3 per-cent interest.*

x	l_x	d_x	p_x	q_x	D_x	N_x
(1)	(2)	(3)	(4)	(5)	(6)	(7)
10	100,000	347	.99655	.00345	74,410	1,895,308
11	99,653	344	.99650	.00350	71,993	1,823,315
12	99,309	356	.99646	.00354	69,650	1,753,665
13	98,953	355	.99640	.00360	67,383	1,686,282
14	98,598	362	.99633	.00367	65,186	1,621,096
15	98,236	368	.99625	.00375	63,112	1,557,984
16	97,868	376	.99617	.00383	60,987	1,496,997
17	97,492	381	.99608	.00392	58,985	1,438,012
18	97,111	392	.99596	.00404	57,043	1,380,969
19	96,719	405	.99584	.00416	55,157	1,325,812
20	96,314	416	.99569	.00431	53,327	1,272,485
21	95,898	427	.99554	.00446	51,550	1,220,935
22	95,471	443	.99537	.00463	49,825	1,171,110
23	95,028	430	.99548	.00452	48,149	1,122,961
24	94,598	418	.99560	.00440	46,536	1,076,425
25	94,180	404	.99571	.00429	44,981	1,031,444
26	93,776	393	.99582	.00418	43,483	987,961
27	93,383	379	.99594	.00406	42,041	945,920
28	93,004	379	.99593	.00407	40,650	905,270
29	92,625	376	.99593	.00407	39,305	865,965
30	92,249	376	.99593	.00407	38,005	827,960
31	91,873	375	.99592	.00408	36,749	791,211
32	91,498	375	.99592	.00408	35,532	755,679
33	91,123	379	.99585	.00415	34,356	721,323
34	90,744	383	.99578	.00422	33,216	688,107
35	90,361	388	.99571	.00429	32,112	655,995
36	89,973	393	.99563	.00437	31,043	624,952
37	89,580	397	.99556	.00444	30,008	594,944
38	89,183	402	.99550	.00450	29,005	565,939
39	88,781	408	.99540	.00460	28,027	537,912
40	88,373	424	.99520	.00480	27,091	510,821
41	87,949	443	.99500	.00500	26,176	484,645
42	87,506	466	.99465	.00535	25,287	459,358
43	87,040	495	.99430	.00570	24,419	434,939
44	86,545	525	.99395	.00605	23,572	411,367
45	86,020	559	.99350	.00650	22,747	388,620
46	85,461	594	.99305	.00695	21,941	366,679
47	84,867	632	.99255	.00745	21,154	345,525
48	84,235	671	.99205	.00795	20,385	325,140
49	83,564	711	.99150	.00850	19,634	305,506
50	82,853	757	.99085	.00915	18,899	286,607
51	82,096	822	.99000	.01000	18,181	268,426
52	81,274	868	.98930	.01070	17,475	250,951
53	80,406	940	.98830	.01170	16,785	234,166
54	79,466	1,009	.98730	.01270	16,105	218,061
55	78,457	1,098	.98600	.01400	15,438	202,623
56	77,359	1,184	.98470	.01530	14,778	187,845
57	76,175	1,275	.98327	.01673	14,128	173,717
58	74,900	1,379	.98160	.01840	13,487	160,230
59	73,521	1,485	.97980	.02020	12,853	147,377

TABLE XXI—continued.

x	l_x	d_x	p_x	q_x	D_x	N_x
(1)	(2)	(3)	(4)	(5)	(6)	(7)
60	72,036	1,585	·97800	·02200	12,227·0	135,150·0
61	70,451	1,690	·97600	·02400	11,610·0	123,540·0
62	68,761	1,808	·97370	·02630	11,001·0	112,539·0
63	66,953	1,915	·97140	·02860	10,400·0	102,139·0
64	65,038	2,042	·96860	·03140	9,808·2	92,331·0
65	62,996	2,154	·96580	·03420	9,223·4	83,107·6
66	60,842	2,270	·96270	·03730	8,648·5	74,459·1
67	58,572	2,343	·96000	·04000	8,083·5	66,375·6
68	56,229	2,513	·95530	·04470	7,534·1	58,841·5
69	53,716	2,627	·95110	·04890	6,987·6	51,853·9
70	51,089	2,727	·94660	·05349	6,452·4	45,401·5
71	48,362	2,830	·94150	·05850	5,930·1	39,471·4
72	45,532	2,923	·93580	·06420	5,420·5	34,050·9
73	42,609	3,034	·92880	·07120	4,924·7	29,126·2
74	39,575	3,170	·91990	·08010	4,441·0	24,685·2
75	36,405	3,373	·90736	·09264	3,966·1	20,719·1
76	33,032	3,317	·89957	·10043	3,493·9	17,225·2
77	29,715	3,234	·89118	·10882	3,051·5	14,173·7
78	26,481	3,123	·88205	·11795	2,640·2	11,533·5
79	23,358	2,986	·87218	·12782	2,261·0	9,272·5
80	20,372	2,820	·86156	·13844	1,914·50	7,358·0
81	17,552	2,633	·85000	·15000	1,601·40	5,756·6
82	14,919	2,423	·83760	·16240	1,321·60	4,435·0
83	12,496	2,196	·82427	·17573	1,074·70	3,360·3
84	10,300	1,958	·80986	·19014	860·04	2,500·27
85	8,342	1,716	·79431	·20569	676·24	1,824·03
86	6,626	1,472	·77787	·22213	521·48	1,302·55
87	5,154	1,237	·75999	·24001	393·83	908·72
88	3,917	1,014	·74113	·25887	290·59	618·13
89	2,903	809	·72119	·27881	209·09	409·04
90	2,094	630	·69925	·30075	146·40	262·64
91	1,464	472	·67742	·32258	99·39	163·25
92	992	345	·65212	·34788	65·37	97·88
93	647	240	·62880	·37120	41·39	56·49
94	407	163	·60000	·40000	25·27	31·22
95	244	104	·57527	·42473	14·72	16·50
96	140	64	·54206	·45794	8·22	8·28
97	76	37	·51724	·48276	4·33	3·95
98	39	19	·50000	·50000	2·17	1·78
99	20	11	·46667	·53333	1·05	·73
100	9	5	·42857	·57143	·48	·25
101	4	3	·33333	·66667	·20	·05
102	1	1	·00000	1·00000	·05	...

Om Rate.

ABSTRACT OF DISCUSSION.

Mr. H. W. MANLY, in opening the discussion, said the paper was important because it went very far to settle a controversy, which had been raging for a good many years, which hitherto had been argued with very few figures, a great deal of heat sometimes, and generally more from a physiological than from an actuarial point of view. The President, in his inaugural address a year ago, referred to the absence of statistics of abstainers as a separate class. He was sure the members all rejoiced that the Directors of the United Kingdom Temperance and General Institution had consented to the publication of their experience, which had been so admirably compiled by their actuary, Mr. R. M. Moore. He confessed to have had some ideas himself in regard to the difference in mortality, but the experience now presented seemed to have upset them. There were two curious features which seemed to come out in the experience. First, there was the comparative heavy rate of mortality under the age of twenty-five—that applied to both sections—and the sudden increase of the mortality after the age of 75 in the abstainers' section. If the experience which was presented in Table 3 were examined, *i.e.*, the experience of the non-abstainers, it would be seen that, commencing with the age of 35, the rate of mortality ran very closely, in fact agreed exactly, with the O^M Table; but the mortality was greater, from 15 to 35, than in that experience. He thought it was necessary, in examining the tables, to keep in mind that it was not only in one section that the increased mortality amongst the younger members appeared; although when one referred to Table 4, where the abstainers were compared with non-abstainers, the abstainers seemed to have a still larger mortality, under 20, as compared with non-abstainers. He did not know that anything very particular arose out of that. If there was a period in a person's life when one would have supposed that alcohol was exceedingly injurious he should have thought it would have been under the age of 20, and it came to them as a startling surprise to be told it was the reverse. He did not know whether the author could give any explanation of that fact. A prominent point in the paper was Table 14, which showed what one might have expected, namely, that there had been a general improvement in the mortality. It was known that the mortality as shown by the O^M Table, *i.e.*, the experience from 1863 to 1893, was lighter pretty well throughout than the H^M Table, which was an experience up to 1863; and they therefore expected to see an improvement in the mortality in this Institution when the period from 1841 to 1888 was compared with 1841 to 1901. The improved mortality came half-way between the H^M and the O^M, so that both the sections had benefited by the advance in sanitary improvements, and also by the improvement of medical science. There was one thing he wished to greatly emphasize, namely, the danger of using ratios for the purpose of comparisons. Figures were sometimes rather dangerous things, especially when used by people who were not skilled in their use, but ratios he thought were perhaps the most dangerous of all. He might illustrate that point by referring to two of the tables. Table 8 gave particulars of the mortality of those who entered in 1889–1901. If the rate of mortality in the abstainers' section from 15 to 19 was taken, the ridiculous result was obtained that it was

infinitely greater than in the non-abstainers'; because there were two deaths out of 1,050 persons exposed, and among the non-abstainers there were no deaths. In the next quinary group the ratio was 24.4, and in the third it was 104.09. If all the numbers under 29 years of age exposed to risk were added together it would be found that the rate of mortality among the abstainers was .25, and among the non-abstainers .37, and instead of the extraordinary figures of 24 and 104, we should have 68 as the ratio in col. 8. Similarly, in Table 9 there was a curious case of one death among the abstainers between 10 and 14, and no deaths among the non-abstainers. Therefore the abstainers' mortality was infinitely greater than the non-abstainers'. In the next age, 15 to 19, there were no deaths among the abstainers, and one among the non-abstainers, so that the result was just the reverse; the deaths among the non-abstainers were infinitely greater than among the abstainers. In comparing ratios it was not sufficient to know what the ratios were, but it was necessary also to compare the figures which were the basis of the calculations. If the weight of the observations were taken into consideration, it would be found that very little importance could be attached to the apparently heavy ratio between the mortality among the abstainers and the non-abstainers at the younger ages: a few accidental deaths would upset all the comparisons. He had said that the paper upset some of his own theories. He had had an idea that the transference of presumably bad cases from the abstainers' section to the non-abstainers' must have had a serious effect upon both; and he thought it was a matter of very great satisfaction, and certainly a great relief, to find that there was nothing of the kind; that, in fact, neither section had been at all influenced by the change; and that the experience of those who were transferred from the abstainers to the non-abstainers had not been at all serious; that their falling from grace had not affected their rate of mortality very much. In confirmation of those figures he had a note of the experience of such cases in another office, which did a similar, but smaller, business, to the office referred to in the paper, where there were two sections, and where they transferred from one class to another as the members became abstainers or otherwise. The transfers from the abstainers class to the non-abstainers class were carefully observed for the ten years from 1890 to 1899 by comparing the expected mortality according to the H^M Table with the actual deaths which occurred, the result being that over the ten years the expected was 36.5 by the H^M Table and the actual 15, showing a proportion of only 41 per-cent. deaths to the expected, while in the whole of the abstainers' class the proportion was 53. In that case the falling from grace resulted in an improvement. He thought Mr. Moore was justified in his conclusions; that non-abstainers assured in his institution were good average lives, and generally equal to the O^M Table; that the abstainers showed a marked superiority over non-abstainers throughout the entire working years of life; and thirdly, that the superiority had not been brought about by the operation of the transfers. Two diagrams were given in the paper, but he thought neither of them was the best way of illustrating the tables. The author had been good enough to show him a curve which he thought was much better than the curve in diagram 2, namely, the curve of the abstainers' q_x compared with the q_x of the

O^m Table. That, to him, was very interesting, because it was at once seen where the two mortality tables met, at between the ages of 70-75. The members must not be led away into too wide a generalization from these comparisons, because, although they were true of the class of lives insured in the author's institution, there were tables in existence which showed even a better vitality than the abstainers. Take the class of the clergy assured in the Clergy Mutual from 1829 to 1887. The experience extended over about the same length of time, although at a different period. He did not suppose that from 1829 to 1887 the clergy of the Church of England could be classed as total abstainers; but it was very curious to note that if the expectation of life as exhibited in Table 22 was compared with the expectation of life amongst the clergy, it came out practically half a year less at all ages. He had another table of expectation of life, namely, the experience of the Equitable, for 30 years, from 1863 to 1893. In comparing that with the abstainers' experience he found that the expectation of life at the younger ages in the abstainers' table was nearly one year longer than in the Equitable; at 35 there was a difference of three-quarters of a year; at 45 one-third of a year; but at 55 it was just the other way—the Equitable experience was slightly better, and increased in that respect to the end of life. At age 70 the Equitable experience gave a quarter of a year longer expectation than the experience in the paper. The class of people represented by these figures he had given were well-to-do merchants or professional men with a little sprinkling of the aristocracy. The experience in itself was very good, but, of course, it did not come up to the experience of the assurable clergy who, for actuarial purposes, were a select class, having a certain income, living moral lives, and with not very much care or anxiety. If it were possible to divide the clergy into two sections, abstainers and non-abstainers, they might possibly show even as startling results as were given in the paper. They had not the means of doing that; but what they had before them, which he thought had been absolutely proved by the figures, was that in the class of people that were insured by the United Kingdom Institution, the abstainers' mortality during the working years of life was a lighter mortality than the non-abstainers in that period, and to a very considerable extent. Finally he wished to say that the Institution whose experience was before them had done a good and noble work in the cause of temperance. It was founded at a time when the teetotaler was considered to be a crank, but the directors persistently held on to their principles, which were now proved to be correct—that the abstainers as a class were better and lived longer than the non-abstainers; they had on each occasion of their periodical investigations compared the deaths in the two classes, and on each occasion a very great advantage in the abstainers' class was shown. In now publishing their complete experience, they had broken down all possible opposition, and removed all doubts as to the value of their figures; and they had, in that way, done more service to the cause they had at heart than any amount of talking would do.

Mr. J. H. BARNES thought he might fairly express from the floor of the house the members' great appreciation of the services that had been rendered to the Institute by the presentation of the paper. It

had helped to crystallize ideas that some members had held before, and he had no doubt they would now be able to understand the general bearing of the subject a great deal better than they had previously. He agreed with Mr. Manly that they should always look to see whether the ratio was based on the experience of two or four deaths out of 1,000, or whether they were dealing with a much larger number. Laws might be deduced, after constant experience, of large numbers of deaths in connection with large numbers of lives, but to attempt to deduce a law from an experience of two or four deaths out of 1,000 lives in a very short space of time was quite out of the question.

Mr. A. BARRAND said that Mr. Manly had compared the mortality of the abstainers with the mortality of the clergy. He had no doubt Mr. Manly intended the members to see clearly that the two classes could not be properly compared at all, that in any proper comparison of results individuals of practically the same class must be compared; and he thought that one of the great merits of the paper was that the author had compared the mortality of abstainers and non-abstainers of practically the same class. A comparison between the members of the United Kingdom Institution and the clergy assured in the Clergy Mutual, was, of course, quite fallacious. They lived under entirely different conditions, and normally would experience quite a different rate of mortality. But the great advantage of the comparison made by the author was that two bodies of lives, not exactly equal, but sufficiently equal for the purpose, taken from precisely the same class, were compared, the only difference between them being that one class consisted of abstainers and the other of non-abstainers; and therefore the comparison was of very much greater value than if those conditions had not existed.

Mr. A. LEVINE reminded the members that there had been other investigations than those presented by the author into the difference of mortality between abstainers' and non-abstainers' sections. He noticed one of those investigations the other day in a report of the New Zealand Insurance Department; and there, somewhat to his surprise, it appeared that, on the whole, the mortality in the abstainers' section had not been any better than the mortality in the other section; and, if he was not mistaken, in the Canadian branch of an office which did a large business in Canada, the mortality experience of the abstainers had been on the whole rather worse than that of the non-abstainers.

Mr. GEO. KING said he had a little hesitation in intervening in the discussion, because he had practically no settled convictions. If there was anything to give settled convictions in a case of the kind, he thought the author's very able paper would do so; but even now he did not feel absolutely convinced, because he thought that possibly, not to the fullest extent, but at any rate to a certain extent, they might be placing the cause for the effect, or the effect for the cause. He thought it was quite possible that those who were total abstainers were so because they were vigorous and active, and did not feel any necessity for stimulants, whereas those who were not total abstainers in the same class might perhaps not feel themselves quite equal in physique. He was very much interested in the class of transfers, and perhaps that might throw a little light on the points he had touched upon. He thought it would be very interesting to know the causes of death amongst the cases of transfer from abstainers to

non-abstainers, because that would help to clear up the question. The transfers from the abstainers to the non-abstainers showed a rather heavier mortality than in the abstainers' class by itself. Was that because those transferees had taken alcohol, or was it not possible that they might have passed to the non-abstainers' class because they had found themselves not so strong and vigorous as they were, and now required a little stimulant? He thought that was a point that wanted further elucidation. Then he wished to say, further, that he thought the author had proved from the experience of his company that, on the average, abstinence from alcohol generally did not do any harm: it might do good: but it did not follow that in every particular case it would do the same. He did not think it should be necessarily assumed that because in the aggregate in the abstainers' class the lives were better than in the non-abstainers', that, therefore, none of the abstainers would be the better for, in moderation, a little alcohol. He merely threw out these ideas to show that, notwithstanding the very strong circumstantial evidence the author had brought forward, he did not think the case was proved right away to the hilt. His own experience in the matter of abstainers' sections had been rather peculiar. Mr. Levine had mentioned New Zealand. Mr. R. P. Hardy and himself had been for many years frequently consulted over the actuarial work of the New Zealand Government, and the matter of the two sections had, on various occasions, come prominently before them. The facts were that in five valuations the total abstainers obtained a better bonus on one occasion than the non-abstainers, on two occasions the total abstainers obtained a smaller bonus, and on two occasions the bonuses were equal. There was never, however, any very great difference between them. It was not due to the way of managing the departments, because the bonuses from general profits were divided among them as if they were one institution, but an investigation on each occasion was made into the profits from mortality, and it was profits from mortality alone that made the differences, whatever differences there were, in the bonuses. So that it could not be that more expenses were thrown on one than on the other, nor any other circumstances whatever, except that among the abstainers' section there was a somewhat larger proportion of endowment assurances, and there was not the same room for gain in the mortality in that class as there was among the whole-life policies. Further, Mr. Levine had spoken of Canada, and said that a similar result was produced there. He had had another total abstainers' section at the Cape of Good Hope. He there took out the experience of a small company over a number of years. The classes were kept absolutely distinct throughout that period, and the result was that, if they had adhered to keeping the departments separate, the abstainers would not have got much more than half the bonus of the non-abstainers. It was, therefore, very remarkable that while in the United Kingdom, according to the author's figures, total abstainers were a much better class, yet it would appear that alcohol in moderation did no harm in the Colonies.

Mr. R. P. HARDY enquired whether the difference at the Cape was due to a larger mortality of the abstainers?

Mr. KING said that the rate of mortality, age for age, or taking

groups of ages, was considerably larger among the abstainers than the non-abstainers.

Mr. HARDY enquired whether there were a large number of cases under observation.

Mr. KING replied in the negative.* It was also only right to mention that the figures were, to a certain extent, influenced by the war. He had no means of ascertaining how many deaths in each section were caused by the war, but he did not see why non-abstainers were less likely to be killed in battle than abstainers.

Mr. D. C. FRASER said that Mr. King had thrown out the interesting suggestion that the superior vitality of abstainers might not be due simply to abstinence from alcohol, but to the fact that through greater vigour of constitution they were able to dispense with strong drink. He wished to point out that, for the purposes of business, it did not matter what the cause of the superior vitality was; there was the fact proved in the paper that abstainers did show a lower mortality. That was a new fact thrown into the arena of business which would have to be taken into account in the future. He wished to add his testimony to that of the previous speakers as to the absolute and unique importance of the paper. There had been a great controversy for many years as to whether total abstinence or moderate drinking was the better; and the paper, which contained all the canons of a scientific examination of the question, unmistakably showed that total abstinence produced the lighter mortality. Two bodies of lives were drawn from the same classes of the community, selected by the same directors for the same period of time, the only difference between the two bodies being the one fact of their relation to the drinking of intoxicants. It might be taken for granted that the non-abstainers had answered the description of moderate drinkers and were selected with great care, and, notwithstanding that, the total abstainers had shown a superior vitality. Mr. King had also referred to the experience of abstainers and non-abstainers in the Colonies. He wished to point out in that connection that in the Colonies the sentiment in regard to strong drink seemed to be entirely different from what it was in this country. He saw lately some figures published by the Statistician of New South Wales, probably the most eminent statistician in the Colonies, which showed that in Australia and New Zealand the average consumption of alcohol was only two-thirds of what it was in this country. It was very probable that the non-abstainers in Australia and New Zealand were superior to the non-abstainers in this country, on account of the lesser consumption of alcohol per head. He did not know the sentiment existing at the Cape of Good Hope; it might be the same as in Australia and New Zealand, but Mr. King had pointed out that the experience there was only a small one, and in small experiences they knew that the figures might run in very curious ways.

Mr. T. P. WHITTAKER, M.P., as a visitor, said he could not pretend to speak as an actuary, but he might be able to deal with one or two of the practical phases of the question as they came before him. When Mr. Hughes made his reference in his presidential address to the lack of information on the subject, it was a matter

* Mr. King informs us that at the close of the period there were 865 policies in force in the abstainers' section, and 2,792 in the non-abstainers'.

of considerable interest to the directors of his office, because for three or four years they had been talking in the office of getting out the figures, and at that time had progressed a considerable way with the work. Therefore he was glad of the indication that when Mr. Moore had got out his paper the Institute of Actuaries would be pleased to receive it. They determined at the very outset to give the Institute the whole of the facts and the complete figures, whatever they might turn out to be. Reference had been made to the fact that the experience showed a somewhat high mortality amongst the younger lives, but he thought it would be misleading for the purpose of the discussion to compare that mortality with the O^M. That was not the point to be considered; the mortality had to be compared in the two sections. If the mortality, high though it might be in the abstainers' section in young life, was compared with the general section, it would be found very much lower than the general section, where it was higher still. The reasons for that high mortality were special, and had to be sought, but the important point was that even there the abstainer came out very much better than the non-abstainer. The mortality ran rather high between the ages of 75 and 79, and he ventured to think the explanation given by the author was a sound one. If their contention that abstinence was conducive to long life was correct, it meant that the lives of somewhat weaker persons were prolonged who, if they had not been abstainers, would have died earlier. but obviously they were not the very best and the very strongest lives. The time came when they had to die, and they seemed to die between 75 and 79, but they were lives which, if they had not been abstainers, would have dropped out probably at 45, 50 and 55. It seemed to him that the important table in the paper was Table 13, which showed that the percentage of the temperance section to the general section indicated a gain of from 25 to 45 per-cent from 25 to 65 years of age, *i.e.*, in the working, active, valuable part of life. That was an important feature from the national point of view. Mr. Manly had referred to the general improvement which had taken place in the health of the population, and expected from that fact that the later mortality would show an improvement on the earlier, and it did. But the important point of Table 14 was not merely that it showed that the mortality was lighter in the later years, but that the improvement in the abstainers' section in the later years was much greater than the improvement of the general; showing that the improvement in mortality generally resulting from greater sanitary knowledge, education and medical experience, gave greater value still to the temperance life, and that, under the improved conditions, the temperance life improved more than the general life did. One cause for that was, that there was more teetotalism in it, *i.e.*, more life abstainers. In the early part of the experiences a very large proportion of the abstainers were men who, in early life, had not been abstainers. He did not mean that they had been heavy drinkers. People were under a great delusion if they imagined that teetotalers consisted of heavy drinkers who had had to abstain. That was not the case at all, but still they had not been abstainers in the early years. He was struck, as the papers from the proposers came through his hands, at the very large proportion of the temperance

section who were life abstainers; and therefore the office was getting a fuller and more complete benefit of the abstinence. Another point was, that a great many of them were the sons of teetotalers; and he suggested that those two reasons accounted very largely for the fact that the improvement in the temperance section during recent years had been greater than in the general, and it supported the contention that abstinence did lead to longer life. There was a practice in existence which possibly tended to make the temperance experience less favourable than it ought to be, namely, as the author said, the tendency to take a somewhat doubtful life on more favourable terms if it was in the temperance section; it was probably thought sometimes that the total abstinence counterbalanced something which, in the general section, would make them write the assured up. He knew that a little blemish was likely to be passed over in some cases because the proposer was an abstainer. Obviously such cases put the temperance section at a slight disadvantage. One point affecting the transfers, which accounted for the large number found in the year 1900-1, was that up to that time the office depended upon the agents to get the declarations signed, but there were a number of abstainers who would not sign the declaration: they thought they were making a pledge, and they had an objection to doing anything of the kind. In that year the company rigidly insisted upon the signing of the declaration, and some of the men who were undoubted abstainers would not sign. The company stuck to its regulations, and that was an explanation of the fact that some of those who had gone out of the temperance section were still abstainers, and went out because they would not sign the declaration. Mr. King had suggested—which was rather a compliment to them—that abstainers were more vigorous and active, and were better lives. Probably he (Mr. Whittaker) knew more of the abstainers in the country than any man in the room, and he doubted if that were so. He doubted whether, apart from their abstinence, they were a particularly careful, healthy and strong lot. They were a restless, strenuous, active, and sometimes a contentious lot, especially in years gone by. Men who took up an unpopular stand in social life were necessarily of that stamp of mind, and he did not think that strenuous, worrying attitude of mind was necessarily conducive to health or longevity. It was the quiet, equable, steady man who allowed nothing to worry him, that lived. The teetotaler was a strenuous, conscientious, active, fighting, worrying, driving sort of fellow; and he was not at all sure that that was particularly conducive to longevity. He thought it was fair to say that, on the whole, teetotalers were about on an average with ordinary steady-going men. A large number of the proposers in the United Kingdom Office came before him, and he would defy anybody who saw them to say which was the abstainer and which was the non-abstainer, unless he had the record before him. They lived in the same towns, they worked at the same occupations, they had practically the same rate of income, they were precisely the same kind of people. They were not comparing them with the general population, but with the general body of the assured in their general section, and, as a rule, the latter were careful, provident, and thrifty people; and he ventured to say that his company obtained a particularly careful lot. The

directors and nearly all the agents were abstainers ; therefore they naturally worked amongst a class of men who either were abstainers, or very closely approximated to it. Another point was that, being abstainers, they were what people would call a little censorious, *i.e.*, particular about liquor-taking. They looked at that question much more strictly than a non-abstainer would, and he thought, therefore, that their non-abstainers were a particularly sober and careful body. Mr. King made some remarks in regard to the question of transfers, and hoped a little further information would be obtained on the subject. He thought his company would follow it up with a good deal of interest ; but suggested that the main point now in connection with the discussion was that the transfers were so few in number that they practically made no difference to the figures. Reference had been made to the experience of the Colonies. Of course, he could speak with no authority or special knowledge there, but he had a strong impression that the difference was a financial one more often than one of mortality. Mr. King had stated that there was a difference of mortality also ; but when one came to the question of endowments the mortality did not tell nearly so much. Whatever might be the case in the Colonies—and he suggested that the experience was small and comparatively short—the paper contained particulars of a fairly large experience extending over 60 years in this country, and the results were clear. Comparison had been made with the Clergy Mutual and the Equitable, but he thought it was perfectly clear that the people in the United Kingdom Office were not of the social class of the Clergy Mutual or the Equitable. The mortality in the temperance section approached very closely to that of the Clergy Mutual and of the old Equitable. The United Kingdom dealt with a small class of people ; the paper stated that the average value of the policy was small. One-third of the policies last year were for £100. Small people necessarily had not the same opportunity of looking after their health as the bigger people ; they did not live in as good dwellings ; they had not the same leisure ; they had frequently more worry ; they had not the same opportunity of having medical attendance ; they could not take a holiday in the same way as the better class people could, and it was not to be expected that they would come out with a mortality equal to that of the clergy, or of the wealthy people insured in the old Equitable. But if by adding to their ordinary conditions total abstinence, they could bring those people up to the standard of a class of people high above them in the social scale, they had accomplished a great deal.

Mr. R. P. HARDY asked, before intruding upon the meeting the few observations which occurred to him, to be allowed to premise that there were present many dear friends of his, in whose regard he hoped to continue to live, and he trusted they would not consider that any of the observations he might feel it his duty to make were in any way directed against the rules of life they, in the exercise of their own discretion, had adopted for their guidance. As the one who had been mainly responsible for the statements put into the mouth of the United Kingdom Institution for years past, he appealed to all present whether he had in any degree misrepresented the results which were laid so fully before them that evening. He was sure the members would desire to congratulate the author on having

emerged from his unnecessarily modest retirement, and on having obeyed the injunction of the President by contributing such an important paper to the Transactions. He might even go further, and say he considered he was highly privileged that, through the instrumentality of the Institute, he was the means of communicating to the world an important series of authentic facts in aid of the solution of a social problem of indisputable gravity. Although the only function with which the Institute was directly concerned was that of the death-rate and its dependencies, namely, the rates of premium, the reserves, and individual profit-rights, the ethics of advanced States had regard to all preventible causes of the waste of the life and energies of the people, among which certainly stood the evident ravages made by alcohol, extending most unhappily to the hitherto exempt Continent of Europe. Hence arose the political question, whether Governments were justified in exercising any control over the liquor traffic, apart from police regulations as to order. Moreover, it was highly desirable, in the interests of both men and their dependent families, that the real effects upon the constitution and prospects of longevity resulting from a constant resort to the various artificial stimulants in common use as lenitives under the pressure induced by our modern civilization, should be dispassionately examined, and that, if any dangerous tendency in the habitual use of such were disclosed, mankind should be warned, so that their daily life could in that, as in other sanitary matters, be framed according to the best rules approved by physiology. Hitherto it had been impossible to secure that necessary philosophic review, and even many of those most anxious to arrive at the truth had been compelled to retreat from a controversy where the unguarded and unverified assertions of philanthropists usurped the place of argument. While even the views of medical experts, based as they were upon their own sanguine interpretation of the limited series of facts falling within their observation, were too often expressed in the most positive language; and further, sometimes stood at direct variance with each other. But to-night they were in a wholly different arena, and one more congenial to the discovery of truth. The appeal now made was neither to their religious principles nor to their feelings, nor to what some individuals might think best for mankind, but rather to a tribunal whose imperturbability would not be led astray by side-issues, and whose impartiality would regard only the weight of the testimony adduced. That tribunal was the body of actuaries there assembled, and the evidence put before that Court was certain numerical results unimpeachable in their origin, and marshalled in due scientific order. Dealing with the broad results, and passing over all the minor points, the members had before them the life record of two distinctly middle-class populations, and during the same period, similarly circumstanced as to race and climate, and as to occupations, and with the elements of time and number in sufficient proportions, the only distinction being that in the one there had been a rigid abstinence from alcoholic liquors, and in the other—though selected with especial care as to habits—the prevailing custom in the use of liquor had been followed. It was indisputable that, if both groups were similar in all respects, they would expect to find the same experience. But, as a fact, a very marked difference was observed, and one too great to be passed over.

It was clearly not accidental, for he noted nearly the same in his investigation in 1882, and they moreover could trace it, though not to the same extent, in the subordinate groupings of the present enquiry. In a word, a notable difference in favour of the mortality of the abstainers forced itself upon their notice wherever they turned. When seeking an explanation of those phenomena, it was, he contended, a valid induction to refer it to the single isolated point presenting itself in the schedules, namely, the sole and prominent distinguishing feature of the groups, or abstinence as generally understood—unless it could be shown that abstinence was merely a name for a combined series of sanitating virtues, effecting proportionate ends, of which it might be taken as one type. He asked the members to see how the case could be put, if technically dealt with. The question was, was abstinence an antecedent, or a single continued act of the will, operating directly, by controlling the immature disintegrating agencies that made for death, or indirectly by releasing a plurality of independent, conjoint, or concomitant forces that retarded the lethal influences when artificially accelerated by vicious custom, in addition to other beneficial results? Or, was it a mere consequent, admittedly a halo upon the individual, but yet only a part of the moral outgrowth of the higher elements of character, congenital or acquired, that promoted the survival of the fittest? The first was the view taken by all persons who had had to deal with those pitiable cases of alcoholic derangement, and also by many medical experts, who having profoundly studied the physiological effects of alcohol, declared that it enfeebled the will and impaired the judgment. The second practically amounted to asserting that abstainers were merely amiable enthusiasts, deluding themselves, and, as Mandeville would have put it, throwing away good liquor for the empty vanity of self-denial. Speaking for himself, the present and other evidence, when reviewed in all its bearings, did not support the last view. In his long observation he had found that abstinence was a great deal more than that—that it stood *per se* amongst human efforts for the mastery of natural malefic impulses—that it was a *primum mobile*, an active energizing principle, originating and sustaining, not subsidiary to nor equipollent with any other inducing virtues. A recent endeavour had been made by one of the members to deny the *prima facie* explanation of the experience, a sample of the dangerous position which was sometimes taken up at the bar, of admitting the facts and denying the inference. The case was half given away when the facts were admitted. They referred it to mere physical strength, such, as it was said—he did not say it—distinguished all persons of high self-control, and which would assert its superiority under all circumstances, here or at the North Pole, in temptation or out of it, or anywhere. To that he replied that no proof, not even the semblance of a proof, had been tendered that the starting point and maintaining power of abstinence was superior physical strength, and that, until an experience had been tabulated under that classification, the argument was an unbiased assertion, and could not be opposed to one drawn from such an analysis as had been presented. It was not mere brute force, but that higher moral strength, induced by intelligently directed habits, which enabled men to neutralize the

evils by which they were surrounded, and to emerge triumphant. The contrary doctrine was undiluted materialism. If the experience was examined in its age incidence it would be seen how effective an armour those habits of life had proved in the day of trial to those who had so wisely equipped themselves. Those habits, from early manhood down to approaching age, had continuously repelled the assaults of long hours, of unwholesome conditions of labour, and of the wear and tear inseparable from the lower grades of commercial life—life not conducted under such pleasant circumstances as actuaries, happily, lived in. It had been found that where the strain was greatest, there the principles told—the bow bent, but did not break. In conclusion, he was not without hopes that the facts laid before the Institute in the paper, though perhaps not complete in the impossible proof that technical logic demanded, might yet arrest their attention, and so far convey to their minds a strong impression of the highly probable truth of the conclusions advanced. It was not too much to say that the members' approval, and even their generous countenance would vastly strengthen the efforts of those devoted men who, having renounced what were to most the chief pleasures of this world, with apostolic zeal, consecrated their lives to strengthening the weak, to upholding the trembling, and to recalling the outcast.

The PRESIDENT (Mr. William Hughes) thought the paper was doubly welcome; firstly, because it introduced the Institute to a new contributor to their transactions, and he hoped it would not be the last time they would have contributions from Mr. Moore. The paper was also welcome as throwing a new light upon what had hitherto been a somewhat obscure subject. The absence until now of any precise information of the kind now disclosed, and the reluctance which had hitherto appeared to prevail amongst conductors of insurance companies which had temperance sections to give any detailed and explicit information as to their methods and their figures, had naturally induced suspicion that the temperance sections were being unduly favoured in some way. In fact, to use the author's expressive word, they were being "nursed." The members now had the opportunity, for the first time, of forming for themselves a judgment on a great many points upon which they had been till now in the dark; and especially it must be very gratifying to them to find that their suspicions—at all events, as far as the institution with which Mr. Moore was connected—had no foundation. But he thought they ought to be very careful as to the general deductions made from the facts put forward. Those facts would repay a great deal of study. No doubt the figures were capable of other explanations than those which would be sought to be put upon them. Enthusiastic temperance advocates would say that the favourable comparison between the two sections was entirely due to abstinence from alcohol. He wished to say that the paper, whatever it proved, did not prove that. The conclusions at which the author had arrived had been stated by him with studied moderation, and fell very far short of proving that total abstinence in itself was conducive to a long life. Of course there were a great number of people who hoped, when they knew the paper was going to be produced, that it would conclusively prove that fact, and he feared there were some people who would say it did prove it, and thereby they would do some injury to their own

cause. Many a good cause had been quite as much damaged by injudicious advocacy as it had been by the open attacks of its enemies; and he wished to say to all those who desired to use the paper and discussion for that purpose to be very careful they did not overdo it. He thought one explanation had been indicated by one of the speakers. Mr. Whittaker had given them a portrait from life of the average teetotaler. It was not a pleasant one. He (the President) would not have dared to have painted the gentleman in such gloomy colours. According to what Mr. Whittaker had said, he would appear to be a very disagreeable person. He was quite sure he was not so bad as Mr. Whittaker painted him, but he would prefer to say that teetotalers were only a section of that part of the community which took particularly good care of themselves in every way. They were the people who always "wrapped up", and were careful not to expose themselves to any sort of influence that might injure their health or tend to shorten their life; they were not addicted to any vices, or to any of those customs and habits falling short of vices, which some careless people were addicted to; and he should say they were a careful-living and generally religious, God-fearing sort of people, and naturally would live a long life. In fact, they conducted life upon a principle exactly the contrary to that expressed by the formula too often adopted by others—"A short life and a merry one." In conclusion, he wished to propose a hearty vote of thanks to Mr. Moore for his most able paper.

The motion having been carried,

Mr. MOORE, in reply, thanked the members very much for the kind way in which the paper had been received. He was well aware that the interest attached more to the materials used in it than the treatment. It had been his endeavour to state the facts impartially. It was simply for him to show what differences arose, and not what the causes of the differences were. He had to deal with two bodies of people, the only distinction between them, as far as his company could ascertain, being that the one body were teetotalers and the other not, and the figures showed the respective rates of mortality. Most of the points which had been raised in the discussion had been answered by other speakers, mainly by Mr. Whittaker. One was the comparison with the Clergy Mutual and the Equitable. He might point out that the average sum assured in the Clergy Mutual was £750, and in the Equitable £1,200, while in the United Kingdom Temperance it was under £300. He thought if they could show, with a much humbler class of people, that the addition of the practice of total abstinence would bring about as favourable a rate of mortality as that among the clergy and the well-to-do people assured in the Equitable, it was a very satisfactory state of affairs. In reply to Mr. Manly's remarks about ratios, personally he would not look at ratios without looking at the figures on which the ratios were based. He had stated the figures on which the ratios were based, so that they could see what the value of the ratios were. Mr. King had spoken of the mortality in New Zealand and at the Cape. He did not know what explanation could be offered, except that there appeared to be a great deal of fluctuation, which would naturally suggest small figures; in addition to which there were a larger number of endowment assurances in the temperance section. He again thanked the members very much indeed.

CORRESPONDENCE.

MORTALITY OF ABSTAINERS AND NON-ABSTAINERS.

[We have much pleasure in finding space for the following letters which have been addressed to us.—Ed. *J.I.A.*]

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the discussion which followed the reading of Mr. Moore's Paper on this subject it was pointed out by several speakers that, in order to obtain a true comparison between the rate of mortality amongst abstainers and non-abstainers respectively, it was essential that the lives brought under observation in the two classes should be similar in all other respects; and it was generally considered that Mr. Moore had been fortunate in obtaining data which possessed this necessary qualification. Had time permitted it was my intention to point out one respect in which the abstainers' experience differed from the non-abstainers' in a sense favourable to the former.

Mr. Moore's tables appear to me to contain ample internal evidence that the new business in the abstainers' section has been increasing very considerably in proportion to the new business in the non-abstainers' section.

I. On reference to the footnote to Table XV, it will be seen that during the period of 19 years from 31 December 1882 to 31 December 1901, the number of policies in force in the abstainers' section *increased* by about 29 per-cent, while the number of policies in the non-abstainers' section *decreased* by about 23 per-cent. In fact, while the total number in the two sections remained nearly the same, the numbers in the separate sections practically changed places.

II. On reference to Tables III and IV it will be seen that the policies "existing" at the close of the observations represent 43·5 per-cent of the entrants in the abstainers' section, but only 29·5 per-cent of the entrants in the non-abstainers' section.

III. On reference to the same tables it will be seen that the average number of years of exposure per new policy entering was 13·7 in the abstainers' section and 14·7 in the non-abstainers' section; while from Table V it appears that, if the first five years of assurance be excluded, the figures are 10·0 and 10·7 respectively. *Ceteris paribus*, the difference might have been expected to be in the reverse direction in view of the lower average entry-age and the lower rate of discontinuance in the abstainers' section.

If the explanation of these figures be, as I have suggested, that the proportionate amount of new business in the abstainers' section has increased, the result must be that in two senses of the word the abstainers' experience is somewhat "newer" than the non-abstainers. That is to say—

I. On the average the exposures must refer to a *later period* of time in the abstainers' section than in the non-abstainers', and there is clear evidence (both in Mr. Moore's paper and in the recent O^M experience) that the rate of mortality is improving as time goes on.

II. On the average the exposures in the abstainers' section must relate to *more recently selected lives* than those in the non-abstainers' section. This point is not entirely met by Mr. Moore's Table V in which the experience of the first five years from entry is excluded.

because it is now generally recognised that the effect of selection is felt for a very much longer period.

It will be observed that whatever effect these differences might have, would be in the direction of exaggerating the divergence between the two sections.

I do not, however, suggest that these considerations would have sufficient effect in this case to seriously alter Mr. Moore's percentages, or the deductions which he draws from them. I refer to them rather as showing (on the point of statistical method) that it is not necessarily sufficient "that the same method and period be used for both sections" since this does not exclude differences, which, in certain circumstances, might be important.

I am, Sir, your obedient Servant,

G. J. LIDSTONE.

1. *Bartholomew Lane*, 30 November 1903.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Replying to Mr. Lidstone's letter of 30 November, the principle involved in his criticism of my paper must be admitted; but I submit that its bearing on the relative mortality of the abstainers and non-abstainers amounts to very little. Indeed, Mr. Lidstone is good enough to admit this in the last paragraph of his letter. If we compare the rates of mortality of the whole-life policies, healthy males, to the end of 1882, when the relative positions of the abstainers' and non-abstainers' sections were practically the reverse of those at the end of 1901, we have the following results:—

*Percentage the Abstainers' Rate of Mortality is of the
Non-Abstainers' Rate.*

Ages	To end of 1882	To end of 1901
(1)	(2)	(3)
20-24	89.5	69.9
25-29	75.5	70.1
30-34	59.6	56.5
35-39	58.1	54.8
40-44	58.8	57.2
45-49	64.1	58.5
50-54	61.6	62.4
55-59	65.2	70.6
60-64	81.7	78.5
65-69	82.1	84.0
70-74	85.7	86.5
75-79	88.6	110.0
80-84	74.9	93.7

I contend that the figures in column (2) support my general conclusions equally with those in column (3).

Some light is thrown on the effect of "selection" in my Tables V to VIII. Moreover, it does not necessarily follow that the selection operates in the same manner in each section. If we omit the first five years of assurance, the superiority of the abstainers' life is rather more evident than in the total experience, as is here shown:—

Percentage the Abstainers' Rate of Mortality is of the Non-Abstainers' Rate, Whole-Life Healthy Males, 1841-1901.

Ages	Total Experience	Excluding first five years of Assurance.
20-24	69.9	45.8
25-29	70.1	58.2
30-34	56.5	51.7
35-39	54.8	52.7
40-44	57.2	55.8
45-49	58.5	55.9
50-54	62.4	60.7
55-59	70.6	69.4
60-64	78.5	76.1
65-69	84.0	83.4
70-74	86.5	86.4
75-79	110.0	109.5
80-84	93.7	93.7

We might be justified in assuming from these figures that if "selection" could be entirely eliminated, the abstainers would come out better still.

I am, Sir, your obedient Servant,

R. M. MOORE.

1, *Adelaide Place, E.C.*, 14 December, 1903.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The object of my letter of 30 November was to call attention to a question of principle, which, as it seemed to me, was in some danger of being overlooked, and I am glad to note that in principle Mr. Moore and I are in agreement. I was, however, careful to disclaim any intention of discussing the quantitative effect of the considerations to which I alluded, and I do not now propose to enter into any such discussion, for which the published data appear to be inadequate.

I should like, however, to suggest to Mr. Moore that, as his paper is likely (for a long time to come) to be the standard source of information on the subject, it would be very desirable to add to his tables a statement giving, for quinquennial periods, the relative numbers of new entrants in the two sections; thus showing clearly the different rates of progression.

Mr. Moore's remarks as to the relative effect of selection in the abstainers' and non-abstainers' experience are very interesting, and encourage the hope that as he is in possession of a unique body of facts he may see his way, at no remote time, to analyze the data according to the duration of assurance. By this means he would not only obtain the most scientific comparison between the rates of mortality in the two sections, but would also secure the means of investigating whether the effect of selection differs in the two sections as he suggests.

In the case of assurance statistics it is hardly possible to *eliminate* the effect of selection, but what I understand Mr. Moore to suggest is that as the duration of the assurance increases the effect of

selection may wear out more rapidly in the case of the non-abstainers than in the case of the abstainers. This hypothesis is certainly consistent with the percentages given in the second table of Mr. Moore's letter, and it offers an interesting subject for further investigation. It may be worth while to point out that if the hypothesis be correct the effect will be to increase and not to diminish the disturbing effect of the differences of duration to which I called attention in my previous letter.

I am, Sir, your obedient Servant,

G. J. LIDSTONE.

1, Bartholomew Lane, E.C., 21 December, 1903.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I regret that owing to pressure of other business I am unable at present to furnish the relative numbers of new entrants quinquennially in the Temperance and General Sections. To do so would involve a very extensive re-sorting and counting of the cards. I hope, however, in a few months to go more closely into the question of "selection" by an analysis of the data, as suggested by Mr. Lidstone.

I did not intend to imply in my letter of 14th inst. that selection may wear out more rapidly in the case of non-abstainers, nor have I any reason to believe that this is so, except that "Abstinence" is an element of selection with the Abstainers, and this always remains with them; in other words, never "wears out." But the other elements are the same in both sections, so far as we know.

Broadly, the aggregate experience shows a certain superiority of the abstainers over the non-abstainers. If we exclude the experience of the first five years of assurance, this superiority is even more evident. At present we have no material to show that either section would show relatively better or worse results if we excluded the first seven or ten years of assurance, but I hope to make this clear shortly.

I am, Sir, your obedient Servant,

R. M. MOORE.

1, Adelaide Place, E.C., 29 December, 1903.

ACTUARIAL NOTES.

I.

STAFF PENSION FUNDS.

On the Construction of Columns for obtaining the value of the Return, on Death or Withdrawal, of the Total Contributions (based on Salary) with Compound Interest at rate j per annum, discounted at rate i per annum. By ERNEST C. THOMAS, F.I.A.

LET it be assumed that the contribution is payable at the beginning of each year in advance.

Let $\frac{1}{(1+i)^x} = v^x$, and $\frac{1}{(1+j)^x} = u^x$, and

Let the salary (as a function of the age) be expressed as s_x .

It will be convenient to follow Mr. Manly's method and base our calculations in the first instance on the whole salary.

Out of l_x persons aged (x), d_x persons will die in the first year, the present value of whose benefit will be $s_x \cdot d_x \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}$; d_{x+1} will die in the second year, and the value of their benefit will be $s_x(d_{x+1} \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}) + s_{x+1}(d_{x+1} \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}})$; and so on. The total value in respect of the l_x persons will therefore be represented by the following expression:

$$\begin{aligned} & s_x(d_x \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}) \\ & + s_x(d_{x+1} \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}) + s_{x+1}(d_{x+1} \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}) \\ & + s_x(d_{x+2} \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}) + s_{x+1}(d_{x+2} \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}) + s_{x+2}(d_{x+2} \times v^{\frac{1}{2}} \times \overline{1+j}^{\frac{1}{2}}) \\ & + \&c. \quad \dots \quad (1) \end{aligned}$$

Summing this expression in vertical columns, we have:

1st col.—

$$= s_x \left\{ d_x \cdot \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + d_{x+1} \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + d_{x+2} \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + \dots \right\}$$

2nd col.—

$$= \frac{s_{x+1}}{(1+j)} \left\{ d_{x+1} \cdot \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + d_{x+2} \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + d_{x+3} \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + \dots \right\}$$

3rd col.—

$$\begin{aligned} & = \frac{s_{x+2}}{(1+j)^2} \left\{ d_{x+2} \cdot \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + d_{x+3} \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + d_{x+4} \left(\frac{1+j}{1+i} \right)^{\frac{1}{2}} + \dots \right\} \\ & \&c., \&c. \quad \dots \quad (2) \end{aligned}$$

It will be seen that by multiplying each of the expressions, within the large brackets, by $\left(\frac{1+j}{1+i} \right)^x$, we obtain functions analagous to the ordinary commutation column \bar{C}_x . Let such a column be formed and represented by an italic capital, thus, C_x , where $C_x = d_x \times \left(\frac{1+j}{1+i} \right)^{x+\frac{1}{2}}$, instead of $d_x \times \left(\frac{1}{1+i} \right)^{x+\frac{1}{2}}$.

From expression (2) we now have—

$$\begin{aligned} \text{value of 1st line} &= s_x \cdot \left(\frac{1+i}{1+j} \right)^x \left\{ C_x + C_{x+1} + C_{x+2} + \dots \right\} \\ \text{,, , 2nd line} &= \frac{s_{x+1}}{(1+j)} \cdot \left(\frac{1+i}{1+j} \right)^x \left\{ C_{x+1} + C_{x+2} + C_{x+3} + \dots \right\} \\ \text{,, , 3rd line} &= \frac{s_{x+2}}{(1+j)^2} \cdot \left(\frac{1+i}{1+j} \right)^x \left\{ C_{x+2} + C_{x+3} + C_{x+4} + \dots \right\} \\ &\&c., \&c. \quad \dots \quad (3) \end{aligned}$$

If we sum the column C_x from (x) upwards and call the

resulting column M_x , expression No. (3) will be reduced to the following :

$$\left(\frac{1+i}{1+j}\right)^x \left\{ s_x \cdot M_x + s_{x+1} \cdot M_{x+1} \cdot \frac{1}{(1+j)} + s_{x+2} \cdot M_{x+2} \cdot \frac{1}{(1+j)^2} + \dots \right\} \quad (4)$$

The next step is to form a column of $(M_x \times s_x)$, which, in accordance with Mr. Manly's notation, we will call M_x^s . Formula No. (4) is now represented as follows :

$$\left(\frac{1+i}{1+j}\right)^x \left\{ M_x^s + M_{x+1}^s \cdot \frac{1}{1+j} + M_{x+2}^s \cdot \frac{1}{(1+j)^2} + \dots \right\} \quad (5)$$

$$= \left(\frac{1+i}{1+j}\right)^x (1+j)^x \left\{ M_x^s \cdot \frac{1}{(1+j)^x} + M_{x+1}^s \cdot \frac{1}{(1+j)^{x+1}} + M_{x+2}^s \cdot \frac{1}{(1+j)^{x+2}} + \dots \right\} \quad (6)$$

$$= \frac{1}{v^x} \left\{ M_x^s \cdot v^x + M_{x+1}^s \cdot v^{x+1} + M_{x+2}^s \cdot v^{x+2} + \dots \right\} \quad (7)$$

If now we form a further column $(M_x^s \times v^x)$ which we will call M_x^{sv} , and sum this from (x) upwards and call the resulting column R_x^{su} , the whole expression reduces to

$$\frac{1}{v^x} \times R_x^{su} \quad (8)$$

Reducing now to the basis of a unit of salary, and dividing also by l_x , we obtain as the value of the Benefit for each member aged x the following simple expression :

$$\frac{1}{v^x \times s_x \times l_x} \times R_x^{su} = \frac{R_x^{su}}{D_x^s} \quad (9)$$

To sum up briefly, the columns required are as follows :

$$(1) C_x = \left\{ d_x \times \left(\frac{1+j}{1+i}\right)^{x+\frac{1}{2}} \right\}$$

$$(2) M_x = \sum C_x.$$

$$(3) M_x^s = (M_x \times s_x).$$

$$(4) M_x^{sv} = (M_x^s \times v^x).$$

$$(5) R_x^{su} = \sum M_x^{sv}.$$

$$(6) D_x^s = v^x \times l_x \times s_x.$$

The last column is one already made familiar to us by Mr. Manly, and will be required for the ordinary benefits of a Pension Fund.

The formula will be equally applicable to withdrawals by the substitution of w_x for d_x , but if more than one set of such

columns is calculated it will be advisable to distinguish them by using Mr. Manly's upper prefixes (*d*) or (*w*) as the case may be, thus dC_x or wC_x , &c.

The various columns will be summed, of course, only up to the year preceding the age of normal retirement.

It will readily be seen that when the rate of accumulation is the same as the rate of discount, *i.e.*, when (*j*) = (*i*), the formula becomes identical with that given by Mr. Manly in his paper on Staff Pension Funds (see *J.I.A.*, vol. xxxvi, p. 237).

C_x becomes d_x ,

M_x „ $\Sigma d_x = l'_x$,

M_x^w „ $l'_x \times v^x \times s_{x:j} = {}^dD_x(1+i)$,

R_x^w „ $\Sigma ({}^dD_x \cdot 1+i) = {}^d\bar{M}_x(1+i)$,

the presence of the factor $(1+i)$ being explained by the fact that in Mr. Manly's formula the contribution is assumed to be payable at the end of the year instead of the beginning.

If the contributions are spread evenly over the year there will be on the average only half a year's contribution in the year of death. The formula in that case over-estimates the benefit by the inclusion of a final payment of s_{x+t} instead of $\frac{s_{x+t}}{2}$, and also by the inclusion of half-a-year's excess interest at the rate (*j*), each payment, that is to say, being accumulated for half a year too much. In order to allow for the excess interest we must multiply our result by $\frac{1}{(1+j)^{\frac{1}{2}}}$; the correction for the half-year's contribution in the year of death will be

$$-\frac{1}{2} \cdot \frac{1}{l_x \cdot s_x \cdot v^x} (d_x \cdot s_x \cdot v^{x+\frac{1}{2}} + d_{x+1} s_{x+1} v^{x+1\frac{1}{2}} + \dots)$$

$$= -\frac{1}{2} \cdot \frac{{}^d\bar{M}_x}{D_x} \text{ see } J.I.A., \text{ xxxvii, p. 193.}$$

The corrected formula is, therefore,

$$= \frac{R_x^w}{D_x(1+j)^{\frac{1}{2}}} - \frac{1}{2} \frac{{}^d\bar{M}_x}{D_x} \dots \dots \dots (10)$$

If there be an interval before the Benefit commences, say 10 years, a convenient way of dealing with the question is to fix an average age at entry, for example 20. In constructing our columns we should then put $C_x=0$ for ages 20 to 29. Consequently the values of M_x for ages 20-30 would be identical. From that point onwards the operations would proceed as before.

If, however, this assumption should be impracticable, owing

to the great divergence in the entry ages, formula (9) would become :

$$\frac{M_{x+10}\{s_x \cdot u^x + s_{x+1} \cdot u^{x+1} + s_{x+2} \cdot u^{x+2} \dots + s_{x+9} \cdot u^{x+9}\} + R_{x+10}^{su}}{D_x^s} \quad (11)$$

In a valuation of existing contracts we should have as the total value of the Benefit in respect of each member :

$$\begin{aligned} & (\text{Past Contributions accumulated at the rate } (j)) \times \frac{M_x^{su}}{D_x^s} \\ & + (\text{Present Contribution}) \dots \times \frac{R_x^{su}}{D_x^s} \end{aligned}$$

The foregoing problem is similar to the one investigated by Mr. H. T. Adlard in a note to the *Journal*, vol. xxxvi, p. 389. His object, however, was to obtain a formula by means of which an isolated value could readily be calculated without recourse to special columns. Moreover, both he and the authorities quoted by him confined their attention to the case of a uniform contribution. The question having recently arisen in connection with the rules of an actual Pension Fund under which the contribution would be dependent upon annual salary varying with the age, and the values of the Benefits would be required for Members of all ages, it became necessary to deal with the problem by means of the columnar method. It is hoped that in these circumstances the above will be found to be a satisfactory solution.

II.

On the Determination of the Rate of Interest in an Annuity-certain. By JOHN SPENCER, F.I.A.

THE following notes on the determination of the rate of interest in an annuity-certain are submitted in the belief that they will be found to throw new light upon a problem which, though not at the present time of any great practical importance, has for two centuries attracted the attention of mathematical investigators, and occupies a definite place in the history of the subject of interest and annuities.

1. Two of the simplest of the approximations involving the use of interest tables, are those attributed to Francis Baily and George Barrett, which are here given :

$$\rho = \frac{i'(a' - a)}{a - nv'^{n+1}} \quad (A) \qquad \rho = \frac{i'(a' - a)}{a' - nv'^{n+1}} \quad (C)$$

where a = the given annuity-value.
 i = the unknown rate at which a is calculated.
 i' = a rate near to i , found by inspection of the tables.
 a' = the corresponding annuity-value.
 n = the term ; and
 $\rho = i - i'$.

The following examples show how far these formulæ may be relied upon.

Given Annuity	i	i'	Error in Value of ρ as found by formula	
			(A)	(C)
$a_{40} = 22.20843$.0325	.0300	+ .000139	— .000074
$a_{50} = 20.30983$.0433	.0400	+ .000129	— .000147
$a_{60} = 17.44985$.0550	.0600	+ .000087	— .000352

Some misconception appears to exist with regard to the relative accuracy of these formulæ. Since Barrett first suggested his formula (C) as an improvement on that of Baily, it has been usually stated without qualification that (C) gives better results than (A), while the method by which the latter approximation is deduced has been condemned on the ground that it involves loss of accuracy as compared with the process followed in obtaining formula (C). The fact is, that the relative accuracy depends upon the term of the annuity under investigation, as is suggested by the above illustrations. This point will be dealt with shortly, but I may at once point out that in the extreme case of a perpetuity, formula (A) is rigidly accurate, since nv'^{n+1} vanishes when n is infinite, and the formula reduces to $i - i'$. On the other hand, formula (C) in this case becomes $(i - i') \frac{i'}{i}$.

2. The initial equation from which the above-mentioned formulæ are derived is :

$$a = \frac{1 - (1 + i' + \rho)^{-n}}{i' + \rho} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Multiplying both sides of this equation by $i' + \rho$ and expanding, we have :

$$i'(a' - a) = (a - nv'^{n+1})\rho + \frac{n(n+1)}{2} v'^{n+2} \rho^2 \\ - \frac{n(n+1)(n+2)}{6} v'^{n+3} \rho^3 + \&c. \quad . \quad . \quad . \quad (2)$$

The terms neglected in deriving formula (A) may be written

$$+ \frac{n(n+1)}{2} v'^{n+2} \rho^2 \left(1 - \frac{n+2}{3} d' \cdot \frac{\rho}{i'} \right. \\ \left. + \frac{(n+2)(n+3)}{12} d'^2 \cdot \frac{\rho^2}{i'^2} - \&c. \right) \quad . \quad . \quad . \quad (3)$$

or, writing y for the series in brackets,

$$+ \frac{n(n+1)}{2} v'^{n+2} \cdot \rho^2 \cdot y.$$

This expression will be referred to as the (A) remainder.

Again, formula (C) is obtained by writing equation (1) in the form

$$a = \left[1 - (1 + i' + \rho)^{-n} \right] \left[1 + \frac{\rho}{i'} \right]^{-1} \frac{1}{i'}$$

whence, after expansion, we have

$$i(a' - a) = (a' - nv'^{n+1})\rho - \left(a' - nv'^{n+1} \right. \\ \left. - \frac{n(n+1)}{2} v'^{n+2} i' \right) \frac{\rho^2}{i'} + \&c. \quad . \quad . \quad . \quad (4)$$

The series discarded from this equation in deducing formula (C) may conveniently be called the (C) remainder, and since

$$a' = \frac{1 - i'^n}{i'} = \frac{1}{i'} \left[\frac{i'^n}{(1 - d')^n} - v'^n \right] \\ = \frac{i'^n}{i'} \left[nd' + \frac{n(n+1)}{2} d'^2 + \frac{n(n+1)(n+2)}{6} d'^3 + \&c. \dots \right]$$

it may be written

$$- \frac{n(n+1)}{2} v'^{n+2} \rho^2 \left[\left(\frac{n+2}{3} d' + \frac{(n+2)(n+3)}{12} d'^2 + \dots \right) \right. \\ \left. - \left(\frac{n+2 \cdot n+3}{12} d'^2 + \dots \right) \frac{\rho}{i'} + \&c. \dots \right]$$

Putting now

$$\frac{1}{x} = 1 + \frac{n+2}{3} d' + \frac{(n+2)(n+3)}{12} d'^2 + \dots$$

the (C) remainder becomes

$$- \frac{n(n+1)}{2} v'^{n+2} \rho^2 \left[\left(\frac{1}{x} - 1 \right) - \left(\frac{1}{x} - 1 - \frac{n+2}{3} d' \right) \frac{\rho}{i'} + \dots \right]$$

which will be found to reduce to

$$- \frac{n(n+1)}{2} v'^{n+2} \rho^2 \left(\frac{1}{x} - y \right) \left(\frac{1}{1 + \frac{\rho}{i'}} \right) \quad . \quad . \quad . \quad (5)$$

An examination of the (A) remainder makes it clear that in

practice this will be positive, the effect of neglecting these terms in formula (A) being to overstate the value of ρ . A contrary result obtains in regard to the (C) remainder, and we are led to the conclusion that the correct value of ρ will be found to lie between the (A) and (C) approximations, and that this may be written

$$\frac{i'(a' - a)}{a - nv'^{n+1} + c(a' - a)} \quad \text{or} \quad i' \left[\frac{a - nv'^{n+1}}{a' - a} + c \right]^{-1} \quad . \quad . \quad . \quad (6)$$

where c is positive and less than unity.

From this point of view we have

$$i'(a' - a) = (a - nv'^{n+1})\rho + c(a' - a)\rho \quad . \quad . \quad . \quad (7)$$

Now, multiplying equation (2) by $(1 - c)$, and adding to the result c times equation (4), we find that

$$i'(a' - a) = (a - nv'^{n+1})\rho + c(a' - a)\rho + (1 - c) \\ \times (\text{A) remainder} + c \times (\text{C) remainder},$$

and equating the right-hand number of this equation and the corresponding expression in equation (7), we arrive at the result

$$c = \frac{(\text{A) remainder}}{(\text{A) remainder} - (\text{C) remainder}} = \frac{y}{y + \left(\frac{1}{x} - y \right) \left(1 + \frac{\rho}{i'} \right)} \\ = \frac{1 + \frac{\rho}{i'}}{1 + \frac{\rho}{xy + \frac{\rho}{i'}}} \quad . \quad . \quad . \quad (8)$$

Formula (8) is exact, but it of course involves the unknown quantity ρ . By means of it, however, we may obtain expressions giving the value of ρ to any required degree of accuracy.

For example, putting $\rho = 0$, we have—

$$c = x = \left[1 + \frac{n+2}{3} d' + \frac{(n+2)(n+3)}{12} d'^2 + \right]^{-1} \quad . \quad . \quad (9) \\ = \frac{\frac{n(n+1)}{2} d'^2}{(1+i')^n - (1+nd')}$$

whence

$$\rho = i' \left[\frac{a - nv'^{n+1}}{a' - a} + \frac{n(n+1)d'^2}{2[(1+i')^n - (1+nd')]} \right]^{-1} \quad . \quad (10)$$

At first sight this looks like a new formula, but it is, in fact,

identical with that given by Mr. J. J. McLauchlan, and known as formula (D), since—

$$\frac{\frac{n(n+1)}{2} d'^2}{(1+i')^n - (1+nd')} = \frac{\frac{n(n+1)}{2} i' v'^{n+2}}{a' - nv'^{n+1}}$$

We may deduce from formula (10) a simple approximation for ρ which, considering the labour involved, will be found to give very good results, except in extreme cases.

From (9) above

$$x = 1 - \frac{n+2}{3} d' + \frac{(n+2)(n-1)}{36} d'^2 + \dots$$

In cases where ni' is not greater than 3 or 4, we may, without serious loss of accuracy, write for this series—

$$x = 1 - \frac{ni'}{3} + \frac{(ni')^2}{36}$$

whence

$$\rho = \left[\frac{a' - nv'^{n+1}}{i'(a' - a)} - \frac{n}{3} \left(1 - \frac{ni'}{12} \right) \right]^{-1} \quad \dots \quad (11)$$

Two examples in illustration of this formula may be given :

$a_{30} = 19$			$i = \cdot 0324252$			$a_{40} = 16\cdot 04612$			$i = \cdot 055$		
			$i' = \cdot 03$						$i' = \cdot 050$		
			$\rho = \cdot 0024252$						$\rho = \cdot 005$		
Formula	Value of ρ	Error	Formula	Value of ρ	Error	Formula	Value of ρ	Error	Formula	Value of ρ	Error
A	$\cdot 0025732$	$+ \cdot 0001480$	A	$\cdot 0052326$	$+ \cdot 0002326$	A	$\cdot 0052326$	$+ \cdot 0002326$	A	$\cdot 0052326$	$+ \cdot 0002326$
C	$\cdot 0023699$	$- \cdot 0000553$	C	$\cdot 0047369$	$- \cdot 0002631$	C	$\cdot 0047369$	$- \cdot 0002631$	C	$\cdot 0047369$	$- \cdot 0002631$
11	$\cdot 0024230$	$- \cdot 0000022$	11	$\cdot 0050001$	$+ \cdot 0000001$	11	$\cdot 0050001$	$+ \cdot 0000001$	11	$\cdot 0050001$	$+ \cdot 0000001$

3. At this point we may investigate a little more closely the relative accuracy of formulæ (A) and (C). Using formula (10) as a standard of comparison, we see from (9) that when n is small the value of x will be but slightly less than unity, and in such a case as this, formula (C), which may be written

$$\rho = i \left[\frac{a - nv'^{n+1}}{a' - a} + 1 \right]^{-1}$$

would obviously give a close approximation. On the other hand, in the case of an annuity for a long term at a high rate of interest, x will be a small fraction, and here formula (A) might very appropriately be employed. At the point at which the two

formulae give equally good results, we should not be far wrong in writing $x = \frac{1}{2}$, and formula (10) would become—

$$\rho = i' \left[\frac{a - nv'^{n+1}}{a' - a} + \frac{1}{2} \right]^{-1} \quad . \quad . \quad . \quad (12)$$

whence, equating this value of x and that given in deducing formula (11), we have, roughly, $ni' = 1.8$, a result which enables us to tell at once whether (A) or (C) will furnish the better approximation in a given case.

The following two illustrations, at widely different rates of interest, form a sufficiently stringent test of formula (12) :

$a_{90} = 38.44489$			$i = .0225$		
$ni' = 1.8$			$i' = .0200$		
			$\rho = .0025$		
			$a_{15} = 8.33458$		
			$i = .0975$		
			$i' = .1000$		
			$\rho = -.0025$		
Formula	Value of ρ	Error	Formula	Value of ρ	Error
12	.002497	-.000003	12	-.002501	-.000001
A	.002663	+.000163	A	-.002470	+.000030
C	.002350	-.000150	C	-.002533	-.000033

4. Returning now to formula (8), I will give two closer approximations, which may be of interest from a mathematical point of view.

Writing
$$c = \frac{x \left(1 + \frac{\rho}{i'} \right)}{\frac{1}{y} + x \frac{\rho}{i'}}$$

and substituting for x and y their respective values, we find that

$$c = x \left[1 - \frac{(n+2)(n-1)}{36} d'^2 \frac{\rho}{i'} \right] \text{ very nearly.} \quad (13)$$

The (A) approximation may, with advantage, be substituted for ρ in this expression, and, writing for $(n+2)(n-1)$, what is practically the same thing, $n(n+1)$, we have, finally :

$$\rho = i' \left[b + x \left(1 - \frac{n \cdot n + 1}{36b} d'^2 \right) \right]^{-1} \quad . \quad . \quad . \quad (14)$$

where, for convenience, b is put for $\frac{a - nv'^{n+1}}{a' - a}$.

This approximation involves very little more labour than formula (10), since the value of $n \cdot n + 1 d'^2$, the numerator in the

new term, will already have been calculated in finding the value of x .

Again, from (8),

$$c = \frac{1 + \frac{\rho}{i'}}{\frac{1}{xy} + \frac{\rho}{i'}} = \frac{\frac{i'}{\rho} + 1}{\frac{i'}{\rho} + 1} \dots \dots \dots (15)$$

Now the true value of

$$\rho = i \left[\frac{a - nv'^{n+1}}{a' - a} + c \right]^{-1} = i' [b + c]^{-1}$$

whence $\frac{i'}{\rho} = b + c$, and formula (15) becomes—

$$c = \frac{b + c + 1}{\frac{b + c}{xy} + 1} = \frac{b + 1}{\frac{b + c}{xy}}$$

Our object now is to transform this expression so as to get rid of the terms involving ρ the unknown. For all practical purposes this may be done by adding $(x-1)$ to the numerator, and $\frac{x-1}{c}$ to the denominator, since, writing for c its value as given in formula (13), and inserting the value of y , we find that—

$$\frac{b+c}{xy} + \frac{x-1}{c} = 2 + \frac{b-1 + \frac{n+2}{3}d'}{x} + \text{other terms which may be neglected.}$$

Hence—

$$c = \frac{b+x}{b-1 + \frac{n+2}{3}d' + 2 + \frac{x}{x}} = \frac{x(b+x)}{b+2x-1 + \frac{n+2}{3}d'}$$

and

$$\rho = i' \left[b + \frac{x(b+x)}{b+2x-1 + \frac{n+2}{3}d'} \right]^{-1} \dots \dots \dots (16)$$

Here, as before, $b = \frac{a - nv'^{n+1}}{a' - a}$, and x may be written—

$$\frac{n(n+1)d'}{2[s'_{n+1} - (n+1)]}$$

Illustrations.

$$a_{30} = 19 \quad i = .032 \, 425 \, 198 \, 8$$

$$i' = .03$$

$$\rho = .002 \, 425 \, 198 \, 8$$

$$a_{40} = 16.558 \, 269 \quad i = 5\frac{5}{15} \text{ per-cent}$$

$$i' = 5 \text{ per-cent}$$

$$\rho = \frac{5}{15} \text{ per-cent}$$

Formula	Value of ρ	Error	Formula	Value of ρ	Error
10	.002 424 936 3	-.000 000 262 5	10	.002 631 202	-.000 000 377
14	.002 425 199 0	+.000 000 000 2	14	.002 631 521	-.000 000 058
16	.002 425 200 2	+.000 000 001 4	16	.002 631 581	+.000 000 002

$$a_{100} = 9.523 \, 370 \, 404 \quad i = .105$$

$$i' = .100$$

$$\rho = .005$$

Formula	Value of ρ	Error
10	.004 999 928 0	-.000 000 072 0
14	.005 000 015 8	+.000 000 015 8
16	.004 999 999 8	-.000 000 000 2

The last example is an extreme case utilized for illustrative purposes by De Morgan in his famous paper on this subject (*J.I.A.*, vol. viii, p. 61). He suggests it as a severe test both as to n and ρ , although it will be observed that the results given above are unusually close. [(A) gives $\rho = .0050007$; (C), .0047625, a considerable error]. In regard to this point, it may be remarked that the accuracy of any formula of the kind here referred to depends not so much on the closeness of the trial rate to the true value, as upon the magnitude of the quantity b , or what is practically the same thing $\frac{i'}{\rho}$. Moreover, in the case of the final illustration, the given annuity differs very slightly from the perpetuity, and the closeness of the result due to the (A) formula is at once accounted for, while the more elaborate formulæ gain in accuracy to a corresponding degree.

REVIEWS.

British Offices Life Tables, 1893. Select Tables.

THIS is the seventh volume of tables published by the Institute of Actuaries and the Faculty of Actuaries, in connection with the completed investigation into the mortality experience of British Life Assurance Companies; and now only one more volume is to come, namely, that which will contain the anxiously awaited explanations of the methods of procedure.* Until this last volume appears, a good

* As we go to press, a copy of this volume has reached us. We hope to deal with it in our next number.—ED. *J.I.A.*

deal which is contained in those already in our hands must be taken upon trust, but that fact does not diminish the confidence which will be felt in the great series of tables due to the labours of the Joint Mortality Experience Committee, and to the public spirit and liberality of the British Life Offices, who subscribed the heavy cost of compilation and publication.

The volume of Select Tables before us is based upon eleven parallel mortality tables, all graduated by Makeham's Law, and yet all interdependent, so that the law of uniform seniority runs transversely across the various tables, as well as downwards through each. The first of these mortality tables is that for lives just admitted to assurance, the second for lives who have been assured exactly one year, and so on, until the eleventh is that for lives who have been insured for ten years or more. The graduation is a masterpiece, due to the erudition and skill of Mr. G. F. Hardy. Makeham's constants s and c are absolutely constant throughout all the eleven tables, and for all ages, but the constants k and g , while constant for all ages in each of the tables, vary from table to table; and yet, in tracing select joint lives across the tables as the duration of their joint assurance increases, the law of uniform seniority prevails, and the two joint lives of different ages may be represented by two joint lives of the same age, just as in an ordinary table graduated on Makeham's hypothesis. Therefore, for the Select Experience, there is no need to prepare extended joint-life tables, but a single table at each rate of interest is all that is required for each number, two, three, &c., of joint lives. In the volume published, tables are given for only two joint lives, but anyone requiring more could without much trouble make the necessary computations.

The foregoing is an imperfect description of the Select Mortality tables provided, and great interest will be felt when Mr. Hardy's full explanation is available. Indications of his methods will be found in *J.I.A.* xxxi, p. 359, and xxxiii, p. 493.

The tables included in the volume of Select Experience are not very numerous. Those not involving interest consist of extended values of l_x , $\log l_x$, d_x , $\log d_x$, q_x , $\log q_x$, μ_x , and $\log \mu_x$ for each of the first ten years of assurance, and for ten years or more. It will be noticed that p_x and $\log p_x$ do not appear, and we think this is a little unfortunate, as in many respects the function p is more important in making calculations than the function q . Of course it can be obtained at once by taking the arithmetical complement of q , but frequently its logarithm is required, and that necessitates reference to a book of logarithms or differencing of the values of $\log l_x$, tedious processes when rapidity of calculation is important.

Monetary Tables are given at $2\frac{1}{2}$, 3, $3\frac{1}{2}$, and 4 per-cent interest, and they include extended D and N columns, with their ultimate values; also Select values of a_x , A_x , and P_x , with Select values of temporary annuities, for all values of the initial age, x , up to 75, and for all durations, n ; and, lastly, Select annuity-values for two lives of equal age, these being calculated to each tenth of the year of age to render interpolation easy. At the end of the volume there is a small table of uniform seniority for two joint lives, identical with that given for the $O^{M(5)}$ mortality in the volume of aggregate tables,

because Makeham's constant, c , is the same for the Select and the $O^{M(5)}$ Tables.

It will be observed that there are many omissions from the foregoing list. For instance, there is no extended table of annuity-values, and the assurance commutation columns are not given. The Joint Committee have provided the material for the preparation of complete tables of Select monetary values, rather than supplied the want themselves. They have done so much that it would be ungracious to complain of their not having done more, but evidently there is room for some of the industrious members of the profession to provide yet another volume of tables, which would prove very useful to the practising actuary.

The Select Tables will probably be largely used for the calculation of premium rates, and it is interesting to compare the rates they produce with those derived from other standards. We, therefore, give the annual premium at 3 per-cent interest at representative ages, by the new $O^{M(5)}$ Select Tables, with the corresponding function by the O^M Aggregate, the $H^{M(5)}$ Select, and H^M Aggregate Tables. It will be observed that the new Select Tables show similar, but, perhaps not so marked, peculiarities as the old, the rates being higher at the younger ages than those by the corresponding Aggregate Table, and lower at the older. Throughout, the $O^{M(5)}$ Select rates are lower than the $H^{M(5)}$, which was to have been expected.

Table of Annual Premiums per-cent.

INTEREST 3 PER-CENT.

Age	$O^{M(5)}$ Select	O^M Aggregate	$H^{M(5)}$ Select	H^M Aggregate
20	1.365	1.306	1.563	1.427
25	1.551	1.524	1.703	1.625
30	1.785	1.790	1.925	1.880
35	2.081	2.116	2.218	2.193
40	2.457	2.524	2.603	2.589
45	2.940	3.046	3.106	3.113
50	3.564	3.730	3.755	3.801
55	4.377	4.641	4.635	4.725
60	5.444	5.872	5.827	5.987
65	6.853	7.557	7.433	7.705

It is not likely that Companies will make their valuations by the Select Tables, more particularly now that the period of selection has been extended from five to ten years; but, nevertheless, it is important to know what effect would be produced by such a valuation. When Mr. G. King, a year ago, read his paper at the Institute of Actuaries, on the Comparative Reserves of Life Offices, the Select Tables were not available; but at the Congress held in September in New York he submitted the figures for a 3 per-cent valuation by them, and the following are the comparative reserves, the O^M aggregate 3 per-cent reserve being taken as 10,000.

Parallel with them we also give the comparative reserves by the Combined O^M and $O^{M(5)}$, the O^M being taken alone for the first five years, and similarly by the Combined H^M and $H^{M(5)}$, all at 3 per-cent interest.

Age of Office	COMPARATIVE RESERVES AT 3 PER-CENT, THE RESERVE BY O ^M AGGREGATE BEING TAKEN AS 10,000		
	O ^M Select	Combined O ^M & O ^{M(5)}	Combined H ^M & H ^{M(5)}
5	11,410	10,000	9,698
10	10,932	10,273	10,333
15	10,633	10,217	10,289
20	10,454	10,166	10,226
25	10,344	10,131	10,182
30	10,273	10,107	10,152
35	10,226	10,092	10,132
40	10,196	10,081	10,119
45	10,177	10,075	10,111
50	10,166	10,071	10,106

It will be observed that the new Select Tables give the highest reserves of all, appreciably higher than those by the Combined O^M and O^{M(5)} Tables.

In conclusion, we must refer to an interesting table appearing at the end of the volume, namely, that giving the single and annual premiums for contingent assurances, the assured life being taken as a select male by the new Tables, and the counter-life a select female by the British Offices Life Annuity Tables. In so far as the counter-life is concerned, we imagine that a better basis for computing the rates could not be chosen, because it is a well-known fact that life tenants, against whose lives such assurances are generally effected, enjoy remarkable longevity; and, even should they be males, no great hardship will be done by employing the female tables. As regards the assured life, however, we doubt very much whether the O^M Select Tables are sufficiently stringent, at least under ordinary circumstances. The volume of Unadjusted Data published by the Joint Mortality Experience Committee gave the experience derived from contingent assurances, and, although the materials were meagre, they yet indicated that the rate of mortality amongst lives assured under contingent policies is high. The rates given in the present volume, must, therefore, be looked upon as minimum rates, and they probably will require somewhat heavy and discriminating loading to render them suitable for practical use. In making this remark we are not running counter to the views of the Joint Committee themselves, because in the volume of explanations yet to come we believe that a note to similar effect will find place.

*A Digest of the Results of the Census of England and Wales in 1901.**

MR. T. G. ACKLAND's introduction to the above-named manual, says: "This Handbook claims to be nothing more than a digest of "the results of the Census for England and Wales, 1901, condensed "from the voluminous folios recently issued by the Census Office", but as Mr. Sanders, the compiler, has succeeded in reproducing in less

* A Digest of the Result of the Census of England and Wales in 1901, compiled by Mr. William Sanders, and produced under the general supervision of Mr. Thomas G. Ackland, F.I.A. London: C. & E. Layton, 56, Farringdon Street, E.C.

than a fiftieth part of their original bulk, the salient features of the elaborate publication of the Census Office, the value and extent of the task he has performed is hardly apparent from this modest statement of its purpose. The fact that the book has been produced under the general supervision of Mr. Ackland himself, is a sufficient guarantee of its completeness and efficiency. Mr. Sanders' explanatory notes, which cover some twenty pages, are exceedingly interesting reading, and will appeal to many who turn from mere statistics with aversion, besides affording the student of social and actuarial questions a useful guide to the wealth of information contained in the book itself. Following these notes are thirty-two admirably arranged tables, relative to the condition of the people, and lastly, a table comparing the scope of the different censuses of Great Britain and Ireland and our Colonies. The following summary of the contents of the tables will give the best idea of the completeness of Mr. Sanders' compilation:—

Tables I to VI show the Population and Number of Houses in England, Wales and the Islands in the British Seas, at each census, from 1851 to 1901 inclusive, and the Population in the administrative County of London at each successive census, from 1801 to 1901.

Tables VII to X give the Area, Houses and Population in 1891 and 1901, of the Metropolitan Boroughs, the Ancient Counties, Administrative Counties and Parliamentary Counties and Boroughs.

In Table XI the Population in 1901 of the City of London and each Metropolitan Borough, of each County Borough, and of each Urban District with over 50,000 inhabitants, is compared with the population of the same localities in 1891.

Table XII, which covers twenty-five pages, compares the population in 1891 and 1901 of County Boroughs, Municipal Boroughs, and other Urban Districts, and gives the area and houses in the latter year.

Then follow Tables showing the number of persons enumerated in Military and Naval Barracks, Quarters, Hospitals and Prisons, and on His Majesty's Ships in Home Waters; in Workhouses, Hospitals and Lunatic Asylums; in Reformatory and Industrial Schools; on board Sea-going and Coasting Merchant Vessels; and on Inland Barges and Boats; in Barns, Sheds, Caravans, Tents, or in the Open-air (XIV to XVII).

Table XVIII throws a great deal of light on the condition of the people as regards over-crowding, and shows the number of persons living in tenements of fewer than five rooms.

In Table XIX, the population of England and Wales is divided according to sex, and grouped at quinquennial ages from 25 to 95, and at groups with shorter age intervals below 25. Table XX gives the same data sub-divided into Urban and Rural Districts. Table XXI shows the condition of the people as to marriage, and Table XXII is especially interesting, as it gives the ages in combination of Husbands and Wives, suggesting valuable material for a study in correlation. We note that one wife of 21 had a husband of 95, one husband of 20 had a wife of 65, and one husband of 65 had a wife over 100! Then follow Tables showing the occupations of Males and Females in ten age groups. Among male occupations, excluding those retired or unoccupied, the largest group is of those engaged in the conveyance of men, goods and messages, while

domestic service accounts for the most numerous group among women. These figures are further divided into Urban and Rural Districts.

Following these are Tables of Birth places, and an enumeration of the Blind, Deaf, Dumb and Insane; and lastly, Tables of Births and Deaths for ten years, and a Table of Emigration for the same period.

Altogether the book will prove fascinating to the student of Statistics, and of great use to busy public men.

*Aus der Praxis der Gothaer Lebensversicherungsbank.**

A brief Review by T. E. YOUNG, Past-President Institute of Actuaries, and WILLIAM R. STRONG, F.I.A.

WE have been favoured by the courtesy of Mr. Gustav Fischer with a copy of a publication recently issued by the Gotha Lebensversicherungsbank, and extending to 520 pages.

The information furnished in the volume must prove of significant import to the members of this flourishing Institution; and we have extracted those portions which appear likely to possess an interest to our Profession. We have appended comparative statistics wherever this course has been feasible.

The Gotha Office was established in 1827, and the occasion of the present compilation is the completion of the seventy-fifth year of its existence, which the volume is intended to celebrate.

The number of the Assured on January the 1st 1902 was 95,923, and the aggregate sum assured amounted to £40,387,360, assessing the mark at 1s.

TABLE I.

The General Rate of Mortality from 1829 to 1878.

The Number of Years observed was 963,973, the Total Deaths 19,980, and the Ratio of Mortality per 1,000, consequently 20·7. The observations principally consisted of Male Lives.

GOTHA MORTALITY EXPERIENCE—MALES				RATES OF MORTALITY PER 1,000	
Ages	No. Exposed to Risk	No. of Deaths	Rate of Mortality per 1,000	OM Table	Fair's Life Table— Males (No. 3)
(1)	(2)	(3)	(4)		
15-20	714·5	3	5·96	3·99	6·60
21-25	717·4	44		7·20	8·81
26-30	4057·4	237	5·84	7·70	9·69
31-35	97948·5	647	6·61	8·89	10·75
36-40	141078·5	1,153	8·17	10·31	12·23
41-45	15685·4	1,582	10·09	11·89	14·36
46-50	148165·	2,008	13·55	14·99	17·36
51-55	128034·5	2,417	18·88	18·93	22·28
56-60	99884·	2,761	27·64	25·70	28·91
61-65	69231·	2,843	41·07	36·25	39·79
66-70	12108·5	2,672	63·46	53·84	57·50
71-75	21253·5	1,953	91·89	79·14	84·18
76-80	8288·	1,104	133·20	117·	121·45
81-85	2339·5	459	196·20	172·9	169·98
86-90	325·5	97	298·	237·5	229·67

* Experience of the Gotha Life Office. Published by Gustav Fischer, at Jena, 1902.

TABLE II.

GOTHA MORTALITY EXPERIENCE—FEMALES (excluding the first 5 years of Assurance)				RATES OF MORTALITY PER 1000	
Ages	No. Exposed to Risk	No. of Deaths	Rate of Mortality per 1,000	O/ Table (excluding the first 5 years of Assurance)	Farr's Life Table— Females (for all years) (No. 3)
(1)	(2)	(3)	(4)		
15-25	109	1	9.17	6.04	7.94
26-30	497.5	4	8.04	7.05	10.19
31-35	1,330.5	11	8.27	8.76	11.16
36-40	2,541	35	13.77	12.16	12.27
41-45	3,687.5	36	9.76	11.80	13.63
46-50	4,678.5	59	12.61	13.09	15.38
51-55	5,422	83	15.31	16.46	18.38
56-60	5,934.5	173	29.15	21.98	25.39
61-65	5,522	202	36.59	30.89	35.57
66-70	4,260.5	223	52.35	47.25	51.70
71-75	2,436	225	92.36	72.58	76.26
76-80	1,042	125	119.95	111.48	111.12
81-85	324.5	57	175.67	173.46	157.36
86-90	62.5	20	320	233.13	215.08

TABLE III.

Age-Groups	GOTHA MORTALITY EXPERIENCE AMONG THE CLERGY		THE MORTALITY EXPERIENCE	
	Rate of Mortality per 1,000		Of the Clergy Mutual Assurance Society (1829 to 1887)	Of the Clergy in England and Wales (deduced by the Rev. John Hodgson)
	From the 6th Year of Assurance	For all Years of Assurance		
	Rate of Mortality per 1,000			
31-35	7·2	5·	3·6	6·4
36-40	5·9	5·6	5·4	7·1
41-45	7·2	7·2	4·9	9·2
46-50	8·3	8·5	8·1	11·3
51-55	14·8	14·2	12·8	18·5
56-60	22·8	22·5	18·5	22·9
61-65	34·	33·9	25·3	34·5
66-70	60·3	59·9	41·3	49·8
71-75	87·4	87·3	64·4	83·
76-80	137·9	137·9	99·3	125·9
81-90	223·6	223·6	184·2	195·3

TABLE IV.

Ages	GOTHA MORTALITY EXPERIENCE			MORTALITY AMONG THE MEMBERS OF THE ROYAL MEDICAL AND CHIRURGICAL SOCIETY (Compiled by F. G. P. Neison)		
	No. Exposed to Risk	No. of Deaths	Rate of Mortality per 1,000	No. Exposed to Risk	No. of Deaths	Rate of Mortality per 1,000
(1)	(2)	(3)	(4)			
26-30	2,662·5	23	8·6	2,198·	9	4·09
31-35	5,230·	39	7·5	2,190·5	14	6·39
36-40	6,701·5	74	11·	1,353·5	20	14·77
41-45	7,165·	87	12·1	811·5	11	13·55
46-50	6,679·	86	12·9	578·5	8	13·83
51-55	5,777·	138	23·9	410·	17	41·46
56-60	4,521·5	136	30·1	200·	7	35·
61-65	3,339·	146	43·7	74·	6	81·08
66-70	2,061·5	144	69·9	18·	4	222·20
71-75	1,035·5	101	97·5
76-80	422·5	57	134·9
81-90	127·5	21	164·7

TABLE V.

Ratio of Mortality per 1,000 (all Ages) from Various Diseases.

Gotha Experience	Clergy Mutual Experience	
Gout
Other Constitutional Diseases
Cerebral Apoplexy	} Nervous System	3·14
Brain Diseases		
Spinal Diseases		
Acute Diseases of the Air Passages and Lungs
Inflammation of the Lungs	} Respiratory Organs	2·57
Pulmonary Consumption		
Chronic Bronchial Catarrh		
Diseases of the Stomach	} Digestive Organs	1·46
Liver Diseases		
Bright's Disease		
Diseases of the Bladder and Urinary Duct	} Urinary Organs	1·24
Skin Diseases		
Diseases of the Bones and Joints		
Suicide	} Violent Deaths and Acci- dents	·43
Accident and Murder		
Typhus	} Infectious Diseases	8·6
Small-pox		
Other Infectious Diseases		
Malignant Growths
Diseases of the Organs of the Circulation	} Organs of the Circulation	2·12
Diseases of the Sexual Organs		
	} Organs of Generation	·03

TABLE VI.—*Diseases of Women. Mortality per 1,000 at all ages.*

Gotha Experience		Clergy Mutual Experience (Males and Females)
Infectious Diseases	2·42	8·6
Diseases of the Nervous System	3·05	3·14
„ „ Respiratory Organs	9·41	2·57
„ „ Organs of Circulation	3·19	2·12
„ „ Digestive Organs	3·00	1·46
„ „ Urinary Organs	·47	1·24
„ „ Organs of Generation	·75	·03
Violent Deaths	·44) 43
Suicide	·12	

TABLE VII.—*Female Lives.*

Ages	GOTHA MORTALITY EXPERIENCE			RATE OF MORTALITY PER 1,000
	No. Exposed to Risk	No. of Deaths	Rate of Mortality per 1,000	Fair's Table (No. 3)
(1)	(2)	(3)	(4)	
15-30	3276·5	31	9·46	8·61
31-40	9238·5	115	12·45	11·70
41-50	13975·5	168	12·02	14·47
51-60	16283·5	346	21·25	21·70
61-70	10763·5	449	41·72	42·80
71-80	3483·	350	100·49	59·70
81-90	387·	77	198·97	173·

TABLE VIII.—*The Rate of Mortality according to Causes of Death among the Assured of the Gotha Lebensversicherungsbank during the period from 1829-1878.*

Estimated and Actual Mortality at all Ages in relation to the Sums Assured, with the Causes of Death, from the 6th Year of Assurance.

		NO. OF DEATHS		Percentage of the Actual to the Estimated Claims
		Estimated	Actual	
Infectious Diseases	(Up to £150 .	428·59	462	107·8
	(£150 to £300	678·02	666	98·2
	(Over £300 .	297·05	271	91·2
Constitutional Diseases	(Up to £150 .	512·02	516	100·8
	(£150 to £300	801·15	776	96·9
	(Over £300 .	358·60	383	106·8
Diseases of the Nervous System	(Up to £150 .	850·63	752	88·4
	(£150 to £300	1,342·56	1,376	102·5
	(Over £300 .	599·67	672	112·1
Diseases of the Breathing Organs	(Up to £150 .	1,604·89	1,967	122·6
	(£150 to £300	2,521·63	2,452	97·2
	(Over £300 .	1,112·	869	78·1
Diseases of the Circulatory Organs	(Up to £150 .	701·32	587	83·7
	(£150 to £300	1,008·78	1,090	98·3
	(Over £300 .	496·33	645	130·
Diseases of the Digestive Organs	(Up to £150 .	508·31	494	97·2
	(£150 to £300	802·85	800	99·6
	(Over £300 .	357·77	375	104·8
Diseases of the Urinary Organs	(Up to £150 .	234·28	202	86·2
	(£150 to £300	369·48	353	95·5
	(Over £300 .	165·30	215	130·
The Aggregate Mortality	(Up to £150 .	5,107·67	5,279	103·4
	(£150 to £300	8,048·18	7,895	98·1
	(Over £300 .	3,573·08	3,631	101·6

FOURTH INTERNATIONAL CONGRESS OF ACTUARIES,
1903.

IN opening the sessional meeting of the Institute of Actuaries, held on 30 November 1903, the PRESIDENT (Mr. WILLIAM HUGHES) said that, before proceeding to the main business of the meeting, he wished to say a few words on the Fourth International Congress which had taken place in New York since the last meeting. There were about 190 members present of numerous nationalities—some from Japan, New Zealand, Australia, and various remote places, showing that interest in actuarial matters was spreading throughout the world. The meetings were very well attended, and the reading and discussion of the papers was sustained with great interest. The subjects discussed ranged over a very wide field indeed; in fact, he thought it might be doubted whether the Congress did not attempt a little too much; and it would be a matter for the consideration of the Permanent Committee of the Congress whether, by limiting the number of subjects to be discussed or by other means, they could not make the work of future Congresses a little more manageable. Some of the subjects discussed were of somewhat limited interest, but nearly all were of a very valuable character; and when the proceedings of the Congress had been printed and circulated he was sure the members would all obtain a great deal of information from reading them. The social side of the Congress was of great value; the opportunities they had of meeting old friends and making new ones in America were very much appreciated. Of their reception by their American and Canadian friends they could not speak too enthusiastically. It was specially gratifying to find with what regard and affection, and almost veneration, the Institute was looked upon by their foreign friends, especially by the Americans. Where the warmth of the reception was so universal, it might seem a little invidious to name anybody individually, but he could not refrain from mentioning the names of Dr. Pierson, the President of the Actuarial Society of America, and Mr. Tatlock, the Organising Secretary, who, in addition to their official duties, were most assiduous in promoting the comfort and convenience of the members of the Congress, and of the ladies who accompanied them. He also wished to refer to Mr. Macaulay, who performed a like service for those who took the journey into Canada. None of those present who made the visit to the United States could ever forget it; and it made them look forward with pleasurable anticipation to meeting the same friends and others at the next Congress to be held in Berlin in 1906.

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INCORPORATED BY ROYAL CHARTER 29TH JULY, 1884.

(Corrected to 31 December, 1903.)

STAPLE INN HALL, HOLBORN, W.C.

G.P.O. TELEPHONE: No. 1710 CENTRAL.

LONDON:

PRINTED BY CHARLES & EDWIN LAYTON,
56, FARRINGDON STREET, E.C.

1904.

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Date of becoming a Fellow.		Date of becoming a Fellow.	
1876	†Ackland, Thomas Gans, F.S.S., Mem. Act. Soc. Amer., 5 & 6 Clement's-inn, Strand, W.C.	1891	†Anderson, William Smith, <i>Gresham Life Assurance Society,</i> <i>St. Mildred's-house, Poultry, E.C.</i>
1871	†Addiscott, Francis, <i>Medical Sickness, Annuity & Life</i> <i>Assur. Soc., 33 Chancery-ln., W.C.</i>	1885	†Andras, Henry Walsingham, F.S.S. (LIBRARIAN), <i>Provident Life Office, 50 Regent-</i> <i>street, W.</i>
1892	Adlard, Alfred Barton, <i>Law Life Assur. Soc., 187 Fleet-</i> <i>street, E.C.</i>	1885	†Ansell, Hubert, <i>Anglo-American Debenture Cor-</i> <i>poration Ltd., 75 Lombard-st., E.C.</i>
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1894	†Aldcroft, William Hancock, <i>Refuge Assur. Co., Oxford-st.,</i> <i>Manchester.</i>	1901	†Austin, Herbert Henry, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1889	†Allen, Arthur Gregory, 13 Fairfax-road, N.W.	1903	†Bacon, James, <i>c/o T. G. Ackland, Esq., 5 & 6</i> <i>Clements-inn, Strand, W.C.</i>
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1899	†Allin, Samuel John Henry Wallis, <i>Mutual Life Insurance Co. of New</i> <i>York, 16, 17 & 18 Cornhill, E.C.</i>	1896	†Baker, Henry James, (SUB-EDITOR OF JOURNAL), <i>Metropolitan Life Assur. Soc.,</i> <i>13 Moorgate-street, E.C.</i>
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1902	†Anderson, Thomas Frederic, F.F.A., <i>Royal Exchange Assurance Cor-</i> <i>poration, Royal Exchange, E.C.</i>		

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1889	†Bell, Frederick, <i>Alliance Assurance Co., Limited</i> <i>(Imperial Life Assurance Fund),</i> <i>47 Chancery-lane, W.C.</i>	1894	†Burn, Joseph, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1867	†Berridge, George William, <i>Dunton-ldg., The Knoll, Becken-</i> <i>ham, Kent.</i>	1881	†Burridge, Arthur Francis, Mem. Act. Soc. Amer. (VICE-PRES.), <i>Equity and Law Life Ass. Soc.,</i> <i>18 Lincoln's-inn-fields, W.C.</i>
1886	†Berry, Berry Alfred, B.A., <i>London Life Association, Ltd.,</i> <i>81 King William-street, E.C.</i>	1887	†Byers, Frederick Timothy Mason, <i>Clergy Mutual Assurance Soc.,</i> <i>2 & 3 The Sanctuary, West-</i> <i>minster, S.W.</i>
1895	†Besant, Arthur Digby, B.A., <i>Clerical, Medical & General Life</i> <i>Assur. Soc., 15 St. James's-</i> <i>square, S.W.</i>	1888	†Calderon, Henry Philip, <i>Law Accident Insurance Society,</i> <i>Limited, 215 Strand, W.C.</i>
1879	Besso, Marco, <i>Via Gregoriana 54 in the Villino</i> <i>Besso, Rome.</i>	1871	†Carment, David, F.F.A., Mem. Act. Soc. Amer., <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>
1894	†Blackadar, Alfred Kimball, M.A., Mem. Act. Soc. Amer., <i>Government Insur. Department,</i> <i>Ottawa, Canada.</i>	1889	†Chatham, James, F.F.A., F.S.S., <i>Scottish Life Assurance Co.,</i> <i>19 St. Andrew-sq., Edinburgh.</i>
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Date of becoming a Fellow.		Date of becoming a Fellow.	
1898	†Cockman, Arthur Charles Road-night, <i>Liverpool & London & Globe Insurance Co., 1 Dale-street, Liverpool.</i>	1897	†Day, William Reginald, <i>Standard Life Association, Ltd., 28 Elizabeth-street, Sydney, Australia.</i>
1884	†Colenso, Francis Ernest, M.A., <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>	1903	†Denham, Walter, F.F.A., <i>City of Glasgow Life Assurance Co., 30 Renfield-street, Glasgow.</i>
1864	†Coles, John, J.P., F.S.S., <i>39 Throgmorton-street, E.C.</i>	1869	Deuchar, David, F.F.A., F.R.S.E., Mem. Act. Soc. Amer., <i>Caledonian Insurance Company, 19 George-street, Edinburgh.</i>
1903	†Collins, Frank Lakeman, <i>Clerical, Medical & General Life Assurance Soc., 15 St. James's-square, S.W.</i>	1883	Deuchar, John Jas. Walker, F.F.A., <i>Norwich Union Life Insurance Society, Norwich.</i>
1882	†Colquhoun, Ernest, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>	1882	Dewey, Thomas Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1875	†Cooke, Thomas Homans, <i>Glendower, Torre Vale, Torquay.</i>	1886	†Dickinson, Arthur Lowes, M.A., F.C.A., <i>54 William-street, New York, U.S.A.</i>
1889	†Cooper, Walter George, <i>Norwich Union Life Insurance Society, Norwich.</i>	Under the Charter.	Docker, Edward, M.A., <i>Dudley-house, Spring-grove, Isleworth.</i>
1902	†Countts, Charles Ronald Vawdrey, <i>Hand-in-Hand Insur. Society, 26 New Bridge-street, E.C.</i>	1887	Douglas, Gordon, F.F.A., <i>Life Association of Scotland, 82 Princes-street, Edinburgh.</i>
1878	†Crisford, George Stephen, <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>	1874	†Duncan, John, <i>Clergy Pensions Institution and Ecclesiastical Insurance Office, 11 Norfolk-street, Strand, W.C.</i>
1903	†Cross, Howard Turner, <i>Economic Life Assurance Soc., 6 New Bridge-street, E.C.</i>	1901	†Dunn, Spencer Grame, <i>Liverpool & London & Globe Insurance Co., 11 Cornhill, E.C.</i>
1889	†Cross, Robert, <i>Atlas Assurance Company, 92 Cheapside, E.C.</i>	1869	Dymond, Joseph John, <i>Friends' Provident Institution, Bradford, Yorkshire.</i>
1864	Curtis, Frank Allan, <i>3 Ennismore-gardens, Salisbury-road, Dover.</i>	1872	Eccles, Yvon Richard, <i>Scottish Amicable Life Assurance Society, 1 Threadneedle-st., E.C.</i>
Under the Charter.	Davies, Griffith, <i>11 Freeland-road, Ealing, W.</i>	1897	†Elder, Kenneth William, <i>Atlas Assurance Company, 92 Cheapside, E.C.</i>
1898	†Dawson, Charles Pearl, <i>Alliance Assurance Co., Ltd. (Imperial Life Assurance Fund), 47 Chancery-lane, W.C.</i>	1901	†Elderton, William Palin, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>
1855	†Day, Archibald (PAST PRESIDENT, 1886-88), <i>Clifton-lodge, St. John's-park-road, Blackheath, S.E.</i>	1898	†Elliott, Charles Alfred, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1885	†Day, Stanley, <i>Marine and General Mutual Life Assurance Society, 14 Leadenhall-street, E.C.</i>		

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1889	† Faulks, Joseph Ernest, B.A., F.S.S., <i>Law Life Assurance Society,</i> 187 Fleet-street, E.C.	1880	† Hardy, George Francis, 7 Broad-street House, E.C.
1897	† Fellows, Rowland Hill, F.S.S., <i>Pelican and British Empire Life</i> <i>Office, 70 Lombard-street, E.C.</i>	1870	† Hardy, Ralph Price, 61 Addison-road, W.
Under the Charter.	Fisher, Richard Charles, 2 Walsingham-rd., Hove, Sussex.	1893	† Harris, Arnold Stoughton, M.A., <i>Clerical, Medical & General Life</i> <i>Assurance Society, 36 Park-row,</i> <i>Leeds.</i>
1892	† Foot, Herbert, B.A., <i>Northern Assurance Company,</i> 1 Moorgate-street, E.C.	1892	† Hart, James Robert, <i>Pelican and British Empire Life</i> <i>Office, 70 Lombard-street, E.C.</i>
1884	Frankland, Frederick William, F.S.S., Mem. Act. Soc. Amer., <i>New York Life Insurance Co.,</i> 346 & 348 Broadway, New York.	1879	Harvey, Chas. J., <i>Colonial Life Insurance Co. of</i> <i>America, 43 Montgomery-street,</i> <i>Jersey City, N.J., U.S.A.</i>
1900	† Fraser, Alexander, Jr., F.F.A., <i>Scottish Life Assur. Company,</i> 19 St. Andrew-sq., Edinburgh.	1888	† Hemming, Arthur George, F.S.S., <i>London Assurance Corporation,</i> 7 Royal Exchange, E.C.
1897	† Fraser, Duncan Cumming, M.A., <i>Royal Insurance Co., Liverpool.</i>	1896	† Henderson, Robert, B.A., <i>Equitable Life Assurance Soc. of</i> <i>the United States, 120 Broadway,</i> <i>New York.</i>
1895	† Fulford, Frederick Wesley, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1864	Hendriks, Augustus, F.S.S., Mem. Act. Soc. Amer. (PAST PRES- IDENT, 1892-94), 6 Observatory-gardens, Kensington, W.
1902	† Gillies, George, <i>Union Assurance Society, 81</i> <i>Cornhill, E.C.</i>	Under the Charter.	Hendriks, Frederick, F.S.S., 7 Vicarage-gate, Kensington, W.
1887	Gillison, John Broth, F.F.A., <i>National Mutual Life Association</i> <i>of Australasia, 76 & 77 Cornhill,</i> <i>E.C.</i>	1883	Hewat, Archibald, F.F.A., F.S.S., <i>Edinburgh Life Assurance Co.,</i> 22 George-street, Edinburgh.
1878	Gordon, Charles, F.F.A., <i>South African Mutual Life</i> <i>Assurance Society, Cape Town,</i> <i>South Africa.</i>	1874	† Higham, Charles Daniel, Mem. Act. Soc. Amer. (EX-PRES.), <i>London Life Association, Ltd.,</i> 81 King William-street, E.C.
1901	† Gordon-Smith, Randolph, F.F.A., <i>Scottish Amicable Life Assur.</i> <i>Soc., 35 St. Vincent-pl., Glasgow.</i>	1898	† Hodgson, William Horsford, <i>Law Life Assurance Society,</i> 187 Fleet-street, E.C.
1882	† Graham, James, F.F.A., <i>Australian Widows' Fund Life</i> <i>Assurance Society, Collins-street-</i> <i>west, Melbourne, Australia.</i>	1899	† Holliday, John, M.A., F.S.S., <i>New York Life Insurance Co.,</i> 124 Rua du Ouvidor, Rio de Janeiro.
1886	Gunn, Niel Ballingal, P.F.A., <i>Scottish Amicable Life Assur. Soc.,</i> 35 St. Vincent-place, Glasgow.	1888	† Hopkins, William Raynes, <i>London and Lancashire Life</i> <i>Assur. Co., 66 & 67 Cornhill, E.C.</i>
1864	Harben, Sir Henry, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1890	† Hovil, Lewis Frederick, <i>National Provident Institution,</i> 48 Gracechurch-street, E.C.

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1871	†Hughes, William, Mem. Act. Soc. Amer. (PRESIDENT), <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901	†Little, James Fulton, <i>Mutual Life Association of Australasia, Sydney, Australia.</i>
1894	†Hutcheson, William Anderson, F.F.A., Mem. Act. Soc. Amer., <i>Mutual Life Insurance Company of New York, Nassau-street, New York, U.S.A.</i>	1899	Low, George Macritchie, F.F.A., <i>Scottish Equitable Life Assur. Society, 28 St. Andrew-square, Edinburgh.</i>
1893	†Hutton, William, F.F.A., <i>Scottish Amicable Life Assur. Soc., 1 Threadneedle-street, E.C.</i>	1899	†Lutt, Harold Edward William, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1903	†Jarman, William Rees, B.A., <i>National Debt Office, Finsbury-pavement-house, E.C.</i>	1898	†Macaulay, Thomas Bassett, Mem. Act. Soc. Amer., <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1869	†Justican, Edwin, F.S.S., <i>Gresham Life Assurance Society, St. Mildred's-house, Poultry, E.C.</i>	1874	McClintock, Emory, Mem. Act. Soc. Amer., <i>Mutual Life Insurance Company of New York, New York.</i>
1902	†Kenchington, Charles William, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1894	†McDonald, John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1897	†Kentish, Owen, <i>Economic Life Assurance Soc., 6 New Bridge-street, E.C.</i>	1864	McGedy, Frank, <i>14 Fitz-George-avenue, Kensington West.</i>
1874	†King, George, F.F.A., Mem. Act. Soc. Amer. (VICE-PRESIDENT AND EDITOR OF JOURNAL), <i>15 Walbrook, E.C.</i>	1883	†McGowan, James, B.A., <i>The Treasury, Cape Town, South Africa.</i>
1887	†Kyd, Thomas, F.F.A., <i>Northern Assurance Company, 1 Union-terrace, Aberdeen.</i>	1885	Mackenzie, Alexander George, F.F.A., <i>47, York-terrace, Regent's-park, N.W.</i>
1882	Lancaster, William John, J.P., <i>South Lynn, Putney-hill, S.W.</i>	1900	†Macnaghten, Stuart Edye, A.C.A., <i>Equity & Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>
1894	†Laughton, Alexander Millar, F.F.A., <i>National Mutual Life Assoc. of Australasia, Limited, Corner of Collins and Queen-streets, Melbourne, Australia.</i>	1901	†Macphail, Donald, F.F.A., <i>Yorkshire Insurance Company, York.</i>
1887	†Lemon, William Kent, Barrister-at-Law, <i>5 Pump-court, E.C.</i>	1870	†Manly, Henry William, Mem. Act. Soc. Amer. (PAST PRESIDENT, 1898-1900), <i>Equitable Life Assurance Soc., Mansion-house-street, E.C.</i>
1896	†Levine, Abraham, M.A., <i>National Mutual Life Assur. Soc., 39 King-st., Cheapside, E.C.</i>	1890	†Marks, Geoffrey (LIBRARIAN), <i>National Mutual Life Assur. Soc., 39 King-street, Cheapside, E.C.</i>
1896	†Lewis, John Norman, F.F.A., <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1900	†Marr, Vyvyan, F.F.A., <i>Edinburgh Life Assurance Co., 22 George-street, Edinburgh.</i>
1892	†Lidstone, George James, <i>Alliance Assurance Co., Limited, Bartholomew-lane, E.C.</i>	1902	†May, Basil, <i>National Mutual Life Assur. Soc., 39 King-street, Cheapside, E.C.</i>

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1897	†May, George Ernest, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1887	†Nightingale, Harry Ethelston, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1851	†Meikle, James, F.F.A., Mem. Act. Soc. Amer., <i>Wardielburn-house, Edinburgh.</i>	1903	†Norris, Charles Arthur, <i>National Mutual Life Association of Australasia, Limited, Melbourne, Australia.</i>
1897	†Miller, Neville, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1901	†Norton, William Ernest, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
1893	†Milner, John William, <i>North British & Mercantile Insur. Co., 61 Threadneedle-street, E.C.</i>	1899	†Parker, Robert Peter, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>
1892	†Milton, Henry, M.A., <i>Law Debenture Corporation, Ltd., 41 Threadneedle-street, E.C.</i>	1864	Pearson, Arthur, <i>Betchworth-house, The Bank, Highgate, N.</i>
1899	†Moir, Henry, F.F.A., Mem. Act. Soc. Amer., <i>Provident Savings Life Assur. Soc., 346 Broadway, New York, U.S.A.</i>	1891	†Phelps, William Peyton, M.A., <i>Equity and Law Life Assur. Soc., 18 Lincoln's-inn-fields, W.C.</i>
1890	†Molyneux, Arthur Ernest, <i>Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street, E.C.</i>	Under the Charter.	Priestley, John George, <i>44 St. German's-road, Forest-hill, S.E.</i>
1901	†Moorhouse, Alfred, <i>Friends' Provident Institution, Bradford, Yorkshire.</i>	1891	†Pulley, William Pritchard, <i>Norwich Union Life Insur. Soc., 71 & 72 King William-st., E.C.</i>
1897	†Moors, Elphinstone MacMahon, M.A., <i>University of Sydney, Australia.</i>	1903	†Rae, Joseph, <i>Finance Department, Town-hall, Upper-street, N.</i>
1896	†Moran, Joseph Flack, <i>Marine & General Mutual Life Assurance Society, 14 Leadenhall-street, E.C.</i>	1899	†Raisin, Arthur Herbert, <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>
1900	†Morgan, Benjamin Charles, M.A., <i>Commercial Union Assur. Co., 24, 25 & 26 Cornhill, E.C.</i>	1897	†Rees, Martin, <i>Law Reversionary Interest Soc., Ltd., 24 Lincoln's-inn-fields, W.C.</i>
1895	†Muter, Percy, <i>New Zealand Government Life Insurance Department, Wellington, New Zealand.</i>	1901	†Reeve, Charles Ernest, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1888	†Nash, Willie Oscar, <i>Law Reversionary Interest Soc., Ltd., 24 Lincoln's-inn-fields, W.C.</i>	1902	†Richmond, George William, <i>Scottish Widows' Fund and Life Assur. Society, 28 Cornhill, E.C.</i>
1883	Nelson, Francis G. P., F.S.S., <i>19 Abingdon-st., Westminster, S.W.</i>	1898	†Robinson, George Frederick, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>
1888	†Newman, Philip Lewin, B.A., <i>Yorkshire Insurance Co., York.</i>	1888	†Rusher, Edward Arthur, F.S.S., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1865	Newton, Algernon, M.A., <i>c/o London & Westminster Bank, 94 & 96 High-st., Kensington, W.</i>	1882	†Ryan, Gerald Hemington, Mem. Act. Soc. Amer., <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>

FELLOWS.

Those marked † are Fellows by Examination.

Date of becoming a Fellow.		Date of becoming a Fellow.	
1898	†Salmon, Richard George, F.S.S., <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1898	Stirling, Robert, F.F.A., <i>Law Union & Crown Insurance Co., 126 Chancery-lane, W.C.</i>
1883	Saunders, Harris Charter Lindon, F.R.A.S., <i>"Marquise," Twickenham.</i>	1892	†Straker, Edward Robert, <i>Pelican and British Empire Life Office, 70 Lombard-street, E.C.</i>
1886	†Schooling, Frederick (HON. SEC.), <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1878	†Straker, Frank Arthur, <i>Legal and General Life Assur. Society, 10 Fleet-street, E.C.</i>
1901	†Searle, George Morley, <i>Sun Life Assurance Society, 60 Charing-cross, S.W.</i>	1902	†Strong, William Richard, <i>London Guarantee & Accident Co., 61 Moorgate-street, E.C.</i>
1901	†Sharman, William Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1884	†Stuart, John Moody, F.F.A., <i>Leeds Permanent Building Soc., Victoria-buildings, Park-lane, Leeds.</i>
1896	†Sim, William Abernethy, F.F.A., <i>Scottish Union and National Insurance Co., 35 St. Andrew-square, Edinburgh.</i>	1900	†Sutherland, John, M.A., <i>Australasian Temperance and General Mutual Life Assurance Society, Swanston-street, Melbourne, Australia.</i>
1875	†Smither, Arthur, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>	1889	†Tarn, Arthur Wyndham, <i>Westminster and General Life Assurance Association, 28 King-street, Covent-garden, W.C.</i>
1881	†Somerville, William Finlay, <i>Liverpool and London and Globe Insurance Co., 7 Cornhill, E.C.</i>	1887	Teece, Richard, F.F.A., F.S.S., Mem. Act. Soc. Amer., <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1877	†Sorley, James, F.F.A., C.A., F.R.S.E., <i>32 Onslow-square, S.W.</i>	1872	Templeton, Col. John M., C.M.G., <i>National Mutual Life Association of Australasia, Melbourne, Australia.</i>
1898	†Spencer, John, <i>English and Scottish Law Life Assurance Assoc., 12 Waterloo-place, S.W.</i>	1886	Tennant, John Bell, <i>Friends' Provident Institution, Bradford, Yorkshire.</i>
1894	†Sprague, Alfred Ernest, B.Sc., M.A., F.F.A., <i>Edinburgh Life Assurance Co., 22 George-street, Edinburgh.</i>	1864	†Terry, James, <i>Hernlee, Lyme Regis, Dorset.</i>
1857	Sprague, Thomas Bond, M.A., LL.D., Hon. Mem. F.A., F.S.S., F.R.S.E. (PAST PRESIDENT, 1882-86), <i>29 Buckingham-ter., Edinburgh.</i>	1889	†Thiselton, Herbert Cecil, F.F.A., Mem. Act. Soc. Amer., <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>
1896	†Stahlschmidt, Louis, <i>Alliance Assurance Co., Limited, Bartholomew-lane, E.C.</i>	1901	†Thodey, Robert, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
Under the Charter	Stevens, Charles, <i>Aberdeen H.C., Preston, Brighton.</i>	1893	†Thomas, Ernest Charles, <i>Gresham Life Assurance Society, St. Mildred's-house, Poultry, E.C.</i>
1888	Stewart, John, F.F.A., <i>City of Glasgow Life Assur. Co., 30 Renfield-street, Glasgow.</i>	1899	†Thomas, Robert Arthur Caradoc, <i>Pelican and British Empire Life Office, 12 Dalhousie-sq., Calcutta.</i>

FELLOWS.

Those marked † are Fellows by Examination.

Date of
becoming
a Fellow.

- 1895 †Thomson, Herbert Archer, B.A.,
Raven Hotel, Hook, Hants.
- 1880 Thomson, Robert,
*Colonial Mutual Life Assurance
Society, Collins - street - west,
Melbourne, Australia.*
- 1876 Thomson, Spencer Campbell, B.A.,
F.F.A.,
*Standard Life Assurance Co.,
3 George-street, Edinburgh.*
- 1893 †Thorne, Alfred Charles,
*Equity & Law Life Assur. Soc.,
18 Lincoln's-inn-fields, W.C.*
- 1891 †Tilt, Robert Ruthven,
*General Reversionary & Invest-
ment Co., Ltd., 26 Pall-mall, S.W.*
- 1902 †Timmer, Thomas,
*Comptroller's Depart., London
County Council, Spring-gardens,
S.W.*
- 1881 †Todd, George, M.A. (HON. SEC.),
*Economic Life Assurance Society,
6 New Bridge-street, E.C.*
- 1894 †Todhunter, Ralph, M.A.,
*University Life Assur. Soc.,
25 Pall-mall, S.W.*
- 1899 †Trommeer, Harold Moltke, M.A.,
*London Life Association, Ltd.,
81 King William-street, E.C.*
- 1878 Turnbull, Andrew Hugh, F.F.A.,
F.R.S.E.,
*Scottish Widows' Fund and Life
Assur. Soc., 9 St. Andrew-square,
Edinburgh.*
- Under the
Charter. Tyndall, William Henry, F.S.S.,
F.R.Met.S.,
*Morlands, Oxford-road, Redhill,
Surrey.*
- 1889 Wallace, Thomas, F.F.A.,
*North British & Mercantile
Insurance Co., Edinburgh.*
- 1888 †Warner, Samuel George,
*Law Union & Crown Insur. Co.,
126 Chancery-lane, W.C.*
- 1893 †Watson, Alfred William,
*Manchester Unity Friendly Soc.,
Nottingham.*

Date of
becoming
a Fellow.

- 1895 †Watson, James Douglas,
*English & Scottish Law Life Assr.
Assoc., 12 Waterloo-place, S.W.*
- 1880 †Whittall, Wm. Joseph Hutchings,
Mem. Act. Soc. Amer.,
*Clerical, Medical & General Life
Assur. Soc., 15 St. James's-sq., S.W.*
- 1864 Wilson, Robert,
44 Talfourd-rd., Camberwell, S.E.
- 1888 †Wilson, Robert, Jr.,
*General Assurance Company,
103 Cannon-street, E.C.*
- Under the
Charter. Winsor, Thomas Boorman,
*81 Shooter's-hill-road, Black-
heath, S.E.*
- 1899 †Winter, Arthur Thomas,
*Pelican and British Empire Life
Office, 70 Lombard-street, E.C.*
- 1897 †Wintle, Laneelot Andrewes,
*Economic Life Assurance Soc.,
6 New Bridge-street, E.C.*
- 1884 †Woods, Ernest, Mem. Act. Soc.
Amer. (VICE-PRES.),
*Westminster and General Life
Assurance Association, 28 King-
street, Covent-garden, W.C.*
- 1902 †Woolmer, Alfred Henry,
*Star Life Assurance Society,
32 Moorgate-street, E.C.*
- 1902 †Workman, William Arthur,
*Equitable Life Assur. Society,
Mansion-house-street, E.C.*
- 1902 †Worthington, William,
Royal Insurance Co., Liverpool.
- 1875 †Wyatt, Frank Bertrand, Mem. Act.
Soc. Amer.,
*Clergy Mutual Assurance Soc.,
2 & 3 The Sanctuary, S.W.*
- 1874 Young, Thomas Emley, B.A.,
F.R.A.S. (PAST-PRESIDENT,
1896-8), Mem. Act. Soc. Amer.,
*108 Eering-road, Stoke New-
ington, N.*

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1900	² Adams, Cecil Francis, <i>New Zealand Accident Insurance Company, Wellington, New Zealand.</i>	1889	(²) Bremner, Thomas William, F.F.A., <i>Mutual Life Insurance Co. of New York, Sydney, Australia.</i>
1869	² Adey, Theodore Henry, <i>Scottish Provident Institution, 17 King William-street, E.C.</i>	1896	(²) Brown, George Andrew, <i>Clerical, Medical & General Life Assurance Society, 1 King William-street, E.C.</i>
1899	³ Adlard, Stanley, A.K.C., <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1899	² Brown, Harold, <i>Scottish Union and National Insurance Co., 3 King William-street, E.C.</i>
1899	² Ansell, George Frederic, <i>National Debt Office, Finsbury-pavement-house, E.C.</i>	1900	³ Brown, Henry, B.A., <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>
1883	² Ashley, John Geo., M.A., <i>War Office, S.W.</i>	1886	Buckley, Thomas John Wesley, <i>9 St. Andrew-street, Holborn-circus, E.C.</i>
1901	² Ashton, William Richard, <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>	1882	Burke, David, F.S.S., <i>Royal Victoria Life Insur. Co., Montreal, Canada.</i>
1881	² Ayling, Charles Stephen, <i>Commercial Union Assur. Co., 20 New Bridge-street, E.C.</i>	1900	² Burnley, Isaac, <i>Australian Mutual Prov. Society, Sydney, Australia.</i>
1903	² Ball, Sidney Robertson, <i>English and Scottish Law Life Assurance Association, 12 Waterloo-place, S.W.</i>	1895	³ Butterfield, William Thos., A.C.A., <i>9 Market-street, Bradford, Yorkshire.</i>
1885	Barton, Arthur, <i>United Kent Insurance Institution, Maidstone.</i>	1876	Carter, Eric Mackay, <i>33 Waterloo-street, Birmingham.</i>
1894	³ Barton, Robert Whitechurch, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>	1899	² Catchlove, Chas. Hamilton Leyland, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1903	² Baxter, Edwin Herbert, <i>Scottish Provident Institution, 17 King William-street, E.C.</i>	1900	³ Chandler, Thomas Richard, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>
1901	² Benjamin, Stanley O., <i>Australian Mutual Provident Society, Sydney.</i>	1898	² Coates, Thomas Linnaeus, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1881	Birks, Edmund Alfred, <i>Yorkshire Insurance Co., York.</i>	1871	Cook, Arthur James, M.J.I., <i>Victoria Mutual Assur. Society, Farringdon-street, E.C.</i>
1873	² Bloek, Robert John, <i>Essex-villa, Chelsham-road, Clapham, S.W.</i>	1899	³ Cook, William Playfair, <i>Guardian Assurance Company, 11 Lombard-street, E.C.</i>
1898	(²) Blount, Edward Thos. J., F.F.A., F.S.S., <i>Standard Life Assurance Co., Shanghai, China.</i>	1897	² Coop, Charles Rowland, <i>United Kingdom Temperance and General Provident Institution, 5 Bennett's-hill, Birmingham.</i>
1873	² Boon, Gerald Inglis, <i>United Legal Indemnity Insur. Soc., Limited, 222 Strand, W.C.</i>		

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.
Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1891	² Coote, Ernest Charles, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1899	² Dougharty, Harold, F.S.S., <i>F.C.I.S., London and Lancashire Life Assurance Company, 66 & 67 Cornhill, E.C.</i>
1900	² Corbett, Edwin Somerville, <i>Australasian Temperance and General Mutual Life Assurance Soc., Adelaide, South Australia.</i>	1902	² Doust-Smith, Ernest Charles, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1871	Connts, Edwin Arthur, <i>North British and Mercantile Insurance Company, Victoria- street, Nottingham.</i>	1881	Dovey, William Roadly, F.F.A., <i>Mem. Act. Soc. Amer., 26 Crouch Hall Road, Crouch End, N.</i>
1900	² Corington, Oliver Henry, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1870	Dowson, John, <i>Royal Insur. Company, Liverpool.</i>
1884	Craig, Robert Alexander, <i>Abstainers' and General Assur. Co., City Buildings, Birmingham.</i>	1898	² Doyle, Arthur James, <i>54 Bourke-st., Sydney, Australia.</i>
1901	² Culley, Alfred Benjamin, <i>Star Life Assurance Society, 32 Moorgate-street, E.C.</i>	1901	² Earle, Arthur Percival, <i>Reliance Life Insurance Co., Farmers' Bank-buildings, Pitts- burgh, Pa., U.S.A.</i>
1901	³ Curjel, H. W., M.A., <i>Royal Insurance Co., Liverpool.</i>	1868	Eaton, Henry William, <i>Liverpool & London & Globe Insurance Company, William- street, New York, U.S.A.</i>
1900	² Curtis, William Allen, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, S.W.</i>	1872	² Evans, William, F.F.A., F.R.S.E., <i>38 Morningside-park, Edinburgh.</i>
1902	² Dawson, Miles Menander, F.S.S., <i>11 Broadway, New York, U.S.A.</i>	1902	³ Falconer, Herbert Francis, F.F.A., <i>Scottish Life Assurance Co., 19 St. Andrew-sq., Edinburgh.</i>
1902	² Denmead, John Charles, M.A., <i>Estate Duty Office, Somerset- house, W.C.</i>	1896	² Featherstonehaugh, William Irwin, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1901	² Diamond, George Frederick, <i>City Mutual Life Assurance Society, Hunter-street, Sydney, Australia.</i>	1903	² Ferguson, Colin C., B.A., <i>Canada Life Assurance Co., Toronto, Canada.</i>
1900	³ Diver, Oswald Francis, M.A., <i>Clerical, Medical & General Life Assur. Soc., 15 St. James'-sq., S.W.</i>	1897	² Findlay, Alexander Wynand, LL.D., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1855	Dix, James, <i>Hurstdale, Wood-la., Highgate, N.</i>	1902	² FitzGerald, Charles R., <i>Home Life Association of Canada, Toronto, Canada.</i>
1901	(²) Donald, Alexander Graham, M.A., F.F.A., <i>Scottish Provident Institution, 6 St. Andrew-square, Edinburgh.</i>	1901	² FitzGerald, William George, <i>260 McLaren-street, Ottawa, Canada.</i>
1881	Donaldson, John, <i>Australian Widows' Fund Life Assurance Society, Collins-street- west, Melbourne, Australia.</i>	1890	(²) Fox, Charles Edward, F.F.A., <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of becoming an Associate.		Date of becoming an Associate.	
1886	Fox, Morris, Mem. Act. Soc. Amer., <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>	1897	² Goodwyn, John, Jr., <i>Norwich & London Accident Insurance Association, Sydney, Australia.</i>
1894	² Fraser, Thomas John, <i>Australian Alliance Assurance Company, Melbourne, Australia.</i>	1902	² Grant, Milton Daniel, B.A., <i>Government Insurance Department, Ottawa, Canada.</i>
1901	(2) Gaff, William Robertson, C.A., F.F.A., <i>3 Crown-court, Old Broad-st., E.C.</i>	1902	² Gray, Robert Alexander, B.A., <i>324 Markham-street, Toronto, Canada.</i>
1873	² Gage, Uriah Woodard, <i>North British and Mercantile Insurance Company, 61 Threadneedle-street, E.C.</i>	1898	² Green, George, M.A., <i>Union Assurance Society, 81 Cornhill, E.C.</i>
1902	² Gaines, John M., <i>New York Life Insurance Co., 346 & 348 Broadway, New York, U.S.A.</i>	1868	Greig, John Andrew, <i>Sun Life Assurance Society, 60 Charing-cross, S.W.</i>
1897	³ Galer, Frederic Bertram, B.A., <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>	1869	Griffith, E. Clifton, <i>4 Carlton-chambers, S.W.</i>
1895	² Galwey, Charles Edmund, <i>New Zealand Government Life Insurance Dept., Wellington, New Zealand.</i>	1903	² Hall, John Bertram, <i>Imperial Life Assurance Co. of Canada, Toronto, Canada.</i>
1893	² Gardiner, Robert Edward, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1893	² Hall, John Francis Edmund, <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>
1885	² Gayford, Herbert Stannard, <i>Northern Assurance Co., 1 Moor-gate-street, E.C.</i>	1869	Hann, Robert George, Mem. Act. Soc. Amer., <i>Equitable Life Assur. Soc. of the United States, 120 Broadway, New York.</i>
1903	² Gemmill, William, <i>7 Royal-terrace, Queen's-park, Glasgow.</i>	1894	² Hardcastle, Edward Edgington, M.A., <i>Union Central Life Office, Cincinnati, Ohio, U.S.A.</i>
1899	³ Gibb, James Burnett, F.F.A., <i>Penn Mutual Life Insce. Co. of Philadelphia, 923 Chestnut-st., Philadelphia, U.S.A.</i>	1900	² Harding, Harry Burnard, <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>
1871	² Glennie, William Gordon, <i>Scottish Union & National Insur. Co., 3 King William-street, E.C.</i>	1896	³ Harris, Frederick Joseph, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1897	² Goggs, Frank Sidney, <i>Scottish Metropolitan Life Assur. Co., Ltd., 25 St. Andrew-sq., Edinburgh.</i>	1897	² Hayercraft, William Melhuish, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1882	Goldman, Leopold, <i>North American Life Assurance Co., North American Life Building, 112-118 King-street-west, Toronto, Canada.</i>	1897	² Hazell, James Stanley, <i>National Provident Institution, 48 Gracechurch-street, E.C.</i>
		1895	² Heness, Leonard Thomas, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>

ASSOCIATES.

Those marked 2 or 3 have passed two or three of the four Examinations of the Institute.

Those marked (2) have been exempted under the Bye-laws from the Examinations in Parts I and II.

Date of
becoming an
Associate.

- 1878 Henry, Alfred, F.C.A.,
*Throgmorton-house, Cophall-
avenue, E.C.*
- 1900 ² Hicks, Arthur Joseph,
*Reversionary & General Securities
Company, Ltd., Craven-house,
Northumberland-avenue, W.C.*
- 1884 Higham, William Samuel,
*Equitable Life Assurance Soc.,
Mansion-house-street, E.C.*
- 1894 ² Hollingworth, Albert Chas.,
*Australian Mutual Provident
Society, Sydney, Australia.*
- 1883 Holt, Edward Hallett,
*Law Life Assurance Society,
187 Fleet-street, E.C.*
- 1898 ² Howell, Chas. Edward, B.A., LL.D.,
*Standard Life Assurance Compy.,
66 Upper Sackville-st., Dublin.*
- 1899 ² Hudson, Alfred James,
*Northern Assurance Company,
1 Moorgate-street, E.C.*
- 1903 ² Humphreys, Henry Thompson,
*Sun Life Assurance Society,
63 Threadneedle-street, E.C.*
- 1875 Hunt, Richard Aldington, F.S.S.,
*Wesleyan & General Assur. Soc.,
Corporation-street, Birmingham.*
- 1893 ⁽²⁾ Hunter, Arthur, F.F.A., Mem.
Act. Soc. Amer.,
*New York Life Insurance Co.,
346 & 348 Broadway, New York.*
- 1902 ² Hunter, Robertson G., Mem. Act.
Soc. Amer.
*New York Life Insurance Co.,
346 & 348 Broadway, New York.*
- 1887 ² Hunter, Samuel,
*Patriotic Assurance Company,
9 College-green, Dublin.*
- 1889 ⁽²⁾ Jacobs, Frederick Job,
*Australian Mutual Provident
Society, Sydney, Australia.*
- 1876 ² James, George Trevelyan,
12 Waterloo-place, S.W.
- 1871 Jellicoe, George Rogers,
*Eagle Insurance Company, 79
Pall-mall, S.W.*

Date of
becomiz an
Associate.

- 1883 Jerman, Richard,
*Commercial Union Assurance
Company, Exeter.*
- 1896 ² Jobson, Alexander,
*Equitable Life Assurance Society
of the United States, Sydney,
Australia.*
- 1894 ² Johannessen, Nikolai Mikal,
*Hygea Life Assurance Company,
Bergen, Norway.*
- 1894 ² Johnston, Frederick H.,
*Prudential Life Insurance Co.
of America, Newark, N.J., U.S.A.*
- 1903 ² Jones, Leonard Alexander Mouat,
*Hand-in-Hand Insur. Society,
26 New-bridge-street, E.C.*
- 1903 ² Jones, Wallace Mouat,
*General Reversionary & Invest-
ment Company, Limited, 26 Pall-
mall, S.W.*
- 1898 ² Kaufman, Henry N., Assoc. Act.
Soc. Amer.,
*Phoenix Mutual Life Insurance
Co., Hartford, Connecticut,
U.S.A.*
- 1876 Kearry, Joseph,
*44 Charwood-street, Belgrave-
road, S.W.*
- 1899 ² Kelly, John Joseph,
*Citizens' Life Assurance Co.,
Sydney, Australia.*
- 1897 ² Kemp, Julian Ernest Sandford,
*Eagle Insurance Company, 79
Pall-mall, S.W.*
- 1902 ² Kilgour, David Errett,
*North American Life Assurance
Co., North American Life
Building, 112-118 King-street
West, Toronto, Canada.*
- 1874 King, Arthur Thomas,
*National Debt Office, Finsbury-
pavement-house, E.C.*
- 1882 ² King, William Alfred,
*Northern Assurance Company,
1 Moorgate-street, E.C.*
- 1902 ² Kitchen, Frederick Harcourt, B.A.,
Broad Clyst, Teddington.
- 1893 ² Laing, William Claud,
*North British and Mercantile
Insurance Company, 61 Thread-
needle-street, E.C.*

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Date of becoming an Associate.		Date of becoming an Associate.	
1897	² Lane, Arthur Vere, B.A., <i>University Life Assurance Soc.,</i> 25 Pall-mall, s.w.	1896	² Macmillan, John Campbell, <i>Northern Assur. Co. and Law</i> <i>Union and Crown Insurance Co.,</i> <i>Apartado Postal No. 872, Mexico.</i>
1899	² Lawton, George Herbert, <i>Clerical, Medical & General Life</i> <i>Assurance Society, 15 St. James's-</i> <i>square, s.w.</i>	1867	Macpherson, Ronald, <i>Law Union & Crown Insurance</i> <i>Co., 126 Chancery-lane, w.c.</i>
1885	Ledward, Archibald Prentice, B.Sc., 29 Langland-gardens, n.w.	1883	² Makeham, William Reed, <i>Alliance Assurance Co., Ltd.</i> <i>(Imperial Life Assurance Fund),</i> 47 Chancery-lane, w.c.
1879	Leitch, Alexander, <i>Scottish Provident Institution,</i> 17 King William-street, E.C.	1883	Mannering, George Willsher, <i>London and Lancashire Life</i> <i>Assur. Co., 66 & 67 Cornhill, E.C.</i>
1897	² Le Maitre, Frank William, <i>Sun Life Assurance Society, 63</i> <i>Threadneedle-street, E.C.</i>	1880	Manwaring, Henry, <i>National Debt Office, Finsbury-</i> <i>pavement-house, E.C.</i>
1885	Leveaux, Arthur Michael, F.S.S., <i>Registry of Friendly Societies,</i> <i>Central Office, 28 Abingdon-</i> <i>street, Westminster, s.w.</i>	1896	² Martin, Sidney George, <i>National Mutual Life Assoc. of</i> <i>Australasia, Ltd., 150 Queen-</i> <i>street, Brisbane, Australia.</i>
1885	² Lidbury, Isaac Stephen, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1897	² Mascall, Alfred John, <i>Standard Life Assurance Co.,</i> 3 Pall-mall East, s.w.
1868	Litchfield, Edward, 92 St. Vincent-street, Glasgow.	1900	² Maunder, George Harvard, <i>Standard Life Assurance Co.,</i> 83 King William-street, E.C.
1876	² Lucey, Herbert, <i>General Assurance Company,</i> 103 Cannon-street, E.C.	1902	(²) Maxwell, Benjamin Bell, F.F.A., <i>Scottish Equitable Life Assur.</i> <i>Society, 28 St. Andrew-square,</i> <i>Edinburgh.</i>
1890	(²) Lugton, Hugh, F.F.A., <i>North British and Mercantile</i> <i>Insurance Co., 61 Threadneedle-</i> <i>street, E.C.</i>	1903	² May, Walter Thomas, <i>Scottish Amicable Life Assurance</i> <i>Society, 1 Threadneedle-st., E.C.</i>
1900	³ McArthur, Harry de C., <i>Economic Life Assur. Society,</i> 6 New Bridge-street, E.C.	1899	² Meade, Gerald Willoughby, <i>North British & Mercantile</i> <i>Insurance Company, 61 Thread-</i> <i>needle-street, E.C.</i>
1867	Macdonald, William Rae, F.F.A., <i>Scottish Metropolitan Life Assur.</i> <i>Co., Limited, 25 St. Andrew-</i> <i>square, Edinburgh.</i>	1896	² Merfield, Percy Henry, <i>Law Life Assurance Society,</i> 187 Fleet-street, E.C.
1882	³ McDougald, Alfred, <i>Pelican and British Empire Life</i> <i>Office, Montreal, Canada.</i>	1874	Miller, John W., F.S.S., <i>Scottish Widows' Fund and Life</i> <i>Assur. Soc., 28 Cornhill, E.C.</i>
1884	Mackay, Alexander, <i>Law Union & Crown Insur. Co.,</i> 126 Chancery-lane, w.c.	1902	² Milligan, Charles Livingstone, <i>Provident Life Office, 50 Regent-</i> <i>street, w.</i>
1901	² Mackenzie, Michael Alexander, <i>Trinity College, Toronto, Canada.</i>	1884	Mills, Daniel Yarnton, <i>Scottish Equitable Life Assur.</i> <i>Society, 28 St. Andrew-square,</i> <i>Edinburgh.</i>

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Date of becoming an Associate.		Date of becoming an Associate.	
1879	Monilaws, William Macgeorge, (AUDITOR), <i>Scottish Provident Institution,</i> 17 King William-street, E.C.	1886	Owen, Evan Frederick, F.S.S., <i>Actuary for Friendly Societies,</i> Melbourne, Australia.
1877	Moon, James, <i>Prudential Assurance Company,</i> 30 Dale-street, Liverpool	1895	² Pagden, Lionel King, <i>Union Assurance Society,</i> 81 Cornhill, E.C.
1877	Moon, John, <i>Prudential Assurance Company,</i> 76 King-street, Manchester.	1864	Panton, Edward Henry, 50 Wood-cle, Forest Hill, S.E.
1879	Moon, Sidney Norman Laming, 74, 76 & 78 William-street, New York, U.S.A.	1901	³ Papps, Percy Charles Herbert, <i>Manufacturers' Life Insurance</i> <i>Co., Toronto, Canada.</i>
1903	² Moore, George Cecil, <i>Imperial Life Assurance Co. of</i> <i>Canada, Toronto, Canada.</i>	1895	² Paradise, William Henry, <i>Australian Mutual Provident</i> <i>Society, Sydney, Australia.</i>
1898	² Moore, Joseph Patrick, <i>Citizens' Life Assurance Co.,</i> <i>Sydney, Australia.</i>	1869	Park, David Francis, C.A., F.F.A., <i>Crédit Foncier of Mauritius</i> <i>(Limited), 39 Lombard-st., E.C.</i>
1871	² Moore, Roderick Mackenzie, <i>United Kingdom Temperance and</i> <i>General Provident Institution,</i> 1 Adelaide-place, London-bridge, E.C.	1884	Park, Leslie John, <i>Colonial Mutual Life Assurance</i> <i>Society, Melbourne, Australia.</i>
1893	² Munro, Donald Alexander, <i>Brook-house, 10 Walbrook, E.C.</i>	1882	² Paterson, William Brockie, F.F.A., Mem. Act. Soc. Amer., <i>Norwich Union Life Insurance</i> <i>Society, Norwich.</i>
1900	² Nash, Alfred Charles, <i>Clerical, Medical and General</i> <i>Life Assurance Society, 15 St.</i> <i>James's-square, S.W.</i>	1898	⁽²⁾ Pearce, Henry John, F.F.A., <i>Scottish Amicable Life Assurance</i> <i>Soc., St. Vincent's-place, Glasgow.</i>
1903	² Neill, Samuel Bennett, <i>London, Edinburgh & Glasgow</i> <i>Assurance Co., Ltd., Insurance-</i> <i>buildings, Farringdon-st., E.C.</i>	1899	² Peele, Thomas, <i>Universal Insurance Company,</i> 77 Briggate, Leeds.
1897	² Newling, Sidney Wallis, B.A., <i>Woodleigh, South Woodford,</i> <i>Essex.</i>	1901	² Penman, William, Jr., <i>Northern Assurance Company,</i> 1 Moorgate-street, E.C.
1903	² Nicholls, Arthur William, <i>Australian Mutual Provident</i> <i>Society, Brisbane, Australia.</i>	1900	² Peters, Charles Furness, <i>L'pool. Victoria Legal Friendly</i> <i>Society, 18 St. Andrew-street, E.C.</i>
1884	Nicoll, John, F.F.A., <i>Life Association of Scotland,</i> 82 Princes-street, Edinburgh.	1895	⁽²⁾ Pierson, Israel Coriell, Mem. Act. Soc. Amer., 141 Broadway, New York, U.S.A.
1900	² Oakley, Henry John Percy, <i>North British and Mercantile</i> <i>Insurance Company, 61 Thread-</i> <i>needle-street, E.C.</i>	1902	² Pigrome, George Davey, <i>Prudential Assurance Company,</i> Holborn-bars, E.C.
1883	Orr, Lewis P., F.F.A., <i>Scottish Life Assur. Co., Ltd.,</i> 19 St. Andrew-sq., Edinburgh.	1899	² Pipe, Sidney Herbert, <i>Independent Order of Foresters,</i> Temple-bldgs., Toronto, Canada.
		1883	Pitts, Thomas, <i>Commercial Union Assurance</i> <i>Company, Exeter.</i>

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Date of becoming an Associate		Date of becoming an Associate.	
1876	Pound, Thomas James, <i>Clerical, Medical & General Life Assurance Soc., 15 St. James's-square, s.w.</i>	1855	Ronald, Thomas Robert, <i>Lar Guarantee and Trust Soc., Ltd., 49 Chancery-lane, w.c.</i>
1890	² Powell, Alfred, <i>Alliance Assurance Company, Limited, Bartholomew-lane, E.C.</i>	1897	² Ryley, Edmund, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1881	Price, William John, <i>Life Association of Scotland, 5 Lombard-street, E.C.</i>	1896	² Sanderson, Frank, M.A., F.F.A., F.S.S., Mem. Act. Soc. Amer., <i>Canada Life Assurance Company, Toronto, Canada.</i>
1869	Pringle, James, C.A., F.F.A., <i>42 Drumsheugh-gardens, Edinburgh.</i>	1884	Schooling, John Holt, <i>Fotheringay-house, Montpelier-row, Twickenham.</i>
1884	Pullar, James, F.F.A., <i>Colonial Mutual Life Assurance Society, Melbourne, Australia.</i>	1899	² Schouten, Pieter, <i>Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 74, Amsterdam.</i>
1881	Purves, Thomas Peter, <i>New York Life Insurance Company, Sydney, Australia.</i>	1873	Scott, Ernest Willem, Mem. Act. Soc. Amer., <i>Algemeene Maatschappij van Levensverzekering en Lijfrente, Damrak, 74, Amsterdam.</i>
1867	Rattray, Patrick, C.A., <i>Gresham-house, 45 West Nile-street, Glasgow.</i>	1861	² Searle, Thomas John, <i>Mansion-house-chambers, Bucklersbury, E.C.</i>
1874	² Ray, Charles Richard, <i>Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.</i>	1900	² Searls, Edwin Richard, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1885	Rea, Charles Herbert Edmund, F.R.A.S., F.S.S., <i>3 & 4 Clement's-inn, w.c.</i>	1900	² Sharpe, Edgar Cecil Engledue, <i>London Life Association, Ltd., 51 King William-street, E.C.</i>
1898	² Reid, Edward E., B.A., <i>London Life Insurance Co., London, Ontario, Canada.</i>	1894	³ Sheppard, Herbert Norman, B.A., Mem. Act. Soc. Amer., <i>Home Life Insurance Company, 256 Broadway, New York, U.S.A.</i>
1901	² Rhodes, Francis, B.A., <i>Royal Insurance Co., Liverpool.</i>	1899	² Sherrieff, Francis Henry, <i>Provident Clerks' and General Mutual Life Assurance Assoc., 27 & 29 Moorgate-street, E.C.</i>
1887	Richardson, Josephus Hargreaves, F.F.A., Mem. Act. Soc. Amer., <i>New Zealand Government Life Insurance Department, Wellington, New Zealand.</i>	1897	² Shimmell, James Edward, <i>8 Osborne-road, Handsworth, Birmingham.</i>
1879	Roberts, Thomas B., <i>Australian Alliance Assurance Company, Collins-street, Melbourne, Australia.</i>	1896	² Shlager, Joseph, <i>Equitable Life Assurance Society of the United States, Mansion-house-chambers, Adderley-street, Cape Town, South Africa.</i>
1878	Robertson, William, F.F.A., <i>29 Stafford-street, Edinburgh.</i>	1903	² Shovelton, Sydney Taverner, B.A., <i>Merton College, Oxford.</i>
1876	Robinson, Andrew, <i>Sunningdale-park, Sunningdale, Berks.</i>		
1902	² Robinson, Hugh Thomas Kay, <i>Clergy Mutual Assur. Soc., 2 & 3 The Sanctuary, Westminster, s.w.</i>		

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Date of becoming an Associate.		Date of becoming an Associate.	
1897	² Slade, Henry, 21 Buckle-y-rd., Brondesbury, N.W.	1898	² Thompson, Thomas Percy, B.A., Pelican and British Empire Life Office, 70 Lombard-street, E.C.
1864	Smith, Howard Samuel, F.F.A., F.S.S., F.C.A., Bank-chambers, 11 Waterloo-street, Birmingham.	1883	² Titmuss, Walter George, Prudential Life Office, 50 Regent-street, W.
1898	² Smith, Robert Parker, Royal Insurance Company, Liverpool.	1902	² Traversi, Antonio Thomas, New Zealand Government Life Insurance Department, Wellington, New Zealand.
1884	Smithett, Edward Henry, "Hillside," Fitzroy-park, High-gate, N.	1883	Tregaskis, George Alfred, Hand-in-Hand Insurance Soc., 26 New Bridge-street, E.C.
1871	Spencer, Robert James, F.S.S., 75 King's-road, Southsea.	1894	² Trenerry, Charles Farley, B.A., University of London, South Kensington, S.W.
1868	Spens, William George, Scottish Amicable Life Assur. Soc., 35 St. Vincent-pl., Glasgow.	1869	² Trew, Edward Bellingham, Law Life Assurance Society, 187 Fleet-street, E.C.
1902	² Spurgeon, Ernest Frank, Prudential Assurance Company, Holborn-bars, E.C.	1891	² Turnbull, A. D. Lindsay, C.A., F.F.A., Scottish Widows' Fund and Life Assurance Society, 9 St. Andrew-square, Edinburgh.
1860	Stark, James, Reversionary Interest Society, 30 Coleman-street, E.C.	1877	² Turpin, William Gibbs, National Debt Office, Finsbury-pavement-house, E.C.
1866	Stark, William Emery, Chapel-walks, Manchester.	1884	Vian, William Collett, Railway Passengers' Assurance Company, 64 Cornhill, E.C.
1878	Stevenson, Charles, 9 Albert-square, Manchester.	1884	Vincent, Frederick James, F.S.S., London, Edinburgh & Glasgow Assurance Co., Ltd., Insurance-buildings, Farringdon-street, E.C.
1880	Stock, Edward James, National Mutual Life Assoc. of Australasia, Melbourne, Australia.	1899	² Vokins, George Alfred, Prudential Assurance Company, Holborn-bars, E.C.
1896	² Stuckey, Jos. James, M.A., Salisbury Chambers, 49a King William-street, Adelaide, South Australia.	1883	² Walker, Davidson, F.F.A., Norwich Union Life Insurance Society, Norwich.
1869	Surenne, David John, F.F.A., 29 Inverleith-road, Edinburgh.	1879	Wall, Walter George, 3 Shrewsbury-road, Birkenhead.
1899	³ Symmons, Frank Percy, Prudential Assurance Company, Holborn-bars, E.C.	1878	Walton, William Gandy, F.F.A., Scottish Provident Institution, 6 St. Andrew-square, Edinburgh.
1882	Tarn, Walter George, Reversionary Interest Society, 30 Coleman Street, E.C.	1902	³ Wandless, John Robert, Northumbria, Hockley, Essex.
1893	² Taylor, Arthur, Westminster and General Life Assurance Assoc., 28 King-street, Covent-garden, W.C.	1902	² Wares, Harold Wallace, Yorkshire Insurance Company, York.
1875	Taylor, J. Wilford, North British and Mercantile Insur. Co., 61 Threadneedle-st., E.C.		

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Date of becoming an Associate.		Date of becoming an Associate.	
1862	Waterhouse, Edwin, M.A., F.C.A., F.S.S., 3 Frederick's-place, Old Jewry, E.C.	1901	³ Wilson, John Sydney, <i>Australian Widows' Fund Life Assurance Society, Melbourne, Australia.</i>
1903	² Watherston, Charles F., B.A., <i>War Office, s.w.</i>	1873	² Windett, Charles, <i>Legal & General Life Assurance Society, 10 Fleet-street, E.C.</i>
1883	² Watson, John Robertson, <i>British Law Fire Insurance Co., 176 West George-st., Glasgow.</i>	1898	³ Wood, Arthur Barton, B.A., Mem. Act. Soc. Amer., <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1894	² Watt, George, <i>Royal Insurance Co., Liverpool.</i>	1903	² Wood, William Archibald Porter, B.A., <i>Canada Life Assurance Co., Toronto, Canada.</i>
1900	⁽²⁾ Watt, James, F.F.A., 18 Moray-place, Edinburgh.	1883	Woodhouse, Lister, A.C.A., F.S.S., <i>City Comptroller, City-hall, Westminster, s.w.</i>
1883	Weall, Bertram, 16 Waldegrave-park, Twickenham.	1877	² Woods, Arthur Biddle, <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>
1902	² Weatherill, Charles, <i>Scottish Office, Whitehall, s.w.</i>	1866	Woods, Bernard (AUDITOR), <i>Metropolitan Life Assurance Society, 13 Moorgate-street, E.C.</i>
1899	³ Weatherill, Henry, <i>National Debt Office, Finsbury-pavement-house, E.C.</i>	1875	Woods, Edward, <i>Victoria Life and General Insurance Co., Market-st., Collins-st. West, Melbourne, Australia.</i>
1894	⁽²⁾ Weeks, Rufus Wells, Mem. Act. Soc. Amer., <i>New York Life Insurance Co., 346 & 348 Broadway, New York.</i>	1879	Wornum, Thornton Selden, <i>Rock Life Assurance Company, 15 New Bridge-street, E.C.</i>
1898	³ Whigham, Charles Frederick, F.F.A., C.A., <i>Messrs. Moncrieff & Horsburgh, 46 Castle-street, Edinburgh.</i>	1903	² Worth, Bertram Oliver, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's-square, s.w.</i>
1884	Whyte, Alexander, <i>c/o Messrs. Lever Bros., Ltd., Port Sunlight, Cheshire.</i>	1893	² Wright, Robert Young Murray, M.A., <i>Royal Insurance Co., 45 Dame-street, Dublin.</i>
1897	² Wickens, Charles H., <i>Government Statistician's Office, Perth, W. Australia.</i>	1871	Yardley, John, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1896	² Wilkinson, Edward Berkeley, 24 Mazilla-gardens, N. Kensington, W.	1873	Young, Alexander Hunter, 60 Market-street, Melbourne, Australia.
1903	² Wilkinson, William Magnay, Jr., <i>Citizens' Life Assurance Co., Sydney, Australia.</i>	1900	³ Young, Arthur Stanley, <i>Metropolitan Life Assurance Society, 13 Moorgate-street, E.C.</i>
1900	² Wilson, George, <i>Standard Life Assurance Company, 3 George-st., Edinburgh.</i>		
870	² Wilson, Henry Edward (AUDITOR), <i>Northern Assurance Co., 1 Moorgate-street, E.C.</i>		

STUDENTS.

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Date of becoming a Student.		Date of becoming a Student.	
1892	¹ Aaron, David Hyam, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1896	¹ Barry, David, <i>Royal Commission on the University of Melbourne, Supreme Court Library, Melbourne, Australia.</i>
1903	¹ Acum, Wilfred Harry, <i>15 Lordship-lane, Wood Green, N.</i>	1900	¹ Bell, Henry Soady, <i>"Netherhall," The Drive, Sidcup, Kent.</i>
1894	¹ Anderson, Adam Thomson, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1898	¹ Bennell, Samuel Thomas, <i>20 Narford-road, Brooke-road, Clapton, N.E.</i>
1886	Arnold, Thomas, Jr., <i>British Equitable Life Assurance Company, Ltd., Queen-street-place, E.C.</i>	1903	¹ Bennett, Reginald, <i>Refuge Assurance Co., Oxford-Street, Manchester.</i>
1896	¹ Ashley, Charles Henry, <i>Refuge Assurance Company, Oxford-street, Manchester.</i>	1898	¹ Bennett, Samuel, <i>National Deposit Friendly Soc., 11 Red Lion-square, Holborn, W.C.</i>
1902	¹ Askwith, Thomas Nowell, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>	1902	¹ Beresford, Cecil, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1901	¹ Atkins, Leonard George, <i>Law Union & Crown Insurance Co., 126 Chancery-lane, W.C.</i>	1902	¹ Biden, Norman Frederick, <i>Standard Life Association, 28 Elizabeth-street, Sydney, Australia.</i>
1899	¹ Baber, Walter Crosbie, <i>Royal Victoria Life Insurance Co. of Canada, Montreal, Canada.</i>	1895	¹ Bigby, Robert Frederick Mitchell, <i>General Assurance Company, 103 Cannon-street, E.C.</i>
1903	¹ Baggs, Henry Ernest, <i>English and Scottish Law Life Assurance Association, 12 Waterloo-place, S.W.</i>	1900	¹ Bingeman, Milton H., <i>Great West Life Assurance Company, Winnipeg, Manitoba, Canada.</i>
1903	¹ Bain, William Algernon, <i>Manufacturers' Life Insurance Co., Toronto, Canada.</i>	1903	¹ Binney, Charles Eardley-Wilmot, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1899	¹ Barnett, Isaac, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>	1891	¹ Bird, Edward William, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>
1899	¹ Barrett, William Goodsman, <i>United Kingdom Temperance and General Provident Institution, 1 Adelaide-place, London-bridge, E.C.</i>	1898	¹ Bishop, Harold Garfield, <i>Northern Assurance Company, 1 Moorgate-street, E.C.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1898	¹ Blake, Frederick Edward, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1903	¹ Bradbury, Algernon Charles, <i>Australian Mutual Provident</i> <i>Society, Melbourne, Australia.</i>
1901	¹ Blake, Francis Seymour, <i>62 Oakhurst-grove, East Dul-</i> <i>wich, S.E.</i>	1899	¹ Brady, John Francis, <i>Citizens' Life Assurance Co.,</i> <i>Sydney, Australia.</i>
1898	¹ Blake, Henry Prince, <i>Union Assurance Society, 81</i> <i>Cornhill, E.C.</i>	1897	¹ Brierley, William Ernest, <i>Refuge Assurance Company,</i> <i>Oxford-street, Manchester.</i>
1895	Blanch, Frederick William, <i>Sun Life Assurance Society,</i> <i>63 Threadneedle-street, E.C.</i>	1902	¹ Brook, Frank A., <i>Refuge Assurance Co., Oxford-</i> <i>street, Manchester.</i>
1902	(1) Blanchard, Norman, B.A., <i>Equity & Law Life Assur. Soc.,</i> <i>15 Lincoln's-inn-fields, W.C.</i>	1894	¹ Brough, Frank, <i>Federal Life Assurance Company,</i> <i>Hamilton, Ontario, Canada.</i>
1901	¹ Blehl, Ernest M., A.M., <i>Security Trust & Life Insurance</i> <i>Co., 26th and Broadway, New</i> <i>York, U.S.A.</i>	1891	¹ Brown, William Heron, <i>Gresham Life Assurance Society,</i> <i>Limited, St. Mildred's-house,</i> <i>Poultry, E.C.</i>
1887	Blossom, James, <i>186 South-view-road, Sheffield.</i>	1889	Buckle, Frederick, <i>Aberfoyle, Valkyrie-road, West-</i> <i>cliff-on-Sea, Essex.</i>
1892	¹ Boddy, Henry Mitchell, <i>Manufacturers' Life Insurance</i> <i>Co., Johannesburg, South Africa.</i>	1901	¹ Caldwell, Richard H., <i>North British & Mercantile In-</i> <i>surance Co., Birmingham.</i>
1903	¹ Bodley, Rupert Frank, <i>Star Life Assurance Society,</i> <i>32 Moorgate-street, E.C.</i>	1903	¹ Capon, Frank Christopher, <i>Prudential Assurance Co.,</i> <i>Holborn-bars, E.C.</i>
1897	Bond, Frederic D., <i>122 South 39th Street, Phila-</i> <i>delphia, U.S.A.</i>	1902	¹ Capon, Geoffrey William, <i>Norwich Union Life Insurance</i> <i>Society, Norwich.</i>
1900	¹ Borrajo, Edward Joseph William, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1903	¹ Carpenter, Thomas B. Boyd, <i>4 Greencroft-gardens, South</i> <i>Hampstead.</i>
1902	¹ Bowerman, Judah Philip, <i>Provident Life and Trust Co.</i> <i>of Philadelphia, 149 Broadway,</i> <i>New York, U.S.A.</i>	1901	¹ Carter, George Stanley, <i>Life Association of Scotland,</i> <i>5 Lombard-street, E.C.</i>
1897	¹ Bowles, Francis Marsh, <i>Pearl Life Assurance Company,</i> <i>London-bridge, E.C.</i>	1899	² Carter, Norman John, <i>Eagle Insurance Company, 79</i> <i>Pall-mall, S.W.</i>
1891	¹ Boyd, Henry Norris, <i>City of Glasgow Life Assurance</i> <i>Company, 21 St. Andrew-square,</i> <i>Edinburgh.</i>	1900	¹ Chambers, John Joseph, <i>100 Linaker-street, Southport.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1902	¹ Chandler, Frederick Joseph, <i>Eagle Insurance Co., 79 Pall-mall, S.W.</i>	1900	¹ Cooper, Bernard Hugh, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903	¹ Cheshire, Harold Frank, <i>Woodside, Shornden, St. Leonards-on-Sea.</i>	1903	¹ Cooper, John James, <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>
1903	¹ Child, Robert Harold, <i>North British and Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>	1902	¹ Corbett, Archibald Gladstone, <i>Australian Mutual Provident Society, Collins-st., Melbourne, Australia.</i>
1901	¹ Chubb, William, <i>Sun Life Assurance Company of Canada, Montreal, Canada.</i>	1903	¹ Cotton, Arthur Sparkes, <i>Scottish Office, Whitehall, S.W.</i>
1901	¹ Clarke, Eustace Edgar, <i>Pelican & British Empire Life Office, 70 Lombard-street, E.C.</i>	1897	² Court, Alexander George Dacus, <i>Coningsby, Hyde-vale, Greenwich, S.E.</i>
1897	¹ Clinton, George, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1901	¹ Coventry, Cameron H., <i>Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.</i>
1902	¹ Clinton, Louis Ernest, <i>Sun Life Assurance Society, 63 Threadneedle-street, E.C.</i>	1894	Cox, Edward William, <i>Canada Life Assurance Co., Toronto, Canada.</i>
1902	¹ Coates, Frederick George, <i>Legal and General Life Assurance Society, 10 Fleet-street, E.C.</i>	1894	Cox, Herbert Coplin, <i>Canada Life Assurance Co., Toronto, Canada.</i>
1901	¹ Cockerton, John Leonard, <i>Pioneer Life Assurance Co., Ltd., Century-buildings, 31 North John-street, Liverpool.</i>	1887	¹ Cross, Henry John, <i>3 Park-rd., Wandsworth-common, S.W.</i>
1895	Cogar, William Edward, <i>New York Life Insurance Co., Trafalgar-square, W.C.</i>	1897	¹ Crump, Percy C., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1898	¹ Collier, Charles Aubrey, <i>46 Crockerton-road, Tooting, S.W.</i>	1897	¹ Dalton, John, <i>London Life Association, Ltd., 81 King William-street, E.C.</i>
1899	¹ Collins, Patrick A., <i>Citizens' Life Assurance Co., Sydney, Australia.</i>	1902	⁽¹⁾ Daman, Gerard William, B.A., <i>Hand-in-Hand Insurance Soc., 26 New-bridge-street, E.C.</i>
1902	¹ Collins, William Ernest, <i>61 Osmund-terrace, Norwood, South Australia.</i>	1891	Daniell, Ferrers Aitken, <i>Royal Exchange Assurance Corporation, Royal Exchange, E.C.</i>
1896	¹ Cook, Henry Milton, <i>Standard Life Assurance Company, Dalhousie-square, Calcutta India.</i>	1889	¹ Davies, Hugh Myddelton, <i>Royal Insurance Co., Liverpool.</i>
		1900	¹ Davies, William Allison, <i>Borough Treasurer's Office, Town Hall, Birkenhead.</i>

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Date of becoming a Student.		Date of becoming a student.	
1899	¹ Davison, Horace Williams, 15 Torrington-square, W.C.	1892	¹ Edwards, Edward Samuel, Australian Mutual Provident Society, Sydney, Australia.
1891	¹ Dawson, Frank Aubrey, Ecclesiastical Insurance Office, Limited, 11 Norfolk-street, Strand, W.C.	1902	¹ Edwards, Thomas Baker, Comptroller's Dept., London County Council, Spring-gardens, S.W.
1902	¹ Deck, James Gilbert, National Provident Institution, 48 Gracechurch-street, E.C.	1892	¹ Eedy, Arthur Malcolm, Citizens' Life Assurance Com- pany, Sydney, Australia.
1902	¹ Denmark, Robert John, Norwich Union Life Insurance Society, Norwich.	1901	¹ Eggleton, Harold Edward, Prudential Assurance Company, Holborn-bars, E.C.
1901	¹ Dent, Ernest Edward, London and Lancashire Life Assurance Company, 66 & 67 Cornhill, E.C.	1900	¹ Elderton, Robert Lapidge, National Provident Institution, 48 Gracechurch-street, E.C.
1896	¹ de Ville, Francis, Clergy Pensions Institution, 11 Norfolk-street, Strand, W.C.	1902	¹ Ellis, Reginald George Gregson, 12 Manson-pl., Queen's-gate, W.
1897	¹ Dick, William Thos., B.A., M.L.A., Newcastle, N.S.W., Australia.	1903	¹ Ellis, Thomas Barnes, Public Works Loan Board, Old Jewry, E.C.
1890	¹ Docker, Leslie, North British and Mercantile Insurance Co., 61 Threadneedle- street, E.C.	1893	¹ Emery, John M., American Union Life Insurance Co., Bowling Green-building, Broadway, New York.
1897	¹ Dorrian, John Christopher, Citizens' Life Assurance Com- pany, Sydney, Australia.	1903	¹ Farmer, Ernest Chattock, London, Edinburgh & Glasgow Assurance Company, Limited, Insurance-bldgs., Farringdon- street, E.C.
1899	¹ Douglas, J. Joseph, Victoria-place, Blackrock, co. Dublin.	1892	¹ Farrell, John, Citizens' Life Assurance Co., 210 Queen-street, Brisbane, Aus- tralia.
1903	¹ Downes, Edward George, c/o Thomas G. Ackland, Esq., 5 & 6 Clement's-inn, W.C.	1902	¹ Farrow, Alfred Ellis, Yorkshire Insurance Company, York.
1901	² Downes, Sidney Cecil, Prudential Assurance Company, Holborn-bars, E.C.	1901	¹ Fielder, William Crowhurst, Atlas Assurance Co., Ltd., 92 Cheapside, E.C.
1903	¹ Ebihara, Kaitaro, 33 Redcliffe-gardens, South Kensington, S.W.	1901	¹ Fisher, John William, B.A., Crown Life Insurance Co. Toronto, Canada.
1897	¹ Ecroyd, Cuthbert W., Friends' Provident Institution, Bradford.		

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Date of becoming a Student.		Date of becoming a Student.	
1896	¹ Fisk, George William Victor, F.S.S., <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1893	¹ Gledstone, W. L., <i>Royal Exchange Assur. Corpora-</i> <i>tion, Royal Exchange, E.C.</i>
1901	¹ Franklin, Herbert Dare, <i>Australian Mutual Provident</i> <i>Society, Melbourne, Australia.</i>	1902	¹ Godsill, Richard Collis, <i>Liverpool Victoria Legal</i> <i>Friendly Soc., 18 St. Andrew-</i> <i>street, E.C.</i>
1902	⁽¹⁾ Frater, George Douglas, B.A., B.Sc., <i>22 Canonbury-villas, N.</i>	1894	¹ Golding, Arthur, <i>40 Allerton-road, Stoke New-</i> <i>ington, N.</i>
1903	¹ Fulford, William John, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1888	¹ Gooding, Harold John, <i>Law Guarantee and Trust Soc.,</i> <i>Ltd., 56 Moorgate-street, E.C.</i>
1890	Gamman, Robert Ebenezer, <i>London Joint Stock Bank,</i> <i>Princes-street, E.C.</i>	1900	¹ Goodman, Gilbert, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1886	Garcke, Emile, F.S.S., M.I.E.E., <i>Ditton-house, near Maidenhead.</i>	1903	¹ Gopp, John Ive, <i>14 Church-hill-road, Waltham-</i> <i>stow, E.</i>
1900	¹ Garner, James, <i>9 Arlington-gardens, Chiswick, W.</i>	1892	Gordon, Alexander, <i>168 Islington, Liverpool.</i>
1901	⁽¹⁾ Gerrish, Frank Wilfred, <i>Minerva-villa, Albert-rd.-south,</i> <i>Buckhurst-hill, Essex.</i>	1896	¹ Gordon, Harry Duncan Lockhart, <i>221 George-st., Toronto, Canada.</i>
1899	¹ Giles, Hylton Lloyd, <i>Pelican & British Empire Life</i> <i>Office, 70 Lombard-street, E.C.</i>	1902	¹ Gordon, Walter Hamilton, <i>45 Braydon-road, Stamford-</i> <i>hill, N.</i>
1895	¹ Gill, James Stewart, <i>Australian Widows' Fund Life</i> <i>Assurance Society, Melbourne,</i> <i>Australia.</i>	1902	¹ Gorham, Edwin Arthur, <i>75 Albion-st. East, Brunswick,</i> <i>Melbourne, Australia.</i>
1900	¹ Gillespie, Joseph Hugh Ross, M.A., <i>Manufacturers' Life Insurance</i> <i>Co., Toronto, Canada.</i>	1897	² Gosset, Thorold, <i>16 Durham-road, Wimbledon,</i> <i>S.W.</i>
1901	¹ Glassford, David Murray, <i>Mutual Life Association of Aus-</i> <i>traliasia, Sydney, Australia.</i>	1902	¹ Gould, W. H., M.A., <i>Canada Life Assurance Co.,</i> <i>Toronto, Canada.</i>
1893	Glasson, George Cornish, <i>Economic Life Assurance Soc.,</i> <i>4 St. Stephen's-chbrs., Baldwin-</i> <i>street, Bristol.</i>	1886	Gover, Frederick Field, F.S.S., <i>10 Lee-park, Blackheath, S.E.</i>
1902	¹ Gleave, Charles Sheldon, <i>Refuge Assurance Co., Oxford-</i> <i>street, Manchester.</i>	1900	¹ Green, James Proctor, <i>Refuge Assurance Co., Oxford-</i> <i>street, Manchester.</i>
		1903	¹ Green, Walter, <i>Ecclesiastical Commission, 10</i> <i>Whitehall-place, S.W.</i>

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1886	Greening, Herbert Joseph, <i>Abstainers & General Insur. Co., City-buildings, Birmingham.</i>	1901	¹ Harpell, James John, B.A., <i>North American Life Assurance Co., North American Life Building, 112-118 King-street- west, Toronto, Canada.</i>
1899	¹ Grigg, Benjamin, <i>Sun Life Assur. Co. of Canada, Montreal, Canada.</i>	1901	¹ Harper, Henry, <i>96 Byron-road, Birmingham.</i>
1902	¹ Gysin, Howard, <i>87 Downs-park-rd., Clapton, N.E.</i>	1903	¹ Harris, Ernest Arthur, <i>40 Lambert-road, Brixton-hill, S.W.</i>
1901	¹ Hall, Arthur F., <i>North American Life Assurance Co., North American Life Build- ing, 112-118 King-street-west, Toronto, Canada.</i>	1889	¹ Harris, Henry, <i>Friends' Provident Institution, Bradford.</i>
1902	⁽¹⁾ Hallett, William Sebastian, B.A., <i>Equitable Life Assurance Soc., Mansion-house-street, E.C.</i>	1897	¹ Harriss, Walter James, <i>Life Association of Scotland, 5 Lombard-street, E.C.</i>
1896	¹ Hallman, M. S., <i>Mutual Life Assurance Company of Canada, Waterloo, Ontario, Canada.</i>	1896	Haskins, George Frederick, A.C.A., <i>18 Walbrook, E.C.</i>
1899	¹ Halloran, George Henry, <i>20 Castlereagh-street, Sydney, Australia.</i>	1894	¹ Hatten, David Leslie, <i>Standard Life Assurance Co., 83 King William-street, E.C.</i>
1901	¹ Hamilton, George Powell, <i>North American Life Assurance Co., North American Life Build- ing, 112-118 King-street-west, Toronto, Canada.</i>	1897	¹ Hay, John Dalziel, <i>Crown Lands Office, Wellington, New Zealand.</i>
1902	¹ Hammant, Francis Clive, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1897	¹ Hepburn, Charles James, <i>Clerical, Medical and General Life Assurance Society, 1 King- William-street, E.C.</i>
1900	¹ Hammond, Reginald, <i>British Equitable Life Assur. Co., Ltd., Queen-street-place, E.C.</i>	1903	¹ Hill, Frank Wilson, <i>Norwich Union Life Insurance Society, Norwich.</i>
1892	Hancock, Arthur Tom, <i>Clerical, Medical & General Life Assurance Society, 15 St. James's- square, S.W.</i>	1896	² Hines, Walter Robert, <i>Norwich Union Life Insurance Society, Norwich.</i>
1903	¹ Hancock, Edwin J., <i>72 Tredegar-road, Bow, E.</i>	1897	⁽¹⁾ Hitchins, William Richmond, B.A., <i>Manufacturers' Life Insurance Company, Toronto.</i>
1902	¹ Hardy, Reginald Herbert, <i>32 Highfield-street, Leicester.</i>	1902	¹ Hodge, Cecil Wilfred, <i>Star Life Assurance Society, 32 Moorgate-street, E.C.</i>
1903	¹ Harley, Brian, <i>Guardian Assurance Co., 11 Lombard-street, E.C.</i>	1896	¹ Hogg, Charles, <i>10 Whitehall-place, S.W.</i>
		1898	² Hooper, George Duncan, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1895	² Horn, Ernest Frederick, <i>Clerical, Medical and General Life Assurance Society, 1 King William-street, E.C.</i>	1903	¹ Jefferson, John Arthur, <i>c/o Thomas G. Ackland, Esq., 5 & 6 Clement's-inn, W.C.</i>
1902	¹ Houston, Charles Cornelius, <i>Metropolitan Asylums Board, Victoria-embankment, E.C.</i>	1895	¹ Jenkyn, John, <i>Squirrel's-heath, Romford, Essex.</i>
1901	¹ Howell, Archibald Rennie, B.A., <i>Royal Insurance Co. of England, Montreal, Canada.</i>	1896	¹ Jepps, John Blacklee, <i>English and Scottish Law Life Assurance Assoc., 12 Waterloo-place, S.W.</i>
1898	Hughes, Arthur J., <i>Crown Life Insurance Co., Toronto, Canada.</i>	1898	¹ Johnston, Arthur Edward, <i>3 Cumnor-road, Sutton.</i>
1900	¹ Hughes, Arthur Sidney, <i>6 Telford-avenue, Streatham-hill, S.W.</i>	1902	¹ Jones, Edward Furnival, A.S.A.A., <i>Hearts of Oak Benefit Society, 17 Charlotte-street, W.</i>
1902	¹ Hughes, Charles, <i>Insurance Department of the State of Connecticut, Hartford, Conn., U.S.A.</i>	1903	¹ Jones, Ernest Stephens, <i>National Debt Office, Finsbury-pavement-house, E.C.</i>
1902	¹ Hugill, Herbert, <i>"Briarfield," Keighley.</i>	1896	¹ Jones, Richard Foxley, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1902	¹ Humphreys, John A., <i>National Mutual Life Assurance Society, 39 King-street, Cheap-side, E.C.</i>	1893	² Kelham, Cyril Stephen, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1902	¹ Humphry, Edmund William, <i>Life Association of Scotland, 5 Lombard-street, E.C.</i>	1902	¹ Kemper, J. M. de Bosch, <i>La Mutuelle Hollandais Insurance Company, 21 Avenue de l'Opera, Paris.</i>
1891	Hunt, Arthur Leonard, <i>Wesleyan and General Assur. Society, 101 Finsbury-pavement, E.C.</i>	1898	Kidson, Leonard Douglas, <i>"Oaklands," 1 Morley-road, Southport.</i>
1902	(1) Jackson, Charles William, M.A., <i>267 University-street, Montreal, Canada.</i>	1894	² Kingsbury, James William, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1902	¹ Jackson, Herbert Moore, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1900	¹ Kirkham, Alfred, <i>229 Chapel-street, Prahran, Victoria, Australia.</i>
1890	² Jackson, Samuel, <i>Scottish Widows' Fund and Life Assurance Society, Liverpool.</i>	1903	¹ Kirsopp, Frederick, <i>Liverpool Victoria Legal Friendly Society, 18 St. Andrew-street, E.C.</i>

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Date of
becoming
a Student.

1899 ¹ Kissan, Edgar Duguid,
Listowel, *The Limes Avenue*,
New Southgate.

1895 ¹ Knight, Alfred Murray,
Bank-house, Chapel-st., Devon-
port.

1902 ¹ Lang, Frederick John,
Royal London Friendly Society,
6 Paul-street, Finsbury, E.C.

1902 ¹ Langstaff, James Miles,
Imperial Life Assurance Co. of
Canada, Toronto, Canada.

1901 ¹ Latham, Bertrand,
Australian Mutual Provident
Society, Melbourne, Australia.

1891 Layzell, Phillip Cuddington,
Prudential Assurance Company,
Holborn-bars, E.C.

1901 ¹ Leigh, Samuel George,
Refuge Assurance Co., Oxford-
street, Manchester.

1894 Leonard, Maurice,
14 Sotheby-road, Highbury, N.

1896 ¹ Ley, James,
Office of the Actuary for Friendly
Societies, Melbourne, Australia.

1889 ¹ Lighton, Harold John,
Law Union & Crown Insurance
Co., 126 Chancery-lane, W.C.

1895 ¹ Littell, Lewis Lloyd,
Standard Life Assurance Co.,
83 King William-street, E.C.

1890 Love, Robert,
Pelican & British Empire Life
Office, 70 Lombard-street, E.C.

1894 Lucey, Frederick Samuel, F.C.A.,
15 George-street, Mansion-house,
E.C.

1903 ¹ Macaulay, Frederick Robertson,
4007 Dorchester-street, West-
mount, Montreal, Canada.

Date of
becoming
a Student.

1888 ¹ McConway, James Robert,
Royal Insurance Company, Liver-
pool.

1903 ¹ McDonald, Charles Joseph Angus,
Australian Mutual Provident
Society, Wellington, New
Zealand.

1903 ¹ Macdonald, Charles Strange,
Confederation Life Association,
Toronto, Canada.

1903 ¹ Macdougall, Graham L.,
Massachusetts Mutual Life In-
surance Co., Springfield, Mass.,
U.S.A.

1902 ¹ Macfarlane, James Allan,
North American Life Assurance
Co., North American Life
Building, 112-118 King-street-
west, Toronto, Canada.

1902 ¹ McGee, Cyril H.
Box 981, St. Thomas, Ontario,
Canada.

1902 ¹ McKellar, John A.,
Equitable Life Assur. Society of
the United States, 120 Broadway,
New York, U.S.A.

1901 ¹ Macmillan, Alexander,
27 Westcliffe-grove, Harrogate.

1903 ¹ Macneill, Murray,
McGill University, Montreal,
Canada.

1897 ¹ McPhail, Frederick Charles,
Colonial Mutual Life Assurance
Society Limited, Melbourne,
Australia.

1903 ¹ Malthy, Charles Hugh,
Scottish Widows' Fund and
Life Assurance Society, 28
Cornhill, E.C.

1903 ¹ Manly, George William,
Clerical, Medical & General
Life Assurance Society, 15 St.
James's-square, S.W.

1903 ¹ Martin, Frederick Charles,
Prudential Assurance Company,
Holborn-bars, E.C.

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Date of becoming a Student.		Date of becoming a Student.	
1902	¹ Maudling, Reginald G., <i>London and Lancashire Life Assur. Co., 66 & 67 Cornhill, E.C.</i>	1902	¹ Morton, Francis, <i>Hand-in-Hand Insurance Soc., 26 New-bridge-street, E.C.</i>
1895	¹ Mayhew, Percy Craske, <i>Westminster and General Life Assurance Assoc., 28 King-st., Covent-garden, W.C.</i>	1902	¹ Muckle, Charles Park, <i>Union Life Assurance Co., Toronto, Canada.</i>
1890	¹ Meikle, Henry George Watson, F.F.A., <i>Oriental Government Security Life Assurance Co., Limited, Bombay, India.</i>	1902	¹ Mullin, Alexander, B.A., 76 Major-st., Toronto, Canada.
1901	¹ Melville, Henry Edward, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>	1903	¹ Myers, Harry Duxbury, A.S.A.A., 64 Deronshire-street, Keighley.
1892	¹ Meyers, Henry Wilson, <i>National Mutual Life Association of Australasia, 76 & 77 Cornhill, E.C.</i>	1896	¹ Neale, Maurice Baldwin, <i>Alliance Assurance Company, Ltd., Bartholomew-lane, E.C.</i>
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1897	Mirams, Arthur Greyford, <i>Australasian Temperance and General Mutual Life Assurance Society, Melbourne, Australia.</i>	1895	¹ Newnham, Ernest Whiffin, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903	⁽¹⁾ Monkhouse, Charles Cosmo, B.A., <i>English & Scottish Law Life Assur. Association, 12 Waterloo-place, S.W.</i>	1903	¹ Northcott, John Arthur, 363 Givens-st., Toronto, Canada.
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1895	¹ Moore, Gerald Leslie, A.C.A., 1 Rosebery-gardens, Muswell-hill, N.	1903	¹ Oates, Percy Tuckfield, 30 High-street, Wimbledon, S.W.
1902	¹ Moore, Hubert Fred, <i>London Assurance Corporation, 7 Royal Exchange, E.C.</i>	1902	⁽¹⁾ O'Connor, William, M.A., M.D., <i>Toronto Technical School, Toronto, Canada.</i>
1903	¹ Moore, Roderick John, 34 Cleveland-mansions, Widley-road, Elgin-avenue, W.	1892	¹ O'Reilly, Anthony James, <i>Government Insurance Department, Ottawa, Canada.</i>
1898	¹ Moore, Stanley, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1897	¹ Osborn, Nathaniel Banner Francis, 11 Bruce-grove, Tottenham, N.
		1893	¹ Owen, Edgar Theodore, F.S.S., <i>Registrar of Friendly Societies and Government Actuary, Perth, Western Australia.</i>
		1901	¹ Papworth, Frederick William, A.S.A.A., <i>Admiralty, S.W.</i>

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Date of becoming a student.		Date of becoming a student.	
1895	¹ Pascoe, William Yeoman Bennett, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1901	¹ Ramsay, Cecil Byron, <i>Mutual Life Insurance Co. of</i> <i>New York, 16, 17 & 18 Cornhill,</i> <i>E.C.</i>
1901	¹ Paton, Albert George, <i>Woodstock, 26 Manor-park,</i> <i>Lee, S.E.</i>	1903	¹ Raynes, Harold Ernest, <i>Legal and General Life Assur-</i> <i>ance Society, 10 Fleet-st., E.C.</i>
1897	¹ Paton, Harry Arthur, <i>Royal Exchange Assurance Cor-</i> <i>poration, Royal Exchange, E.C.</i>	1898	¹ Reynell, Guy Courtenay, <i>National Mutual Life Assurance</i> <i>Society, 39 King-st., Cheapside,</i> <i>E.C.</i>
1896	² Penny, Charles Augustus, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>	1903	¹ Reynolds, William Daniel, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1901	¹ Petter, Herbert, <i>British Workman's and General</i> <i>Assurance Co., Broad-street-</i> <i>corner, Birmingham.</i>	1894	¹ Richards, Gilbert P. A., <i>Oak Cottage, Bulwer-road, New</i> <i>Barnet.</i>
1902	¹ Phillips, Tom Herbert Bartholomew, 2 Edith-road, W.	1894	³ Rietschel, Hermann Julius, <i>Sun Life Assurance Society, 63</i> <i>Threadneedle-street, E.C.</i>
1898	Poort, Willem Anthonie, Phil. Nat. Doct., <i>Algemeene Friesche Levens-</i> <i>verzekerings Maatschappij Leeu-</i> <i>warden, Leeuwarden, Holland.</i>	1902	¹ Robertson, Aubrey Charles, <i>London Assurance Corporation,</i> <i>7 Royal Exchange, E.C.</i>
1903	⁽¹⁾ Porter, Frank, B.A., <i>Mansfield-house, Canning Town, E.</i>	1901	¹ Robertson, A. W. L., <i>Guardian Assurance Co., 11</i> <i>Lombard-street, E.C.</i>
1893	¹ Pownall, Herbert Wilfred, <i>Australian Mutual Prorident</i> <i>Society, Adelaide, Australia.</i>	1903	¹ Robertson, Bernard, <i>Prudential Assurance Company,</i> <i>Holborn-bars, E.C.</i>
1897	Proctor, William, Jr., <i>Refuge Assurance Company,</i> <i>Oxford-street, Manchester.</i>	1903	¹ Robinson, Ernest William, <i>Standard Life Association, Ltd.,</i> <i>28 Elizabeth-street, Sydney,</i> <i>Australia.</i>
1902	¹ Proud, Charles Winton, <i>c o Messrs. Whiteheads & Coles,</i> <i>39 Throgmorton-street, E.C.</i>	1896	¹ Robinson, Frederick Charles, <i>Royal Exchange Assur. Corpora-</i> <i>tion, Royal Exchange, E.C.</i>
1886	¹ Quick, John Richard, <i>Equity & Law Life Assur. Soc.,</i> <i>18 Lincoln's-inn-fields, W.C.</i>	1893	¹ Roll, Frederick James, <i>Pearl Life Assurance Company,</i> <i>London-bridge, E.C.</i>
		1893	¹ Roodenburch, Bartholomeus Adrianus, <i>Verzekeringsbank Victoria,</i> <i>689 Prinsengracht, Amsterdam.</i>

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a Student.

- 1895 ¹ Ross, Christopher Watson,
*c/o Messrs. M. Moss & Co.,
Flinder's-lane, Melbourne, Aus-
tralia.*
- 1901 ¹ Rountree, Arthur FitzGerald,
*The Rectory, Stretford, near
Manchester.*
- 1895 Rowley, James Edward, A.C.A.,
7 Waterloo-street, Birmingham.
- 1895 ¹ Rudd, Alfred James,
*Australian Widows' Fund Life
Assurance Society, Melbourne,
Australia.*
- 1899 ¹ Rutter, Edward Valentine,
*Pelican & British Empire Life
Office, 70 Lombard-street, E.C.*
- 1894 Salter, George Ferry, Mem. Act.
Soc. Amer.,
*123 N. 16th-street, E. Orange,
N.J., U.S.A.*
- 1902 ¹ Sare, Thomas Henry,
*Commercial Union Assur. Co.,
24, 25 & 26 Cornhill, E.C.*
- 1892 ¹ Savery, Robert S. B.,
*Gresham Life Assurance Society,
Giselastrasse, No. 1, Vienna.*
- 1897 ¹ Scott, Alexander Lewis,
*Australian Mutual Provident
Society, Melbourne, Australia.*
- 1900 ¹ Searle, Arthur Joseph,
*English & Scottish Law Life
Assurance Association, Limited,
12 Waterloo-place, S.W.*
- 1888 Sewell, Richard, C.A., F.F.A.,
63 Threadneedle-street, E.C.
- 1886 ¹ Sharp, Joseph Benjamin,
*Clerical, Medical and General
Life Assurance Society, 15 St.
James's-square, S.W.*
- 1896 Shawyer, John William,
*Law Union & Crown Insurance
Co., 126 Chancery-lane, W.C.*
- 1902 ¹ Shrubsole, Stanley Smith,
*Prudential Assurance Company,
Holborn-bars, E.C.*

Date of
becoming
a Student.

- 1896 ¹ Shute, Oxenham Bent,
*National Provincial Bank of
England, 53 Baker-street, W.*
- 1892 ¹ Simpson, William Murray,
*North British and Mercantile
Insurance Company, 61 Thread-
needle-street, E.C.*
- 1891 ¹ Sindall, Alfred John,
*London and Lancashire Life
Assurance Co., 66 & 67 Cornhill,
E.C.*
- 1888 ² Slimon, William James, F.F.A.,
2 James'-place, Leith.
- 1902 ¹ Smith, Septimus Wontner,
*Equitable Life Assurance Soc.,
Mansion-house-street, E.C.*
- 1903 ⁽¹⁾ Smith, Thomas Cooper, B.A.,
*Hand-in-Hand Insurance Soc.,
26 New-Bridge-street, E.C.*
- 1903 ¹ Smith, William,
*Standard Life Association, Ltd.,
28 Elizabeth-street, Sydney,
Australia.*
- 1902 ¹ Smither, Herbert Buxton,
*University Life Assurance Soc.,
25 Pall-mall, S.W.*
- 1903 ¹ Sneddon, Andrew William,
*Australian Mutual Provident
Society, Sydney, Australia.*
- 1900 ¹ Somerville, Walter Harold,
*Mutual Life Assurance Company
of Canada, Waterloo, Ontario,
Canada.*
- 1897 ² Stamp, Horatio E.,
*Prudential Assurance Company,
Holborn-bars, E.C.*
- 1903 ¹ Stanford, Harold William,
*London Assurance Corporation,
7 Royal Exchange, E.C.*
- 1903 ¹ Stebbings, George Warne,
37 Leigh-road, Highbury, N.
- 1901 ¹ Steffensen, Johan F.,
*"Nordisk" Re-insurance Co.,
3 Romersgade, Copenhagen.*

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Date of becoming a Student.		Date of becoming a Student.	
1898	² Stewart, Lionel William, <i>Alliance Assurance Co., Ltd., Bartholomew-lane, E.C.</i>	1895	¹ Thistlethwaite, William, <i>4 Warren-terrace, Wakefield.</i>
1886	² Stirling, James, <i>Scottish Imperial Insurance Co., 183 West George-st., Glasgow.</i>	1900	¹ Thomson, Frederick Robert T., <i>Kent-house, Church-end, Finch- ley, N.</i>
1903	¹ Story, Cyril, <i>Norwich Union Life Insurance Society, 71 & 72 King William- street, E.C.</i>	1902	¹ Thwaites, Frederick George, <i>Norwich Union Life Insurance Society, Norwich.</i>
1888	¹ Stott, Walter, <i>Royal Insurance Co., Liverpool.</i>	1897	¹ Tipping, Oswald, <i>Trustees', Executors', and Agency Co., Limited, 412 Collins-street, Melbourne, Australia.</i>
1893	¹ Streeter, Theodore Edward, <i>The Rectory, Orcheston St. Mary S.O., Wilts.</i>	1901	¹ Todhunter, Joseph, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1903	¹ Strong, Allan Wilnot, <i>Sun Life Assurance Co. of Canada, Montreal, Canada.</i>	1902	¹ Tope, Maurice William, <i>National Mutual Life Assurance Society, 39 King-street, Cheap- side, E.C.</i>
1902	¹ Strong, William Boughton, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1897	¹ Touzel, Philip Duncan, <i>Australian Mutual Provident Society, Melbourne, Australia.</i>
1889	¹ Stuckey, Reginald Robert, <i>Australian Mutual Provident Society, Adelaide, S. Australia.</i>	1903	¹ Townley, Ebenezer William, <i>National Mutual Life Assurance Society, 39 King-st., Cheapside, E.C.</i>
1902	¹ Sturt, Herbert Rothsay, <i>c/o C. H. E. Rea, Esq., 3 & 4 Clement's-inn, Strand, W.C.</i>	1897	¹ Townshend, Edward Villiers, <i>North British and Mercantile Insurance Co., 7 Tithebarn-street, Liverpool.</i>
1903	¹ Suddaby, William A., <i>Liverpool Victoria Legal Friendly Society, 38 Lambeth- road, S.E.</i>	1901	¹ Tregaskis, George, <i>Sun Insur. Office, 40 Chancery- lane, W.C.</i>
1902	¹ Sudell, Alfred Henry, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1902	¹ Tully, Arthur Patrick Thomas, <i>The Mill-house, Wimbledon- common, S.W.</i>
1902	¹ Sugars, Robert Morrison, B.A., <i>Gresham Life Assurance Society, St. Mildred's House, Poultry, E.C.</i>	1891	Tyler, Edgar Alfred, F.S.S., <i>9 Old Jewry-chambers, Bank, E.C.</i>
1901	¹ Sutcliffe, Charles Ernest, <i>Hand-in-Hand Insurance Soc., Manchester.</i>	1903	¹ Vance, John Gabriel, <i>Citizens' Life Assurance Co., 24 & 25 King William-st., E.C.</i>
1895	Taylor, Leopold Victor, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>	1903	¹ Warren, Lloyd, A. H., <i>Balderson, Ontario, Canada.</i>

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Date of becoming a Student.		Date of becoming a Student.	
1903	¹ Watson, Alexander R. D., <i>Devonport, Auckland, New Zealand.</i>	1899	¹ Winstauley, Charles William, <i>North British & Mercantile Insurance Co., 61 Threadneedle-street, E.C.</i>
1900	¹ Watt, Arthur W., <i>Sun Life Assur. Co. of Canada, Montreal, Canada.</i>	1903	¹ Wolfenden, Edgar Sydney, <i>Australian Mutual Provident Society, Sydney, Australia.</i>
1898	¹ Webb, Lloyd, <i>Hand-in-Hand Insurance Soc., 26 New-bridge-street, E.C.</i>	1895	¹ Wood, David James, <i>Commercial Union Assurance Co., 24, 25 & 26 Cornhill, E.C.</i>
1902	¹ Wellisch, Frederick, <i>Australian Mutual Provident Society, Sydney, Australia.</i>	1901	¹ Wood, Roland Stuart, <i>Liverpool & London & Globe Insurance Co., 7 Cornhill, E.C.</i>
1893	¹ Welman, Arthur Joseph, <i>Legal & General Life Assurance Soc., 15 Tithebarn-st., Liverpool.</i>	1902	¹ Woodhouse, David Alfred, <i>Refuge Assurance Co., Oxford-street, Manchester.</i>
1902	¹ White, Wilfred Clare, <i>Trinity College, Toronto, Canada.</i>	1896	¹ Woodhouse, Hubert Allen, <i>Union Assurance Society, 81 Cornhill, E.C.</i>
1886	¹ Williams, David, <i>181 Queen Victoria-street, E.C.</i>	1900	¹ Woolston, Paul Livingston, B.S., <i>Spectator Company, 95 William-street, New York, U.S.A.</i>
1894	¹ Williams, Frederick Alfred, <i>Hurstpierpoint, Horchurch, Essex.</i>	1894	¹ Wyatt, George Matthew, <i>Law Guarantee & Trust Society, 49 Chancery-lane, W.C.</i>
1895	¹ Williams, Henry Samuel Walter, <i>North British and Mercantile Insurance Co., Dunedin, New Zealand.</i>	1894	¹ Wylie, Samuel Brown, A.M., <i>112 N. Broad-st., Philadelphia, U.S.A.</i>
1900	⁽¹⁾ Williams, Lewis, B.A., <i>Hand-in-Hand Insurance Soc., 26 New-bridge-street, E.C.</i>	1886	Yeatman, Alexander Alfred, <i>2 Gresham-buildings, E.C.</i>
1903	¹ Wilson, Arthur Benjamin, <i>Australian Mutual Provident Soc., Wellington, New Zealand.</i>	1895	¹ Yeldham, William James, <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1901	² Wiltou, Herbert George, <i>Norwich Union Life Insurance Society, Norwich.</i>	1903	¹ Young, Henry J., <i>Prudential Assurance Company, Holborn-bars, E.C.</i>
1894	¹ Windett, Sydney V., <i>Eagle Insurance Company, 79 Pall-mall, S.W.</i>	1897	¹ Younger, R. Hugh, <i>Hand-in-Hand Insurance Soc., 26 New-bridge-street, E.C.</i>

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Member of the Dutch Actuarial
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l'Association des Actuaire Belges;
Membre Correspondant de l'Institut
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 viii. Baross-utca, 10.*

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 surance Questions; Member of the
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Herr Dr. Gottfried Schaertlin,
*Direktor der Schweizerischen Lebens-
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United States.**NEW YORK.**

Mr. David Parks Fackler,
*Ex-President of the Actuarial Society
 of America (1891-93); Consulting
 Actuary, 35 Nassau-street.*

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By Order of the Council.

November, 1899



VOL. XXXVIII.

PART IV.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

No. CCXVI.—APRIL 1904.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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FARRINGTON STREET.

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BERLIN: CARLSTRASSE 11. MELBOURNE: MCCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

PRINTED BY C. & E. LAYTON, FARRINGTON STREET, E.C.
TO WHOM ALL COMMUNICATIONS FOR THE EDITOR SHOULD BE SENT, POST PAID.

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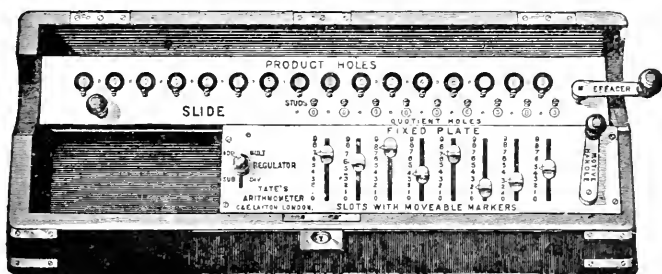
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Opinions of Users of Tate's Arithmometer.

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"I have had a constant experience with various arithmometers during the last thirteen years, and I say without hesitation that the Tate Machine is the most reliable and the best made of all others in the market."

From the *Registrar-General, Dublin*, in 1887 :—

"The Machine (supplied in 1885) appears to work satisfactorily."

From the *Pearl Life Assurance Company* in 1887 :—

"I have had the advantage of about two years' hard work out of one of Tate's Arithmometers, and I am quite satisfied that the Machine, which does not as yet show the slightest indication of wear, has invariably produced correct results."

From the *Wesleyan and General Assurance Society* in 1887 :—

"The Arithmometer which you supplied to us in 1885 has given every satisfaction."

From the *Australian Mutual Provident Society* in 1888 :—

"Since 1884 Tate's Arithmometer has been employed during our annual valuation, and at other times sufficiently to enable us to form a good opinion of its worth. We can speak in the highest terms of the Machine."

From the *Mutual Life Assurance Office* in 1889 :—

"I have much pleasure in stating that the Tate's Arithmometer with which you supplied us in 1884 has been in constant use ever since, and that it works admirably."

From the *Rock Life Assurance Company* in 1889 :—

"The Tate's Calculating Machine supplied to us by you in 1885 continues to work well, and to give satisfaction."

From the *India Office, Whitehall, S.W.*, in 1889 :—

"Tate's Arithmometer supplied by you for this Office in the beginning of 1887, has, in every respect, proved most satisfactory. It is now in excellent working order, and the results obtained I find to be absolutely reliable."

From the *Insurance Department, State of New York*, in 1889 :—

"The 'Tate's Calculating Machine' furnished this Department by you in 1885 has been, since that time, and is now, used almost constantly, and has given the best possible satisfaction. It has never needed repair, even to the replacing of a broken spring, and, from present appearance, is likely to work well for several years to come."

From J. M. CRAIG, Esq., Actuary, *Metropolitan Life Insurance Company, New York*, in 1891 :—

"The work produced by the use of 'Tate's Arithmometer,' both as to accuracy and speed, is worthy of the highest commendation. There are four of them in use in this Office, and we would feel it a loss without them."

From J. DOWSON, Esq., *Royal Insur. Co., Liverpool*, in 1891 :—

"I have used Tate's Arithmometer now more than six years, and have never found it in fault."

From RUFUS W. WEEKS, Esq., Actuary, *New York Life Insurance Company*, in 1891 :—

"The 'Tate's Arithmometer' has been in use by the Actuary's Department of this Company for about ten years, and we keep a number of the machines in constant use, and find them a very valuable aid to our work."

From J. C. REMINGTON, General Manager, *Mutual Life Association of Australasia, Sydney*, in 1894 :—

"The machine arrived in good order last year, and has given great satisfaction."

From JOHN B. LUNGER, Esq., Actuary, *The Prudential Insurance Company, Newark, N. J.*, in 1891:—

"Our two 'Tate Arithmometers' are in use almost constantly. Mechanically the machines seem perfect; although run at a high rate of speed, there has not been a single breakage, and each machine is apparently in as good condition as when delivered."

From C. A. LOVELAND, Esq., Actuary, *North-Western Mutual Life Insurance Company, Milwaukee*, in 1891:—

"About two years ago the North-Western purchased a 16-place 'Tate Arithmometer,' which has since been in almost constant use in my department. I do not hesitate to say that it has given perfect satisfaction. From the knowledge I now have, I would select the Tate machine in preference to all others."

From ISRAEL C. PIERSON, Esq., Actuary, *Washington Life Insurance Company of New York*, in 1891:—

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"The best machine of the many I have tried."

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From the Superintendent, Insurance Department, Springfield, Illinois, in 1895:—

"Some time ago this Department purchased from you a Tate's Arithmometer. We have had occasion to use it a good deal, and I am pleased to advise you that it has given entire satisfaction. It saves a surprising amount of labour, and I have come to regard it as an indispensable equipment for the mathematical work of the Department."

From the Commissioner, Government Life Insurance Department, Wellington, in 1896:—

"The 1889 machine has been in nearly constant use for seven years, and has been worked for months at a time at an average speed of 160 per minute, and, except a broken spring that was replaced in a few minutes, has not once been out of gear; it has proved, in fact, as near a perfect machine as could well be made."

From HENRY DE SMIDT, Esq., Under Colonial Secretary, Cape Town, in 1896:—

"I wish to state for the information of Messrs. C. & E. Layton that the Arithmometer supplied to this department by them, 1890, has proved to be wholly satisfactory. I venture to say it has been in more constant use and has done more work than any instrument used for a like period. It has been in constant use for all Statistical, Actuarial, and other Government purposes, and I have never known it fail in a single instance to the slightest extent. It has remained in perfect adjustment, and is still in as perfect a condition as it was when first received here."

From T. B. MACAULAY, Esq., *Sun Life Assurance Company of Canada, Montreal*, in 1899:—

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From JAMES R. WRIGHT, Esq., *Sun Life Assurance Company of Canada*, 1902:—

"We have now two of the Arithmometers in use, both of which, we are pleased to say, give us entire satisfaction."

[ENTERED AT STATIONERS' HALL.]

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXXVIII. — PART IV.

APRIL 1904.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: 8, RUE LAMARTINE, 8.
BERLIN: KARLSTRASSE 11. MELBOURNE: McCARRON, BIRD & CO.
NEW YORK: THE SPECTATOR COMPANY.

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LONDON :
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGTON STREET.

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NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The Income Tax as Affecting Life Offices, with special reference to some recent decisions. By JOSEPH ERNEST FAULKS, B.A., F.I.A., Joint Actuary of the Law Life Assurance Society.

[Read before the Institute, 21 December 1903.]

“IN this world,” said Benjamin Franklin, “nothing is certain but death and taxes”, a remark with which all taxpayers, and not least those responsible for the management of life offices, will feel at the present time in cordial agreement. The great increase in the last few years in the rate of income tax undoubtedly presses very hardly upon the tax-paying community, and it is probably, in part, owing to this increase that there have recently been several cases before the Courts on the subject of the tax, which are not without interest to those engaged in the business of life assurance. To some of these I propose to direct your attention to-night.

The mode in which life offices are assessable to the tax was some years ago uncertain, but the various decisions alluded to further on have done much to clear the ground. Perhaps we can as yet hardly apply to the measure of their liability to the tax the motto—which by many years of investigation the profession has earned the right to apply to the purely actuarial side of its work as indicating the goal which is being more and more nearly approached—the motto *Certum ex incertis* recently adopted by this Institute. But we can, at any rate, say that, putting

together the various decisions, inconsistent though some of them at first sight seem with each other, we are arriving at a clearer idea of the way in which a life office has in the existing state of the law to bear its share of a national burden.

The *Journal* contains two papers on the subject, by Mr. Bailey and the late Mr. Newbatt respectively. Each of those papers was written as the outcome of a great legal struggle, and each was written by a Member of the Institute of the highest standing in the profession, and, moreover, by one who had been more or less in the position of a protagonist in that preceding struggle. It indicates, perhaps, some temerity to choose for the subject of a paper one that has already been treated in such masterly fashion, but a great deal happens in thirteen years. The interval of time is long, and if the interval in knowledge of the subject and lucidity of expression is, as I am afraid it is, even longer, I can only claim your indulgence.

I have adopted, in writing this paper, the plan of discussing, in the first place, several points relating to the tax, which are not, perhaps, of the greatest importance, and leaving to the end of the paper any remarks on the general lines of the liability of offices to the tax. I trust that in this way the paper may not prove altogether without value—as bringing under notice some questions which have not been very fully dealt with in previous papers—and may be, in a small way, of assistance to students endeavouring to understand, as regards this particular subject, something of “the lawless science of our law—that wilderness of single instances.”

ABATEMENT IN RESPECT OF LIFE ASSURANCE PREMIUMS, &c.

The Act of 1799, and others of the older Income Tax Acts, contained certain provisions for an abatement of tax in respect of premiums paid for life assurance, but the Act of 1842, the foundation of the present law, did not contain any such provision. The Act of 1853, however, contained a section, No. 54, dealing with the point, the wording of which is as follows:—

Any person who shall have made insurance on his life or on the life of his wife, or shall have contracted for any deferred annuity on his own life or on the life of his wife, in or with any insurance company which shall become registered under any Act to be passed in the present session of Parliament for that purpose, and which shall comply with the requirements of such Act . . . shall be entitled to deduct the amount of the annual premium paid by him for such

insurance or contract . . . from any profits or gains in respect of which he shall be liable to be assessed under either of the Schedules (D) or (E) of this Act, . . . or if such person shall be assessed to duties under any of the Schedules contained in this Act and shall have paid such assessment or shall have paid or been charged with any of the said duties by deduction or otherwise, such person on claim made to the commissioners for special purposes and on production to them of the receipt for such annual payment and on proof of the facts to the satisfaction of the said commissioners shall be entitled to have repaid to him such proportion of the said duties paid by such person as the amount of the said annual premium bears to the whole amount of his profits and gains on which he shall be chargeable under all or any of the Schedules of this Act: Provided always that no such abatement allowance or repayment as aforesaid shall be made in respect of any such annual premium beyond one-sixth part of the whole amount of the profits and gains of such person so chargeable as aforesaid, nor shall any such deduction or abatement entitle any such person to claim total exemption or any relief from duty on the ground of his profits and gains being thereby reduced below £100 or £150 as the case may be.

(The present limits for exemption or abatement, of course, differ from those referred to in the closing words of the section.)

By the Income Tax (Insurance) Act, 1853, the benefits of the above-recited section were extended to persons insuring in or with any insurance company existing on 1 November 1844, or registered pursuant to the Joint Stock Companies Act, 1844; and by the Income Tax (Insurance) Act, 1855, to persons insuring in or with any legally established friendly society *provided that the premiums payable in respect of such insurances shall not be made for shorter periods than three months*; and by the Income Tax Act, 1859, as regards deferred annuities, to persons contracting for deferred annuities with the Commissioners for the Reduction of the National Debt. By the Revenue Act, 1903, the proviso printed above in italics was, in effect, repealed.

To this statement of the legislation on the subject the following notes may be appended:—

(1) Claims for repayment of Tax under the provisions of the sections above referred to must, as in the case of claims for repayment generally, be made within three years after the end of the year of assessment to which the claim relates (Income Tax Act, 1860).

(2) "Insurance on his life." There would appear to be nothing in the wording of the section to exclude from its

operation an accident policy—at any rate so far as regards fatal accidents—and claims for abatement in respect of such policies are, I believe, allowed by the authorities. A child's endowment policy would not, however, be within the Act, and in the House of Commons, in May last, Mr. Ritchie intimated that he was disinclined to extend the exemption in this direction—a decision which will, I think, be generally approved.

(3) "Insurance Company." This does not include a foreign company, even though that company was in existence on 1 November 1844, and has an office in England. The benefit of the provisions applies only to companies over which Parliament has jurisdiction, and a foreign company cannot be registered under the Act of 1844 or similar Acts (*Colquhoun v. Heddon*, confirmed on appeal).

(4) "Annual premium paid by him."

- (a) There can be no doubt that half-yearly and quarterly premiums are within the Act, and it would seem that monthly and even weekly premiums would also be within it, provided, prior to the passing of the Revenue Act, 1903, they were payable to an insurance company within the Act, and not to a friendly society (see the Act of 1855). Apparently, however, from a question and answer in the House of Commons, in 1890, the point is not free from doubt.
- (b) Notwithstanding the use of the word "Annual", I understand that a single premium would be within the Act, provided, of course, that the other conditions laid down were duly fulfilled, *e.g.*, that the single premium was not more than one-sixth of the year's income.
- (c) If a policyholder is himself entitled to the commission on his premium, the proper course would appear to be to claim in respect of the full premium, entering the commission, of course, as an item of income. It has not been without hesitation, however, that the Inland Revenue Authorities have accepted this view as correct.
- (d) The question of "half-credit" policies in connection with the abatement provisions may be briefly referred to. Where only the portion of the full premium paid in cash by the assured has been brought into

account it would seem only just that the assured should have the right, on paying off the arrears of premium constituting the debt on the policy, of bringing such arrears into account for the year of repayment subject to the usual conditions. I am informed that half-credit arrears so repaid have in fact been passed by the authorities as a proper deduction on the same basis as premiums. The recent case of *Hunter v. The King*, however, carries the matter somewhat further. In that case the assured under such a policy claimed to deduct the full amount of his premium, including the one-half thereof not paid in cash. The claim was allowed by the Judge of first instance in June 1902, but disallowed in the Court of Appeal. The case is, I understand, being taken to the House of Lords, and it will therefore be desirable to abstain, at the present time, from any more detailed reference to it or criticism of the arguments adduced. One point may, however, I think, be referred to. In the Court of first instance, the Judge, in upholding the deduction of the full premium, laid considerable stress upon the fact that in this particular case the assured had signed an agreement to pay to the office issuing the policy the principal sums to be advanced by the office (that is, the halves of the premium not paid in cash by the assured), this fact importing the personal liability of the assured for the debt. If his decision should eventually be upheld, and on similar grounds, it must not therefore, I think, be too hastily assumed that the same rule will necessarily apply to half-credit or reduced-payment policies where the sole security for the advances is the policy itself and the assured is under no personal liability.

The provisions of the Acts relative to abatement for life assurance premiums have been by no means universally accepted by the insurance profession as perfect. Originally designed it is presumed as a species of bribe to encourage thrift, they apply, as mentioned by Mr. Bailey (*J.I.A.*, xxviii, 304). to many cases where thrift has little enough to do with the effecting of the assurance.

ANNUITIES.

It has no doubt occurred to most persons when making even a cursory study of the Income Tax Acts, to speculate as to what the framers of the earlier Acts exactly understood in their own minds by the expression "annuity", so frequently appearing in conjunction with, and subjected to the same charge as, "interest of money." Had they chiefly in view perpetual annuities, such as the consolidated annuities payable by the State, from which have developed the modern "Consols"? Or did they intend to denote by the term not only such annuities as those last mentioned but also all other annuities which, from an ordinary business point of view, although not in theory, perhaps, may be considered to be income (as, for instance, a life annuity, purchased by an annuitant with the view of obtaining during his life a larger income than he could obtain by the investment of the purchase money, and enjoyed by him as income in toto, with no thought of setting aside any portion of it to replace the purchase money at his death)? Or, finally, did they have in view any series of annual payments extending over a term of years, certain or uncertain? However this may have been, "annuities" are distinctly charged with the tax, and as to the construction of the word and the charge several questions have arisen.

Section 102 of the Act of 1842 enacts that upon all annuities, payable within or out of Great Britain, either as a charge on property or a personal debt under a contract, income tax shall be charged; but that where such annuities are payable out of profits or gains brought into charge by virtue of the Act, the assessment is not made upon the recipient but upon the whole of the profits, without deducting the annuity, of the person paying the annuity, who is authorized to deduct the tax as against the annuitant; tax being charged directly on the annuitant where the annuity is payable out of profits not charged by the Act. The Act of 1853, Section 40, enacted that every person liable to the payment of an annuity should be entitled on making any such payment to deduct and retain thereout the rate of duty then payable. The Revenue No. 1 Act, 1864, provided, with reference to the last-mentioned section, that the deduction, instead of being at the rate in force at the time of payment, should be a proportionate amount of the several rates, if more than one, chargeable on the annuity or the source thereof during the period of accrual. And the Customs and Inland Revenue Act, 1888, Section 24, enacted that upon

payment of any annuity charged with income tax under Schedule D, and not payable or not wholly payable out of profits or gains brought into charge to such tax, the payer should deduct thereout the rate of income tax in force at the time of such payment, and should render an account to the Inland Revenue of the amount so deducted or deducted out of so much of the annuity as is not paid out of profits or gains brought into charge as the case may be.

Life assurance companies are interested in the charging of income tax on annuities in two ways, as being the recipients of annuities for terms certain in connection with loan and investment transactions, and also as being the payers of many annuities both for lives and for other statuses. I proceed to deal with the subject from each of these points of view.

TERMINABLE ANNUITIES.

It is one of the elementary propositions of actuarial science that in the case of an annuity for a term of years each payment of the annuity may readily be divided into two portions, the one representing interest and the other repayment of capital. A similar division may of course be made of the successive payments of a life annuity, but besides being more technical in its character it would, I think, lead us astray in connection with any enquiry as to the incidence of income tax. A life annuity is, generally speaking, purchased to provide a larger income than could otherwise be obtained, and no provision is made by the recipient either directly or indirectly for the replacement of the purchase money at his death. At his death the capital is gone; it cannot be made to bear any capital tax, such as estate duty, and it will not in future produce income subject to taxation. It would appear, therefore, only just that income tax should be charged upon the full annuity payments during the currency of the annuity.

But other considerations arise if we turn to a different class of annuities. If, for instance, an advance on mortgage is made at a given rate of interest, and the principal sum, instead of being made repayable by equal annual instalments, is made repayable together with the interest by an annuity of fixed amount extending over a term of years, it is usual to insert in the deed a schedule shewing how each annuity payment may be divided into interest and principal, and tax is paid without question on the interest portion, that being income, but not upon the principal portion, that being *pro tanto* a repayment of an advance. In the

case of *Leeds Permanent Benefit Building Society v. Mallandaine*, the Society were held to be assessable to tax upon the *interest portions* of weekly payments, consisting partly of principal and partly of interest, received by them in respect of advances made by them to borrowers upon security of house property, they not having allowed the borrowers to make any deduction for tax. But if the transaction is not a simple mortgage transaction a different view has been taken. We have, for instance, the cases of *Blake v. Imperial Brazilian Railway Company* and *Nizam State Railway Company v. Wyatt*. In each of these cases the total sum received in the shape of an annuity under the guarantee of a foreign Government, although only in part devoted to payment of interest on capital, the remainder being applied by way of a sinking fund, was held to be liable to income tax.

The point has been fully considered in the recent case of the Great Indian Peninsula Railway Company Annuity (*Scoble and others v. Secretary of State in Council for India*). It will be remembered that the Indian Government, on becoming entitled to take over the railway, had the option of paying therefor either by a lump sum down or by an annuity extending over a term of years. The latter method having been selected, the Secretary of State, in paying over to the annuity trustees the first two instalments of the annuity, deducted income tax upon the whole of each instalment, disregarding any division of the instalment into principal and interest. As a matter of fact, in the case of the "B" annuities, the capital was actually being re-created by means of the operation of an outside sinking fund, and in the case of the "A" annuities, paid by the trustees in full, there can be no doubt that at any rate in the majority of cases the capital is also being re-created either by an outside sinking fund or in some practically equivalent way. In the Court of first instance it was held that tax was rightly deducted from the whole annuity, a decision which was apparently at variance with that in *Foley v. Fletcher*. In that case, decided in 1858 (the Crown, however, it may be noted, not being represented), it was decided that on a sale of property in consideration of a certain sum payable part down and part by equal half-yearly instalments over a term of years, such instalments did not constitute an annuity liable to income tax. In the judgment of Phillimore, J., in the Great Indian Peninsula Railway case, *Foley v. Fletcher* is distinguished on the ground that the instalments were instalments of purchase money pure and simple, and did not contain any interest portion as do the

periodical payments of "annuities" subjected to tax, and that the amount of the original purchase money had been adjusted to allow for the fact that a portion of it would be receivable only over a term of years. If such a distinction had been upheld it would seem to result that if I want $\text{£}p$ cash as the purchase money of a certain property, and accept in lieu of such cash an annuity of $\frac{p}{a_n}$ the whole of each periodical payment of this annuity is liable to tax; whereas, if I increase $\text{£}p$ in the ratio of say $\frac{n}{a_n}$ and call $\frac{np}{a_n}$ the purchase money, and make this payable by n instalments of $\frac{p}{a_n}$ each, these payments escape taxation altogether. The reason for a difference between the two transactions is not obvious.

On the Great Indian Peninsula case, however, being carried to the higher Courts, judgment was given against the Crown both in the Appeal Court and in the House of Lords, with the result that in this case tax is held to be deductible only from so much of each annuity payment as represents interest, and not from that portion of it (of course a constantly increasing proportion) which represents a repayment of capital. Satisfactory as the decision is, it is obvious, on reading the judgments, that considerable hesitation was felt in coming to the conclusion actually arrived at, owing to the difficulty of ascertaining exactly what is to be understood by the word "annuity." Actuarially the process of dividing the instalments of an annuity into capital and income would be precisely the same in the case of the Great Indian Peninsula Railway Annuity, as in the case which not infrequently occurs of an annuity for a term certain purchased from a life office. Yet there can, I think, be no doubt from the judgments that in the latter case tax would be held to be payable on the whole of the annuity instalments. The one is the case of the purchase of an annuity with nothing else—no antecedent debt—involved. In the other, there was an actual debt of the amount of the purchase money, and the fact of its being liquidated by means of a series of annual payments is no sufficient reason for disregarding the fact of its original existence. It is clearly recognized in the judgments that the fact of a series of payments being uniform in amount, involving therefore a constantly increasing amount of capital repayment, does not necessarily differentiate it for income tax purposes from

a series of payments which, the capital repayments being uniform, is a decreasing one. In other words, the fact that a series of payments is called an annuity, which is, generally speaking, only done where the payments themselves are uniform, is not sufficient to render those payments liable to tax on the full amount thereof. Regard must be had to the particular circumstances of each transaction, with the view of ascertaining whether it is in reality the conversion of capital into income, or the repayment of a debt, or payment of purchase money, by instalments including both principal and interest. "An annuity means where an income is "purchased with a sum of money and the capital has gone and "has ceased to exist, the principal having been converted into an "annuity." Where this is the case tax must be paid on the full instalments of the annuity.

In this connection it should be remembered that no provision is made in the income tax returns for the running off of the term of leasehold property—the annual value being taken into account, however short the term, nor in the case of income derived from mining property is any deduction allowed to be made on account of depreciation by the running off of the life of the mine. In the case of machinery or plant, however, used for the purposes of any trade or manufacture, the profits derived from which are chargeable with tax, a deduction for depreciation has been allowed since the Customs and Inland Revenue Act of 1878.

It should be added that it appears that, notwithstanding the House of Lords' decision in the Great Indian Peninsula Railway Company's case, the Inland Revenue Authorities have, in the case of the similar annuities of the East Indian Railway Company, refused to admit a claim for exemption from tax on so much of the annuity payments as represents capital. Of the precise grounds for their action I am not at present aware, except that it is stated that the contract for purchase in the case of that Company is not on all fours with the contract in the case of the Great Indian Peninsula Railway Company. Even in the case of the latter Company, it appears that the trustees did not consider it safe to distribute the refunded tax among the annuitants without the directions of the Court.

ANNUITIES PAYABLE BY LIFE OFFICES.

The position, as regards income tax, of annuities payable by Life Offices under their annuity contracts has been greatly affected by the Customs and Inland Revenue Act of 1888, section 24.

Upon reference to the provisions of the Acts shortly set out above, it will be seen that, under the Act of 1842, the payer of the annuity was charged with the tax and authorized to recoup himself by deduction as against the annuitant only where the annuity was payable out of profits or gains brought into charge by virtue of the Act. Now the case of *The Gresham Life Assurance Society v. Styles* (reported in *J.I.A.*, vol. xxx) decided that annuities payable by Life Offices under ordinary annuity contracts were not payable out of profits or gains, and so far as the Act of 1842 went, therefore, no charge would apparently have been raiseable against a Life Office, but the recipient of the annuity would have had to be charged directly. The Act of 1853, while authorizing the payer of *any* annuity to deduct and retain thereout the income tax, did not throw upon him any fresh responsibility to do so, and, in the case of annuities not payable out of profits, it would seem that the payer, though authorized, was not bound to deduct the tax, and if he did, in fact, deduct it, was not bound to account to the Crown for it. The Act of 1888, however, which now rules the point, makes it incumbent upon the payer of an annuity not payable out of profits to deduct the tax and account for it to the Crown. It follows, therefore, that a Life Office, in paying an annuity under one of its ordinary annuity contracts, is bound by this section to deduct tax as against the annuitant, and this, according to the opinion of the Inland Revenue Authorities, whether the annuitant resides within or without the United Kingdom. In accounting to the Crown, however, for the tax so deducted, it is permissible to assume that annuities are payable in the first place out of the taxed interest yielded by the annuity fund, whether such fund has a separate existence in the accounts of the office or not, and that only the balance of the annuities is payable out of something that is not profits. If the interest on the annuity fund has already been charged with tax, it follows that the amount to be paid over by the office to the authorities will be the tax deducted against the annuitants less the tax on the interest earned by the annuity fund. It is difficult to see how, on the wording of the various Acts, a construction more favourable to the offices can be hoped for.

It not infrequently happens that an annuity payable by a Life Office constitutes practically the only income of the annuitant, as, for instance, in the case of a retired business man with no dependents and a small capital, who prefers to relieve himself of trouble and increase his income by investing that capital in the

purchase of an annuity, or in the case of an aged servant, for whose support an annuity is purchased in accordance with the testamentary provisions of his or her deceased employer. Where such is the case, and the amount of the annuity is under the figure at which the income tax commences to be chargeable—at present £160 a year—it has always seemed to be a great and unnecessary hardship on the annuitant that the office should be compelled to deduct income tax from the annuity payments, and account for it to the revenue authorities. Of course, in such cases the tax deducted could always eventually be recovered from the authorities on their being satisfied that it had actually been deducted, and that the annuitant's income was under the taxable limit, but the recovery is a somewhat troublesome process, particularly to those not especially conversant with business. It is, therefore, I think, extremely satisfactory to note that arrangements have been made by the revenue authorities with more than one office under which such annuities may, if certain formalities are observed, be paid in full, without deduction of tax. A declaration has to be annually signed by the annuitant, on a form provided for the purpose, that his or her income is under the taxable limit, and a list of all annuities so paid in full has to be annually forwarded to the revenue authorities, accompanied by the declarations. The provision does not apply to annuities payable to married women, owing to the assessment being made in such cases on the joint income of husband and wife.

The procedure is, perhaps, a little more troublesome to the office than the deduction of tax from all annuities in all circumstances, but I think that the saving of trouble to the annuitants in such cases more than outweighs this, and that the new arrangement should, therefore, be welcomed as heartily by the offices themselves as it undoubtedly is by the annuitants affected.

It may at first sight seem that the arrangement as to accounting to the Inland Revenue authorities in respect of the tax on annuities is the direct contrary to the practice already referred to of, in the case of a loan repayable by instalments, allowing tax on the interest only, and not on the principal repaid. For each payment of a life annuity may be divided in the same way as each payment of an annuity-certain into principal and interest. But any argument founded upon this—such, for instance, as that the life office should not be liable to pay over to the Government any of the tax deducted from the annuities, inasmuch as part of the annuity payments represents

interest which has already been taxed, and part repayment of capital which is not taxable—seems to me to be misleading. In the case of a life annuity the combined effect of the deduction and allowance is that the Government receives from one source or another the tax upon the whole annuity payment. Now take the case of land subject to a terminable rent-charge, and, for simplicity, suppose that the annual value of the land is exactly equal to the rent-charge. The owner is assessed upon the full value of the land, but against the owner of the rent-charge can deduct tax on the interest portion only. The effect is that the authorities receive altogether the tax upon the annual value of the land. In either case in one way or another the whole of the income is taxed, which is what the Acts are designed to effect; for there can, I think, be no doubt that a life annuity is distinctly taxable as income. Its purchase has nothing to do with the repayment of any antecedent debt or the carrying out of any separate or distinct contract, and must be regarded as the conversion of capital into income, with a result that, as would appear from the remarks quoted above, the whole of the annuity payment is subjected to tax. I have never heard it contended in the converse case of the purchase of a life interest by an assurance company—where the necessary policy of assurance is actually set up in the books of the purchasing office, and the income, assumed to be received under deduction of tax, actually divided into premium and interest—that the office would have any good claim for a rebate of tax on the ground that the premium on the policy represents a repayment of capital, and should not therefore be taxable.

CHANGE IN RATE OF TAX.

It will be interesting here to consider the effect of a change in the rate of income tax as regards payment of annuities, and for this purpose interest and rents, generally speaking, may be coupled with annuities, it being granted that all interest on money is taxable as such. Where income tax is increased from, say 1s. to 1s. 2d. in the £, such increase takes effect from the 6th April, and the position as regards payment of interest, &c., payable after the 6th April in respect of a period extending back beyond such date is defined by the Inland Revenue authorities to be as follows:

- (1) The deduction to be proportionate, in respect of the period of accrual, on mortgage interest and ground

rents and dividends on the shares of companies in the United Kingdom.

- (2) The deduction to be not proportionate, but at the rate in force at the time the amount becomes payable, on dividends from public funds or on shares in foreign or colonial companies, on official salaries and pensions, and on interest paid by municipal corporations to creditors on rates.

A reference to the sections in the various Acts, the effect of which is set out above, will show the reason, as I understand it, for this distinction. The Revenue Act of 1864, authorizing deduction of tax at proportionate rates, is overruled by the Customs and Inland Revenue Act of 1888, so far as regards interest, &c., which is not payable or not wholly payable out of profits or gains brought into charge under the Acts. It follows, therefore, that the method of a proportionate deduction applies to interest on mortgages of property in the United Kingdom (as the annual value of the mortgaged property will have been taxed), to ground rents (for a similar reason), or to dividends yielded by the shares in British companies which have paid tax in respect of their profits. Where, however, the whole of the sources from which the interest, &c., becomes payable have not been brought into charge, the tax must be at the rate in force at the time when the interest, &c., becomes payable, and the instances given above of such cases are readily seen to follow this requirement.

As annuities payable by life offices under their contracts are not wholly payable out of profits or gains brought into charge, the deduction therefrom must be at the rate of tax in force at the time when the payment becomes due. But, it will be observed, in accounting for the tax so deducted, and claiming the benefit of the interest earned on the annuity fund, the tax actually paid on such interest would, it would seem, be the figure to be brought into account.

The above statement, it is believed, represents the legal position, but it is, I think, difficult to see any logical reason for the distinction drawn. Why should a person drawing his income as an annuity from a life office—or, for that matter, from dividends on the public funds—be taxed on a different basis from another who derives his income from interest on moneys advanced on mortgage? That he, the former, will benefit by this arrangement on a decrease of the rate of tax, although he is

a loser by an increase, is not a sufficient reason for its continuance, and it seems to be very unfortunate that in the Act of 1888 the wording of the Act of 1853 should have been repeated, notwithstanding the change that had previously been effected by the Act of 1864.

FOREIGN INTEREST.

Still assuming that interest on money is taxable as such, we may consider the position as regards what may be shortly called "foreign interest", namely, interest on investments made outside the United Kingdom, where such interest is not remitted home immediately on its being received. Up to last year the decisions on this point were extremely conflicting, the Scotch Courts having, speaking generally, taken a view of constructive remittance totally different from that taken by the English Courts. A recent decision of the House of Lords, however, has greatly simplified matters.

In the first place "foreign trading" must be distinguished from "foreign interest." In other words the case of a trading company, carrying on business abroad and assessable as regards income tax on the profits of the business, is not on all fours with that of a life office confining its assurance operations to this country and merely investing some of its surplus funds in foreign countries. Speaking broadly, the effect of the various decisions is that an English company carrying on business abroad, either directly or by means of a sub-company, is liable to be assessed for income tax under the first case of Schedule D of the Act of 1842 upon the whole of the profits made, and not only upon the amount remitted to this country. So in the case of the *Norwich Union Fire Assurance Company v. Magee*, the company carrying on business in America, and having made for the purposes of such business an investment in America, the interest derived from which was brought into account in the company's books but not remitted home, it was held that the interest on such investment formed a portion of the profits of the company on its trading, assessable as above-mentioned under the first case of Schedule D.

With respect to what I have designated as "foreign interest" as distinguished from "foreign trading", the 4th case of Schedule D of the Act of 1842 enacts that as regards interest arising from securities in the British dominions outside the United Kingdom and foreign securities (except such as are chargeable under Schedule C), the duty shall be computed on

a sum not less than the full amount of the sums which have been or will be received in the United Kingdom in the current year. In deciding what is meant by the expression "received in the United Kingdom" the Courts have frequently had to consider the doctrine of "constructive remittance." Until the House of Lords' decision above referred to we had on the one hand the following Scotch cases:—

1. *Forbes v. Scottish Widows' Fund*,
Forbes v. Scottish Provident,
in each of which cases it was held that interest on sums invested in Australia not specifically brought home but reinvested abroad was not assessable under the 4th case, though duly included in the revenue accounts. These companies it will be observed are both mutual, and no profits were divisible among policyholders in the year of assessment.
2. *Standard Life Assurance Company v. Allan*,
in which case it was again held that interest reinvested abroad was not assessable under the 4th case. The company in this case is proprietary.
3. *Scottish Provident v. Allan*,
in which case it was held that remittances in respect of Australian investments must be considered to be remittances of interest and assessable to tax as such, unless it was evident either from particular or general circumstances that such remittances were actually in respect of principal and not interest.

On the other hand we had the following English cases:—

1. *Universal Life Assurance Society v. Bishop*,
in which case Indian interest included in the revenue account, although not specifically remitted home, was held liable to tax under the 4th case as being "constructively" received in the United Kingdom. It should be observed that this was the case of a proprietary office, not only investing but also transacting business abroad, and the case of *Norwich Union Fire v. Magee* was followed, although in that case tax was assessed on profits as distinguished from interest. The decision was subsequent to *Forbes v. Scottish Widows* and *Forbes v. Scottish Provident*, which cases were, however, distinguished by Counsel

for the Crown on the ground that in those cases there was no constructive remittance as the question was one of pure investment, there being no obligations to meet abroad, and by one of the deciding judges on the ground that in neither of those cases was the interest received abroad treated as part of the divisible profits. It might have been held if the decision had been against the Crown, as regards the 4th case of Schedule D, that the office was assessable under the 1st case, *i.e.*, in respect of "foreign trading" as distinguished from "foreign interest" as in the case of *Norwich Union Fire v. Magee*.

2. *Gresham Life Assurance Society v. Bishop*,

in which case it was held in the Queen's Bench Division that, all foreign interest included in the revenue account although not specifically remitted home was taxable. In the "case stated" separate reference was made to countries where the Society merely invested without transacting business, and to countries where the Society both invested and transacted business, and in each instance to the various ways in which the interest on investments was dealt with, namely, by reinvestment in the same country, remittance to another foreign country, remittance home, &c. The decision was confirmed on appeal, *Forbes v. Scottish Provident* being distinguished on the ground that in that case "the foreign dividends were left where they were in foreign parts—they were not brought over and brought into account in making up the profit and loss account, and no dividends had been paid out of that profit to the shareholders", that there merely an account had been prepared in which these dividends had been introduced, but that "nothing further than that had been done."

With a boldness which it is pleasing to note has, not for the first time, received its due and proper reward, the appellants in the last-mentioned case, the Gresham Life Assurance Society, decided upon carrying the case to the House of Lords. Judgment was delivered in May, 1902, when the appeal was allowed and the

judgments of the courts below reversed. The doctrine of "constructive remittance" received very severe treatment in the judgments, and would appear to have sustained its death blow. Lord Brampton said in his judgment: "If a constructive receipt 'is the same thing as an actual receipt, I see no reason for the 'use of the word constructive at all. If it means something 'differing from or short of an actual receipt, then it seems to 'me that a constructive receipt is not recognized by the statute, 'which, in using the word 'received' alone, must be taken to 'have used it having regard to its ordinary acceptance."

This decision is in accordance with that in *Standard Life v. Allan*, which was decided subsequently to *Universal v. Bishop*, and the earlier stages of *Gresham v. Bishop*, and in which those two cases were referred to by the Scotch judges as having been decided to some extent upon a misapprehension of the decision of the Scotch Courts in a case (*Scottish Mortgage Company of New Mexico v. McKelvie*) where the doctrine of constructive remittance was first enunciated, but where the circumstances and the entries in the accounts were of a special nature.

It is apprehended, therefore, that according to the present state of the law, it may be regarded as settled that, to render foreign interest assessable to tax under the 4th case of Schedule D, there must be an actual receipt of such interest within the United Kingdom, and that where such interest is not remitted home but is either reinvested abroad or applied abroad in discharging other obligations of the company, it is not liable to tax.

CAPITALIZED INTEREST.

An interesting point, not, perhaps, of very great importance, arises in connection with loans—as, for instance, some loans on reversionary interests—where there is a provision that the interest should not be actually paid by the mortgagor, but capitalized. In the account kept by the mortgagee as against the mortgagor, should income tax be deducted from the periodical sums accruing by way of interest, and being added to the principal of the loan? There appears to be no decision directly bearing upon the point. It has been decided that a debtor making a payment of interest without deducting tax loses the right to deduct from future payments the tax in respect of such former payment, but the creditor receiving such interest in full would no doubt have to render an account to the Crown in respect of the tax thereon.

The only case of practical importance to life offices is that of loans upon reversions, and having regard to the provisions of Section 24 of the Customs and Inland Revenue Act, 1888, it seems doubtful whether in the final event the question is not merely one of rates of tax, namely, whether tax should be periodically deducted from the interest payments according to the rates from time to time in force, or should be deducted on final repayment of the accumulated advance from so much thereof as represents interest at the rate in force at the time of such repayment. It would be interesting to have the point definitely settled, but in the absence of any judicial pronouncement it is I believe generally, though not invariably, regarded as the better course to deduct tax from each interest payment becoming due before adding it to principal.

FRACTIONAL INTEREST.

There appears to be some difference of opinion as to the incidence of the tax on interest for short periods of time. Mr. Newbatt, in his address of 1890, refers to the then recently-decided case of *Goslings and Sharpe v. Blake*, and it will not, therefore, be without interest to attempt to state the position.

The Act of 1842, Section 100, Schedule D, 3rd case, deals with the duty to be charged in respect of sundry profits of an uncertain annual value, and the second rule refers, *inter alia*, to all discounts and all interest of money, not being annual interest payable or paid by any person whatever. Section 102 of the Act, as mentioned above with respect to annuities, authorizes collection by deduction instead of by direct assessment where annuities are payable out of profits already charged, but in this connection couples with annuities *yearly* interest of money; where the interest is not payable out of profits already charged, "or where any interest of money shall not be reserved or charged or payable for the period of one year", the assessment is to be made directly upon the recipient. Section 40 of the Act of 1853 again couples with annuities any *yearly* interest of money, but Section 24 of the Customs and Inland Revenue Act, 1888, refers to payment of "any interest of money or annuities." (The effect of these sections, so far as material to the present point, is set out above, *s.v.* Annuities).

It would seem, therefore, that any person paying "non-yearly" interest is bound to deduct tax only where such interest is not payable out of profits charged with tax (in which case he

of course must account for it to the Crown). Where the interest is payable out of profits charged with tax, the person paying has no authority to deduct tax, but the assessment must be made directly on the payee.

Interest accruing *de die in diem* allowed by bankers at current rates varying from time to time on sums left with them on deposit will, therefore, be payable in full, but the recipient will have to account to the authorities for tax on such interest. The expression "yearly interest" would cover any interest for a fractional period at a fixed rate per-cent per annum (as in the case of interest for odd days on repayment of a mortgage), or on a bank deposit for a fixed period at a fixed rate, (at any rate if the fixed period be not less than a year and probably in any case; see *Bebb v. Bunny*.) It will be seen that as I understand tax is payable on all interest, even fractional interest at current rates on bank deposits, and that *Goslings and Sharpe v. Blake* deals rather with the question of the collection of such tax, namely, by deduction or by direct assessment, than with the question of the liability of the interest to tax in the hands of one party to the transaction or the other.

PROFITS FROM REVERSIONS AND SECURITIES.

If an office be taxed upon the basis of profits as distinct from interest, there can I think be no doubt that the profits upon which tax would be payable would include profits made upon the realization of securities and also profit on reversions. See, as to the former, *Northern Assurance Company v. Russell*. With regard to profits on reversions, the authorities have on more than one occasion sought to impose tax thereon in cases where the office in question has been paying tax on the interest basis, but hitherto without success. There has not, as far as I am aware, been any judicial decision on the point, but it seems to be only reasonable that if the authorities elect to tax upon a certain basis they should not be in a position to depart from that basis for isolated items—in other words, that if profits on reversions are taxable, then taxation generally should be on the basis of profits and not on the basis of interest.

THE GENERAL QUESTION.

Since the two great cases of *Last v. London Assurance Corporation* and *Clerical, Medical and General v. Carter*, there has been no decision of great importance affecting the broad

principles of the liability of assurance companies to income tax as distinguished from details in the application of those principles. The reports of those cases, together with the papers written *à propos* to them by Mr. Bailey and Mr. Newbatt respectively, and the discussion on, and correspondence arising out of, the latter paper, contain a full statement of those principles, to which there would seem to be but little to add.

The law as it stands, and as it stood before the decision of these cases, is, in my opinion, stated with all the clearness and accuracy which one would have, knowing other writings of the author, expected in Dr. Sprague's letter to the *Insurance Record*, reprinted in *J.I.A.*, vol. xxviii. Employing for the moment the expression "life office" to designate a purely life assurance office, whether mutual or proprietary, tax must be paid upon all interest received, within the United Kingdom, by a life office, and although such tax is as regards the greater part of the interest collected by deduction, if any interest is actually received in full the office must account to the authorities for the tax on such interest. If—which is not likely to happen in ordinary cases—the profits of the company for the year, that is the proper proportion of the surplus ascertained on valuation, should exceed the interest income, then no doubt tax would be payable on the difference, at any rate if the company were proprietary.

Subsequent to the decision in *Clerical, Medical and General v. Carter*, pleas that a life office is assessable to tax on profits and not on interest have been raised in *Gresham v. Bishop*, and also, I believe, in *Standard v. Allan*. On careful examination of the various judgments in the former case I find no sign of any leaning on the part of the Courts to this contention.

It remains to mention two other cases, *New York Life Insurance Company v. Styles*, and *Equitable Life Assurance Society of U.S.A. v. Bishop*. It appears to me to be somewhat difficult to arrive at the exact circumstances of these cases, but they are both cases dealing with the taxation of profits and not with the taxation of interest. Presumably there was in each case no question as to the liability to pay tax on any interest received in the United Kingdom, but, owing to remission of surplus income to the United States for investment there, the interest received in the United Kingdom did not amount to the profits made in the United Kingdom, the amount, and the liability to tax, of which became therefore important in spite of *Clerical, Medical and General v. Carter*. The former case was taken to the House

of Lords (and in this connection it may be well to point out that the decision given in the Queen's Bench Division as reported *J.I.A.*, xxvii, 32, was eventually overruled), with the result that it was held that the company being a mutual one the whole surplus or profits was not liable to tax, but only so much thereof as arose from non-participating policies, annuities, &c., that is from transactions with persons not members of the Society. In the second case the profits of the British branch were held liable to tax on the ground that the company in question was not a purely mutual one, and that the facts that its share capital was very small in proportion to its operations, and that the shareholders only drew thereon a comparatively small dividend, did not suffice to take it out of the category of proprietary companies.

It would seem, therefore, that if at any time, through legislation or otherwise, it became necessary to consider the liability to taxation of a life office from the "profits" point of view instead of the "interest" point of view, or if in any case the annual profits, or share of valuation surplus, exceeded the taxable interest so that tax were demanded on the difference, something might turn on the question of whether the life office was a mutual or a proprietary one. It must not, however, be forgotten that in *New York v. Styles* the House of Lords' decision, overruling the previous decisions, was not unanimous, and "abstract justice" would, in my opinion, be opposed to the contention that the policyholders' share of the surplus of a proprietary office should be subject to taxation while the surplus of a mutual office should be exempt. It is to be both expected and hoped that, if occasion should arise, means would be found to ensure the taxation of the two classes of offices on similar bases. So long, however, as the decision, as I believe the right decision, in *Clerical, Medical and General v. Carter* holds, and the circumstances of offices in general remain as they are, the question is not of great practical importance.

It may be pointed out that the present method of taxation, that, namely, of charging tax on interest and not on profits, presses in reality more hardly upon a proprietary life office than upon a mutual office. There is a certain amount of force in the contention that, the life assurance fund being formed by the accumulation of the premiums paid by the assured for their policies, the interest on that fund should be subject to tax in the same way as would have been the case if the assured, instead of paying premiums, had invested equal sums in mortgages or stocks

and shares, and the fact that with a view to encourage life assurance the State has thought it desirable to grant an abatement of income in respect of premiums hardly seems fair grounds for contending that interest on premiums should escape taxation, and thus secure an additional preferential treatment when compared with other forms of savings. But the shareholders in a life office are in a different position to the assured. They are members of a corporation trading for profit, and there does not seem to be any real reason for their being treated differently as regards income tax from shareholders in any other trading company. The true measure of their liability to tax would appear to be the profits which they make, or in other words (assuming no profit or loss on the investments representing the shareholders' capital) the interest earned by their capital plus the annual sum, or the share of the quinquennial sum, transferred to their account from the surplus that would have arisen in the life assurance fund had no income tax been payable on interest earned by that fund. Thus an office having a shareholders' capital of £100,000 earning £4,000 a year interest, and an assurance fund of £2,000,000 earning £80,000 a year interest now pays tax on £84,000 interest. Suppose that on an annual valuation the declared profits are £50,000, one-tenth of which goes to the shareholders. If income tax had not been payable on the interest of the assurance fund the surplus would of course have been larger, say, taking tax as 1s., £54,000. Even if nine-tenths of the £80,000 be held liable to tax as belonging to the policyholders, why should the shareholders who are merely trading for profit be called upon to pay in effect tax upon £12,000 (namely, $\frac{1}{10}$ of £80,000 + £4,000), while the whole profit they have made by their trading is represented before charging any tax at all by £9,400 (namely, $\frac{1}{10}$ of £54,000 + £4,000)? Even if a policyholder is to be taxed on interest, why should a shareholder be taxed in respect of anything more than the profits he makes?

So far consideration has been given only to the case of a life office. Where the company concurrently with its life assurance business carries on fire or marine assurance business also, the position taken up by the Revenue Authorities is, it is understood, a different one. Notwithstanding the decision in *Clerical, Medical and General v. Carter*, such offices, I gather, continue to be taxed on the lines laid down in *Last v. London Assurance Corporation*, and *Scottish Union and National v. Smiles*. They are assessed on a profit basis, namely, on the profits of their fire or other business

plus the annual (or proper proportion of the quinquennial) surplus of their life business, but are allowed to deduct, from the total, interest on which tax has already been paid. It follows that they are apparently in a distinctly better position in all ordinary circumstances than a purely life office, whether mutual or proprietary, as the effect is that on their life business as well as on their other branches they pay tax on profits only instead of on the whole interest received, unless in any particular year an assessment on the interest basis would yield better results for the revenue than an assessment on the profits basis, in which case the authorities would, it is apprehended, have the right to elect to charge under the fourth case of Schedule D instead of under the first case.

Why an insurance office transacting life and other business should be treated for income tax purposes more favourably as regards its life business than an office transacting life business only is not apparent to me. It may fairly be anticipated that sooner or later the taxation of both classes of office will be placed on the same basis, and, while from one point of view it may be hoped that the "profits" basis should be made applicable to both classes, it is I think rather to be expected that uniformity will be secured by the alternative process.

The difficulty which must be felt by anyone acquainted with life assurance business in contending that life offices should be taxed on the "profits" basis, or, in other words, that the annual (or share of quinquennial) surplus should form the measure of taxation, arises from the fact that a life office valuation on modern lines is conducted not merely with the object of disclosing profits at the present time, but also with the object of ensuring as far as may be a continuance of a not widely dissimilar rate of profit in the future. If the "profits" basis were employed we should, I imagine, soon have an energetic revenue official testing the case of a strengthening of the valuation basis, and the consequent withdrawal from the surplus and absorption by the reserves of a perhaps considerable sum. Even where no increase in stringency of basis takes place, it has always seemed to me difficult to contend that a valuation at a very low rate of interest, however desirable that may be both as a measure of extra precaution against future possibilities and also as a means of maintaining future bonuses, is a proper one for the ascertainment of taxable profit. As an alternative we should probably be driven to two valuations, one for the ascertainment of divisible surplus and the other for the

ascertainment of taxable surplus, and conducted on what may be briefly described as a "solvency" basis. The dangers, to say nothing of the inconveniences, of such a course will at once be apparent, and it may be doubted whether it is not well to be content with taxation on the interest basis lest a worse thing befall.

CONCLUSION.

In the Budget discussion of this year Mr. Ritchie, the then Chancellor of the Exchequer, stated that in his view certain of the provisions of the Income Tax Acts and practice required amendment, and promised that a Commission should be appointed to consider the subject. Whether in the present state of political affairs that Commission will ever actually come into being seems somewhat problematical, but if it should do so life offices may not improbably desire that the question of their liability to tax should be considered. In the event of any opportunity being given for a statement of their views it will of course be for the representatives of the offices, together with the various professional bodies, to determine what suggestions should be put forward, and in particular whether any attempt should be made to abolish taxation on an interest basis and substitute taxation on a profits basis. The different treatment now accorded to a purely life office and a fire and life office is of course a strong point to be urged in this connection, and I have no doubt that the reasonableness of both being taxed on the same basis would be apparent to the Commissioners. This would not, however, necessarily imply recourse to a profits basis in both cases, as the Commissioners might consider it fairer to tax a fire and life office on the interest basis as regards its life fund, and on a profits basis as regards its other trading. In any event it is to be hoped that all offices, whether proprietary or mutual, and whether life only or fire and life, would be able to agree to some uniform course of procedure in the event of its being possible to bring the subject of their taxation before the Commission.

In my own personal view it is improbable that taxation on the interest basis will be got rid of, and I should imagine it more likely to be extended.

It is to be hoped that the Commission will be able to do away with the different methods of charging tax on a change of rate and substitute a uniform procedure therefor.

I would also suggest that the taxation of interest as it affects the shareholders in proprietary life offices is one worthy of the attention of such a Commission.

POSTSCRIPT.—Judgment was given in the House of Lords on the 15 March 1904, in the case of *Hunter v. The Attorney General on behalf of His Majesty the King*, referred to in the preceding paper, but not therein discussed at length owing to the appeal to the Lords being pending at the date when the paper was read. The appeal was dismissed, one of the Lords, out of six who heard the appeal, dissenting. The result is that the law is settled that an assured under what may be shortly described as a “half-credit” policy can bring into account for income tax purposes—even where there is a personal liability on him for payment of the portion of premium not actually paid in cash, and, *à fortiori*, where there is no such liability—only such part of the premium as he actually pays in cash. It is understood that should he, later on, pay off in cash the arrears of premiums constituting the debt on the policy he could then bring such arrears into account, subject to the usual restrictions. The case is not yet fully reported, but a remark by Lord Robertson, implying that the real question to be ascertained was by how much the Appellant’s income had been diminished in respect of his assurance, may be placed on record. Taking all the Courts together, two Judges have been in favour of allowing the deduction of the full premium, and eight against allowing it.

ABSTRACT OF THE DISCUSSION.

Mr. A. D. BESANT thought that anyone who had carefully studied the paper would feel a deep debt of gratitude to the author for having epitomized the leading features of the income tax laws, and the chief decisions that had been given in the Courts in regard to their bearing upon assurance companies. The subject was a most difficult and confusing one, and hitherto the materials for studying it as a whole had not been easily available, but the paper gave exactly what was wanted, and, in addition, a most valuable record of the existing condition of the law. In the few remarks he had to make he would follow the author in dealing first of all with some points of detail, and then with the main question as to what basis should be used for assessing the tax. Turning first to page 300, where the author dealt with the allowance on a single premium, although the word used in the Act was “annual”, he could confirm what Mr. Faulks had said, because in his own office he had had a correspondence with the Board of Inland Revenue, who admitted that abatement might be claimed in respect of a single premium, provided, of course, that the abatement did not exceed one-sixth of the whole year’s income. Turning next to a much more important question, the author dealt in section (d) with the subject of the abatement on “half-credit” or discounted-bonus policies. He felt a certain amount of diffidence in

dealing with the question, on account of the case of *Hunter v. the King* being pending in the House of Lords; but he thought it might be interesting to the meeting if he (Mr. Besant) mentioned that in the office with which he had the honour to be connected they had issued discounted-bonus policies for 10 or 12 years, where only 75 per-cent. of the ordinary with-profit premiums were paid in cash and the odd 25 per-cent. of the premium remained as a debt. At the outset the income tax authorities only allowed abatement in respect of the reduced premiums, 75 per-cent., and said that when a bonus was declared they would deal with the question of the other 25 per-cent., but would not decide anything definitely at the time. In 1897, when the first bonus was declared upon the policies, the company claimed abatement on behalf of their assured in reference to the past five quarter premiums which had been paid up by the declaration of the bonus. First of all the authorities said "No you cannot have any abatement at all in respect of those past payments." A long correspondence ensued, and after nearly a year the company managed to secure abatement on the whole. They could only get the abatement on what was paid in cash year by year; but when the five deferred instalments were discharged by the application of a bonus, then the abatement allowance was claimed in respect of them in that particular year. That decision held good until 1902, when one of the policyholders made the usual claim. A certificate was sent to him stating that the payment had been actually made, and the Board of Inland Revenue absolutely refused to allow anything at all. The company therefore took the question up once more, but "*Hunter v. the King*" had then gone into the first Court and nothing could be done one way or the other: the Inland Revenue authorities would not allow the abatement, which they had allowed for five years before; and as "*Hunter v. the King*" was still going on they remained in the unfortunate position of being unable to tell the policyholders on what basis they could claim abatement. As a matter of fact, he believed surveyors in different parts of the country dealt with the matter in different ways, some allowing it on the full tabular amounts, but most of them only allowing it on the 75 per-cent. actually paid. He thought possibly other members present might be able to shed some more light on the question, and be able to render help; but he did not expect, until "*Hunter v. the King*" was finally settled that any definite decision would be given. A very interesting point was the general question of the ethics of the abatement system, as to whether it should be allowed at all. Mr. Bailey many years ago set out the objections on that point very fully; but, whatever the objections might have been then, they had been enormously strengthened since by the increasing number of endowment assurances which were now effected, more especially as so many of the endowment assurances were for short terms of years. He wished to mention that, as far as he could see, while the abatement system held, the life assurance companies of this country enjoyed a measure of protection which no other British industry did. As to the main question of the basis upon which income tax ought to be assessed, he thought they ought to clear their minds very definitely as to what was the law of the land upon the one side, and what might be their

own individual opinions of what was fair and equitable on the other. In regard to the law of the land, he thought the author had set it out so fully that he could not say much more about it, but he thought he might epitomize it. In the case of "*Last v. the London Assurance Corporation*", he thought the general practice seemed to have been settled that in the case of a mixed company the profits were to be the basis of the taxation; and, of course, if the profits were less than the interest income of the company then the interest became the basis. Then in the case of the "*Clerical, Medical and General v. Carter*" it was decided that interest *quâ* interest was taxable. Putting those same facts in other words, he thought they had, as the result of those two judgments, a definite assurance that, in the first place, all interest was taxable, and secondly, that if the total surplus or profits of the company exceeded the interest, then tax must be also paid upon such excess. That seemed to be the law as it stood to-day. He had not overlooked the fact that the decision in the case of "*New York v. Styles*" might possibly have modified this statement of the law in the case of a mutual company, but he must confess to a great feeling of doubt in his own mind as to whether a mutual company—simply because it was a mutual company—would be allowed to escape on that account from any taxation to which on all other accounts it would have been liable. He now came to the general question; was it fair, was it equitable? On the whole, while the existing laws of income tax held, he must say he felt inclined to feel that it was. The argument was somewhat difficult to put, and he thought perhaps he could make his meaning more clear if he took a definite concrete example of a somewhat extreme nature. Supposing a life insurance company invested the whole of its funds in purchasing, say, an electric lighting company or some trading concern, then at the end of the year the balance sheet of that company would show a certain amount of profit resulting from the trading of the year, and the Government would naturally—and, as he thought, equitably—claim tax upon that amount, which, in the ordinary way, would be divided in the form of dividends. He could not see what avail it would be to argue against the Government that, because the life insurance company employed such dividends for its own purposes in its own business, the Government should thereby be debarred from getting as much income tax as it would otherwise have had. Putting the case in general terms, the mere payment of interest to anybody implied the making of interest profits liable as such to taxation, and the subsequent employment of such interest, it seemed to him, did not affect the question. He thought most present would agree with what the author mentioned near the end of the paper, that perhaps it would be better to submit to the evils they had in the taxation of their interest less worse things befell them. Even Mr. Newbatt, although he argued so sturdily that the surplus should be the basis of taxation, gave them a most solemn warning that even if they won that contention in the Courts they were still open to attack, because new legislation might be at once brought into force which would destroy the whole advantage which had been purchased with so much difficulty by legal action.

Mr. A. H. BAILEY said he had read the paper with great interest, because it gave a summary of the decisions of various Courts on different points. The real question which concerned the Institute was that if the Commission mentioned in the paper was eventually formed, the members of the Institute should frame some opinion of their own to lay before the Commission, as to what changes should be made. One matter which had particularly struck him during his long experience was the absurdity of allowing an abatement of the premium. He thought the Institute might obtain some statistics of claims as to the amounts that were paid to the families of the persons whose lives were assured and the amount that was paid to creditors. If those figures were obtained for two or three years, he thought it would be of great advantage. Assignments for marriage settlements must be distinguished from others; but, speaking from an experience of fifty years, he believed it would be found that more money was paid to creditors than to the families of persons whose lives were insured. If those figures were obtainable, it would be a strong argument for showing that the absurdity of allowing the deductions should cease, and perhaps the Government might be disposed to carry his suggestion out, because it would really increase their revenue. The curious part of the matter was, that whereas they allowed the deduction from the premium a man paid, as soon as he was dead all his capital was taxed under the Estates Duties Act. He thought what was urgently needed was a change in the different rates of tax that were levied on the different schedules. It was not right that schedules D and E should be paid at the same rate as schedules A and C, and reform was urgently needed in that respect. Scientific finance was eminently the profession of actuaries, and he thought they might do a great deal if they considered the matter, and placed before the public a more just method of levying direct taxes.

Mr. J. B. GILLISON said he had recently taken out a few particulars in relation to the Income Tax Acts of the different Australian States, and he thought some of those States had attempted to solve one or two of the difficulties mentioned. On page 298 the clause of the Act in regard to the abatement of life insurance premiums was given, and the companies were specified whose policyholders were entitled to that abatement. Unfortunately, Colonial companies were not included. In reference to abatements, he found that in the State of New South Wales there was an abatement allowed up to £50, if the premiums for insurance were on the man's own life, or on the life of his wife, or for a deferred annuity, or for other provision for wife and children, or for a fidelity or guarantee bond. In the State of Victoria there was an abatement up to £50 allowed from the taxable income of the amount of the premiums paid on any insurance on his own life for his own benefit, or for the benefit of his wife and children, with any company carrying on the business of life assurance in Victoria. In Queensland there was an abatement up to £50 in respect of any policy of insurance on his own life or the life of a child, or for the benefit of wife and children. In South Australia, so far as he could learn, there was no abatement; in Tasmania there was no abatement; and in Western Australia

there was no income tax. He wished to draw attention, first of all, to the fact that there was no distinction made in the three first-mentioned States in regard to English companies. There were several English companies which did business in Australia, or rather, he might say, had done business in the past, and they had a large number of policies in existence, but it was of no consequence to the Government there whether the policies were with an English, American, or Colonial company: the man who paid the premium received (for the purpose of encouraging thrift) the abatement; and if the system of abatement was continued in this country that was the principle he would like to see adopted. In several of the States, until quite recently, mutual life assurance companies were entirely exempt from taxation by way of income tax. In the State of New South Wales they were exempt in every way, except as regards income from mortgages; and he had no doubt there were certain cases there where, if they were taxed upon profits, it would be larger than the interest on the mortgages. In the State of Victoria until 1903 they were entirely exempt from all taxation; they were also exempt from putting receipt stamps on the receipts; indeed, in every way the mutual life insurance companies had been treated as being friendly societies for the benefit of the people. The average sum assured was comparatively small, roughly about £240 or £250 per policy, and it was recognized by the Legislature that life offices were institutions which deserved some consideration, and they were therefore practically exempted from taxation in any shape or form. But, in consequence of the desire to obtain a little more money for the revenue, in 1903 life insurance companies were compelled to pay a tax under the Income Tax Act. The maximum tax, the produce of property, was 1s. 4d. in the £, but the life insurance companies were charged 1s. in the £ on 30 per-cent of their premium income—evidently an endeavour to obtain by a rough and ready method the amount of profit supposed to be earned; that was to say, it was practically assumed that the profits of the companies would be 30 per-cent of their premium income, and they were taxed at the rate of 1s. per £ on it. In Queensland the incomes of mutual life insurance companies were exempted. In South Australia mutual life insurance companies paid on their periodical surplus. It was estimated by the actuary from year to year between one investigation and another, and when the investigation took place, and the actual amount was ascertained, an adjustment was made. In Tasmania, the life insurance companies paid 1s. in the £ on 20 per-cent of their premium income. In Western Australia—this might perhaps not be considered an income tax—the life assurance companies paid 1 per-cent on the premiums received. With regard to the rates of tax, in New South Wales the rate of tax was 6d. in the £ all round, with an abatement of £200. In Victoria, the tax was divided under two heads—income from personal exertion, and income produced from property. The tax started under the heading “Income from Personal Exertion” at 4d. in the £ up to £500, with an abatement of £100; from £500 to £1,000 it was 5d.; up to £1,500 it was 6d.; up to £2,000, 7d.; and over £2,000, 8d. in the £. Income which was produced from property was charged double the above-mentioned

rates. That was in 1903—since then he believed some reduction had been made on income from personal exertion. Incomes not exceeding £125 were exempted. In Queensland there was an income tax on individuals up to £100 a year of 10s., up to £150 of £1, above £150 £1 for the £150, and on the remainder 6d. per £ from personal exertion and 1s. in the £ from property. In South Australia, from personal exertion, up to £800 a year it was 4½d., above £800 a year 7d.; from property up to £800 it was 9d., and above £800, 1s. 1½d. There was an abatement of £135 up to £400. In Tasmania there was a tax of 6d. from personal exertion and 1s. from property. In Western Australia there was no tax. In New Zealand income tax was levied on all incomes above £300 at the rate of 6d. in the £ on the first taxable £1,000, and 1s. in the £ over £1,000. Life assurance premiums up to £50 allowed as abatement. Life assurance companies did not appear to be liable for income tax on either interest or profits, but they paid a license fee and graduated land tax on mortgages and landed property.

Mr. J. DOUGLAS WATSON said he would confine himself to a point connected with the abatements allowed for life premiums. He was surprised to see in the paper that the author suggested that these abatements had been allowed as a means of encouraging life assurance. He had always understood from the works of economists such as Mill and McCulloch that the abatements were made with a view, if possible, to meeting the difference in the quality of incomes arising from personal exertions, and from invested capital. He had therefore taken the trouble to look at the debates which took place when the tax was originally introduced, and he found the question was mainly debated on the alleged necessity for distinguishing between these two kinds of incomes. In 1798, he did not find any reference to life assurance, but subsequently in 1842, when the income tax was again imposed, he found that one of the speakers, Sir William James, made the following remarks: "A graduated scale would be liable to objection, though he thought some allowance ought to be made in the case of incomes arising from professions. Suppose the case of a man with £400 a year arising from land, he had not the same anxiety for making a provision for his family as a professional man. Suppose the latter also had an income of £400, but out of that he paid £60, £70 or £100 as an insurance upon his life, in his opinion that portion of his income ought to be exempted from taxation." He did not find anywhere in the debates any sign of a desire on the part of the Legislature to benefit life assurance, but rather of an attempt by allowing these abatements to equalize these two sources of income, or in other words to make, by taxation, two different classes of people equal, which personally he considered an unsound use of fiscal impositions.

Mr. C. D. HIGHAM thought it might be of interest to the meeting if—carefully avoiding any argument as to a matter *sub judice*—he mentioned the broad fact that was being fought in "Hunter v. the King", for the case had been practically narrowed to the point whether, if a man found cash for a portion of his premium, and borrowed the rest, he had "paid" the whole premium or not. The Crown maintained that he had not if he had borrowed

part from the office; admitting, it was understood, that if he had borrowed from his banker, or father, or some other insurance company, he had "paid" it, but not if he had borrowed from the office itself, at any rate if on the policy in question. Everything had been done to get the hearing in the House of Lords expedited, but he had been informed recently that it could not come on before February. The decision did not concern any one company, or any special policies, but all the many occasions on which a man borrowed part of a premium from the company which had issued the policy. Mr. Besant had spoken of protection. He did not think it could be called protection because the premiums of American life offices were not allowed the rebates in respect of income tax, for the authorities made the distinction that the whole of the interest of those companies had not been subject to tax here; and he agreed with Mr. Watson, that these allowances were not because of any desire to encourage life assurance. He had previously mentioned to the Institute his own theory that personal incomes under Schedules D and E ought to have one-sixth knocked off before taxation, just as with the rateable value of a house, there was a one-sixth deduction for repairs; and he was then in correspondence with the Chancellor of the Exchequer on the subject, so that whatever was done, or not done, the Chancellor would know what the views of some of them were. With regard to the relative amounts paid to assignees and to the family, Mr. Bailey would agree with him that that depended very much on the office. He was very glad the author had stated the advantages as to the tax which offices with fire and life and other businesses, obtained as against offices for life assurance only. He wished the authorities would decide to tax profit, because, if that was done, he thought they would be able to prove that a mutual life office did not make any profit at all except on any non-participating business. If a man gave his clerk £50, out of which to pay his premium, and told him to bring back the balance, he did not call that balance profit, but change; and when a man was charged a premium, with avowedly a portion to come back, that was not profit either. There was much to be said for the author's position of being content with the present arrangement for fear of getting something worse, but, if there were a change, they would make a brave fight to show that mutual life assurance did not make profit at all.

Mr. H. W. MANLY wished to refer to one point mentioned in the paper in connection with the question of interest obtained from bankers on deposits. The bankers themselves did not deduct income tax from the interest which they paid, nor when they received any interest did they allow the income tax off it. It being the business of bankers to deal with money they were excluded, so far as their business was concerned, from the Income Tax Acts, which seemed to be quite right. The point raised was the following. His company had received interest from money deposited with the bankers, from which income tax was not deducted; on the other hand, they had paid interest upon loans obtained from the bankers, which interest was paid in full. He was asked to make a return of the amount of interest which had been paid to the office on which

income tax had not previously been deducted, and he claimed to set against that the interest which he had paid in full, but was informed by the Income Tax Surveyor that he could not do that. He then saw the Inspector upon the matter, who pointed out to him the clause in the Act, which said that the income tax must be paid on all the interest received in full *without any deduction whatever*. The Inspector added that the next time there was an alteration in the Act perhaps they might get the authorities to take a different view of the matter, but for the time being he could not allow the deduction.

Mr. A. LEVINE thought that if the Commission in regard to the amendment of the Income Tax Acts was appointed, it might be desirable for the Institute to point out to them a method by which the Inland Revenue authorities could considerably increase the present income obtainable from the income tax. A very large portion of the income tax which was retained by borrowers from life offices never got to the Inland Revenue Office at all. For instance, income tax retained by policyholders, with incomes below £160, on the interest of a large number of small policy loans was neither received in the life office nor by the Crown; and if, therefore, so far as regards the interest on the mortgages, and on policy loans, and other loans made by life offices, the Inland Revenue were to change the method of collection, so as to make the life office receive the interest in full, and at the end of each year account for the income tax, he thought the Inland Revenue would gain considerably by the change.

Mr. J. H. BARNES expressed surprise at the statement made by Mr. Manly with reference to the action of the Commissioners in the case referred to, because he knew that another Commissioner living not very many hundred yards away took a different view of the Act, and applied for the tax upon the amount of interest received in excess of the amount that was paid. He did not know that in the particular case mentioned a rebate was ever obtained when the amounts paid to and fro went the other way; but there seemed every reason why the interest that was paid to the bankers should be allowed to be deducted from that received when reckoning the tax. The policyholder, for example, was allowed to deduct his income tax in paying the premium to the office, and the provisions of the Act which allowed that deduction should, in the same way, allow the life office to deduct the income tax from the payment made to the banker. For certain reasons that course was not found convenient, but the tax should be recoverable by the payer of interest, who was obviously as much entitled to deduct the tax from his interest payment as any other borrower.

Mr. H. E. W. LUTT said that, reverting to Mr. Bailey's idea as to the deduction of the tax allowed on life assurance premiums, it seemed to him that whatever was the origin of the allowance it was generally now regarded as an encouragement to thrift. If the tax was taken on the premiums throughout, when the sum assured was paid at death, if the Government wished to encourage thrift they might possibly allow estate duty at a smaller rate on the policy which was payable to the estate of the assured, and then all the trouble connected with assignees would be obviated.

Mr. F. B. WYATT hoped that a suggestion made by Mr. Faulks near the end of the paper would not be allowed to drop—namely, “In any event it is to be hoped that all offices, whether proprietary or mutual, and whether life only, or fire and life, would be able to agree to some uniform course of procedure in the event of its being possible to bring the subject of their taxation before the Commission.” He had heard that the Commission referred to was not likely to be shelved altogether, and therefore hoped the Council would bear the subject in mind, and be prepared to get, if possible, delegates appointed to the Commission, so that the views of insurance companies could be properly expressed. There were some matters which the author had not touched upon in regard to income tax. The first was the fact that income tax was supposed to be payable in advance. Merchants and traders were called upon to make estimates of their income, but he thought the country should not have to depend too much on the honesty of the trader. The country lost many thousands of pounds by people making estimates based on the average of the previous five years. If the Government could get rid of the plan whereby income tax was payable in advance, and make it payable on the actual fact, it would be a great improvement. The trader at the end of the year should make a declaration as to what his profits were. If he had not made any profit, but had lost, he would be allowed to carry his account on to future years. That was a mere suggestion, which, he admitted, was rather a big one. The next point he wished to refer to was to a change in the rate of income tax. When the rate changed, the Chancellor of the Exchequer should suggest some good working principle, because the Inland Revenue authorities did not agree. He would mention a case to illustrate his meaning. There were two local authorities, one in the North and one in the South, dealing with his company. The company deducted income tax in accordance with the instructions issued, but the local authorities took opposite views. The company forwarded copies of the instruction which had been received from the Inland Revenue, and one of the local authorities sent it back with a copy of a letter received from the Department exactly stultifying what the printed order said. His company had been in the habit of borrowing money from their bankers nearly every year, they had also large amounts on deposit, and he had filled in the return as follows: The amount of interest received from the bankers was so much; the amount they had paid on a temporary advance was so much; the balance was so much, on which they were assessable, and that had always been accepted. During the year he had had the difficulty that the interest paid on loans greatly exceeded the interest received from deposits, so he made a note on the return that not only was nothing payable, but the company were entitled to a refund of income tax, and that the company applied for it. The surveyor replied that he was very glad the company were going to do so, but that he, as surveyor, had no instructions or authority to make any return.

Mr. T. G. ACKLAND thought the tendency of the paper was rather to take the legal view, and to support the legal decisions—no doubt most properly—and frequently in the direction that would be least favourable to the actuary and the insurance companies. With regard

to the abatement on premiums, he supposed it might be understood, although the author did not tell them so specifically, that Government assurances shared with other assurances in the premiums being subject to abatement. He could confirm the remark that an accident policy, including payment in the event of fatal accident, had been admitted, though rather reluctantly, as subject to the abatement, and also that an endowment on a child's life had not been so admitted, after some little correspondence and discussion. He believed also that, in the case of endowment assurances, the authorities held, and no doubt rightly, that the element of whole-life assurance in the combined assurance was alone subject to the abatement, and that the premium had to be divided up for this purpose. With regard to single premiums, it appeared to him that, from the actuary's point of view, it was inequitable that the single premium, paid in one sum in a particular year, should be subject to the condition that it must not exceed one-sixth of the taxable income of the person paying it. He should have thought that equitably the single premium should be distributed over lifetime, and the annual equivalent accepted if within the assigned limit. The difficult point of annuities was dealt with very fully by the author, who had laid before the members very clearly the distinction in the case of terminable annuities and life annuities. He could not see why the payments of a life annuity should not be subject to the same division into capital and interest in certain cases, as a term annuity. They all knew, for example, that there were cases—although perhaps they did not very frequently arise—where a private speculator bought a life interest of a fixed uniform sum per annum, and actually purchased a life policy with a portion of the annuity, to recoup his capital upon the early death of the life tenant, and that would surely be regarded as a return of capital in the strictest sense of the word. He was surprised to find the remark in the paper as to the attitude of the Revenue authorities in reference to foreign annuitants, and he would like the author to tell them his authority for the statement. According to the paper it was the opinion of the Inland Revenue authorities that the tax was to be deducted against the annuitant, whether he resided within or without the United Kingdom. He was under the impression that, consequent upon well-known cases in which questions of annuities were raised, it had been decided by the Revenue authorities, without the Courts being asked to adjudicate upon the matter, that the question of foreign annuitants was not to be so raised. It certainly seemed to be inequitable that a company paying large annuities, in Italy, for instance, where the annuitant was subject to a considerable tax, approximately amounting to 20 per-cent of his income as returned, should have to pay a tax on account of the foreign annuitants which, in the nature of things, it would be quite impossible to recover; and he was under the impression that the Crown did not press that view; but perhaps some change of attitude had recently taken place. With reference to foreign investments, the author said in the paper: "Speaking broadly, the effect of the various decisions is that an English company carrying on business abroad, either directly or by means of a sub-company, is liable to be assessed for income tax." This passage read as if the

author were giving the broad decision referring equally to a life assurance company and a trading company, but he imagined that the paragraph could refer only to a trading company, because, taken more widely, it did not appear to be consistent with what followed. With regard to the general question, and what had been rightly called the great decisions in "*Last v. London Assurance Corporation*", and "*Clerical, Medical and General v. Carter*", he thought they must not come to the conclusion that the methods followed in assessment in those cases were entirely in opposition to one another, and in this he thought Mr. Besant in his opening remarks agreed with him. In the case of "*Last v. London Assurance Corporation*", it was only true in a sense that the assessment was on profits. On the figures that were produced in that case, it turned out, after adding together the combined profits of the fire, life, and marine offices, that the interest which had already suffered tax by deduction exceeded the total amount of those profits, so that the company did in the end pay on interest, and presumably (unless the relative figures were since altered) was still paying the tax on an interest basis. In the case of a company where the combined profits exceeded the interest, that would not be the case, but he imagined that such a case would not very frequently arise, and that as a rule the taxation might be said to be on an interest basis, rather than on a profit basis. On the general question of the taxability of interest, he regretted that the case of "*Clerical, Medical v. Carter*", was not carried to the House of Lords. He felt that there was a resulting weakness in the position of the assurance companies relative to income tax, and although in the view of the author and others interest as such was taxable, and although no doubt there was a charging section in the Act of 1853 which was, he supposed, the crux of the whole question, one would have liked to have seen the question carried forward to appeal, bearing in mind that the House of Lords had been disposed to take such a practical and commonsense, and, on the whole, favourable, view of the position of life assurance companies relative to income tax. He thought there was immense force in the argument in the paper, as to the position of the shareholder in reference to that matter. It seemed to be clear from the argument, and from the figures that were set forth in that part of the paper, that the shareholder in a life assurance company was treated inequitably under the present system of taxing interest as such. The intention of the original Income Tax Acts was to tax profit incomes; and interest as such, and annuities as such, should consistently only be subject to taxation so far as they were of the nature of profit income, and if by imperfect drafting or otherwise the wording of the later Acts led to legal decisions in an adverse sense, it was unfortunate, and he thought inequitable, for the assurance companies and their shareholders.

Mr. FAULKS, in reply, said he thought he only had four points to reply to. Mr. Ackland referred to the expression "company" on page 311. He meant a trading company, including a fire company; for instance, as shown by the case of the Norwich Union Fire Insurance Company, which was referred to in the same paragraph. Further, he thought that in the case of an English life office

transacting business abroad, as distinct from investing abroad, if the company were to be assessed under the first case of Schedule D the rule would apply. With regard to the statement on page 307, as to the tax on annuities, he was afraid he could not say offhand where he obtained the statement that the opinion of the Inland Revenue authorities was that an office was bound to deduct tax against an annuitant, even when he resided outside the United Kingdom. He had several notes bearing on the subject, and it came from one of those; certainly subsequent in date to the introduction of the Act of 1888, which first made it compulsory on offices to deduct tax. Mr. Wyatt had said there were several other points in the income tax law which might have been dealt with. He had thought of referring to one or two of those, more particularly perhaps to one question which Mr. Wyatt touched upon indirectly—the question of the three years' average. That was a very curious thing. For instance, it had a considerable bearing on the tax on salaries. The tax on the salaries of the officials of life assurance companies was assessed on the year's salary, because they were supposed to be holding a public office, but he did not believe that was ever the intention of the Act. A public office was, he thought, intended to mean only an office in the Civil Service. The following curious result arose—not in a life insurance office, but in a bank. The officials of a private bank and the officials of a joint-stock bank were taxed as regards their salaries on an entirely different basis. Why they should be he could not say. One was, or should be, taxed on the average of three years; and the other on the salary for the particular year, which seemed to him very anomalous. The only other point was the remark made by Mr. Higham, that he would prefer that offices should be taxed on the profit basis, and that he hoped then to be able to go on to show that a mutual office did not make a profit. He thought that would be a very good thing, provided it were admitted also that, so far as regarded the life assurance fund of a proprietary office, that life assurance fund could make no profit; otherwise a distinction between proprietary and mutual offices would at once arise, and that was one of the things, in his opinion, which they ought to keep as much as possible in the background. The proprietors' share in the surplus might be taxed, but the policyholders' share of the surplus in a proprietary office should not be subjected to taxation, if the whole surplus in the case of a mutual office were exempt. (Mr. Higham signified concurrence in this view).

On the Graduation of the Rates of Sickness and Mortality Presented by the Experience of the Manchester Unity of Oddfellows during the period 1893-97. By JOHN SPENCER, F.I.A., Assistant Actuary of the English and Scottish Law Life Assurance Association.

[NOTE.—It has been suggested that the references to the subject of graduation contained on pages 62-3 of the Account of the Manchester Unity Experience, 1893-97, should be amplified, in order to deal with the methods of graduation at greater length than would have been appropriate in a work intended primarily for non-actuarial readers. The following account has accordingly been prepared by Mr. SPENCER, whose valued collaboration in this section of the work I am glad to have the opportunity of adequately acknowledging.—A. W. WATSON.]

I.—RATES OF SICKNESS.

IN considering the various methods of graduation open to us, we were impressed, having regard to the large number of series to be dealt with, by the great facility and rapidity with which formulæ of summation can be applied with the certainty that the results obtained will conform closely to the unadjusted facts. For the Sickness Tables the summation method is especially suitable, since the average number of weeks (or rate) of sickness at any age is necessarily separable for practical purposes into "periods of attack", *e.g.*, in the case of the Manchester Unity Experience, 1893-97, "First 3 Months", "Second 3 Months", "Second 6 Months", "Second 12 Months," and "After 2 years", the successive values of each of which must form a regular series, whilst the sum of any two sections forming a "period of attack" in general use (*e.g.*, First 6 Months=First 3 Months+Second 3 Months) must be equally regular in progression. It is obvious that if a summation method be adopted, *and the same formula be used throughout*, a combination of any two or more of the graduated series will be itself a graduated series, and will, indeed, be identical with the result of an independent graduation of the combined unadjusted rates.

A graphic process independently applied to each sectional series of rates would doubtless produce entirely satisfactory results as regards that series, but that the result of a combination of two or more series would be equally satisfactory could hardly be expected. Examination of the various sectional curves did

not suggest the existence of any well-known law which would have facilitated the work of graduation, although in a Memoir* contributed to *Biometrika*, I have shown that from age 20 to age 80 the "All Periods" rate of Sickness in the important A. H. J. Group (Normal Occupations) closely follows a curve of the Makeham type, *i.e.*, the Force of Sickness at age x may be expressed in the form $A + Be^x$.

It was decided that the summation formula to be adopted for the Sickness Tables should have as restricted a range as possible, subject to its yielding a sufficient degree of smoothness in the graduated results, and the following, involving 15 terms, was employed :

$$\begin{aligned} u'_0 &= \frac{S_{5,4,4}}{320} (-3u_{-2} + 3u_{-1} + 4u_0 + 3u_{+1} - 3u_{+2}) \\ &= \frac{1}{320} \{ 74u_0 + 67(u_{-1} + u_{+1}) + 46(u_{-2} + u_{+2}) + 21(u_{-3} + u_{+3}) \\ &\quad + 3(u_{-4} + u_{+4}) - 5(u_{-5} + u_{+5}) - 6(u_{-6} + u_{+6}) - 3(u_{-7} + u_{+7}) \} \end{aligned}$$

N.B.—The symbol $S_{5,4,4}$ may be read, first, sum the series in fives; second, sum these results in fours; and third, again sum these results in fours.

This formula may, in point of smoothness, be compared with Mr. J. A. Higham's formula,† though the range of the latter is greater, 17 terms against 15.

The working process is as follows:—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Ungraduated Rate of Sickness	Divide by 8 = u	Sum in 3's	$u_{-2} + u_{+2}$	$(3) - (4)$	(5) multiplied by 3	$(6) + (2)$	Sum in 5's	Divide by 40	Sum in 4's	Sum in = Graduated Rate

As regards the ends of the Table, it may be mentioned that throughout the rule adopted was to insert average rates at ages below 16. At the concluding ages the ungraduated rates tended to a constant, and at the point at which by reason of this the graduated series became unsatisfactory the average ungraduated rate was substituted for each age up to the limit. This further adjustment was only required as a rule at the ages over 90.

* Graduation of a Sickness Table by Makeham's Hypothesis, (*Biometrika*, vol. iii, p. 52).

† *J.I.A.* xxiv, 46.

The validity of the formula for the purpose to which it was applied is attested by the following abstracts of the "Expected" and "Actual" Weeks of Sickness:—

ABSTRACT "A."

"Whole Society" Sickness Rates, "All Periods" of Sickness combined.

Ages	Expected weeks of Sickness by Graduated Rates	Actual weeks of Sickness	Expected less Actual
16- 29	933,059	929,861	+ 3,198
30- 39	927,348	927,532	- 184
40- 49	1,001,619	1,001,806	- 187
50- 59	1,244,160	1,245,873	- 1,713
60- 69	1,369,410	1,366,345	+ 3,065
70- 79	1,215,861	1,216,835	- 974
80- 89	323,311	323,790	- 479
90-100	10,325	10,356	- 31
	7,025,093	7,022,398	+ 2,695

ABSTRACT "B."

Differences between Actual and Expected Weeks of Sickness in each period of attack ("Whole Society" rates).

Ages	First 3 months	Second 3 months	Second 6 months	Second 12 months	After 2 years
16- 29	+ 1,086	+ 1,251	- 87	+ 1,299	- 351
30- 39	- 39	- 117	- 1,394	+ 940	+ 426
40- 49	+ 1,128	- 1,387	+ 452	- 393	+ 13
50- 59	+ 408	- 291	- 348	- 437	- 1,045
60- 69	+ 23	- 610	+ 288	+ 1,308	+ 2,056
70- 79	+ 256	+ 67	+ 85	- 622	- 760
80-100	+ 16	- 56	- 80	- 402	+ 12
	+ 2,878	- 1,143	- 1,084	+ 1,693	+ 351
Total Actual Weeks of Sickness	2,741,303	646,694	620,003	658,291	2,356,103

The ungraduated rates of Sickness exhibited in the case of the E.F. Group of Occupations ran somewhat irregularly, and in this class it was found desirable to employ a more powerful summation formula than that used in the graduation of the other Sickness Tables. The formula chosen, which, in point of

smoothness, compares with Mr. G. F. Hardy's Friendly Society formula,* though the latter involves 17 terms only as against 19 in the present case, is here given :

$$\begin{aligned}
 u'_0 &= \frac{S_{7:5:5}}{175} (-u_{-2} + 3u_0 - u_{+2}) \\
 &= \frac{1}{175} [31u_0 + 29(u_{-1} + u_{+1}) + 24(u_{-2} + u_{+2}) + 17(u_{-3} + u_{+3}) \\
 &\quad + 8(u_{-4} + u_{+4}) + 2(u_{-5} + u_{+5}) - (u_{-6} + u_{+6}) \\
 &\quad - 3(u_{-7} + u_{+7}) - 3(u_{-8} + u_{+8}) - (u_{-9} + u_{+9})]
 \end{aligned}$$

This formula leads to a very simple and rapid working process, as follows :

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ungraduated Rate	Divide by 7 = " "	(2) multiplied by 3	$u_{-2} + u_{+2}$	(3) - (4)	Sum in 7's	Multiply by 4 cutting down	Sum in 5's	Sum in 5's = Graduated Rate

II.—MORTALITY TABLES.

The only objections to the summation method in the case of aggregate Mortality Experiences such as those with which we had to deal are :

1. The inapplicability, strictly speaking, of the process at the beginning and the end of the Table, owing to the range of the formula employed ; and
2. The circumstance (applying to most of the summation formulæ hitherto used, that the graduated values of the function selected for treatment do not present such a smooth curve as is obtained in the case of a table based on a mathematical law of mortality, or by a graphic process.

The objection last mentioned applies especially to Woolhouse's formula and, in a lesser degree, to that suggested by Mr. J. A. Higham, though one or two other formulæ which have been suggested, notably Mr. G. F. Hardy's Friendly Society formula,

* *J.I.A.* xxvii, 277.

and the process described by Dr. Karup in his paper appearing in the Transactions of the International Congress of Actuaries in London in 1898, show a considerable improvement in this respect.

With the view, however, of finding a formula which should produce even smoother results than have hitherto been possible in a mechanical graduation, investigation was made by the writer on the lines suggested in Mr. G. F. Hardy's paper on Graduation Formulæ (*J.I.A.*, xxxii, 371), a valuable contribution to the literature of the subject of Graduation by summation. In addition to those described above a number of formulæ were obtained which do not appear to have been previously given, the one eventually chosen for graduation of the Mortality Tables being as follows :

$$u'_0 = \frac{1}{350} \{ 60u_0 + 57(u_{-1} + u_{+1}) + 47(u_{-2} + u_{+2}) + 33(u_{-3} + u_{+3}) \\ + 18(u_{-4} + u_{+4}) + 6(u_{-5} + u_{+5}) - 2(u_{-6} + u_{+6}) - 5(u_{-7} + u_{+7}) \\ - 5(u_{-8} + u_{+8}) - 3(u_{-9} + u_{+9}) - (u_{-10} + u_{+10}) \}$$

This formula, which is correct to third differences and involves 21 terms, is believed to be much more powerful than any hitherto given, and it is not perhaps too much to claim that, in point of smoothness of the results obtained, it compares very favourably with the Graphic method, this circumstance and the simplicity of the working process marking it out as specially suitable for use in graduating Tables such as those with which we had to deal. The range of the formula is somewhat greater than in the case of several of the formulæ previously employed, a fact accounting to some extent for its greater power. Superiority as regards smoothness is not, however, a necessary consequence of wider range.

The formula may be written as follows :

$$u'_0 = \frac{S_{7.5.5}}{350} (-u_{-3} + u_{-1} + 2u_0 + u_{+1} - u_{+3})$$

The following example of the working process may be of interest :

21-Term Formula. Illustration of Working Process.

Age (x)	(1) Ungraduated q_x	(2) Divide by 7 = u	(3) Sum in 3's	(4) (2) + (3)	(5) $u_{-3} + u_{+3}$	(6) (4) - (5)	(7) Sum in 7's	(8) Divide by 5	(9) Sum in 5's	(10) Graduated q_x = Sum in 5's cutting down as far as necessary
20	00431	62
1	00409	58	181	239
2	00429	61	179	240
3	00422	60	197	257	128	129
4	00530	76	208	284	129	155
25	00505	72	214	286	136	150
6	00459	66	209	275	140	135	997	199
7	00499	71	212	283	160	123	1,034	207
8	00526	75	226	301	157	144	1,043	209	1,055	...
9	00563	80	239	319	158	161	1,069	214	1,101	...
30	00587	84	249	333	167	166	1,130	226	1,154	00582
1	00595	85	261	346	182	164	1,223	245	1,220	00614
2	00647	92	273	365	189	176	1,302	260	1,294	00648
3	00669	96	295	391	195	196	1,375	275	1,370	00682
4	00746	107	312	419	203	216	1,439	288	1,438	00716
35	00760	109	327	436	213	223	1,508	302	1,501	00749
6	00778	111	338	449	215	234	1,566	313	1,559	...
7	00828	118	350	468	238	230	1,616	323	1,620	...
8	00846	121	358	479	246	233	1,667	333
9	00836	119	371	490	256	234	1,745	349
40	00916	131	387	518	272	246
1	00956	137	413	550	282	267
2	01014	145	436	581	280	301
3	01076	154	461	615
4	01134	162	477	639
45	01124	161

The rate of mortality was the function selected for graduation, the adoption of q_x instead of l_x having the effect of reducing the labour involved. Mr. Woolhouse claimed that his formula should be applied to the numbers living, but there seems no practical objection to the values of q_x being used. Indeed Dr. Karup came to the conclusion (Trans. 2nd Int. Act. Congress, p. 95) after careful investigation in a particular case, that the employment of q_x as the function to be graduated led to better results than the use of the values of l_x .

Special consideration had to be given to the treatment of the ends of each table. In certain cases at the beginning of the table it was found desirable to substitute for the unadjusted rates, which, owing to paucity of data, often ran irregularly, average rates obtained by grouping the observations for several consecutive ages. Moreover, in order to arrive at graduated rates from age 16, this

being the commencing age in the published tables, it was necessary to insert hypothetical rates of mortality for ages 6 to 15. It was usually found that the adoption at these ages of a uniform value of q_x , corresponding with or slightly less than that used at age 16, led to satisfactory results, but when this was not the case a simple graphic process was employed to graduate the rates at the initial ages, a junction being made with the principal curve at some convenient point.

As regards the concluding ages, striking irregularities, due to lack of data, were removed and appropriate alternative rates substituted, while, in order to utilize the formula up to the final age, a series of 10 hypothetical rates was inserted, commencing with unity at the limiting age and proceeding roughly by second differences to 4 or 5 at an age ten years beyond. This procedure is of course quite arbitrary, but reference to the graduated rates in the various tables at the concluding ages will show that satisfactory results were arrived at.

It may be mentioned that, in applying a summation formula to graduate rates of mortality, Dr. Karup has suggested the adoption of unity as the quantity to be inserted at and above the limiting age. This leads in the graduated results to rates of mortality proceeding by diminishing first differences, but while such rates may be found to exist at ages over 100 in the case of a table based on Makeham's Law (which of course places no theoretical limit to the duration of life), assuming a very large radix to have been taken, their progress is unlike that of the rates shown at the final ages in most published tables, and the suggested alternative process appears to lead to results more in accordance with probability. For the purpose of graduation by summation the successive values of q_x may be treated as a series of numbers without particular reference to their meaning when inserted in the Mortality Table, and from this point of view there is no objection to using terms of a value exceeding unity.

The result of the application of the formula above given to the "Whole Society" Experience may be summarized as follows :—

Ages	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation
16-29	4,501	4,517	-16	-16
30-39	5,153	5,130	+23	+7
40-49	6,060	6,072	-12	-5
50-59	7,666	7,674	-8	-13
60-69	7,553	7,536	+17	+4
70-79	6,049	6,036	+13	+17
80-89	2,013	2,035	-22	-5
90 & over	93	93	0	-5
	39,058	39,093	-5	-5

APPENDIX.

Graduation of the O^M Table by the 21-Term Formula.

In the preceding description of the process by which the Mortality Tables were graduated, the statement has been made that the results arrived at compare very favourably, as regards smoothness, with those obtained by the use of the graphic method, and that in this respect the new formula is believed to be more powerful than any summation formula previously used for graduation. It may be of interest to apply the formula to graduate some well-known table on the lines mentioned above. For this purpose we require a table in the graduation of which a summation formula is applicable, and, one, which, at the same time, has already been graduated by some other process, in such manner as to provide a stringent test of smoothness and reliability. Obviously, however, a table based upon a mathematical Law of Mortality, such as Makeham's, would not furnish a valid standard, since the perfect smoothness of the curve, in a table of this kind, can hardly be ascribed to the skill of the graduator. The conditions laid down are admirably met by the O^M Table, of which, fortunately, we have a graduation by a master of the subject. In this case, as Mr. G. F. Hardy says, it was necessary to abandon Makeham's formula pure and simple, and "the only object in view was to obtain a perfectly smooth curve, and to represent as nearly as possible the ungraduated facts."

I have graduated the O^M unadjusted rates of mortality by the new process, and give in the following table the graduated rates of mortality for ages 10 to 79, together with the differences of the q_x column, in order that the smoothness of the tables may be seen. The table has not been continued beyond age 79,

since from that point the official graduation is practically a Makeham curve.

It may be stated that the actual ungraduated rates of Mortality have been used throughout for purposes of the new graduation, with the exception that at all ages below 15 the rate used was .00327, this being the average rate deduced from the observations for ages 10 to 14.

O^M TABLE.

Age	GRADUATION BY 21-TERM FORMULA		MR. G. F. HARDY'S GRADUATION		Age	GRADUATION BY 21-TERM FORMULA		MR. G. F. HARDY'S GRADUATION	
	q_x	$\Delta q_x \times 10^5$	q_x	$\Delta q_x \times 10^5$		q_x	$\Delta q_x \times 10^5$	q_x	$\Delta q_x \times 10^5$
10	.00328	1	.00338	3	45	.01157	57	.01153	60
11	.00329	3	.00341	4	46	.01214	61	.01213	64
12	.00332	4	.00345	5	47	.01275	69	.01277	68
13	.00336	6	.00350	4	48	.01344	76	.01345	77
14	.00342	8	.00354	6	49	.01420	85	.01422	82
15	.00350	9	.00360	7	50	.01505	94	.01504	91
16	.00359	10	.00367	8	51	.01599	102	.01595	98
17	.00369	11	.00375	8	52	.01701	110	.01693	106
18	.00380	13	.00383	9	53	.01811	117	.01799	119
19	.00393	14	.00392	12	54	.01928	124	.01918	127
20	.00407	15	.00404	12	55	.02052	132	.02045	139
21	.00422	15	.00416	15	56	.02184	143	.02184	154
22	.00437	15	.00431	15	57	.02327	154	.02338	167
23	.00452	15	.00446	17	58	.02481	170	.02505	184
24	.00467	14	.00463	18	59	.02651	190	.02689	198
25	.00481	15	.00481	19	60	.02841	214	.02887	218
26	.00496	17	.00500	23	61	.03055	240	.03105	239
27	.00513	19	.00523	21	62	.03295	267	.03344	259
28	.00532	23	.00544	25	63	.03562	296	.03603	283
29	.00555	27	.00569	26	64	.03858	324	.03886	310
30	.00582	32	.00595	25	65	.04182	352	.04196	336
31	.00614	34	.00620	28	66	.04534	381	.04532	363
32	.00648	34	.00648	29	67	.04915	410	.04900	399
33	.00682	34	.00677	29	68	.05325	445	.05299	436
34	.00716	33	.00706	32	69	.05770	477	.05735	472
35	.00749	31	.00738	33	70	.06247	516	.06207	516
36	.00780	32	.00771	33	71	.06763	556	.06723	558
37	.00812	32	.00804	34	72	.07319	606	.07281	611
38	.00844	36	.00838	39	73	.07925	655	.07892	656
39	.00880	38	.00877	38	74	.08580	714	.08548	716
40	.00918	42	.00915	41	75	.09294	783	.09264	779
41	.00960	45	.00956	45	76	.10077	865	.10043	839
42	.01005	48	.01001	47	77	.10942	942	.10882	913
43	.01053	50	.01048	51	78	.11884	1,023	.11795	987
44	.01103	54	.01099	54	79	.1290712782	...

It will be conceded that in point of smoothness the new graduated rates leave little to be desired, even as compared with Mr. Hardy's exacting standard.

The comparison of Expected and Actual Deaths in the case of the new graduation is also eminently satisfactory, as will be seen from the following table:

O^M TABLE.*Comparison of Expected and Actual Deaths.*

Age Group	GRADUATION BY 21-TERM FORMULA				MR. G. F. HARDY'S GRADUATION	
	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation	Deviation	Accumulated Deviation
10-14	10	10	0	0	+ 1	+ 1
15-19	98	97	+ 1	+ 1	- 3	- 2
20-24	785	806	- 21	-20	- 40	- 42
25-29	2,642	2,615	+ 27	+ 7	+ 72	+ 30
30-34	5,196	5,202	- 6	+ 1	0	+ 30
35-39	7,589	7,557	+ 32	+33	- 19	+ 11
40-44	9,623	9,731	-108	-75	-131	-120
45-49	11,607	11,526	+ 81	+ 6	+ 85	- 35
50-54	13,707	13,670	+ 37	+43	- 19	- 54
55-59	15,542	15,594	- 52	- 9	+ 26	- 28
60-64	17,044	17,093	- 49	-58	+175	+147
65-69	17,719	17,677	+ 42	-16	+ 2	+149
70-74	16,117	16,150	- 33	-49	-117	+ 32
75-79	12,241	12,197	+ 44	- 5	- 27	+ 5

In view of the satisfactory character of the new O^M graduation, and of the equally good results obtained in the case of the Manchester Unity Mortality Tables, it is, I think, clear that, failing the applicability of Makeham's Hypothesis, the method now suggested is specially suitable for adoption when one has to deal with an aggregate experience showing a well-distributed set of observations, and it would appear to render unnecessary in such a case the skilled labour involved in the use of the graphic method. As regards smoothness of results obtained, the process could hardly be greatly improved upon, while, like most other summation formulæ employed in practice, it may be relied on automatically to reproduce within very narrow limits the actual deaths recorded. There may, in some cases, be a small amount of error in this last respect, but this would be quite insignificant as compared with the total number of deaths, and would have no appreciable effect on monetary values. Moreover, the total deviation could, without impairing the smoothness of the curve, easily be got rid of, as was done by Mr. G. F. Hardy in graduating the recently published British Offices Tables.

By the same process of reasoning that leads to equation i, (*Text-Book*, part II, chapter xix, art. 3), we can find the value of a life interest to allow for income tax on the interest portion only.

For an annuity of $\frac{P_x}{1-t} + d$, a purchaser could give the vendor $v - P_x$, and pay the first premium, P_x , to the insurance office. His total investment would be v , and the sum assured would be 1. At the end of each year he would receive, after deduction of tax, $(\frac{P_x}{1-t} + d)(1-t)$, or $P_x + d(1-t)$, and this would supply him with the premium, P_x , in full, then due, and with his interest, less tax, $d(1-t)$. For an annuity of 1 gross, or $(1-t)$ net of tax, he can therefore give the vendor

$$\begin{aligned} a' &= \frac{v - P_x}{\frac{P_x}{1-t} + d} = \frac{1 - (P_x + d)}{\frac{P_x}{1-t} + d} \\ &= \frac{1 - (\frac{P_x}{1-t} + d)}{\frac{P_x}{1-t} + d} + \frac{tP_x}{\frac{P_x}{1-t} + d} \\ &= \frac{1}{\frac{P_x}{1-t} + d} - 1 + \frac{tP_x}{P_x + (1-t)d} \quad \dots \dots \dots (ii) \end{aligned}$$

The total investment will be $\frac{v}{\frac{P_x}{1-t} + d}$, and the sum assured will

be $\frac{1}{\frac{P_x}{1-t} + d}$. The annual premium will be $\frac{P_x}{\frac{P_x}{1-t} + d}$, and the annual

interest, less tax, will be $\frac{d(1-t)}{\frac{P_x}{1-t} + d}$. These two together are

$\frac{P_x + d(1-t)}{\frac{P_x}{1-t} + d} = (1-t)$, which is the net income assumed above.

Thus, by formula ii, the purchaser will receive at the end of each year during the life of the life tenant, the premium in full then due, and the year's interest, less tax, on his investment. The

sum assured being $\frac{1}{P_x + d}$, it will return the investment

$\frac{P_x}{1-t} + d$ on the death of the life tenant, and provide $\frac{d}{\frac{P_x}{1-t} + d}$,

the interest in full for the year of death. The purchaser must therefore account to the Inland Revenue authorities for the tax on the interest for the last year.

The last term in expression ii is very small. For instance, if we assume an effective premium of £3 per-cent, remunerative interest at 5 per-cent, and income tax at 1s in the £, expression ii gives 11·647 as the value of the life interest, towards which the last term contributes only ·01994, or ·171 per-cent. Without sensible loss of accuracy we may therefore write

$$a' = \frac{1}{\frac{P_x}{1-t} + d} - 1 \quad \dots \dots \dots \text{(iii)}$$

It will be seen, therefore, that to find the value of a life interest under the conditions named, we have merely to enter a conversion table inversely with $\frac{P_x}{1-t}$, instead of with P_x .

The foregoing investigation applies equally to annuities-certain. We have

$$a_n = \frac{1}{P_{n+1} + d} - 1 \quad \dots \dots \dots \text{(iv)}$$

where P_{n+1} is the sinking fund to be invested at the beginning of each year, to redeem a debt of 1 at the end of $(n+1)$ years, or, in other words, the premium for a leasehold redemption policy maturing in $(n+1)$ years. To allow for income-tax we have merely to write

$$a'_n = \frac{1}{\frac{P_{n+1}}{1-t} + d} - 1 \quad \dots \dots \dots \text{(v)}$$

It is true that the usual custom is, when a rent-charge or an annuity-certain is purchased, to include in the conveyance a schedule separating the annual payments into principal and interest, and for the purchaser to allow income-tax only on the interest portion. But circumstances might arise under which even an

annuity-certain would be subject to deduction of income-tax on its whole amount, and then formula v would be applicable.

The only difficulty in using formulas iii and v is, to fix the value of t . In former times it was not unusual, in calculations of various kinds, to assume an average rate of 1s. in the £, which was thought to be on the safe side, but at the present day the safety of that course would appear to be doubtful.

*"The possible Association of the Consumption of Alcohol with Excessive Mortality from Cancer." * By ARTHUR NEWSHOLME, M.D., F.R.C.P. LOND., Medical Officer of Health of Brighton.*

PART II of Dr. Tatham's decennial supplement to the 55th report of the Registrar-General, published in 1897, contained extremely valuable statistics relating to the relative death-rates and what are known as the "comparative mortality figures" of men engaged in different occupations. These statistics dealt not only with deaths from all causes in conjunction, but also from certain diseases; and the latter figures throw important light upon the influence of occupation on the mortality, for instance, from tuberculosis and cancer.

In commenting on these figures in 1899¹ I drew attention to the fact that the comparatively low mortality figure for medical men (43) and other instances which were adduced did not lend support to the theory that excessive mental strain and anxious work provoked cancer. The following remarks were added: "The low cancer figure for coal miners (38) indicates that they enjoy a relative immunity from cancer as well as from phthisis. Can any reasonable hypothesis be framed to explain why the manipulation of coal underground should be so much less irritating than its manipulation overground? (coal-heavers' comparative figure 56). *Can it be that we have to look to intemperate habits as being a main factor at work rather than the particular occupation?* The figures for commercial travellers (63), coachmen and grooms (58), seamen (60), brewers (70), innkeepers (53), ditto in London (70), and butchers (57) certainly point in this direction. The figures for chimney sweepers (156) appear, however, to stand in a separate and independent category."

My attention was again drawn to the same suspicion at a later date, and in October 1902, I communicated with Mr. T. P. Whittaker, M.P., the Managing Director of the United Kingdom Temperance and General Provident Institution (spoken of throughout this paper as the Institution), and through his kindness and that of Mr. R. M. Moore, the Actuary of the Institution, the figures summarized in Tables I and II came into my hands in November 1902. After working out the death-rates from cancer in standard populations for abstainers and non-abstainers (from alcohol) respectively, and finding that the former were startlingly lower than the latter, I postponed publishing the results, first, because of paucity of data, and secondly,

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because at the time it appeared impracticable to eliminate a fallacy arising from the unknown proportion of males and females in the sections of the Institution under comparison.

The second source of error can now be practically dismissed in the light of Mr. Moore's paper read before the Institute of Actuaries on 30 November 1903.² This paper gives the experience, so far as deaths from all causes are concerned, during the years 1841-1901 of 864,953 male lives at risk, of whom 398,010 were abstainers, and 466,943 non-abstainers; while the corresponding number of females exposed to risk in 1841-1901 were 34,013 abstainers and 67,056 non-abstainers. It may be added that of the females at risk, 19,777 abstainers and 42,495 non-abstainers were over 45 years of age (that is, of the special cancer ages).

The figures supplied to me gave the number of lives at risk in each of three successive quinary valuation years, 1891, 1896, and 1901; and the number of deaths from all causes, and copies of the death certificates from malignant disease were also supplied for the same years. In each set of figures ages were given, and in the mortality figures sex was also indicated.

From these data Table I has been prepared. It will be noted that the abstainers' section is somewhat larger than the non-abstainers' section; but if 45 and over be taken as the special cancer age, then the number of years of life at risk is 27,411 in the abstainers' and 29,968 in the non-abstainers' section.

The last two columns in Table I deal with deaths from cancer. The total number of deaths from cancer given in this table is 125. But the total number of deaths from cancer in the death returns for these three years was 146. The 21 excluded are deaths of females from cancer. This exclusion obviously needs justification. Either exclusion or inclusion is a possible source of error, as the number of female lives at risk in these three years, classified according to age, was unascertainable. Now, during 1841-1901 the proportion of male to female lives at risk was as 8 or 9 to 1; the proportion of abstainers to non-abstainers was among men as 398 is to 467, among women as 34 is to 67. It is, I think, fair to assume that the same proportions of men to women and of abstaining to non-abstaining women holds approximately good for the figures in Table I. Among the women included in the lives at risk in this table 21 deaths from cancer occurred. It is well known that the registered mortality from cancer is much higher among women than among men. Thus in 1901 the death-rate from cancer per million living aged over 35 years, corrected for differences in the age-constitution of the respective populations, was 2.12 among men and 2.90 among women in England and Wales.³ Consequently, if the 21 deaths from cancer among women were left in Table I without a corresponding statement (unobtainable) of the female lives at risk a considerable error would arise. On the other hand, by leaving the female lives at risk out of the same table the death-rate among men in the two sections will be understated. This is of minor importance if the proportion between the two sections is not disturbed. Such freedom from disturbance cannot be secured; but it can be secured that the disturbance shall influence results towards a conclusion opposed to that which will be eventually reached.

The distribution of the 21 female deaths from cancer was 3 in the abstainers' and 18 in the non-abstainers' section. But we have seen that the proportion of female abstainers' lives was to female non-abstainers' lives as one to two (34 to 67). It follows that by altogether excluding the female deaths from cancer, and leaving in the female lives, the abstainers' section is unfairly handicapped in comparing its experience respecting cancer with that of the non-abstainers' section.

The first source of error—that involved in the paucity of the data—will be discussed later.

TABLE I.

Ages of Insured Persons or of Persons Dying in each Group	AGGREGATE POPULATION OR YEARS OF LIFE AT RISK IN THE THREE YEARS— 1891, 1896, AND 1901		DEATHS DURING THE THREE YEARS 1891, 1896, AND 1901			
			From All Causes		From Malignant Disease in Males	
	Abstainers' section	Non- Abstainers' section	Abstainers' section	Non- Abstainers' section	Abstainers' section	Non- Abstainers' section
Under 24	1,490	321	4
25-34	5,926	2,980	13	20
35-44	10,184	7,055	44	52	1	4
45-54	11,884	10,764	120	149	14	12
55-64	9,182	10,882	170	283	13	25
65-74	4,912	6,500	227	399	13	33
75 & upwards	1,433	1,822	180	269	6	4
	45,011	40,324	758	1,172	47	78

From the data given in Table I it is desired to obtain the death-rate in each section from all causes and from cancer respectively. This can be stated in terms of the number living at each age-period 25—, 35—, 45—, &c. ; and this is the method adopted in Mr. Moore's paper referred to above. This method, however, does not give the *idée d'ensemble* secured by the method adopted in Table II, and is open to objection, as regards cancer, that the number of deaths at each age-period is too small to be free from accidental variations. In Table II the number of deaths occurring among the lives at risk at each age-period being given, the number that would occur in a standard million, aged 25 and upwards, is calculated. By this means death-rates in a standard population, aged 25 and upwards, are obtained, which are strictly comparable. The standard population chosen for this purpose is that of the English Life Table No. 3, Persons, in order that the results may be comparable with those given by Mr. George King, F.I.A., and the present writer.⁴ In Table II, as printed, only the cancer death-rates are given, but similar rates were calculated for deaths from all causes. *The death-rate from all causes in the three years 1891, 1896, and 1901 was 17·13 among the abstainers and 23·52 per 1,000 lives at risk among the non-abstainers;*

while the death-rate from malignant disease was 0·95 among the former and 1·34 per 1,000 among the latter. In other words, if the death-rate among non-abstainers in each instance be stated as 100, that of abstainers from all causes was 72·8, and from cancer was 70·9.

The italicized sentences give the main conclusion to be derived from Tables I and II. In discussing the validity of this conclusion we must revert to the already mentioned fallacy of paucity of data, and consider whether, apart from this, the abstainers' and non-abstainers' sections are truly comparable.

TABLE II.

Death-rate per Standard Million in (A) Abstainers' and (B) Non-Abstainers' Section of the United Kingdom and General Provident Institution from Cancer.

A.—ABSTAINERS' SECTION.							
Age	Number Lives at Risk	—	Number in a Standard Million of Population	—	Actual Number of Deaths from Cancer among the Lives at Risk	—	Calculated Number of Deaths from Cancer
25	5,926	is to	260,259	as	0	is to	0
35	10,184	„	232,106	„	1	„	22·8
45	11,884	„	199,912	„	14	„	235·5
55	9,182	„	158,812	„	13	„	224·8
65	4,912	„	102,196	„	13	„	270·5
75 & upwards	1,433	„	46,715	„	6	„	195·6
	43,521	„	1,000,000	„	47	„	919·2
B.—NON-ABSTAINERS' SECTION.							
25	2,980	is to	260,259	as	0	is to	0
35	7,055	„	232,106	„	4	„	131·6
45	10,764	„	199,912	„	12	„	222·9
55	10,882	„	158,812	„	25	„	364·9
65	6,500	„	102,196	„	33	„	518·8
75 & upwards	1,822	„	46,715	„	4	„	102·6
	40,003	„	1,000,000	„	78	„	1,340·8

The labour involved in extracting the death certificates was so great that returns for only three years could be obtained. These dealt with a population at risk of 85,335 persons, among the predominant male portion of whom 125 deaths from cancer occurred. It is possible that the next series of extracts would give somewhat different results. It is highly improbable that they would go far

towards transposing the two cancer death-rates. The trustworthiness of this small basis of cancer death-rates is confirmed by the corresponding death-rates for all causes. As already stated, Mr. Moore has compared death-rates from all causes among abstainers and non-abstainers for 60 years, including 864,953 male lives at risk. These are not given for standard populations as in Table II, but for each quinary age-period separately. Taking the ages 35 to 75 the ratio of the non-abstainers' to the abstainers' mortality varies from 100 to 54·8 up to 100 to 86·5. The proportion between the abstainers' and non-abstainers' general death-rate is, therefore, not very different from that during the three years in Table I, and it is reasonable to suppose that the same proportion would be likely to hold good for cancer if this could be tested. The figures as to cancer, however, indicate the necessity for further investigation on the same lines, and do not justify any dogmatic conclusion until this further statistical investigation has been made. It should be noted, also, that the ratio of the abstainers' to the non-abstainers' death-rate is almost identical, whether the total death-rate or the death-rate from cancer be taken. It cannot therefore be claimed, assuming that more extensive data confirm those in Tables I and II, that alcohol has a greater influence in increasing the death-rate from cancer than in increasing that from the aggregate of other causes of death.

Such a remarkable difference in the corrected death-rate from all causes and from cancer among abstainers and non-abstainers respectively, suggests the desirability of ascertaining whether the data are untrustworthy, either by transfer of lives from one section of the Institution to the other, or because of other sources of error. From Mr. Moore's paper we learn (p. 215) :

"Persons are eligible for the temperance (abstainers') section who do not take alcohol as a beverage in any form. The continued adherence of the assured to the principles of abstinence is checked by an annual declaration to that effect . . . People who do not come within these conditions are eligible for the 'general section' only, and are herein described as 'non-abstainers.' If an abstainer cease to abstain he is transferred to the general section; and on the other hand, assurers in the general section who become abstainers are generally eligible for transfer to the temperance section. The effect of these 'transfers' on the mortality of the two sections respectively will be dealt with later on in this paper, but it may be here stated that such transfers are not made if the assurers be known to be in bad health or of intemperate habits, and transfers from the temperance section are not made in the cases of persons over 70 years of age. Moreover, the 'transfers' are few in number."

Mr. Moore adds: "The same care is exercised in admitting a new assurer into either section as regards family history and personal condition. But possibly if there were any unfavourable information on either of these points the case may have a better chance of acceptance if the life be an abstainer. It should thus be made clear that the temperance section has never been favoured or nursed with the view of securing more favourable mortality results, either by the admission of unexceptionable lives only in the first instance or by the removal or transfer of inferior or doubtful lives."

The same point is further dealt with on p. 238 *et seq.* of the same paper, in which Mr. Moore answers the suggestion that "when an abstainer's health deteriorates, if he takes alcohol, he is transferred to the non-abstainers' section to die, and his death is included in the latter section." On this point Mr. Moore says: "Now (a) an abstainer is allowed to take alcohol temporarily as a medicine *bonâ fide*, and (b) he is not transferred if he be known to be in bad health or of intemperate habits. (In the latter case (b) a record is made in the Institution's registers that he is to receive non-abstainer's rate of bonus only). But, apart from these facts, the transfers have been so few that their effect can be but insignificant."

Table XV in Mr. Moore's paper shows that during the period 1847-1901, only 2,685 transfers to non-abstainers and 1,465 to abstainers occurred. These form a very small percentage of the total number of lives at risk, and by experimental tables combining the experience of the transfers with that of the non-abstainers and of the abstainers respectively (Tables XVIII and XIX in his paper), Mr. Moore is able to establish satisfactorily that the superiority of the abstainers' over the non-abstainers' section has not been brought about by the operation of the "transfers", and that, in fact, this element in the problem may be dismissed from consideration.

Nor can it be said that the non-abstainers come from any stratum of society distinct from that supplying the abstainers. The average sum assured in the two groups is approximately equal throughout the whole experience.

There appears, therefore, to be good reason for accepting the above death-rates from cancer in the two sections as stating, subject to the remarks on paucity of data, the true relative position of the two, or at least as not understating the mortality among the abstainers. In view of the preceding remarks about the 21 deaths from cancer among women insured in the Institution, the relative mortality from cancer among the abstainers is possibly overstated.

It may be added that among the 125 deaths from cancer, 4 male deaths in the abstainers', 4 male deaths in the non-abstainers' section, and no female deaths, were certified to be due to sarcoma. These deaths have not been excluded, and it is likely that a number of other deaths from cancer were due to sarcoma and not to carcinoma. This pathological source of error is inherent in nearly all cancer statistics available, and in those contained in Table I like the rest. There is, however, no apparent reason why this error will differ in the two classes compared.

The cancer statistics of the Institution were further analyzed as to site of cancer; but the figures thus obtained did not indicate any special irritating effects of alcohol on the œsophagus or stomach. The influence of alcohol in favouring or provoking cancer, if any, must be chiefly an indirect one. The view suggested by the figures in Table I is supported by occupation statistics of the Registrar-General and by other figures tabulated in Germany. Evidently, however, if alcohol exerts an irritating effect, this is only one of a number of conditions favouring cancer, and alcohol cannot be elevated to the position of a primary cause. This is at present unknown.

The Directors of the United Kingdom Temperance and General

Provident Institution and of similar associations would be conferring a public boon if they gave facilities for the investigation of their experience as to cancer on a large scale.

REFERENCES.

- ¹ The Statistics of Cancer, *Practitioner*, April 1899, p. 379. ² *On the Comparative Experience among Assured Lives of Abstainers and Non-Abstainers from Alcoholic Beverages* by Mr. R. M. Moore (*J.I.A.*, vol. xxxviii, p. 213).
³ 64th Ann. Rep. of Registrar-General of England and Wales, p. lxii.
⁴ On the Alleged Increase of Cancer, by Geo. King, F.I.A., and A. Newsholme (*Proc. Royal Soc.*, vol. liv, and *J.I.A.*, vol. xxxvi, p. 129).

ACTUARIAL NOTE.

On Joint Life Annuities. By H. W. CURJEL, M.A.

ON pages 208 *et seq.* of the *Text-Book*, Part II, an approximate method of determining joint life annuities where at least one of the lives is under 28 is given, which yields good results down to age 15; accurate results for ages down to 10 can, however, be obtained with little labour by the help of the tables given below, which are calculated at 3 per-cent. The method is applicable to any table in which l_x is the sum or difference of two corresponding terms of two series which follow Makeham's Law, if the value of the constant c is the same in both (*vide J.I.A.*, vol. xxxii, p. 309).

In Mr. King's graduation of the H^M Table, let $l_x = \lambda_x^1 - \lambda_x^2$, where λ_x^1 = the number living according to the normal series and λ_x^2 = the number living according to the complementary series; then $\lambda_x^1 = ks^x g^{c^x}$ and $\lambda_x^2 = \kappa \sigma^x \gamma^{c^x}$, and we have

$$l_{xy} a_{xy} = \lambda_{x-y}^1 a_{x-y}^1 - \lambda_{x-y}^2 a_{x-y}^2 - \lambda_{x-y}^1 a_{x-y}^2 + \lambda_{x-y}^2 a_{x-y}^1$$

$$\text{or} \quad a_{xy} = \frac{\lambda_{x-y}^1}{l_{xy}} a_{x-y}^1 - \frac{\lambda_{x-y}^2}{l_{xy}} a_{x-y}^2 - \frac{\lambda_{x-y}^1}{l_{xy}} a_{x-y}^2 + \frac{\lambda_{x-y}^2}{l_{xy}} a_{x-y}^1$$

$$\text{Now, } x \text{ being } < y, a_{x-y}^1 \text{ will be } a_{z-x}^1 \text{ if } c^{z-x} = \frac{\log \gamma + c^{y-x} \log g}{\log g \gamma},$$

which is only slightly greater than unity, since $\log g$ is small compared with $\log \gamma$; hence $z-x$ will be small; in fact, if $y-x=50$, $z-x=1$. $z-x$ is tabulated in Table II.

Also $a_{x-y}^2 = a_{u-x}^2$, if $c^{u-x} = \frac{\log g + c^{y-x} \log \gamma}{\log g \gamma}$, which is very little less than c^{y-x} ; hence $y-u$ is very small, in fact, for the maximum value of $y-x (=19)$, $y-u$ is less than .01, and may therefore be ignored and we may write $u=y$. If w are the equal ages equivalent to xy as determined by the seniority table

(*J.I.A.*, xxxvi, p. 394; and *Text-Book*, Part II, p. 564), the formula becomes—

$$a_{xy} = \frac{\lambda_{xy}^{(1+1)}}{l_{xy}} a_{wz}^{(1)(1)} - \frac{\lambda_{xy}^{(2+1)}}{l_{xy}} a_{zz}^{(2+1)} - \frac{\lambda_{xy}^{(1+2)}}{l_{xy}} a_{yy}^{(1+2)} + \frac{\lambda_{xy}^{(2+2)}}{l_{xy}} a_{zz}^{(2+2)}$$

of which at least two terms vanish if y is greater than 28. In most cases z may be taken equal to x .

Example: If $x=10$ and $y=17$, $w=14\cdot05$ and $z=10\cdot01$

$$\begin{aligned} \text{then } a_{10;17} &= \frac{\lambda_{10}^{(1)}}{l_{10}} \cdot \frac{\lambda_{17}^{(1)}}{l_{17}} a_{14\cdot05;14\cdot05}^{(1)(1)} - \left(\frac{\lambda_{10}^{(1)}}{l_{10}} - 1 \right) \frac{\lambda_{17}^{(1)}}{l_{17}} \cdot a_{10\cdot01;10\cdot01}^{(2)} \\ &\quad - \frac{\lambda_{10}^{(1)}}{l_{10}} \left(\frac{\lambda_{17}^{(1)}}{l_{17}} - 1 \right) a_{17;17}^{(1)(2)} + \left(\frac{\lambda_{10}^{(1)}}{l_{10}} - 1 \right) \left(\frac{\lambda_{17}^{(1)}}{l_{17}} - 1 \right) a_{14\cdot05;14\cdot05}^{(2)(2)} \\ &= (1\cdot0291 \times 1\cdot0087 \times 19\cdot488) - (0\cdot291 \times 1\cdot0087 \times 4\cdot155) \\ &\quad - (1\cdot0291 \times 0\cdot0087 \times 1\cdot881) + (0\cdot0087 \times 0\cdot291 \times 1\cdot460) \\ &= 20\cdot230 - 12196 - 01684 + 0004 \\ &= 20\cdot092 \end{aligned}$$

In Table I are given $a_{xx}^{(1+2)}$, $a_{xx}^{(2+2)}$, $D_{xx}^{(1+2)}$, $N_{xx}^{(1+2)}$, $D_{xx}^{(2+2)}$, $N_{xx}^{(2+2)}$, as also $\frac{\lambda_x^{(1)}}{l_x}$, $D_{xx}^{(1)(1)}$, $N_{xx}^{(1)(1)}$, $a_{xx}^{(1)(1)}$, as far as age 29, from which point $N_{xx} (= N_{xx}^{(1)(1)})$, $D_{xx} (= D_{xx}^{(1)(1)})$, and $a_{xx} (= a_{xx}^{(1)(1)})$ are tabulated in the *Text-Book* and $l_x = \lambda_x^{(1)}$; it is unnecessary to give $\frac{\lambda_x^{(2)}}{l_x}$, which $= \frac{\lambda_x^{(1)}}{l_x} - 1$. The N's and D's are given with a view to the calculation of temporary annuities.

The British Offices Life Annuity Tables, 1893, furnish an example of tables to which the above method can be applied, for, if the indices M, (1), (2), are used to denote that the symbols refer to values for male lives (p. 1) and female lives, series (1) and (2) respectively (p. 43), the ratios $\beta_t^M : \beta_t^{(1)} : \beta_t^{(2)} (= c : 1 : 8\cdot91259)$ are independent of t . Hence, if

$$(\beta^M + \beta^{(1)})c^x = \beta^M c^x + \beta^{(1)} c^y \text{ or } \mu_z^M + \mu_z^{(1)} = \mu_x^M + \mu_y^{(1)}$$

$$\text{and } (\beta^M + \beta^{(2)})c^x = \beta^M c^x + \beta^{(2)} c^y \text{ or } \mu_u^M + \mu_u^{(2)} = \mu_x^M + \mu_y^{(2)},$$

we have for all values of t , including zero,

$$\begin{aligned} a_{[x]+t, [y]+t}^{\text{M F}} &= \frac{\lambda_{[y]+t}^{(1)}}{l_{[y]+t}^{\text{F}}} a_{[x]+t, [y]+t}^{\text{M (1)}} + \frac{\lambda_{[y]+t}^{(2)}}{l_{[y]+t}^{\text{F}}} a_{[x]+t, [y]+t}^{\text{M (2)}} \\ &= \frac{\lambda_{[y]+t}^{(1)}}{l_{[y]+t}^{\text{F}}} a_{[x]+t, [z]+t}^{\text{M (1)}} + \frac{\lambda_{[y]+t}^{(2)}}{l_{[y]+t}^{\text{F}}} a_{[x]+t, [u]+t}^{\text{M (2)}}; \end{aligned}$$

and if

$$(\beta^1 + \beta^2)c^u = \beta^1 c^x + \beta^2 c^y \text{ or } \mu_u^1 + \mu_u^2 = \mu_x^1 + \mu_y^2$$

and

$$(\beta^1 + \beta^2)c^v = \beta^2 c^x + \beta^1 c^y \text{ or } \mu_v^1 + \mu_v^2 = \mu_x^2 + \mu_y^1$$

and

$$2c^z = c^x + c^y \text{ or } 2\mu_z^1 = \mu_x^1 + \mu_y^2 \text{ or } 2\mu_z^2 = \mu_x^2 + \mu_y^1,$$

$$\text{or } 2\mu_z^F = \mu_x^F + \mu_y^F \text{ or } 2\mu_z^M = \mu_x^M + \mu_y^M$$

then in the same way

$$\begin{aligned} a_{[x]+t}^F \cdot a_{[y]+t}^F &= \frac{\lambda_{[x]+t}^{(1)} \cdot \lambda_{[y]+t}^{(1)}}{l_{[x]+t}^F \cdot l_{[y]+t}^F} a_{[z]+t}^{(1)} + \frac{\lambda_{[x]+t}^{(1)} \cdot \lambda_{[y]+t}^{(2)}}{l_{[x]+t}^F \cdot l_{[y]+t}^F} a_{[y]+t}^{(1)} \cdot a_{[z]+t}^{(2)} \\ &+ \frac{\lambda_{[x]+t}^{(2)} \cdot \lambda_{[y]+t}^{(1)}}{l_{[x]+t}^F \cdot l_{[y]+t}^F} a_{[x]+t}^{(2)} \cdot a_{[z]+t}^{(1)} + \frac{\lambda_{[x]+t}^{(2)} \cdot \lambda_{[y]+t}^{(2)}}{l_{[x]+t}^F \cdot l_{[y]+t}^F} a_{[z]+t}^{(2)} \cdot a_{[y]+t}^{(2)} \end{aligned}$$

By means of which, if the requisite functions were tabulated (it will be sufficient, in cases where two lives are involved, to give values for equal ages), joint life annuities for two lives could easily be calculated for any differences of ages. The formulas for more than two lives are obvious, but become cumbersome if more than two female lives are involved. Of course the fact that $\lambda^{(1)}$ increases with the age in the earlier part of the table does not make the method less applicable.

TABLE I.
H^M 3 per-cent. Text-Book Graduation.

x	$\frac{\lambda_x^{(1)}}{l_x}$	$a_{x,x}^{(1)(1)}$	$a_{x,x}^{(1)(2)}$	$a_{x,x}^{(2)(2)}$	$D_{x,x}^{(1)(1)}$ 100,000	$N_{x,x}^{(1)(1)}$ 1,000,000	$D_{x,x}^{(1)(2)}$ 10,000	$N_{x,x}^{(1)(2)}$ 10,000	$D_{x,x}^{(2)(2)}$	$N_{x,x}^{(2)(2)}$ 10
10	1.0291	20.047	4.159	2.667	78,797	157,970	22,252	92,553	6,283,900	1,675,970
11	1.0266	19.917	3.725	2.299	75,522	150,417	19,588	72,965	5,080,500	1,167,920
12	1.0238	19.782	3.333	1.982	72,380	143,179	16,838	56,127	3,917,000	776,218
13	1.0207	19.641	2.980	1.708	69,366	136,243	14,101	42,926	2,866,700	489,548
14	1.0176	19.496	2.662	1.470	66,472	129,596	11,477	30,549	1,981,600	291,388
15	1.0144	19.346	2.374	1.264	63,696	123,226	9,053	21,496	1,286,800	162,708
16	1.0114	19.191	2.115	1.085	61,031	117,123	6,901	14,595	780,330	84,675
17	1.0087	19.030	1.881	.929	58,474	111,276	5,066	9,528	439,010	40,774
18	1.0064	18.864	1.669	.792	56,019	105,674	3,569	5,958	227,480	18,026
19	1.0045	18.693	1.478	.673	53,662	100,308	2,404	3,554	107,720	7,254
20	1.0030	18.516	1.306	.569	51,399	95,168	1,541	2,012	46,219	2,632
21	1.0019	18.333	1.150	.479	49,226	90,245	.936	1,076	17,798	852
22	1.0011	18.144	1.010	.400	47,139	85,531	.535	.541	6,088	244
23	1.0006	17.950	.883	.332	45,135	81,018	.287	.253	1,829	61
24	1.0003	17.750	.768	.272	43,210	76,697	.143	.110	477	13
25	1.0002	17.543	.663	.221	41,361	72,560	.063	.043	106	2
26	1.0001	17.331	.563	.178	39,584	68,602	.028	.018	20	...
27	1.	17.112	.457	.140	37,876	64,814	.018	.018	3	...
28	1.	16.888	.312	.102	36,234	61,191	.007	.018
29	1.	16.657	34,656	57,726	.001

TABLE II.

For finding z so that $e^z \log \gamma = e^x \log \gamma + e^y \log g$.

$y - x$	$z - x$	$y - x$	$z - x$
1	·00	26	·11
2	·01	27	·12
3	·00	28	·13
4	·00	29	·14
5	·01	30	·16
6	·01	31	·17
7	·01	32	·19
8	·01	33	·21
9	·01	34	·23
10	·02	35	·26
11	·02	36	·28
12	·02	37	·31
13	·03	38	·34
14	·03	39	·37
15	·03	40	·41
16	·04	41	·45
17	·04	42	·49
18	·05	43	·54
19	·05	44	·59
20	·06	45	·64
21	·06	46	·70
22	·07	47	·77
23	·08	48	·84
24	·09	49	·92
25	·10	50	1·00

REVIEWS.

British Offices Life Tables, 1893.—An Account of the Principles and Methods adopted in the Compilation of the Data, the Graduation of the Experience, and the Construction of Deduced Tables. Assured Lives and Annuityants.

THIS volume, which completes the series proposed to be published by the Joint Committee, and thus brings to a conclusion the great work upon which they have been engaged for the last ten years, contains a detailed account by Mr. Ackland of the principles and methods adopted in the compilation and tabulation of the data, a memorandum by Mr. G. F. Hardy on the graduation of the Experience, and two shorter notes by Mr. Baker and Mr. Chatham on the processes employed in the construction of the monetary and other tables. The contributors to the volume owed it to themselves to render an account of their labours, for, without such an account, no one who had not been intimately associated with the work could have formed an adequate conception of the immense amount of time and thought they have devoted to it. At the same time, in discharging this debt to themselves, they have considerably increased the indebtedness of the profession to them, for the resulting volume is the most interesting and—from the educational point of view—the most valuable of the series.

In the introduction to the book the task of compiling and arranging the data is described as one of "great magnitude and delicacy." No student of Mr. Ackland's memorandum can fail to be impressed with the justice of this description. The memorandum will, in fact, be a revelation to anyone who may have supposed that all, or practically all, is said when it is stated that the Policy Year and Nearest Age at Entry method was followed, and that the withdrawals were dealt with by a modification of the nearest duration method. As the reader follows Mr. Ackland in his discussion of such questions as the methods of dealing with defective data, the determination of the average age at entry, the elimination of duplicates, the fractional exposures of withdrawals and terminations, the processes of scheduling and the treatment of the data of transferred companies, the multiplicity and complexity of the points both of principle and detail arising in an investigation of such magnitude become apparent, and the period that elapsed between the delivery of the cards at Staple Inn and the publication of the Unadjusted Experience ceases to be a matter for surprise. It is possible that a certain amount of work, which can now be seen to have been avoidable, may have been caused by the fact that the original instructions to the contributing companies were not sufficiently explicit, *e.g.*, in regard to the assumption to be made as to the dates of death of annuitants where the exact dates were unknown, the dates of cessation of risk to be given in cases of lapse, and the treatment of transferred risks; it seems probable also that a very considerable addition to the labour involved in eliminating duplicates and scheduling the data must have been entailed by the decision of the Joint Committee—an unfortunate decision as some may think in the light of subsequent events—to compile the Aggregate Tables on the basis of one case only on the same life at the same current age being retained instead of by combination of the Select Tables; but even if in these respects the investigation could have been expedited, there remain many points on which much time, thought, and labour must necessarily have been expended. Of these the most important were the investigation of the average age at entry, and the method to be adopted in dealing with the duration of the withdrawals. Mr. Ackland's discussion of these two points, with the relative appendices, constitute the most interesting and instructive part of his memorandum. As regards the age at entry, preliminary investigations were made for both the annuitants and the assured lives—based as regards the former on the whole of the data, and as regards the latter on the cases arising in decennial years of birth—with the result that the Nearest Age method (which, as is of course known, was that ultimately adopted for the entire Experience) was estimated to understate the true average age at entry by about 15 and 18 days respectively for male and female annuitants, and to overstate it to an extent varying from 5 to 13 days for different classes of assured lives, the estimated error for the Whole-Life males being approximately $8\frac{1}{2}$ days. As Mr. Lidstone pointed out recently, the Nearest Age method might be expected to overstate the true average entry-age of assured lives because the entrants at any given age next birthday are in general distributed over the year in an ascending curve, and for a similar reason it might be expected to

understate the average entry-age of annuitants. In the particular case under consideration the resulting errors appear to have been somewhat over-estimated owing to a uniform distribution over each month of entry having been assumed in the calculation of the average age (the large numbers of annuitants, for example, entering in the first month of the year of age having been credited with half a month each, which is probably too much), and their true values would consequently be even less than those given by Mr. Ackland. However this may be, the deviations are so trifling that although on what may be called the "wrong" side in each case they are quite unimportant. It may be noticed incidentally that the average interval between the age of purchasing an annuity and the age last birthday, although rather less than that between the age at effecting an assurance and the age next birthday, is not so much less—or rather was not so much less in 1863–1893—as might have been expected from the fact that annuities are purchased at so much older ages with consequent relatively greater differences between the rates for successive ages. The second of the two points mentioned above—that relating to the Fractional Exposure of Withdrawals—is of exceptional interest and importance. The course adopted was to schedule the withdrawals at durations 0–2, 2–6, 6–8 and 8–12 months respectively in separate groups (this appears to have been practically necessary owing to the different ways in which the companies had interpreted the instructions in regard to the days of grace, and it proved in the end to be a very convenient arrangement), to refer the first two groups and one-half the third to the beginning of the year, and the other half of the third with the fourth to the end, and then to transfer $\frac{1}{12}$ th of the whole from the beginning to the end, *i.e.*, in effect to give the first group 1 month's exposure, the third 7 months (with the result of approximating very closely to the actual duration of the yearly and half-yearly lapses), the second 1 month (or 3 months *less* than the central duration of the group) and the fourth 13 months (or 3 months *more* than its central duration). This method satisfies the essential condition of giving integral durations, and at the same time appears to have been convenient in application, and approximately accurate. It was adopted after exhaustive examination of an extensive body of experimental data, its applicability was confirmed *ex abundanti cautela* by investigation of three more excerpts from the Experience (summaries of all being given) and finally its *rationale* is convincingly and lucidly explained in Appendix M. A point of a rather different kind arose in connection with the maturing of endowment assurances, in respect of which—owing to the greater number of maturities occurring at the round-figure quinquennial ages—the Nearest Duration method would have had the effect of overstating the exposures in the year before these ages and overstating it in the year after. This difficulty was got over by transfer of certain proportions determined by experimental investigation from the one year to the other. It is interesting to notice here how the average interval between the age at maturing and the end of the policy-year, as shown by Appendix N, differs from the corresponding difference at inception, as shown by Appendix C—the difference being due presumably to the inclusion in the latter of endowment assurances maturing on the policy anniversary. Minor

points which may be just mentioned in connection with the compilation of the Unadjusted Experience are, the calculation of the select annuity-values for comparison with the Government Annuity-values—here the method adopted of interpolating between the values of a' does not appear quite happy on account of the fact that the successive values of p are all affected with different errors, and it would perhaps have been more satisfactory to calculate the q 's for central ages more accurately to start with (the whole plan of calculating central values of q by dividing grouped deaths by grouped exposures seems open to question)—the publication of specimen working sheets and cards (a most useful feature) and the use of Greek experience symbols (thus avoiding the double use of e and d). It may be noticed that the meaning attached to the expression “Mean Age Method” in Appendices I and C appears unusual, that O^M in the summary on p. xii should apparently be O^M and O^{NM} , also that the use of the symbol \bar{e}_x for $e_x + \frac{1}{2}$ seems open to objection in view of the magnitude of the resulting error at the older ages.

The graduation of such an Experience as the British Offices 1893 is a very different matter from the graduation of a single table. It involves questions of principle as well as questions of method, and Mr. Hardy's memorandum is valuable, not less perhaps for the light it throws on the general principles of the science of graduation, than for its elegant exposition of the elasticity and adaptability of Makeham's formula—not less for its breadth of view than for its refinement of method. At the outset Mr. Hardy enunciates the principle (with immediate reference to the Annuity Experience, but with equal application to the whole of the data) that “the object of a graduation is something more than merely to remove the accidental irregularities in the ungraduated rates which are inherent in all statistical data. In the present instance, at any rate, the object is to provide a basis for the construction of financial tables which, while adhering closely to all important features of the original experience, shall at the same time be both safe and convenient in practical use. The uses which the graduated table are to subserve must therefore be borne in mind in any proposed treatment of the data.” This principle has been applied with considerable freedom. In obedience to it the male annuity experience below age 55 has been rejected because it exhibits (as appears from a comparison of the ungraduated expectations with the *Text-Book* values) an abnormally low vitality which it would be undesirable to retain “in a graduated table intended as a basis for monetary tables”; similarly in the construction of the Select Tables (*i.e.*, the two Annuity Tables and the O^{MU}) the Truncated Aggregate Tables were abandoned because (owing mainly to the exclusion of further duplicates in their compilation) the result of effecting a junction with them after either 5 or 10 years would have been to under-estimate the annuity-values at date of purchase or entry; and again (to mention a minor application of the principle) the value of the constant in μ as originally computed for the male annuitant table was arbitrarily reduced, mainly because the graduated rates of mortality at the younger ages in the male annuitant table would otherwise have materially exceeded the corresponding rates in the assurance table—an undesirable feature in view of the fact that “it

has in recent years been not uncommon for annuities to be granted and assurances to be effected simultaneously upon the same individuals." The result, as regards the Male Annuitant Table, is rather curious—the whole Table being based on the first 5 years' experience from age 55 upwards joined on to a hypothetical ultimate table constructed from the ungraduated select expectations five years after purchase, so that before age 55 it appears to be, in effect, an extension (unsupported by data, except in so far as it can be considered to derive support from the assurance experience) of the table representing the experience after that age. In these circumstances, although the publication of the table from age 20 will probably be considered sufficiently justified by the practical considerations that the values for ages 20–55 are on the safe side, and that the usefulness of the tables as a whole would have been materially impaired by their omission, an explanatory note might perhaps have been inserted. The hypothetical nature of the earlier part of the table of course explains incidentally the considerable differences which appeared between the graduated select values and Mr. Ackland's at the younger ages. Another—and for future guidance a very important—result of the application of the general principle mentioned above, is that it raises a doubt whether the advantages (if any) accruing from the exclusion of duplicates in the compilation of the Aggregate Tables—other than those already excluded in the compilation of the Select Tables—are at all commensurate with the relative expenditure of labour. The truncated aggregate experience (both annuity and assurance) having been found unsuitable for the construction of the Select Tables, the sole results of the single-life aggregate experience are the O^M and $O^{M(5)}$ Tables in which various degrees of selection are blended in unknown and fortuitous proportions. It is not clear that an aggregate formed by combination of the Select Tables would not have been equally useful for practical purposes, while it would at least have had the merit of being a compound of known or ascertainable elements. It is interesting to notice in Mr. Hardy's Table of the mean values of ungraduated curtate expectations (Table XVI) that the values of the expectations according to the truncated aggregate table obtained by combining the Select Tables excluding the first 10 years run very close to the $O^{M(5)}$ values, so that the former table—which forms the trunk of the Select Tables—would have practically served the same purposes as the $O^{M(5)}$, although in neither can the effect of selection be said to have entirely disappeared. A curious feature exhibited by Table XVI is that at attained ages 40–64 increased duration of assurance appears to be accompanied after a certain point (varying from 10 to 15 years from the date of entry) by an *improvement* in vitality. Mr. Hardy attributes this to the superiority throughout life of the entrants at the younger ages (particularly at 20–29). An alternative explanation would be that suggested by Mr. King in his discussion of the very similar feature exhibited by the H^M experience (*J.I.A.*, xix, p. 397). The greater importance attached to family history in the selection of entrants at the younger ages might conceivably render them superior in later life to entrants at older ages, and it is to be remembered that the younger entrants who come under observation at the older attained ages in the O^M experience were

selected many years ago when family history was perhaps more considered than it is now, but in view of the fact that all entrants up to age 44 appear ultimately to become better lives than entrants at subsequent ages, it certainly seems as if the true explanation of the feature in question is to be found in some cause of a more general nature than the superiority of the younger entrants. Reference has been made above to the fact that the ultimate table for the select annuity tables was obtained by constructing a hypothetical mortality table from the ungraduated select expectations of life five years after entry, whereas for the $O^{[M]}$ Tables it was found that a combination of the Select Experience after the first 10 years—the resulting aggregate differing of course from the $O^{[M]}$ only in the inclusion of a number of duplicated periods of risk—would give an ultimate table with which a fairly satisfactory junction could be effected after 10 years. It would be interesting to know whether the latter expedient could not have been adopted in the case of the annuity tables. Its adoption, however, if in some ways advantageous had it been practicable, would have involved the loss of one of Mr. Hardy's happiest achievements, namely, the construction of the hypothetical graduated mortality table reproducing the ungraduated select expectations five years after purchase. Some of the details of the method employed in the construction of this table are rather obscure (it does not appear, for example, how the values of $\frac{de}{dA}$ were obtained, nor is it clear why the

relative weights assigned to the ungraduated expectations should be the reciprocals of the average deviations), but the general process adopted, namely, the expression of e_x in the form $f(A - \delta A, B - \delta B)$ —approximate values of A and B having been first obtained from a preliminary graduation of the truncated aggregate experience—and the approximate determination of the values of δA^* and δB by expanding to first powers and equating to zero the sums of the weighted deviations and their accumulations can be followed with the aid of Table V, and an explanation is also given of the method by which the average errors in the unadjusted expectations were estimated. An alternative and more obvious course of procedure would apparently have been to graduate an unadjusted hypothetical mortality table deduced directly from the unadjusted expectations; possibly the irregularity of the values of e for consecutive ages, or the difficulty of determining the weights to be assigned to the successive values of q deduced in this way from a number of entirely separate tables, may have rendered this impracticable. The process of determining the constants in the Select Annuity Tables for the first five years is passed over, practically without description, as having been necessarily of a somewhat tentative character, but from the objects kept in view and the form of the functions employed, it appears to have been very similar to that followed in the case of the $O^{[M]}$ Table, and briefly described in the account of the construction of that Table; students of the memorandum on reaching p. 134 would therefore do well to refer to pp. 155–161.

In the case of the two series constituting the Female Annuity

* The value of δA in § 16 p. 131 is given as .241 instead of .00241.

Table, the problem of graduation appears to have been considerably complicated in consequence of Mr. Hardy having set himself the heroic task of securing such a connection between the male and female tables as would admit of the values of joint-life annuities for any combination of ages being obtained from tables of values for equal ages. It is unnecessary to state that the task was successfully accomplished, and from the formulas given on p. 137, it may be inferred that it was at one time in contemplation to publish tables of equal-age values by each combination of the three tables (namely, the Male Table, the Female Table, 1st series, and the Female Table, 2nd series), taken two at a time. This has not, of course, been done—the joint-life annuity tables having been constructed without any reference to the subtle relations between the male and female tables—so that, for the moment, this feature of the graduation is barren of practical result, although its possibilities may perhaps be developed later on. However, the fact that it should have been practicable to maintain these relations between three analyzed tables, consistently with such close adherence to the observations as is shown by Tables XIII–XV, is a remarkable testimony to the adaptability of Makeham's formula, and to Mr. Hardy's exceptional powers of adaptation. Mr. Hardy's co-ordination of the various absolute constants and functions of t involved in the expressions for the select functions is, indeed, a veritable *tour de force*. It is easy to verify the facts that the final relation, printed at the foot of Table VIII, secures the desired result, and to follow the successive artifices by which this relation was arrived at, but the process, as a whole, shares, with many other fine pieces of mathematical work, the characteristic of leaving the student under the impression of having assisted—as a spectator—at a conjuring performance. Two very neat devices, of which special mention may be made, are the making of the B of the Male Table equal to the B_c of the Female Table (1st series)—which, of course, rendered it necessary that ψ_t should be identically the same for the two tables—and the representation of the μ of the Female Table (2nd series) by a section of the male μ with the addition of a constant.

Of the methods employed in the construction of the $O^{M(5)}$ and $O^{(M)}$ Tables, it is unnecessary to write in detail, as they appear to have been the same in all essential respects as those applied to the Annuity Experience, while in regard to the O^M Table very little information is given—the origin of the supplementary curve being dismissed with the brief statement that when the differences between the O^M and $O^{M(5)}$ unadjusted values of $\text{colog } p_x$ were set out graphically it was clear from the nature of the curve that a double frequency-curve of the form adopted would, with suitable co-efficients, well represent the observations—and, in view of the fact that the sole object aimed at was the construction of a smooth curve representing as nearly as possible the ungraduated data, and that the successful attainment of this object is amply demonstrated by Table XX, the only point perhaps upon which further information would be of interest is as to the cause of the maxima of ϕ at ages 29 and $66\frac{1}{2}$. As a final instance of the subordination of the theoretical to the practical throughout the graduation, it may be noted that

the sum of the accumulated deviations in the O^M and $O^{M(5)}$ Tables was not made exactly zero, as this would have necessitated making A appreciably less (and consequently a at the young ages more) than in the annuity table.

R. T.

Experience of 34 Life Companies upon 98 Special Classes of Risks.

Compiled and Tabulated by the Actuarial Society of America, New York, 1903.

THE readers of Mr. D. P. Fackler's recent paper, printed in the *Journal* (vol. xxxvii, p. 1) on the above subject, will have been awaiting the publication by the Actuarial Society of America of the results of the investigation there described with feelings of unusual interest. The magnitude of the experience will be realized when it is remembered that Mr. Fackler estimated that the manipulation of the data would necessitate the writing of over $2\frac{1}{4}$ million cards, and when it is mentioned that the tabulated deaths exceed 140,000.

As the methods of collecting the data have been already fully explained in the paper mentioned above, it is unnecessary to deal with this part of the subject again; and we may pass at once to the consideration of the figures contained in the volume now under review, which gives for each class tables showing:

- (1) The Exposures and Deaths for each age at entry and duration;
- (2) The Actual and Expected Deaths for grouped ages at entry and all durations;
- (3) A Summary of the Actual and Expected Deaths for grouped ages at entry and all durations; and
- (4) A similar table of the Actual and Expected Deaths for grouped ages at entry, but excluding the first five years of Insurance.

The first of these tables gives the raw material in such a way as to be available for use by any Actuary desiring to investigate in detail any special point.

With regard to the remaining three tables, the standard adopted in calculating the expected deaths has in each case been Farr's Healthy English Male Life Table, excepting that for ages 15 to 21 inclusive, the Committee adopted, in lieu of the mortality shown by that table, eleven-tenths of the mortality shown by the $O^{M(5)}$ Table, and at ages 52 to 61 inclusive five-sixths of the mortality shown by the same table.

Having adopted the above-mentioned standard, the committee allowed for the immediate effects of selection by reducing the expected deaths for the first five insurance years as calculated from the standard table by certain fixed percentages, varying according to the age at entry and the year of insurance.

The selection of the standard mortality table and the method of allowing for the effects of entrance selection appear to have been carefully considered by the Committee, but many Actuaries may feel somewhat doubtful as to whether the methods adopted are, on the whole, the most suitable, having regard to the nature of the experience

TABLE I.—*Showing Actual Deaths as percentage of Expected Deaths by Normal Table*

Description of Class Occupations	ALL AGES AND ALL DURATIONS		ALL AGES EXCLUDING FIRST FIVE YEARS		ALL DURATIONS, BUT ENTRANTS GROUPED ACCORDING TO AGE AT ENTRY ACTUAL DEATHS AS PERCENTAGE OF EXPECTED			
	Actual Deaths	Actual deaths as percentage of Expected	Actual Deaths	Actual deaths as percentage of Expected	Ages at Entry			
					15-28	29-42	43-56	57-7
	(1)	(2)	(3)	(4)				
Army Officer (8) *	255	116	148	103	110	121	109	125
Brewer or Employee (31)	796	135	514	138	134	133	135	152
Butcher (22)	1,822	89	1,017	89	74	89	99	102
Cattle dealer (27)	838	96	448	95	84	95	99	110
Contractor (24)	1,537	97	841	98	93	90	103	118
Distiller or Employee (32)	205	107	146	112	94	111	114	88
Electrician (13)	209	81	64	80	79	83	71	217
Fire department (City) (11)	63	112	29	123	133	116	83	71
Glass worker (17)	107	113	58	114	142	103	104	105
Hotel keeper (not attending bar) (28)	1,188	122	685	121	122	116	131	118
House painter (19)	526	85	298	83	67	89	93	84
Labourer (23)	1,188	124	576	111	98	120	145	168
Merchant Marine Officer (30)	133	134	55	110	179	137	112	98
Naval Officer (9)	156	136	89	117	141	145	117	123
Physician (12)	5,643	102	3,409	103	100	94	114	123
Pilot (41)	61	91	37	95	90	92	82	108
Police Force (10)	757	117	415	115	108	118	120	102
Potter (18)	37	78	29	102	77	76	81	77
Printer (20)	775	80	428	79	77	79	92	56
Puddler, foundry, &c., worker, &c., not machinist (16)	671	90	398	90	68	83	113	104
Railway locomotive engineer (37)	220	127	80	105	127	124	147	23
Railway locomotive fireman (38)	61	160	15	123	174	143	111	500
Railway mail clerk (35)	65	80	25	64	83	83	69	67
Railway passenger conductor (33)	207	100	98	93	109	89	137	64
Sailing vessel officer (40)	408	153	208	126	209	154	131	194
Saw-mill worker (14)	295	87	148	76	108	78	90	108
Seaman or Fisherman (42)	193	108	101	91	130	100	108	93
Steel grinder (15)	36	120	13	99	118	123	130	67
Tailor (21)	1,405	88	862	89	73	80	104	107
Theatrical occupation (25)	101	129	51	139	140	112	164	107
Travelling salesman (26)	1,780	80	915	80	78	75	97	111
Wine or liquor seller (abstainer) (29)	411	120	271	128	108	118	127	111
Wine or liquor seller (non-abstainer) (30)	1,704	131	1,045	136	116	129	140	121
Total	23,853	101	13,516	101	91	97	113	111

* The figures in brackets correspond to the number of the class in question, as given in the original investigation.

TABLE I—*continued.*

Description of Class	ALL AGES AND ALL DURATIONS		ALL AGES EXCLUDING FIRST FIVE YEARS		ALL DURATIONS, BUT ENTRANTS GROUPED ACCORDING TO AGE AT ENTRY			
	Actual Deaths	Actual deaths as percentage of Expected	Actual Deaths	Actual deaths as percentage of Expected	ACTUAL DEATHS AS PERCENTAGE OF EXPECTED			
					Ages at Entry			
Personal and Family History	(1)	(2)	(3)	(4)	15-28	29-42	43-56	57-70
Asthma (history of) (46)	1,028	101	668	102	70	97	118	97
Blood spitting (history of) (55)	814	108	546	107	137	111	105	85
Cancer (family history of, one death) (75)	4,680	89	2,830	90	78	81	97	105
Dyspepsia (light weight) (58)	775	94	468	86	127	87	95	103
Dyspepsia (light weight excepted) (57)	3,528	84	2,198	85	71	75	95	111
Height (history of) (49)	168	155	117	180	...	137	165	172
Height, above 6 feet 3 inches (73)	267	99	158	101	95	88	114	151
Hepatic Colic or Gall-stone (history of) (52)	416	107	251	102	99	91	135	86
Hip joint disease (history of) (56)	133	128	82	140	126	139	106	121
Inflammation of Bowels, Peritonitis, or Appendicitis (54)	1,030	83	614	87	66	79	101	103
Insanity (one case in family history) (76)	1,315	83	855	87	78	75	97	87
Intemperance (history of cured) (45)	299	125	145	116	140	127	122	110
Otorrhea (or running from ear) (51)	195	79	90	87	67	81	87	88
Pulse (intermittent or irregular) (43)	240	88	158	90	76	73	117	89
Pulse (slow, below 60) (44)	436	70	274	74	63	68	75	77
Renal Colic, Calculus or Gravel (history of) (53)	495	107	259	110	90	106	101	139
Rheumatism (Inflammatory once) (47)	3,442	97	2,183	100	80	97	102	108
Rheumatism (Inflammatory more than once) (48)	575	108	381	109	95	110	108	114
Syphilis (history of) (50)	397	133	241	142	105	134	153	102
Weight, light, and parent dead, below 70, of any Lung disease (71)	1,869	100	1,120	93	130	97	101	91
Weight, light, and not included under (58) or (71). (72)	12,452	92	7,849	88	93	90	93	94
Weight, normal, and both parents reached 75 (70)	8,020	88	4,928	85	102	85	88	95
Weight, normal, and both parents dead, below 60 (69)	12,260	103	7,559	103	100	98	109	113
Weight, normal, and parent dead, below 70 of Apoplexy or Paralysis (68)	5,945	94	3,661	98	81	87	109	106
Weight, normal, and parent dead, below 70 of Consumption, Phthisis, or Tuberculosis (65)	5,382	87	3,439	88	94	83	88	92
Weight, normal, and parent dead, below 70 of Kidney or Bright's disease (66)	1,475	82	826	88	78	78	98	73
Weight, normal, and parent dead, below 70 of Heart disease (67)	3,269	85	1,922	89	74	82	98	96
Weight, heavy, and girth of abdomen greater than chest expanded (64)	536	133	297	147	78	116	147	144
Weight, very heavy, and girth of abdomen greater than chest expanded (61)	419	152	262	187	116	135	167	164
Weight, heavy, and neither parent dead, below 70 (62)	2,312	99	1,447	109	74	91	119	121
Weight, very heavy, and neither parent dead, below 70 (59)	1,060	118	699	134	73	113	144	132
Weight, heavy, and one parent or both dead, below 70 (63)	5,465	120	3,570	134	87	109	139	126
Weight, very heavy, and one parent or both dead, below 70 (60)	2,324	147	1,522	167	104	130	172	172
Total	83,021	96	51,640	97	87	91	103	104

TABLE II.

Showing Actual Deaths as percentage of Expected Deaths for Entrants between the ages 29 and 56, and excluding first five years of insurance.

Occupation	Actual No. of Deaths	Percentage
Railway mail clerk (35)	16	63
Sawmill worker (14)	125	79
Travelling salesman (26)	639	81
Seaman or Fisherman (42)	74	84
Printer (20)	301	85
Pilot (41)	26	88
House painter (19)	252	90
Tailor (21)	696	92
Electrician (13)	37	93
Butcher (22)	817	93
Contractor (24)	696	95
Cattle dealer (27)	381	96
Railway passenger conductor (33)	84	96
Puddler, foundry, &c. worker, &c., not machinist (16)	334	96
Glass worker (17)	37	97
Army Officer (8)	120	98
Physician (12)	2,841	103
Potter (18)	21	105
Merchant Marine Officer (39)	48	111
Railway locomotive engineer (37)	71	111
Distiller or Employee (32)	110	114
Naval Officer (9)	69	115
Steel grinder (15)	11	115
Sailing vessel Officer (40)	173	117
Labourer (23)	451	118
Police Force (10)	360	119
Hotel keeper (not attending bar) (28)	577	123
Wine or liquor seller, abstainer (29)	223	128
Fire department (City) (11)	25	129
Railway locomotive fireman (38)	7	132
Wine or liquor seller, non-abstainer (30)	894	138
Brewer or Employee (31)	418	139
Theatrical occupation (25)	46	147
Total	10,980	103

generally, and in particular to the fact that a very large proportion (39 per-cent) of the total deaths took place during the first five years of assurance. It is difficult, however, to suggest *à priori* any more satisfactory method of procedure which could have been adopted with any certainty that better results would have been attained, and this question could indeed only have been settled by embarking upon a somewhat laborious examination of the alternative methods and

TABLE II—*continued.*

Personal and Family History.	Actual No. of Deaths	Percentage
Pulse slow (below 60) (44)	206	74
Dyspepsia (light weight) (58)	391	83
Dyspepsia (light weight excepted) (57)	1,787	85
Normal weight (both parents reached 75) (70)	3,997	85
Normal weight (parent dead, below 70 of Consumption, Phthisis, or Tuberculosis) (65)	2,073	88
Light weight, not included under (44) or (71) (72)	6,428	88
Cancer (family history of one death) (75)	2,227	89
Insanity (one case in family history) (76)	693	89
Normal weight (parent dead, below 70 of Kidney or Bright's disease) (66)	653	92
Inflammation of Bowels, Peritonitis, or Appendicitis (history of) (54)	492	92
Light weight (parent dead, below 70 of any lung disease) (71)	946	92
Normal weight (parent dead, below 70 of Heart disease) (67)	1,576	93
Pulse (intermittent or irregular) (43)	120	93
Otorrhea (or running from ear) (51)	71	95
Rheumatism (inflammatory once) (47)	1,755	101
Normal weight (parent dead, below 70 of Apoplexy or Paralysis) (68)	3,028	101
Height above 6 feet 3 inches (73)	131	102
Normal weight (both parents dead, below 60) (69)	6,179	103
Hepatic Colic or Gall-stone (history of) (52)	212	106
Blood spitting (history of) (55)	473	108
Renal Colic, Calculus, or Gravel (history of) (53)	233	108
Asthma (history of) (46)	563	108
Rheumatism (inflammatory more than once) (48)	330	111
Heavy weight (neither parent dead, below 70) (62)	1,163	112
Intemperance (history of cured) (45)	124	117
Heavy weight (one parent or both dead, below 70) (63)	3,039	136
Very heavy weight (neither parent dead, below 70) (59)	579	144
Hip joint disease (history of) (56)	62	147
Syphilis (history of) (50)	205	149
Heavy weight (girth of abdomen greater than chest expanded) (64)	242	151
Very heavy weight (one parent or both dead, below 70) (60)	1,332	171
Gout (history of) (49)	92	171
Very heavy weight (girth of abdomen greater than chest expanded) (61)	216	197
Total	42,248	98

tables available. It would, however, appear from a general examination of the figures that the standard actually adopted somewhat over-estimates the expected deaths at the younger ages at entry, and somewhat under-estimates the expected deaths at the older ages.

It is to be hoped that various papers, with subsidiary investigations based upon the experience, will be published in the near future; and, although the "newness" of the experience will

necessarily detract from the practical value of these investigations, for some purposes, interesting results could, no doubt, be obtained by forming a Mortality Table based upon the experience as a whole, and comparing any separate class which it is desired to investigate with the table so prepared. It would also probably be found that more satisfactory results could be obtained by comparing "expectations of life" than by comparing actual and expected deaths.

In the meantime, with a view to giving a general bird's-eye view of the broad results of the investigation, we append a table (Table I) showing for each Class the Experienced Deaths as a percentage of the Expected Deaths, the latter being calculated by Farr's Healthy English Table, modified as explained above. In order to keep this table within reasonable limits certain unimportant classes have been omitted.

The percentages given in Table I are of some interest, and will repay careful study. At the same time, for reference purposes, probably the following shorter table (Table II), confined to entrants between the ages of 29 and 56 (which comprise the usual insuring ages), will be found more useful.

In considering the percentages contained in the above tables, the importance of the following remarks extracted from the Introduction cannot be over-estimated: "It must not be forgotten that the facts . . . given relate to the respective classes of risks among lives selected for insurance, and do not relate to the same classes among the general population. For example, it is not conceivable that, among the general population, those who have had at least one parent dying of consumption are above the average of the others in vitality. If this is found to be the case as regards that particular class of insured lives, it indicates only that such persons of that class as have actually been accepted for insurance have been selected so carefully as to secure for acceptance only those who are peculiarly good representatives of the class. If, on the other hand, the results appear only moderately bad upon a class of risks heretofore accepted with great circumspection, it is to be inferred that, had such circumspection not been exercised, the results would have been still worse. We trust earnestly that this warning may be borne in mind as applying, and intended to apply, to each one of the classes under consideration."

The necessity for the above warning is brought out very forcibly in examining Table II, where the various classes are arranged, so to speak, in "order of merit." For example, it would seem from the above table that the mortality among "physicians" is, on the whole, less favourable than among "house-painters", but at least two explanations of this result are possible, one, that in the mass the conditions of life among house-painters are more favourable to longevity than among physicians, and the other, that, in selecting lives for insurance more care has been exercised to eliminate doubtful risks in the former case than in the latter. Again, turning to the classes distinguished by some defects in personal or family history, it cannot reasonably be supposed that the mortality among "dyspeptics" as a class is so markedly below normal as would appear, or that it is more healthy to be a "dyspeptic light-weight" than a "normal light-weight", although this would seem to be indicated by a cursory

examination of the tables. In fact, it must constantly be borne in mind that all the figures given merely show the *relative* positions in the scale of various classes of insured lives who have probably been subjected to medical and office selection of very varying intensity.

The broad features of the Experience which seem to stand out most noticeably are that, as regards Occupations, connection with the sale of intoxicants in any form is very unfavourable from a Life Insurance point of view and, as regards personal condition, that persons of unusual heavy weight must be regarded as requiring specially careful selection.

The British Actuary who is thinking of employing the results of the American investigation as a guide in dealing with the corresponding British Classes of risk, will have to consider carefully how far the conditions in the two countries are comparable, both as regards the measure of special risk run by persons engaged in cognate occupations, and as regards the significance of defects in personal and family history. In this connection it may be pointed out (as the fact is not mentioned in the volume) that in every case the occupation is scheduled as at the date of entry, and no attempt has been made to allow for changes in occupation after the issue of the policy. This point is interesting, inasmuch as changes of occupation are stated to be very much more frequent in America than in England.

Making all proper allowances, however, for these points, it is quite clear that the results disclosed are of singular interest and value to the actuarial profession, and the Actuarial Society of America and its officers are to be congratulated not only upon having gathered together an Experience of unique magnitude, but upon the celerity with which they have published the results, and the clear and interesting way in which those results are presented.

J. D. W.

*Sickness and Mortality Experience (1893-97) of the Manchester Unity of Oddfellows.**

THE rise and progress of the Manchester Unity of Oddfellows illustrate the success which may be achieved by private effort and public spirit. The Manchester Unity dates from distant times, and in its early days its methods were empirical and tentative; but it was under sagacious management, and very soon the discovery was made that, if the organization was to be permanent, it must be placed on a sound basis. Against great opposition to begin with, the experience of the Society was extracted for the three years 1846-8, and published in 1850. That was followed by a second investigation for the five years 1856-60, and again by a third, published in 1872, for the five years 1866-70; and this last has been the basis of calculation used by the Society, and by a very large number of other friendly societies, up to the present time. It became, however, gradually apparent that that basis was not to be entirely trusted, and it was

* An account of an Investigation of the Sickness and Mortality Experience of the I.O.O.F., Manchester Unity, during the five years 1893-1897, by Alfred W. Watson, F.I.A., F.S.S., with tables. Published by the Grand Master and Board of Directors, 97, Grosvenor Road, Chorlton-upon-Medlock, Manchester, and by C. & E. Layton, 56, Farringdon Street, London, E.C.

found necessary in the computation of rates and in valuations to make adjustments, which were often arbitrary, and which, although they may have produced reasonable results, were not satisfactory. The tabulated rates of sickness were evidently too low, and at the older ages the amount of experience was so small as not to yield ratios worthy of confidence. Such being the case, the Society determined to make a fresh investigation on a much more extensive scale, and the volume before us is the result.

From many points of view this new volume is of the greatest interest and importance. From the standpoint of the social reformer it is invaluable. A most thorough investigation has been made into the question of how sickness and mortality rates are affected by locality and by occupation, and although it was possible to do this only in the most broad and general way, yet much light is thrown on the social conditions of the workers of the United Kingdom. Then, the book will be beyond price not only to the Manchester Unity itself, but to all friendly societies in the Kingdom, and abroad. The material is so extensive that much sub-division has been practicable, and we imagine that every society, no matter what its constitution and what its own experience, will find in the new book tables which can safely be used in its valuations. Lastly, the actuarial student will discover in the volume much of value, and will be well repaid by giving it a most careful perusal.

A lengthy review is not necessary, because four years ago Mr. Watson himself (*J.I.A.*, vol. xxxv, page 268) described with great fulness the methods being followed in the compilation of the experience, and gave forecasts of the results. To that paper we refer our readers, as it provides an admirable epitome of the investigation as now finally published.

It will be sufficient if we here say that the data were kept separate for three geographical areas, each of which was sub-divided into rural and urban, according to the density of population. The first area may be called Non-Manufacturing, and comprises the North-Western, Eastern, and Southern portions of England, North Wales, and such parts of South Wales as lie outside the mining districts. Area 2, Textile, includes the textile districts of Lancashire, Yorkshire, and Cheshire; and area 3, Manufacturing, includes portions of Derbyshire, Leicestershire, Northamptonshire, Nottinghamshire, Staffordshire, Warwickshire, the Metropolitan Area, Monmouthshire, and part of South Wales, and the mining districts of the North-East and North-West of England, and also the small experience obtained from Scotland and Ireland.

As regards occupation, each area was examined in connection with seven urban, and eight rural, groups of employments. For a description of these we must refer to pages 30-32 of the book itself, and to page 294 of Mr. Watson's paper above referred to. For practical use it would not have been possible to base mortality, sickness, and monetary tables upon each of these sub-divisions, combined, as would have been necessary, with each other in almost an infinity of ways. On a very careful analysis it was found that, broadly, the rate of mortality was influenced mostly by geographical situation, while the rate of sickness was influenced principally by

occupation. Twelve mortality tables, therefore, were prepared and graduated, namely, three for each area—urban, rural, and combined urban and rural; two for all areas combined—rural and urban; and one, rural and urban, for the whole Society. Also for sickness, after most minute research, the experience was collected into four broad groups of occupations, namely, Group A.H.J.—Agricultural and General; B.C.D.—Outdoor Building, Railway, Seafaring, and Outdoor Labour generally; E.F.—Quarry, Iron, Steel, Chemical, &c., workers; and Group G—Mining Workers; and these again were all combined to give the experience of the whole Society. Group G was not made further use of, but the rates of sickness in each of the other groups were combined with the various mortality rates.

As regards duration of sickness, separate columns are given for the first three months, second three months, second six months, second twelve months, after two years, and all durations; and monetary tables have been calculated at four rates of interest, namely, 3 per-cent, $3\frac{1}{4}$ per-cent, $3\frac{1}{2}$ per-cent, and 4 per-cent. The mortality rates, rural and urban together, of each area were, in computing the monetary tables, separately combined with the sickness rates of each of the three occupation groups, A.H.J., B.C.D., and E.F., and also the whole mortality experience of the Society was combined with the whole sickness experience; and, for mortality alone, monetary tables are also given for each area, and for the whole Society, for rural and urban populations separately. Should this large number of combinations not prove sufficient for practical purposes, anyone so disposed can make other combinations for himself from the original facts so copiously supplied.

The graduation of the tables, mortality and sickness, was of necessity a matter of some delicacy, because not only had each table to be made in itself as perfect as possible, but the tables had to be co-related in consequence of the combinations between them which were intended. In the volume very little is said on this important point, although the formulas actually used in the graduation are given; but Mr. John Spencer, F.I.A., who collaborated with Mr. Watson in the work of graduation, has kindly responded to our invitation to supply full information, and his interesting paper on the subject appears in our present issue.

Perhaps the most important advantage which the new experience possesses over all that have gone before is the large body of facts included at the older ages. The "Exposed to Risk of Sickness" at ages over 70 amounted to no less than 70,565, against 10,507 in the experience of 1866-70, and 25,990 in Sutton's *Registered Friendly Societies*, 1876-80. At last, therefore, we have trustworthy data for dealing with sickness in old age.

The main features of the new experience are the great increase in sickness claims which appears to have prevailed of recent years, and the reduction in the rates of mortality at the younger and middle ages, although in old age the rates of mortality do not appear to have fallen, but rather, perhaps, to have increased. It is shown, moreover, that high rates of mortality do not coincide with high rates of sickness, but that there is even a tendency for the contrary conditions to prevail. These results will probably not cause surprise, because

all those who have had much to do with friendly societies have found similar tendencies indicated.

Without going into further detail, we may perhaps call attention to the methods by which the important function "Exposed to Risk" was arrived at. This is explained on page 13. In former experiences the exposed to risk was the same for both sickness and mortality, but, on account of the business methods of the Manchester Unity, Mr. Watson has departed from the prevailing custom. Possibly other friendly societies may have different rules as regards coming into benefit, and cessation of benefit on secession, and, therefore, in using the new tables this point must be borne in mind. Probably the financial results will not be much affected, but the point should have attention. We notice that in calculating the exposed to risk of sickness, the members dying in each year of age were assumed to be at risk until the middle of the year, but that, although perfectly legitimate, and also convenient under certain circumstances, is an unusual method of treatment. It seems to us quite correct, in taking out mortality or sickness experience, to treat *withdrawals* as at risk for half a year. The "Exposed to Risk" at age x means the number at risk between age x and age $x+1$, and, seeing that those who withdraw in that year may, after withdrawal, suffer in that year from illness or may die, we get the correct rates by treating them as at risk for a fractional period only. But the case is different when in preparing sickness tables we look at the deaths. Presumably a man cannot be ill after death, and, therefore, the total sickness between ages x and $x+1$ is generally calculated as if those who die in that year of age were at risk for a full year. The point is a small one, and different methods of treatment will not produce much difference in the results, but, nevertheless, it is one that deserves mention. We observe from page 82 that the monetary tables have been so computed as to agree with the assumption that the deaths are at risk of sickness for the whole year, because the factor $\frac{1}{2}p_x$ is involved. This additional factor is not necessary when the statistical tables are prepared in the usual way.

Not the least useful feature of the work is the very copious index provided.

In conclusion, we congratulate the Directors of the Society on their public spirit in enabling this work to be carried through, and we more particularly congratulate Mr. Watson on his own preponderating share in the undertaking. His name will go down in history with those of Mr. Henry Ratcliffe and his own late esteemed father, Mr. Reuben Watson, and he will receive the gratitude of future generations. The former investigations were invaluable in their day, and as complete as the available facts allowed of, but Mr. A. W. Watson has admirably availed himself of his greater opportunities, and has built for himself a lasting monument. No doubt at some future time a fifth investigation will be found to be desirable, but when that time comes the full explanations which Mr. Watson has given of the methods he has followed will be of the greatest assistance to those who may then be entrusted with the work.

*The Amendment of the Basis of Calculation of the
Gotha Life Office.**

A brief Review by T. E. YOUNG, Past-President of the Institute of Actuaries,
and W. R. STRONG, F.I.A.

FOLLOWING the work entitled *Aus der Praxis der Gothaer Lebensversicherungsbank*, reviewed on p. 292 of vol. xxxviii of the *Journal* (January 1904), we have received from Dr. M. Karl Samwer a copy of the important work prepared by Dr. Johannes Karup, dealing with the modification of the basis of calculation of the Gotha Life Office. This consists of two volumes bound together, containing respectively 170 and 544 pages. The paging of the second volume is distinguished by an asterisk. The preparation of the book, which must have involved immense labour, was commenced so far back as 1893, and some four years were necessary for the collection of the material. From 1897 onwards the work progressed with greater rapidity, and the result is a monument of industry, and is full of interesting information. The first section of the first volume is mainly explanatory of the methods adopted and results obtained. The second section commences with an elaborate explanation of the notation employed. This, while following to some extent the notation of the Institute, is amplified by the addition of numerous symbols specially designed to describe particular functions and to apply to insurance against invalidity. It may be noted that in accordance with German custom $N_x = D_x + D_{x-1} + D_{x-2} + \dots$ instead of $N_x = D_{x-1} + D_{x-2} + D_{x-3} + \dots$ as usually employed in this country, while the symbol δ is used for $\frac{1}{\sqrt{1+i}}$ and is explained to be distinct from the symbol \hat{c} , the force of interest. The notation relative to insurance against the risk of invalidity is copious, and deserving of careful study, involving as it does the adoption of a large number of specially devised symbols.

The second volume is devoted to Tables, of which it contains altogether 148. On pp. 500* and 501* appears the following comparison between the Gotha Experience and the mortality shown by the O^M Table.

Age attained	YEAR OF INSURANCE									
	1st		2nd to 3rd		4th to 5th		6th to 7th		8th & upwards	
	Mortality per-cent for one year		Mortality per-cent for one year		Mortality per-cent for one year		Mortality per-cent for one year		Mortality per-cent for one year	
	O ^M	Gotha	O ^M	Gotha	O ^M	Gotha	O ^M	Gotha	O ^M	Gotha
20-24	·26	·36	·51	·48	·61	·53	·64	·54	·64	·54
30-34	·36	·33	·52	·46	·61	·54	·70	·58	·80	·60
40-44	·52	·54	·73	·73	·86	·84	·94	·90	1·10	·96
50-54	·89	·90	1·29	1·31	1·47	1·50	1·50	1·61	1·75	1·76
60-64	1·66	1·63	2·37	2·46	2·58	2·80	2·62	3·31	3·35	3·77

* *Die Reform des Rechnungswesens der Gotha Lebensversicherungsbank a. G.*
Dr. Johannes Karup. Published by Gustav Fischer. Jena 1903.

The following comparison of the average duration of life which appears on p. 18 will be of interest, although the Author, while mentioning it as the most popular, describes it as the most unsuitable measure of mortality.

Age	New Gotha Experience after expiry of 7 years of Insurance	O ^M (Ungraduated) Whole-Life With-profit Insurances Males, completed Insurance of 7 years and upwards
25	39.05	38.17
30	35.04	34.47
35	31.02	30.79
40	27.07	27.10
45	23.28	23.52
50	19.60	20.01
55	16.17	16.67
60	13.09	13.54
65	10.33	10.69
70	7.92	8.19
75	5.90	6.08
80	4.31	4.36

The comparison between mortality among males and females in the same two tables, given on p. 60, is also of interest.

Age	MORTALITY PER-CENT FOR ONE YEAR					
	Gotha Whole-Life Insurances			O ^M English Table, With Profit, 1863-1875		
	Males	Females	Δ	Males	Females	Δ
15-25	.50	.48	- .02	.46	.60	+ .14
26-30	.43	.88	+ .45	.53	.84	+ .31
31-35	.52	.59	+ .07	.69	.85	+ .16
36-40	.69	.56	- .13	.84	1.09	+ .25
41-45	.95	.88	- .07	1.06	1.12	+ .06
46-50	1.24	1.00	- .24	1.35	1.22	- .13
51-55	1.81	1.22	- .59	1.81	1.57	- .24
56-60	2.72	1.93	- .79	2.49	2.16	- .33
61-65	3.96	3.01	- .95	3.54	3.02	- .52
66-70	5.90	4.89	- 1.01	5.30	4.69	- .61
71-75	8.86	8.36	- .50	7.82	7.24	- .58
76-80	13.29	10.65	- 2.64	11.60	11.13	- .47
81-85	18.83	17.27	- 1.56	17.19	17.35	+ .16
86-90	27.05	21.43	- 5.62	23.52	23.31	- .21

A Table of the reserves derived from the new Gotha Tables is given on p. 36*. These reserves are divided into the "old" and "new" methods. The old method assumes that payment of the claim is made at the end of the year in which death occurs. The new method that payment immediately follows the death.

The actual reserves required by the Gotha Tables under these conditions are shown below, and we have added values calculated on the same lines from the O^M Experience.

Reserve at 3 per-cent Interest for an Insurance of 100.

ENDOWMENT ASSURANCES, MATURING AT AGE 90.

Table	YEAR OF INSURANCE								
	Method	1	3	5	10	20	30	40	50
AGE AT ENTRY 30.									
New Gotha	New	1.55	4.53	7.50	15.20	32.23	50.26	67.15	81.06
Old Gotha	Old	1.15	3.50	5.86	12.19	27.59	44.49	63.76	81.86
OM . . .	New	1.28	3.90	6.59	13.72	29.70	47.29	64.49	79.39
AGE AT ENTRY 40.									
New Gotha	New	2.19	6.35	10.44	20.84	41.84	61.50	77.71	...
Old Gotha	Old	1.57	4.86	8.37	17.54	36.78	58.73	79.34	...
OM . . .	New	1.73	5.27	8.92	18.47	38.80	58.71	75.94	...

It will be noted that at every period in the illustrative table the new Gotha Table gives higher reserves for the above ages at entry than the OM Table.

*Insurance.**

ANY work from the pen of a Past President of the Institute of Actuaries, and especially when that Past President is Mr. T. E. Young, must command attention, and the book in our hands is a good illustration of this truism. Its scope is wide. There are eleven chapters treating of Life Assurance, and four of Fire, Marine, Accident, and other special Assurance, and the contents of each chapter are much more comprehensive than the heading would seem to imply. From one point of view the writing is lucid, but, from another, it is, in certain parts, a little difficult, and requires close attention on the part of the student, yet, in saying this, we do not pass adverse criticism. The book is one that cannot be skimmed over, it must be read; but conscientious reading will be well rewarded. In many places it teems with controversial points, but that is all the better, because independent thought is thereby stimulated, and, if the student does not always come to the same conclusion as the author, he will, nevertheless, profit by a thorough consideration of the author's views, and of the reasons which have led to them.

Space does not permit us to make a lengthy analysis of the book, nor would that course be advantageous, because the book is one that must be taken as a whole. Yet it may interest our readers to call attention briefly to one or two cases where the matter is what we have ventured to call controversial.

Mr. Young deals, in some detail, with the question of the Limit of Risk, because "a very considerable risk upon an individual life

* Insurance—A practical exposition for the Student and Business Man. By T. E. YOUNG, B.A., F.R.A.S., Ex-President of the Institute of Actuaries, &c. London, Sir Isaac Pitman & Sons, Limited.

should be avoided on account of the probability of a heavy loss occurring which would exceed the bounds of prudential administration", and he proceeds to express the opinion, which he deems to be soundly feasible, that "the premiums upon the assurances of maximum amount should be regarded as constituting the fund from which the claims upon those policies should be discharged, so that the amount of premiums annually receivable upon such assurances should prove sufficient for the satisfaction of their demands by death"; and, further on, "policies effected for an amount in each instance equal to the settled limit must be considered as constituting a distinct class and must prove adequate in number to provide their own claims from the source of their separate premiums." Now we fear that if this opinion be literally construed, there is not a single life company in the Kingdom which does not, in its usual practice, run counter to it. No company obtains such a number of policies of its full limit as would justify it in standing on these policies alone; that is, an office re-assuring the whole of the full limit risks of any company and not having any other business on its books, could not escape violent fluctuations in the incidence of mortality. Yet British Companies are not much troubled by such fluctuations. The fact is, our life companies are so stable that it is inconceivable that they could be endangered by the falling in of a "very considerable risk", or could even be embarrassed in obtaining the necessary ready money to meet it. The method of deciding on the maximum risk to be held is, we imagine, generally settled by quite other considerations. It is not a question of solvency or of convenience, but only a question of how the next bonus would be affected by a few large claims during the inter-valuation period. From this it follows that a company making an annual valuation and bonus distribution cannot properly hold so large a maximum of risk as one valuing quinquennially. In the case of the former, fluctuations are all focussed on a single year; in the case of the latter they are spread over five years, and there is a very much greater opportunity for averages to adjust themselves. No doubt Mr. Young had this point in view, but we have ventured to amplify it a little so as to make it quite clear.

The construction of office premiums receives a good deal of attention from Mr. Young, and he gives elaborate formulas to be used in the calculations; but we venture to think that these formulas, besides being intricate to work, would yield results impossible from the point of view of competition. Mr. Young first shows how to compute a non-participating rate by adding to the net premium a percentage for renewal commission and a constant for cost of administration, and then a further loading, spread over the whole term of the policy, and varying according to the age at entry, for initial expenses; and from this non-participating rate he passes to the participating by providing additional loading for bonus. He assumes a reversionary bonus at uniform rate, compounded at quinquennial intervals, and proceeds by means of the D, N, M, and R columns to find its annual equivalent. The process, though algebraical, and good from an educational point of view, would lead to futile results, because principles are involved which mere algebra does not disclose. A company managed with reasonable care pays

larger bonuses than the premiums are theoretically capable of providing, and defrays besides the expenses of management. The cost of a compound reversionary bonus at the rate of j per unit per annum may be found briefly as follows. Let the premiums be computed at rate i , then the value of the sum assured and

bonuses is $\frac{1}{l_x} \left[d_x \frac{1+j}{1+i} + d_{x+1} \frac{(1+j)^2}{(1+i)^2} + \&c. \right]$ the general term being

$\frac{1}{l_x} \cdot d_{x+n} \frac{(1+j)^{n+1}}{(1+i)^{n+1}}$. Now let $\frac{1+j}{1+i} = \frac{1}{1+J}$, so that $J = \frac{i-j}{1+j}$; then

the value of the sum assured and bonuses is A'_x , when A'_x is calculated at rate J . To obtain the annual premium we divide by $1+a_x$ taken

at rate i , so that the premium loaded for bonus only is $P'_x = \frac{A'_x}{1+a_x}$.

This is an exact formula under the conditions assumed.

If a compound bonus of 25s. per-cent per annum be assumed, and if the calculations be made at $3\frac{1}{2}$ per-cent, $J = .02222$, and if we take instead $2\frac{1}{4}$ per-cent, we shall assign too small a value to A'_x . But we have found the value of an annual bonus compounded yearly, whereas the bonus is usually compounded only quinquennially, and there is generally no interim bonus in the first quinquennium. The errors in opposite directions neutralize each other, and the formula gives rates correct at all insuring ages to within about 2*d*. per-cent.

If we take the formula at $3\frac{1}{2}$ per-cent interest to produce a compound bonus of 25s. per-cent per annum, and use the O^M Table, and if we compare the results with the actual rates of an office which for long years past has declared compound bonuses averaging about 30s. per-cent per annum, we have the following:

Age	By Formula	Actual Rate	Difference
20	1.848	1.858	+ .010
40	3.201	3.225	+ .024
60	6.696	6.725	+ .029

Thus the company in question is able to pay a 30s. compound bonus and defray all its expenses, out of rates theoretically calculated to pay only 25s. bonus without expenses. To take another example—if we have to provide a bonus of 30s., A' must be taken at 2 per-cent when i is at $3\frac{1}{2}$ per-cent. There is a company which has paid for years past a compound bonus well over 30s. per-cent per annum, and its rates compare as follows with the theoretical 30s. bonus rates calculated as above.

Age	By Formula	Actual Rate	Difference
20	2.027	2.017	— .010
40	3.403	3.296	— .107
60	6.917	6.971	+ .054

Here again the actual office premium would seem to be capable of providing only a bonus of 30s. without a margin for expenses, and

yet, in practice, considerably larger bonuses are paid, and expenses are covered as well.

Very similar results would be obtained at the principal assuring ages by using the O^M Select Tables.

The author incidentally refers to the question whether a percentage addition to the net premiums will compensate for the same percentage addition to the rate of mortality, but he does not give a solution, although he works out an illustration which leads to an inconclusive result. In one of his earlier papers, the late Mr. Meikle, whose loss so recently we have had to deplore, gave an empirical rule. He said that if the rate of mortality throughout life be increased κ per-cent., the uniform whole-life annual premium will be increased $\frac{1}{2}\kappa$ per-cent. How accurate this very broad generalization is may be judged by an extreme illustration. If we double the force of mortality throughout life, it is the same thing as if we take two joint lives of equal age. By the O^M Table at $3\frac{1}{2}$ per-cent $P_{30} = .01677$ and $P_{30:30} = .02548$; so that by doubling the force of mortality the premium is increased by 51.9 per-cent. The approximation is, therefore, quite sufficiently close for all practical purposes.

Mr. Young, quoting others, refers to the negro as an illustration of very special risks, but why is he so hard on him? On p. 52 the negro is spoken of almost as if he were outside the pale of life assurance, while on pp. 53 and 54 a negro life rated up fifteen years is taken as being equivalent to a European life. But companies transacting business in the West Indies are not accustomed to differentiate between negroes and Europeans, and recent statistics we have had before us, which included a large but unknown proportion of negroes and of persons of mixed blood, showed that the rate of mortality exceeded that of the H^M Table by only about ten to twenty per-cent according to age. Therefore, by Mr. Meikle's formula, given above, the premiums on lives, whether negro or British, residing in the healthier islands of the West Indies, should be, say, ten per-cent above home rates, although considerably higher premiums are usually charged. It is probably an accident that the negro is specifically alluded to, and that Mr. Young merely desired, by an extreme illustration, to enforce the doctrine of the author whom he cites.

We have referred to these few points, not with any thought of being captious, but to emphasize our remark that Mr. Young's book gives food for thought. In conclusion, we would say that some of the chapters, such as that on the Investments of a Life Office, will prove very useful to the aspiring student. The treatment is eminently practical, and the information given from the stores of a man of wide experience cannot be obtained elsewhere.

THE INSTITUTE OF ACTUARIES.

THE PATRIOTIC FUND.

The Forty-First Report of the Royal Commissioners of the Patriotic Fund has been published in the form of a Blue-book. The Funds now administered by the Commissioners, are 21 in

number, the oldest being the Russian War Fund of 1854, and, the most recent, the Transvaal War Fund. The total amount of all the Funds at the end of 1902, was £1,271,547. The Blue-book contains the Report made last year, under the direction of the Council of the Institute of Actuaries, upon seven of these Funds, of which copies are as follows :

STAPLE INN HALL,

LONDON.

2 April 1903.

SIR,—In response to the Commissioners' desire for valuations of certain Funds under their administration, the Council are pleased to be again able to offer their services. Particulars of seven Funds have been submitted to us, six of which are for re-valuation and are now receiving our consideration. The Indian Mutiny Relief Fund is before us for the first time, and as we understand you are especially desirous of dealing with this at the earliest possible date, we have given it our first attention, and now submit our Report thereon.

As in the other valuations which we have already submitted, our calculations are based on the particulars received from you of the numbers and ages of the recipients, the benefits to be secured, and the assets applicable to meet the liabilities for the same.

In the valuations of all pensions we have used the Tables deduced from the Experience of Government Annuitants (four years after purchase) and published in 1884, distinguishing males from females. We have taken 3 per-cent as the rate of interest, which is the same rate as in the previous valuations.

The contribution for management has been assessed on the same basis as that assumed in previous valuations, and necessitates a reserve of £1,500.

The allowances to female beneficiaries require a reserve of £28,854, and the allowances to male beneficiaries a reserve of £3,272. A further sum of £666 is reserved to provide the allowances for those Widows whose cases are still under consideration. The total reserve required is £34,292.

The total assets of the Fund consist of £23,400 Canada Inter-Colonial Railway 4 per-cent guaranteed Bonds which are repayable this year at par, and have consequently been valued at £23,400 cash. It will thus be seen that there is a deficiency of £10,892.

The Commissioners will appreciate the serious position in which the Fund is placed, and will see the urgent necessity for supplementing the assets at the earliest possible opportunity, if it is desired to avoid the painful alternative of reducing the benefits now being conferred. It must be noted that the amount stated as the deficit is the extra amount which is required at the present moment to be held and accumulated at compound interest, in order to meet the liabilities of the Fund. Any delay in making good this deficit will therefore increase the difficulties of the situation.

INDIAN MUTINY RELIEF FUND.

31 December, 1902.

LIABILITIES.	£	ASSETS.	£
£1,442 per annum to 89 Widows and Daughters of Non-commissioned Officers and Men	12,005	Canada Inter-Colonial Railway, 4 per-cent. Guaranteed Bonds, 1903, £23,400 at par	23,400
£1,535 per annum to 40 Widows and dependent relatives of Officers	16,849	Balance (deficit)	10,892
£387 per annum to 11 male beneficiaries	3,272		
£95 per annum to 4 Widows whose cases are still in suspense	666		
Management expenses, and Commission on Post Office Orders	1,500		
	<u>£34,292</u>		<u>£34,292</u>

We are, Sir,

Your obedient Servants,

W. HUGHES, *President.*

F. SCHOOLING,

GEO. TODD,

On behalf of the
Council of the Insti-
tute of Actuaries.

To COLONEL J. S. YOUNG,

*Secretary,**Royal Commission of the Patriotic Fund.*

STAPLE INN HALL,

LONDON, W.C.

29 April 1903.

SIR,—We are in receipt of your letter in reference to the Council's Report upon the Valuation of the Indian Mutiny Relief Funds.

With regard to the estimate of cost of management and commission on Post Office Orders, we took note of the reference in your letter of the 6th March to the transfer of the management of the Patriotic Funds to the Patriotic (General) Fund, and the note there made that the Patriotic (Russian War) Fund would in future be charged 4 per-cent on expenditure as its proportion of expenses of management.

We assumed that the same arrangement would hold with reference to the Indian Mutiny Relief Funds, and our estimate is therefore as low as we considered it would be reasonable to anticipate.

In the valuation of the liabilities we naturally made the same assumptions, as to mortality and interest, as in our previous valuations of other Funds. Consequently the estimate we have made is strictly conformable to those made in respect of all the Funds which we have dealt with at your request.

We should like to call your attention to the fact that the results of the valuations as set out in our last Report still hold good as representing the liability at the 31st December last.

An actuarial valuation must necessarily be made so as to represent the actual state of the Fund at a specified date; and it is manifestly impossible to modify any such valuation by taking into account any particular circumstances that have subsequently arisen, without also reconsidering every other circumstance affecting the valuation.

It may be sufficient for your guidance at the moment to state that the present value of the ten annuities, amounting to £232. 6s. per annum, which you refer to as having been discontinued, is £2,726. The estimate of future expenses of management may be consequently reduced from £1,500 to £1,350, and the total deficiency may now be looked upon as being £8,016 instead of £10,892, as shown in our Report of the 2nd instant.

We are, SIR,

Your obedient Servants,

W. HUGHES, *President.*

F. SCHOOLING

GEO. TODD,

} *Honorary*

} *Secretaries.*

*On behalf of the
Council of the Insti-
tute of Actuaries.*

To COLONEL J. S. YOUNG,

Secretary,

Royal Commission of the Patriotic Fund.

STAPLE INN HALL,

LONDON, W.C.

18 May 1903.

Re TRANSVAAL WAR FUND.

SIR,—The Council of the Institute of Actuaries have the pleasure to submit their Valuation of the Benefits now being granted in connection with the Transvaal War Fund, and an estimate of the cost of the additional benefits which it is the desire of the Commissioners to grant, as indicated in your letter of the 6th March.

The results are presented in the four schedules annexed to this Report.

Schedule 1 exhibits—

- (a) The number of beneficiaries in each class;
- (b) The annual amounts of the benefits;
- (c) The capital amount required to provide these benefits.

Schedule 2 exhibits the particulars of Schedule 1 in columnar form, so as to enable the Commissioners to see readily the total amount of reserve required for each class of benefit, and also for each class of beneficiaries.

Schedule 3 is a Balance Sheet, referring solely to benefits now being granted, and corresponding exactly to that appended to the Council's Report of 10 June 1902.

Schedule 4 shows the cost of the additional benefits, classified in detail.

It will be seen from Schedule 3 that if the benefits are still to be limited to those classes which were included in the valuation of 1902, the Fund would show the very considerable surplus of £69,698.

If, however, the whole of the additional benefits, as set forth in the Schedules supplied, are granted, the additional liability will be £301,087, and therefore the surplus of £69,698 will require to be supplemented by the sum of £231,389.

In Schedule 2, the benefits provided for in the Balance Sheet (Schedule 3) are set forth in columns 6, 7, and 8, and if it should be considered impracticable or unadvisable to grant the whole of the additional benefits proposed, the remaining columns (2, 3, 4, and 5) of Schedule 2 show at a glance what is the liability involved in each class of benefit to each class of beneficiaries, and what consequently will be the effect of disallowing any particular benefit.

The additional benefits are very largely dependent on re-marriage, and we have, therefore, taken considerable pains in estimating the probable rates at which re-marriage will take place. It seems desirable to point out that, as the greater proportion of re-marriages take place in the first few years following widowhood, it must not be expected that the annual relief to the Fund from this source will be maintained at the rate at first experienced.

The rate of interest adopted has been 3 per-cent, as on the occasion of the last Valuation, and the remarks we then made in reference to the disposition of the securities are still applicable.

It will be noticed that the same amount has been reserved for management expenses as in the previous valuation, and that no addition has been made in respect of the proposed additional benefits. It was anticipated at the time of making the first valuation that there would be considerable additions to the number of beneficiaries, and it is thus unnecessary to alter the estimate then made, even if the whole of the increased benefits are given.

During the year 1902, the following changes took place in the numbers of the Widows and Children on the books of the Funds.

Officers' Widows—

As on 31 December, 1901	28	
Added in 1902	13	
	<hr/>	41
<i>Deduct—</i>		
Re-married	2	
Having Army Pensions, &c.	7	
	<hr/>	9
		<hr/>
On books 31 December 1902.		32

Officers' Children—

As on 31 December 1901	36	
Added in 1902	12	
	<hr/>	48
<i>Deduct—</i>		
Children of 9 Widows above	10	
Boy over age	1	
	<hr/>	11
		<hr/>
On books 31 December 1902.		37

Widows of N.C.O.'s and Men—

As on 31 December 1901	2,917	
Added in 1902	716	
		3,633
<i>Deduct—Struck off Books.</i>		
As having Warrant Officers' Widows' Pensions	14	
As having Post Office Annuities and Army Pensions (of whom 106 re-married)	603	
Found to be unworthy	53	
Died	25	
Ineligible from various causes	97	
Re-married	258	
		1,050
On books 31 December 1902		<u>2,583</u>

Of these, 1892 are in receipt of allowances for themselves or their Children, and 681 are not, as having Army or other Pensions; 10 cases were not then decided.

Children of N.C.O.'s and Men—

As on 31 December 1901	2,935	
Additions during 1902.	360	
		<u>3,295</u>

We are, SIR,

Your obedient Servants,

W. HUGHES, *President.*

F. SCHOOLING,)

GEO. TODD,)

Honorary } *On behalf of the*
Secretaries. } *Council of the Insti-*
tute of Actuaries.

To COLONEL J. S. YOUNG,

Secretary,

Royal Commission of the Patriotic Fund.

STAPLE INN HALL,

LONDON, W.C.

5 June 1903.

SIR,—The Council of the Institute of Actuaries have now the pleasure to complete their Report on the Funds submitted for their valuation as at 31 December 1902.

Reports on the Indian Mutiny Fund and the Transvaal War Fund have been already forwarded to you, the Funds now dealt with are :

Patriotic (Russian War, 1854–1856) Fund.

Patriotic (General) Fund.

Soldiers' Effects Fund.

Rodriguez Fund.

Royal Naval Relief Fund.

The principles on which our calculations are based are the same as those set out in detail in our report of the 28 April 1902.

Detailed Schedules in the form adopted in previous Reports are

appended, together with notes of those points to which we think it desirable to draw special attention.

In bringing this valuation to a close we would again express our satisfaction at having been able to be of service to the Commissioners.

We are, SIR,

Your obedient Servants,

W. HUGHES, *President.*

F. SCHOOLING, } *Honorary* } *On behalf of the*
GEO. TODD, } *Secretaries.* } *Council of the Insti-*
 } *tute of Actuaries.*

To COLONEL J. S. YOUNG,

Secretary,

Royal Commission of the Patriotic Fund.

The Schedules appended to these Reports are very voluminous, setting out, in the case of the Transvaal War Fund, the nature and extent of the benefits already granted and their present values, together with a detailed statement of the cost of proposed extended benefits. A balance sheet of each of the other Funds is also given, with notes of certain points to which it seemed desirable to call attention.

In the following letter of acknowledgment Colonel Young said :

53, CHARING CROSS,

LONDON, S.W.

13 August 1903.

GENTLEMEN,—I am desired by His Royal Highness the Duke of Cambridge, President of this Commission, and on behalf of the Patriotic Fund Commissioners, to transmit to you copy of the Forty-First Report of the Commissioners to His Majesty the King, recording, in Paragraph 7, pages 12 and 13, the deep debt of gratitude, not only the Commissioners, but all interested in the welfare of our Sailors' and Soldiers' Widows and Orphans, are under, to the President, Council, and Honorary Secretaries of the Institute of Actuaries, for the laborious and exhaustive services they have so patriotically rendered in the examination of the Assets and Liabilities of the various Funds administered by the Commissioners, and in affording most valuable Reports of the financial condition of these Funds.

I have the honour to be, GENTLEMEN,

Your most obedient Servant,

J. S. YOUNG, Colonel,

Secretary.

THE HONORARY SECRETARIES,

Institute of Actuaries,

Staple Inn Hall,

Holborn, W.C.

VOL. XXXVIII.

PART V.

JOURNAL
OF THE
INSTITUTE OF ACTUARIES.

No. CCXVII.—JULY 1904.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: S, RUE LAMARTINE, S.
BERLIN: CARLSTRASSE 11. MELBOURNE: McCARRON, BIRD & Co.
NEW YORK: THE SPECTATOR COMPANY.

PRINTED BY C. & E. LAYTON, FARRINGDON STREET, E.C.
TO WHOM ALL COMMUNICATIONS FOR THE EDITOR SHOULD BE SENT, POST PAID.

Price 2s. 6d.

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OF THE

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"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—Bacon.

VOL. XXXVIII. — PART V.

JULY 1904.

LONDON:

CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: 8, RUE LAMARTINE, 8.

BERLIN: CARLSTRASSE 11.

MELBOURNE: McCARRON, BIRD & CO.

NEW YORK: THE SPECTATOR COMPANY.

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LONDON :
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGTON STREET

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NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

A Comparison of the Various Methods of Grouping Whole-Life Assurances for Valuation. By DUNCAN C. FRASER, M.A., F.I.A., Actuary to the Royal Insurance Company.

[Read before the Institute, 25 January 1904.]

INTRODUCTORY.

1. **I**N making a valuation of the ordinary whole-life assurance contracts of a company according to the methods in general use at the present day, there are two separate sets of questions to be considered.

In the first place, the basis of valuation has to be chosen; that is to say, we have to select the table of mortality and the rate of interest which are to be employed. The questions that arise in this connection are of great importance, and for their settlement we have the assistance of the powerful instrument devised by Mr. King—the Model Office and its Valuations by various Tables of Mortality and at various Rates of Interest.

2. If the birthdays of the lives assured all fell on 1 January, and if all assurances were effected on that day, the conditions of the Model Office would be realized, and no further questions of any consequence would remain. But, in practice, birthdays and dates of entry are distributed with more or less irregularity over the whole year, and these facts lead to a second set of questions as to the methods by which the liability on the chosen basis may be best ascertained. In order that the calculations may be completed with the greatest economy of time and labour, it is usual to group the policies for valuation, and from the great diversity in the methods of grouping employed by different actuaries, it is evident that the choice of a satisfactory method is not the least difficult of the questions which the practical actuary has to decide; and, moreover, it is a question in regard to which

little assistance is given by the *Journals* and *Text-Books* of the Institute. I have therefore endeavoured to compare the different methods in actual use by applying them to the calculation of the reserves of a Model Office, with the special object of ascertaining what grouping is most suitable to be employed when it is desired to make an exact net premium valuation. In connection with the subject of grouping, it will also be necessary to refer incidentally to the question of adjustments.

Some preliminary matters must first be disposed of in relation to the Model Office and to the manner in which the dates of entry of policies and the birthdays of the lives are to be considered as distributed over the year.

MODIFICATION OF MR. KING'S MODEL OFFICE.

3. The publication of the New Institute Experience supplies a very tempting body of facts for the formation of a new Model Office; but upon consideration it appeared that for the particular object I had in view there was no special advantage to be gained by resorting to the New Experience, while on the other hand, an examination of the Model Office derived by Mr. King from the Old Institute Experience, called by him in his recent paper Model Office No. 1, showed that it could be adapted with very great facility to my purpose. In deciding to adopt that basis for my calculations, I have also been strongly governed by the feeling that I should secure a great advantage by establishing a connection between my results and those already obtained by Mr. King, which are so familiar to all members of the Institute and which have been so influential on actuarial practice.

It will be useful to examine with some minuteness the construction of Mr. King's Model Office No. 1.

For each of the ten central Ages at Entry, 20, 25, to 65, a table of "Exposed to Risk" and "Terminated" was deduced from the Old Institute Experience, the number "Terminated" in any year being the sum of the numbers who "Died" and "Discontinued." In Mr. King's words, "The probabilities of 'termination' were then easily calculated, and all that remained was to choose a suitable radix for each central age at entry, and multiply continuously by the arithmetical complements of these probabilities, slightly graduated to remove the more glaring anomalies. The radices which naturally presented themselves were the total number entering at the five ages which were

“combined to form each column of the table, because in using “them it was at once secured that the proper proportion entered “the hypothetical office at each age.”

4. Mr. King's treatment of the year of entry requires special notice. As an example, I take the actual figures for age at entry 40.

For this age the observations for “Current Ages at Entry” 39, 40, 41, 42, 43, were combined, the total number of entrants being 17,610

By reference to the Institute Experience it will be found that the actual number of terminations in the Calendar Year of Entry was 249

The number remaining at the end of the Calendar Year of Entry was therefore 17,361

This, however, was not the number used by Mr. King. The observations for the Calendar Year of Entry covered a fractional period only; but for the purpose he had in view it was desirable to avoid fractions, and he consequently introduced the assumption that in the Model Office all the Policies were effected on the 1st January of the Year of Entry, and to obtain a whole year's terminations the number given by the Institute Experience was doubled. From the number entering 17,610
he therefore deducted 498

and so obtained as the number remaining at the end of the Calendar Year of Entry 17,112

This is the number that appears in the Model Office against duration “1”, under Age at Entry 40.

5. Now since it is essential to my purpose to face the difficulty presented by the fractional durations in the Calendar Year of Entry, it is necessary to discard the arbitrary assumption made by Mr. King, and to return to the actual facts of the observations.

Thus, out of 17,610 entrants, I should have to deal with 17,361 survivors, instead of 17,112, at the end of the Calendar Year of Entry; and as the probabilities of termination in the succeeding years are unaffected, the survivors at the end of each successive year would have to be increased in the ratio of 17,361 to 17,112. On this principle a new model office could be

formed. But it will come to the same thing if the number of entrants is reduced in the ratio of 17,112 to 17,361, and by this device the figures of the Model Office can be left unaltered, the radix for each age at entry alone being varied.

6. There still remains, however, a point to be settled in regard to the age at entry itself. The assumption of the Old Institute Experience was that the "current age at entry", that is the age next birthday, was exactly attained at the end of the Calendar Year of Entry. In the above example, accordingly, the 17,610 entrants at current ages 39, 40, 41, 42, 43, were treated by Mr. King as exactly attaining the average central age of 41 at the end of the Calendar Year of Entry; and consistently with the assumptions on which the Model Office was constructed he regarded them as having entered at the age of 40 at the beginning of that year.

Here again it is necessary for me to discard the assumption which was convenient for Mr. King's purpose, and to deal with the difficulty presented by the fact that the lives entered at varying intervals in the current year of age.

The following Table shows the alterations made in Mr. King's Model Office :—

TABLE NO. 1.
Showing the Modifications made in Mr. King's Model Office.

Mr. King's Age at Entry	Mr. King's radix at each Age	Age at Entry now taken as between Ages	Radix at each Age now taken as
20	8,882	20 and 21	8 615
25	22,631	25 " 26	22,251
30	26,821	30 " 31	26,398
35	23,462	35 " 36	23,113
40	17,610	40 " 41	17,357
45	12,044	45 " 46	11,913
50	7,945	50 " 51	7,847
55	4,473	55 " 56	4,418
60	2,516	60 " 61	2,477
65	1,087	65 " 66	1,073
Total Entrants	127,471	Total Entrants	125,462

STATISTICS OF DATES OF ENTRY, ETC.

7. We cannot proceed further without more precise information as to dates of entry and ages at entry. Here the Institute experience fails us; but to fill the gap I am enabled to present statistics drawn from the experience of the Company with which I am connected.

The statistics refer to 17,174 ordinary participating whole of life assurances in force at a given date. The following Table shows the number entering in each month, sub-divided according to the number of months to run until next birthday; and in cases where an interval of six months occurs between the month of entry and the month of birth, a further distinction is made between those entering in the first and in the second half of the year of age.

TABLE No 2.

Showing out of 17,174 Policies the number effected in each month, sub-divided according to the number of months to run until next Birthday.

Month of Entry	DIFFERENCE BETWEEN MONTH OF ENTRY AND MONTH OF NEXT BIRTHDAY												Totals		
	0	1	2	3	4	5	6	6	7	8	9	10		11	12
	Entered less than 6 months before n.b.d.							Entered more than 6 months before n. b.d.							
1	101	50	28	16	21	24	10	18	29	27	23	19	13	4	383
2	249	126	84	82	61	64	15	79	59	66	66	42	37	8	1,038
3	375	159	99	102	94	72	34	85	100	90	93	48	31	6	1,388
4	317	146	96	82	74	72	35	67	78	72	74	54	29	7	1,203
5	316	154	98	78	93	74	27	76	90	79	64	58	25	6	1,238
6	311	153	82	78	79	81	24	74	67	68	67	61	26	2	1,173
7	312	137	85	71	63	65	37	64	72	74	64	52	29	8	1,133
8	323	120	103	85	63	64	40	67	72	68	66	44	27	7	1,149
9	319	151	98	89	67	61	27	126	87	79	78	42	25	7	1,256
10	378	160	123	108	97	80	55	136	90	101	73	56	32	5	1,494
11	427	190	111	101	86	74	44	107	95	94	102	65	29	4	1,529
12	513	575	384	408	332	315	174	154	351	314	290	204	164	42	4,490
Totals	3,941	2,121	1,391	1,300	1,130	1,046	522	1,053	1,190	1,132	1,030	745	467	106	17,174

Average Date of Entry, 14 August.

Average Date of Next Birthday, 9 December.

8. I assume that dates of entry and dates of birth fall in the middle of the month, with the exceptions, that in the case of December Policies the average date of entry is taken, after investigation, to be the 25th December, and that where the next birthday falls in the month of entry the date of entry is assumed to be one day before the middle of the month.

On these assumptions the interval between the average date of entry and the end of the year is $\cdot 382$ of a year, or a fraction over 139 days, and the average date of entry is therefore the 14th August. The average interval between entry and next birthday is $\cdot 321$ of a year, or a fraction over 117 days, and this makes the average date of the next birthday the 9th December.

9. According to the above data the average date of entry is 20 weeks before the end of the year; and the Old Institute Experience, by assuming that all policies were effected 26 weeks before the end of the year overstated the average duration in the year of entry by 30 per-cent—a considerable error. On the other hand, the average age at the end of a year was understated by three weeks—a difference of very small consequence.

10. Table No. 3 is given in order to show the distribution of the birthdays among the different months of the year. In the second column the actual number of birthdays falling in each month is shown; and the third column contains the corresponding numbers for months of a uniform length of 30 days, so that a more accurate comparison can be made of the relative frequency of the birthdays at different seasons.

The average date of birth is the 30th June, and the birthdays are distributed over the year with a degree of regularity which is quite sufficient to justify for practical purposes the usual assumption of a uniform distribution. But it is interesting, at the same time, to notice that the facts of the table are not quite consistent with the hypothesis of an even distribution. The deviations from the average number of birthdays in a month of 30 days are given in the fourth column of the table, and the average deviation, irrespective of sign, is 45, while a calculation by Mr. G. F. Hardy's well-known formula indicates that it should, on the assumption of an even distribution, be 29 only. A brief inspection of the table is sufficient to show that the magnitude of the average deviation is explained by the existence of a well-marked maximum period in the months of February, March and April, and an equally well-marked minimum period in the months of August, September and October. It would be a mistake to lay too much

stress upon observations which relate to policies and not to lives; but these features of the table appear to be confirmed by the Registrar-General's returns, account being taken of the fact that the latter relate not to dates of birth but to dates of registration.

TABLE No. 3.

Showing the Distribution of the Birthdays among the various Months of the Year.

Month of Birth	Number of Birthdays in each Month	Corresponding Number of Birthdays in a Month of 30 days	Deviation from Average
1	1,436	1,390	- 21
2	1,346	1,430	19
3	1,562	1,512	101
4	1,485	1,485	74
5	1,449	1,403	- 8
6	1,425	1,425	14
7	1,426	1,380	- 31
8	1,393	1,348	- 63
9	1,336	1,336	- 75
10	1,385	1,340	- 71
11	1,438	1,438	27
12	1,493	1,445	34
Totals	17,174	16,932	+ 269 - 269

Average number of Birthdays in a month of 30 days 1,411

Average deviation irrespective of sign 45

Calculated average deviation by formula $\frac{4}{3} \sqrt{n.p.q} = \frac{4}{3} \sqrt{16,932 \times \frac{1}{12} \times \frac{11}{12}} = 29$.

EXACT VALUATION OF MODEL OFFICE.

11. Before entering upon any discussion of methods of grouping and group Valuations, I shall at once apply the statistics of Table No. 2 to make an exact Valuation of the Model Office, that is a Valuation which will be identical in its results with an individual Valuation of the Policies which gives effect to the exact duration and to the exact age of entry. This Valuation will be useful as a standard of reference.

It is no part of my object to raise any question as to the basis of Valuation, and in all my calculations I shall suppose that the Valuation is to be what is ordinarily described as a Net Premium Valuation by the H^M Table of Mortality with three per-cent interest, this basis being selected simply because it may fairly be considered as representing the average standard of British Companies at the present time.

12. The exact Valuation can be obtained without difficulty by the following method.

If the interval between the end of the Calendar Year of Entry and the date of Valuation be n years, a unit policy effected the fraction k before the end of the year of entry will be of duration $n + k$, and its value, obtained by simple interpolation, is

$${}_{n+k}V_z = (1 - k) \times ({}_nV_z + P_z) + k \times {}_{n+1}V_z$$

where z is the exact age at entry. It can therefore be replaced, without altering the liability, by two fractional policies, one for k effected at the beginning of the year of entry, and one for $1 - k$ effected at the end of the year. In this way all the policies can be replaced by two groups considered as effected respectively at the beginning and at the end of their years of entry. Supposing the number of unit policies effected at the periods k_1, k_2, k_3, \dots before the end of the year to be m_1, m_2, m_3, \dots the number considered as effected at the beginning of the year will be $m_1k_1 + m_2k_2 + m_3k_3 + \dots$, which is equal to the fraction $\frac{m_1k_1 + m_2k_2 + m_3k_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ of the whole number of policies. But

this is the fraction which represents the period between the average date of entry and the end of the year, and its value has already been stated to be $\cdot 382$. We may therefore consider $\cdot 382$ of the assurances to have been effected at the beginning of the year of entry, and $\cdot 618$ of the assurances to have been effected at the end of the year of entry.

By a similar argument it can be shown that each of these groups may be further sub-divided into two groups considered as effected respectively at the beginning and at the end of the year of age current at entry, the respective proportions being $\cdot 321$ and $\cdot 679$.

13. The result may be represented in tabular form as follows:—

Date of Entry	Effected at Age of last Birthday	Effected at Age of next Birthday	Totals
1 January . . .	$\cdot 123$	$\cdot 259$	$\cdot 382$
31 December . . .	$\cdot 198$	$\cdot 420$	$\cdot 618$
Totals . . .	$\cdot 321$	$\cdot 679$	1·000

Denoting the Valuation factor employed by Mr. King in his Valuations of the Model Office by the symbol ${}_{n+1}V_x$, it will be apparent from the above Table that for a Valuation which is to give effect to exact dates of entry and exact dates of birth we must use the complex factor

$$\cdot 123 {}_{n+1}V_x + \cdot 259 {}_{n+1}V_{x+1} + \cdot 198 ({}_nV_x + P_x) + \cdot 420 ({}_nV_{x+1} + P_{x+1}).$$

14. The exact Valuation can therefore be obtained if we have made Valuations of the Model Office by the simple factors ${}_{n+1}V_{x+1}$, ${}_{n+1}V_x$, ${}_nV_{x+1}$, and ${}_nV_x$; and if we have the Net Premiums recorded for the factors P_x and P_{x+1} . The necessary data will be found in Table No. 8, at the end of the paper.

It will be observed that the correct adjustment for unequal distribution of the Premium Income has been automatically included in the above Valuation.

15. I assume that the Model Office pays Claims at the moment of death. This requires an adjustment for early payment of Claims, which is made by multiplying the total liability by the factor 1.015.

16. A few years ago the suggestion was made that an adjustment is required for delay in payment of premiums, since they are not received so as to be available for investment by the Company until an average period of six weeks or two months after the date when they are nominally due. But I think it is quite clear that this adjustment is not theoretically necessary. It is a fundamental principle that at the moment when the policy is effected and before the first premium is paid the value of the Net Premiums must be equal to the value of the risk undertaken. Thus if the sum assured is payable at the moment of death we have the equation $\bar{P}_x \times (1 + a_x) = A_x \times \left(1 + \frac{i}{2}\right)$.

Now if it is assumed that the premiums, including the first, are not available for investment until two months after they are due we ought to use the factor $\frac{1 + a_x}{1 + \frac{i}{6}}$ instead of $(1 + a_x)$; and to

preserve the truth of the equation it follows that instead of the Net Premium \bar{P}_x we must use $\bar{P}_x' = \bar{P}_x \left(1 + \frac{i}{6}\right)$. At any Valuation date, therefore, if we substitute the factor $\frac{k + a_{x+n}}{1 + \frac{i}{6}}$ for

$(k + a_{x+n})$, we must remember that the reason for which this change is made necessitates a corresponding increase in the Net Premium valued, with the result that the value of the Net Premium remains unaltered.

Having included adjustments for unequal distribution of Premium Income and for early payment of Claims, I consider therefore that the Valuation may appropriately be described as exact. The calculated Reserves for the end of each Quinquennial period will be found in Table No. 9.

TABLE No. 4.—Showing, out of 17,174 Policies, the number effected in each month, sub-divided according to the months in which the next birthdays fall.

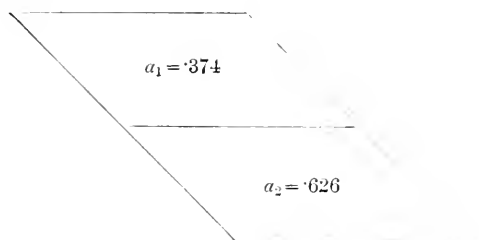
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FURTHER ANALYSIS OF STATISTICS.

17. In Table No. 4 the statistics already given in Table No. 2 are re-stated, so as to bring all cases where the next birthdays fall in the same month into the same vertical column. The vertical columns of Table No. 2, which contain those cases for which the interval until the next birthday is the same, are now arranged in diagonal lines sloping downwards from left to right.

The figures of Table No. 4 are placed in a geometrical diagram which is divided into symmetrical portions, and it is necessary to examine closely the characteristics of the policies which appear in the different divisions of the figure.

The diagram is bisected by a horizontal line which separates the policies effected in the first half of the year from those effected in the second half.

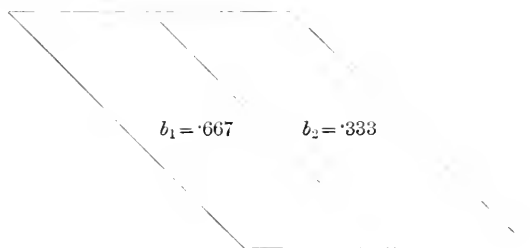


The numbers of these two classes of policies are 6,423 and 10,751, and if we denote the respective proportions by a_1 and a_2 we have

$$a_1 = \frac{6,423}{17,174} = .374$$

$$a_2 = \frac{10,751}{17,174} = .626$$

18. The diagram is also bisected by a sloping line which separates policies effected within six months of the next birthday from those for which the interval is more than six months. The numbers of

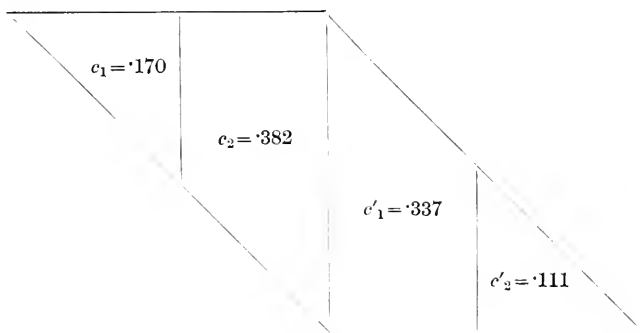


these two classes are 11,451 and 5,723, and if we denote the respective proportions by b_1 and b_2 we have

$$b_1 = \frac{11,451}{17,174} = .667$$

$$b_2 = \frac{5,723}{17,174} = .333$$

19. The diagram is a third time bisected by a vertical line which separates the policies for which the next birthday after entry falls in the year of entry from those for which it falls in the year after entry. Two other vertical lines enable us to distinguish the next birthdays which fall in the first and second half of each year.



The three verticals therefore divide the policies into four sections, of which the numbers and proportions (denoted by c_1 , c_2 , c'_1 , c'_2) are as follows

Date of Next Birthday	Number	Proportion
In first half of year of Entry .	2,917	$c_1 = .170$
In second " " " " .	6,560	$c_2 = .382$
In first half of year <i>after</i> Entry .	5,786	$c'_1 = .337$
In second " " " " .	1,911	$c'_2 = .111$

20. The three systems of classification described above depend respectively upon

- The dates of Entry ;
- The intervals between Entry and next birthdays ;
- The dates of next birthdays ;

and these systems may be combined. Thus the first and second classifications can be used together, the diagram being then divided as follows :—

22. The following is a synopsis of the scheme of notation by which the proportion of policies appearing in any portion of Table No. 4 can be readily referred to; only such symbols and values being inserted as we shall subsequently have occasion to use.

TABLE No. 5.

Showing the Number and Proportion of Policies appearing in each section of Table No. 4, and the Symbol by which each proportionate number is denoted.

Characteristics of each Section of Policies	Actual Number of Policies	PROPORTIONAL NUMBER	
		Symbol	Values
Whole number of Policies	17,174	...	1
Effected in 1st half of year	6,423	a_1	·374
„ „ 2nd „ „	10,751	a_2	·626
Effected less than 6 months before n.b.d.	11,451	b_1	·667
„ more „ „ „ „	5,723	b_2	·333
Next birthdays falling in—			
1st half of year of entry	2,917	c_1	·170
2nd „ „ „ „	6,560	c_2	·382
1st half of year after entry	5,786	c'_1	·337
2nd „ „ „ „	1,911	c'_2	·111
Date of entry. 1st half Interval until n.b.d. < 6 months }	4,386	a_1b_1	·252
Date of entry. 1st half Interval until n.b.d. > 6 months }	2,087	a_1b_2	·122
Date of entry. 2nd half Interval until n.b.d. < 6 months }	7,115	a_2b_1	·415
Date of entry. 2nd half Interval until n.b.d. > 6 months }	3,636	a_2b_2	·211
Date of entry. 1st half Interval until n.b.d. < 6 months N.B.D. 2nd half of year of entry }	1,419	$a_1b_1c_2$	·082
Date of entry. 2nd half Interval until n.b.d. > 6 months N.B.D. 1st half of year after entry }	1,725	$a_2b_2c'_1$	·100

GROUP VALUATIONS OF THE MODEL OFFICE.

23. The following list indicates briefly the various groupings I have used for the Valuations of the Model Office. The verbal description of each grouping is accompanied by a symbolic description in Mr. Whittall's useful notation.

TABLE No. 6.

Showing the various Groupings employed in the Valuations of the Model Office.

No.	Age at Entry	Duration	Valuation Age	IN MR. WHITTALL'S NOTATION		
				Age at Entry	Duration	Valuation Age
1	Exact .	Exact .	Exact .	x	t	$x + t$
2	Nearest .	Exact	(x)	t	...
3	N.B.D. .	Exact	$\{x\}$	t	...
4	N.B.D. .	Nearest	$\{x\}$	(t)	...
5	N.B.D. .	Curtate	$\{x\}$	\underline{t}	...
6	N.B.D. .	Curtate + $\frac{1}{2}$...	$\{x\}$	$\underline{t} + \frac{1}{2}$...
7	N.B.D. .	Curtate + 1	...	$\{x\}$	$\underline{t} + 1$...
8	N.B.D.	N.B.D. .	$\{x\}$...	$\{x + t\}$
9	N.B.D.	Nearest .	$\{x\}$...	$(x + t)$
10	N.B.D.	L.B.D. + $\frac{1}{2}$	$\{x\}$...	$x + t + \frac{1}{2}$
11	Mean Age	Curtate + $\frac{1}{2}$	L.B.D. + $\frac{1}{2}$	$ x$	$\underline{t} + \frac{1}{2}$	$x + t + \frac{1}{2}$
12	Nearest .	Nearest	(x)	(t)	...
13	Nearest	Nearest .	(x)	...	$(x + t)$
14	...	Nearest .	Nearest	(t)	$(x + t)$
15	Nearest .	Curtate + $\frac{1}{2}$...	(x)	$\underline{t} + \frac{1}{2}$...

24. With the exception of the first, which has been introduced as a standard of reference, and with the exception of No. 14, which was introduced by Mr. King as a method for deducing the "Exposed to Risk", but which might equally well be used for valuation purposes, each of the above groupings has been in actual use within recent years for Valuations under the Life Assurance Companies Acts in this country. It is not claimed that the list is exhaustive, but I believe that every system of grouping which is in common use among those offices which aim at making an ordinary net premium Valuation has been included.

25. In the case of every grouping except the first the Valuation has been made both with and without adjustments. This leads to four separate Valuations for each grouping as follows:—

- (a) *Valuation without Adjustments.* By this is meant a Valuation in which no adjustment is made for early payment of claims, and in which the proportion of current premiums reserved is 50 per-cent.

- (b) *Valuation with Premium Adjustment only.* Where the Valuation is made by Exact Durations the adjustment for unequal distribution of Premium Income is made by reserving the full premiums on $\cdot 618$ of the policies; in all other cases it is made by reserving the full premiums on $\cdot 626$ of the policies, this being the proportion effected in the second half of the year.
- (c) *Valuation with Claim Adjustment only.* This Valuation is obtained by multiplying the results of the "Valuation without Adjustments" by $1\cdot 015$, and so providing for payment of claims at the moment of death.
- (d) *Valuation with both Adjustments.* This Valuation is obtained by multiplying the "Valuation with Premium Adjustment only" by $1\cdot 015$.

26. I shall now take the groupings one by one in order to show how the Valuation formula for each may be obtained.

In all the formulas, $n+1$ means the duration as measured from the beginning of the year of entry, and therefore the duration measured from the end of that year is n . The age last birthday at entry is denoted by x , and the age next birthday, or the office age, is therefore $x+1$.

In every case the term which has reference to the proportion of current premiums reserved is omitted.

According to the above notation Mr. King's Valuation formula was simply ${}_{n+1}V_x$.

(1) *Exact Ages at Entry and Exact Durations.*

Omitting the terms which include only P_x or P_{x+1} multiplied by a fraction, the Valuation formula already given becomes—

$$\cdot 259 {}_{n+1}V_{x+1} + \cdot 123 {}_{n+1}V_x + \cdot 420 {}_nV_{x+1} + \cdot 198 {}_nV_x$$

(2) *Nearest Ages at Entry;*

Durations . . . Exact.

Since Exact Durations are used we treat $\cdot 382$ of the policies as effected on the 1st January, and $\cdot 618$ as effected on the 31st December.

Since Nearest Ages at Entry are used, those policies entering within six months of the next birthday, namely, b_1 or $\cdot 667$ of the whole, are taken as entering at age $x+1$; and those entering within six months of the last birthday, namely, b_2 or $\cdot 333$ of the whole, are taken as entering at age x .

The formula is therefore

$$\cdot 382(b_1 \times_{n+1} V_{x+1} + b_2 \times_{n+1} V_x) + \cdot 618(b_1 \times_n V_{x+1} + b_2 \times_n V_x)$$

which becomes, when numerical values are substituted for b_1 and b_2

$$\cdot 255_{n+1} V_{x+1} + \cdot 127_{n+1} V_x + \cdot 412_n V_{x+1} + \cdot 206_n V_x$$

(3) *Ages at Entry N.B.D.*

Durations . . . Exact.

Since Exact Durations are used we treat $\cdot 382$ of the policies as effected on the 1st January and $\cdot 618$ as effected on the 31st December.

The age at entry is in every case $x+1$.

The Valuation formula is therefore

$$\cdot 382_{n+1} V_{x+1} + \cdot 618_n V_{x+1}$$

(4) *Ages at Entry N.B.D.*

Nearest Durations.

The ages at entry are in all cases $x+1$. Policies entering in the first half of the year are treated as if effected on the 1st January, and therefore have duration $n+1$. Policies effected in the second half of the the year are treated as if effected on the 31st December and have duration n . The relative proportions of these two classes being a_1 and a_2 the formula is

$$a_1 \times_{n+1} V_{x+1} + a_2 \times_n V_{x+1}$$

or, with numerical coefficients,

$$\cdot 374_{n+1} V_{x+1} + \cdot 626_n V_{x+1}$$

(5) *Ages at Entry N.B.D.*

Durations . . . Curtate.

This system represents the old Institute method of grouping, the office age at entry being assumed to be exactly attained at the end of the year of entry. The valuation age, in fact, is the same as if dates of entry and days of birth all fell on the 31st December. The valuation formula is therefore

$$_n V_{x+1}.$$

Theoretically the complete valuation formula on the above assumptions should be $_n V_{x+1} + P_{x+1}$ a full year's premiums being reserved; but in practice, as is well known, the usual proportion only is reserved on account of current premiums.

(6) *Ages at Entry N.B.D.**Durations . . . Curtate + $\frac{1}{2}$.*

The assumption here is that all entries take place in the middle of the year, and at the office ages. The result is the same if half the policies be assumed to enter at the beginning and half at the close of the year of entry, the office age in both cases being exactly attained at the date of entry. The valuation formula is therefore simply

$$\cdot 5_{n+1}V_{x+1} + \cdot 5_nV_{x+1}$$

(7) *Ages at Entry N.B.D.**Durations . . . Curtate + 1.*

In this case the valuation ages are obtained on the very severe assumptions that entry takes place at the commencement of the year of entry, and that the office age is exactly attained at the date of entry.

The valuation formula is therefore

$$_{n+1}V_{x+1}$$

Theoretically the assumptions of this method would lead to the conclusion that no reserve should be retained on account of the current premiums; but in practice the usual reserves are made.

(8) *Ages at Entry N.B.D.**Valuation Ages N.B.D.*

Applying the rule given under the last grouping the duration is $n+1$ when the age next birthday after entry falls in the year of entry, that is in $c_1 + c_2$ cases; and the duration is n when it falls in the year after entry, that is in $c'_1 + c'_2$ cases. The valuation formula is therefore

$$(c_1 + c_2) \times _{n+1}V_{x+1} + (c'_1 + c'_2) \times _nV_{x+1}$$

or, with numerical coefficients,

$$\cdot 552_{n+1}V_{x+1} + \cdot 448_nV_{x+1}$$

(9) *Ages at Entry N.B.D.**Nearest Ages at Valuation.*

The formula in this and in other cases where the valuation age is determined without reference to the duration is most easily obtained by observing the following simple rule:—

The duration is

$$n + 1,$$

or $n,$

or $n - 1,$

according as the valuation age at the close of the year of entry is,

- (1) greater than the age of entry by one year ;
- or (2) equal to the age of entry ;
- or (3) less than the age of entry by one year.

Applying this rule to the present case, the duration will be found to be

$$\begin{array}{ll} & n+1, \\ \text{or} & n, \\ \text{or} & n-1, \end{array}$$

according as the next birthday after entry falls,

- (1) in the first half of the year of entry, which happens in c_1 cases ;
- or (2) between the middle of the year of entry and the middle of the next year, which happens in $c_2 + c'_1$ cases ;
- or (3) in the second half of the year after entry which happens in c'_2 cases.

As the age at entry is in all cases $x+1$ the formula is therefore

$$c_1 \times {}_{n+1}V_{x+1} + (c_2 + c'_1) \times {}_nV_{x+1} + c'_2 \times {}_{n-1}V_{x+1}$$

or, with numerical coefficients,

$$\cdot 170 {}_{n+1}V_{x+1} + \cdot 719 {}_nV_{x+1} + \cdot 111 {}_{n-1}V_{x+1}$$

(10) *Ages at Entry N.B.D.*

Valuation Ages (L.B.D. + $\frac{1}{2}$).

The effect is the same if we say that valuation ages are taken as N.B.D. for half the policies and L.B.D. for the remaining half. By substituting L.B.D. for N.B.D. we reduce the duration by one year, since the ages at entry suffer no change ; and by reference to the last case it is readily seen that the formula will be

$$\begin{aligned} (c_1 + c_2) \times (\cdot 5 {}_{n+1}V_{x+1} + \cdot 5 {}_nV_{x+1}) \\ + (c'_1 + c'_2) \times (\cdot 5 {}_nV_{x+1} + \cdot 5 {}_{n-1}V_{x+1}). \end{aligned}$$

Since $c_1 + c_2 + c'_1 + c'_2 = 1$, this may be written

$$\cdot 5 \times (c_1 + c_2) \times {}_{n+1}V_{x+1} + \cdot 5 \times {}_nV_{x+1} + \cdot 5 \times (c'_1 + c'_2) \times {}_{n-1}V_{x+1}$$

or, with numerical coefficients,

$$\cdot 276 {}_{n+1}V_{x+1} + \cdot 5 {}_nV_{x+1} + \cdot 224 {}_{n-1}V_{x+1}$$

- (11) *Ages at Entry* = Mean Ages ;
Durations = Curtate + $\frac{1}{2}$;
Valuation Ages = *L.B.D.* + $\frac{1}{2}$.

The grouping here is by years of birth for the determination both of ages at entry and valuation ages.

The age at entry is $x+1$ or x according as the next birthday falls in the year of entry or in the next year; the relative proportions being (c_1+c_2) and $(c'_1+c'_2)$.

The effect of the assumption as to duration is the same as if half the policies were effected at the beginning and half at the end of the year.

The valuation formula is therefore

$(c_1+c_2) \times (\cdot 5_{n+1}V_{x+1} + \cdot 5_nV_{x+1}) + (c'_1+c'_2) \times (\cdot 5_{n+1}V_x + \cdot 5_nV_x)$
 with numerical coefficients this is

$$\cdot 552 \times (\cdot 5_{n+1}V_{x+1} + \cdot 5_nV_{x+1}) + \cdot 448 \times (\cdot 5_{n+1}V_x + \cdot 5_nV_x)$$

or $\cdot 276_{n+1}V_{x+1} + \cdot 224_{n+1}V_x + \cdot 276_nV_{x+1} + \cdot 224_nV_x$

It will be noticed that the valuation ages are the same, but the durations greater than in the last case.

- (12) *Nearest Ages at Entry.*

Nearest Durations.

The proportion of policies effected in the first half of the year and within six months of the next birthday is a_1b_1 ; for these policies the duration is $n+1$, the age at entry is $x+1$, and the corresponding portion of the valuation formula is therefore

$$a_1b_1 \times {}_{n+1}V_{x+1}.$$

The proportion of policies effected in the first half of the year and with an interval of more than six months before the next birthday is a_1b_2 ; for these policies the duration is $n+1$, the age at entry is x , and the corresponding portion of the valuation formula is therefore $a_1b_2 \times {}_{n+1}V_x$. Dealing similarly with the policies effected in the second half of the year we obtain two additional terms $a_2b_1 \times {}_nV_{x+1}$ and $a_2b_2 \times {}_nV_x$ and the whole formula is therefore

$$a_1b_1 \times {}_{n+1}V_{x+1} + a_1b_2 \times {}_{n+1}V_x + a_2b_1 \times {}_nV_{x+1} + a_2b_2 \times {}_nV_x$$

or, giving numerical values to the symbols,

$$\cdot 252_{n+1}V_{x+1} + \cdot 122_{n+1}V_x + \cdot 415_nV_{x+1} + \cdot 211_nV_x.$$

- (13) *Nearest Ages at Entry.*

Nearest Ages at Valuation.

Consider first the proportion of policies effected within six months of the next birthday, = b_1 . The nearest age at entry

is the age next birthday, or $x+1$. If in all these cases the next birthday fell between the middle of the year of entry and the middle of the following year, the nearest age at 31st December after entry would be $x+1$ and would coincide with the age at entry, giving n for the duration.

The portion of the formula relating to these policies would then be $b_1 \times {}_nV_{x+1}$.

The above supposition however is not quite true. In c_1 cases the next birthday falls in the first half of the year of entry, and the valuation age at the 31st December after entry will in these cases exceed the age at entry by one year, giving duration $n+1$. In the remaining $b_1 - c_1$ cases the duration remains n , and the part of the formula relating to policies effected within six months of the next birthday is therefore

$$c_1 \times {}_{n+1}V_{x+1} + (b_1 - c_1) \times {}_nV_{x+1}$$

Similarly for policies effected more than six months before a birthday the relative portion of the formula is

$$(b_2 - c'_2) \times {}_{n+1}V_x + c'_2 \times {}_nV_x$$

The whole formula is therefore

$$c_1 \times {}_{n+1}V_{x+1} + (b_1 - c_1) \times {}_nV_{x+1} + (b_2 - c'_2) \times {}_{n+1}V_x + c'_2 \times {}_nV_x$$

or numerically

$$\cdot 170 {}_{n+1}V_{x+1} + \cdot 222 {}_{n+1}V_x + \cdot 497 {}_nV_{x+1} + \cdot 111 {}_nV_x.$$

(14) *Nearest Durations.*

Nearest Ages at Valuation.

This is Mr. King's well-known method of grouping, the ages at entry being the nearest ages at nearest 31st December to date of entry.

For the a_1 policies effected in the first half of the year of entry the duration is $n+1$. The age at entry is $x+1$ for c_1 of these policies, and x for the remainder; giving the terms

$$c_1 \times {}_{n+1}V_{x+1} + (a_1 - c_1) \times {}_{n+1}V_x$$

Similarly for the policies effected in the second half of the year of entry we obtain the terms

$$(a_2 - c'_2) \times {}_nV_{x+1} + c'_2 \times {}_nV_x$$

and the whole formula is

$$c_1 \times {}_{n+1}V_{x+1} + (a_1 - c_1) \times {}_{n+1}V_x + (a_2 - c'_2) \times {}_nV_{x+1} + c'_2 \times {}_nV_x$$

or numerically

$$\cdot 170 {}_{n+1}V_{x+1} + \cdot 204 {}_{n+1}V_x + \cdot 515 {}_nV_{x+1} + \cdot 111 {}_nV_x$$

TABLE NO. 7.—Showing the Formulas for Various Groupings.

No.	DESCRIPTION OF GROUPING	SYMBOLIC COEFFICIENTS OF FORMULAS				NUMERICAL COEFFICIENTS OF FORMULAS				
		$n+1 V_{x+1}$	$n+1 V_x$	$n V_{x+1}$	$n V_x$	$n-1 V_{x+1}$	$n+1 V_x$	$n V_{x+1}$	$n V_x$	$n-1 V_{x+1}$
1	a. Age at Entry b. Duration c. Valuation Age	·259	·420	·198	...
2	a. Exact b. Exact c. Exact	·255	·412	·206	...
3	a. Nearest b. Nearest	·382	·618
4	a. N.B.D. b. Exact c. N.B.D.	·374	·626
5	a. Nearest b. Nearest	a_1	...	a_2	1·000
6	a. N.B.D. b. Curtate c. Curtate + $\frac{1}{2}$	·500	·500
7	a. N.B.D. b. Curtate + 1 c. N.B.D.	1·000
8	a. N.B.D. b. Curtate + 1 c. N.B.D.	$c_1 + c_2$...	$c'_1 + c'_2$	·552	·448
9	a. N.B.D. b. Nearest c. Nearest	c_1	...	$c_2 + c'_1$...	c'_2	·170	·719	...	·111
10	a. N.B.D. b. Curtate + $\frac{1}{2}$ c. L.B.D. + $\frac{1}{2}$	$\frac{1}{2}(c_1 + c_2)$...	$\frac{1}{2}$...	$\frac{1}{2}(c'_1 + c'_2)$	·276	·500	...	·224
11	a. Mean b. Curtate + $\frac{1}{2}$ c. L.B.D.	$\frac{1}{2}(c_1 + c_2)$	$\frac{1}{2}(c'_1 + c'_2)$	$\frac{1}{2}(c_1 + c_2)$	$\frac{1}{2}(c'_1 + c'_2)$...	·276	·276	·224	...
12	a. Nearest b. Nearest	$a_1 b_1$	$a_1 b_2$	$a_2 b_1$	$a_2 b_2$...	·252	·415	·211	...
13	a. Nearest b. Nearest	c_1	$b_2 - c'_2$	$b_1 - c_1$	c'_2	...	·170	·497	·111	...
14	a. Nearest b. Nearest	c_1	$a_1 - c_1$	$a_2 - c'_2$	c'_2	...	·170	·515	·111	...
15	a. Nearest	$\frac{1}{3} b_1$	$\frac{1}{3} b_2$	$\frac{1}{3} b_1$	$\frac{1}{3} b_2$...	·3335	·3335	·1665	...

(15) *Nearest Ages at Entry.**Durations . . . Curtate + $\frac{1}{2}$.*In b_1 cases the age at entry is $x+1$; and in b_2 cases it is x .

The effect of the assumption as to durations is the same as if half the policies are taken as entering on the 1st January and half on the 31st December. The valuation formula is therefore

$$b_1 \times \cdot 5({}_{n+1}V_{x+1} + {}_nV_{x+1}) + b_2 \times \cdot 5({}_{n+1}V_x + {}_nV_x)$$

$$\text{or } \frac{1}{2}b_1 \times {}_{n+1}V_{x+1} + \frac{1}{2}b_2 \times {}_{n+1}V_x + \frac{1}{2}b_1 \times {}_nV_{x+1} + \frac{1}{2}b_2 \times {}_nV_x$$

which becomes numerically

$$\cdot 3335 {}_{n+1}V_{x+1} + \cdot 1665 {}_{n+1}V_x + \cdot 3335 {}_nV_{x+1} + \cdot 1665 {}_nV_x.$$

The various valuation formulas are summarized in Table No. 7.

27. The three groupings (12), (13), and (14) which depend upon different applications of the "Nearest" method of obtaining the valuation facts are connected in an interesting manner. It will be found on examination that out of the eight triangular sections into which Table No. 4 is divided, the policies which occupy six of the sections, constituting 81·8 per-cent of the whole number, are valued identically by the three methods. The differences in the valuations apply only to policies in the sections $a_1b_1c_2$ and $a_2b_2c'_1$, that is to policies identified as follows:—

	$a_1b_1c_2$	$a_2b_2c'_1$
Date of Entry . . .	First half . . .	Second half
Interval until N.B.D. . .	Less than 6 months . .	More than 6 months
Date of N.B.D. . . .	Second half of year of entry	First half of year after entry

The exact differences between valuations by the three groupings are indicated in the following Table:—

No. of Grouping	CHARACTERISTICS			VALUATION FACTORS FOR	
	Age at Entry	Duration	Valuation Age	Section $a_1b_1c_2 (= \cdot 082)$	Section $a_2b_2c'_1 (= \cdot 100)$
12	Nearest	Nearest	...	${}_{n+1}V_{x+1}$	${}_nV_x$
13	Nearest	...	Nearest	${}_nV_{x+1}$	${}_{n+1}V_x$
14	...	Nearest	Nearest	${}_{n+1}V_x$	${}_nV_{x+1}$

Thus grouping (13) increases the duration of $a_2b_2c'_1$ policies by one year, and diminishes the duration of $a_1b_1c_2$ policies by one year; while grouping (14) makes similar changes in the ages at entry.

28. Grouping (9) differs from grouping (13) by taking the ages at entry as the ages N.B.D., the valuation ages being the nearest ages in each case. The formula for this grouping is

$$c_1 \times {}_{n+1}V_{x+1} + (c_2 + c'_1) \times {}_nV_{x+1} + c'_2 \times {}_{n-1}V_{x+1}$$

Since $c_1 + c_2 + c'_1 + c'_2 = 1 = b_1 + b_2$

it follows that $c_2 + c'_1 = b_1 + b_2 - c_1 - c'_2$

and therefore the formula can be written

$$c_1 \times {}_{n+1}V_{x+1} + (b_1 - c_1 + b_2 - c'_2) \times {}_nV_{x+1} + c'_2 \times {}_{n-1}V_{x+1}$$

The formula for grouping (13) is

$$c_1 \times {}_{n+1}V_{x+1} + (b_1 - c_1) \times {}_nV_{x+1} + (b_2 - c'_2) \times {}_{n+1}V_x + c'_2 \times {}_nV_x$$

The difference between the two formulas is

$$(b_2 - c'_2) \times ({}_{n+1}V_x - {}_nV_{x+1}) + c'_2 \times ({}_nV_x - {}_{n-1}V_{x+1})$$

Grouping (9) therefore is derived from grouping (13) by increasing the age at entry by one year and diminishing the duration by one year in the case of $b_2 = .333$ of the policies; that is by adopting Dr. Sprague's modification of the net premium method of valuation for one-third of the policies.

29. There is a similar connection between groupings (10) and (11).

No. of Grouping	CHARACTERISTICS		Formula
	Age at Entry	Valuation Age	
10	N.B.D.	L.B.D. + $\frac{1}{2}$	$\cdot 5 \times (c_1 + c_2) \times {}_{n+1}V_{x+1} + \cdot 5 \times (c_1 + c_2 + c'_1 + c'_2) \times {}_nV_{x+1} + \cdot 5 \times (c'_1 + c'_2) {}_{n-1}V_{x+1}$
11	Mean	L.B.D. + $\frac{1}{2}$	$\cdot 5 \times (c_1 + c_2) \times {}_{n+1}V_{x+1} + \cdot 5 \times (c'_1 + c'_2) \times {}_{n+1}V_x + \cdot 5 \times (c_1 + c_2) \times {}_nV_{x+1} + \cdot 5 \times (c'_1 + c'_2) \times {}_nV_x$

The difference between the two formulas is

$$\cdot 5 \times (c'_1 + c'_2) \times ({}_{n+1}V_x - {}_nV_{x+1}) + \cdot 5 \times (c'_1 + c'_2) \times ({}_nV_x - {}_{n-1}V_{x+1})$$

Grouping (10) is therefore derived from grouping (11) by increasing the age at entry by one year and diminishing the duration by one year in the case of $c'_1 + c'_2 = .448$ of the

policies; that is by adopting Dr. Sprague's modification of the net premium method of valuation for nearly 45 per-cent of the policies.

30. For the purpose of analyzing grouping No. (5) it is convenient to invent a new method of grouping, which we will call No. (16). Let us suppose that the valuation ages are obtained on the assumption of the old Institute Experience, namely, that the ages next birthday at entry are exactly attained at the end of the year of entry; and let these valuation ages be used in combination with nearest durations. The effect is that policies entering in the first half of the year of entry are treated as if effected on the 1st January at the ages L.B.D., and that policies entering in the second half of the year of entry are treated as if effected on the 31st December at the ages N.B.D.

The valuation formula for grouping (16) is

$$a_1 \times {}_{n+1}V_x + a_2 \times {}_nV_{x+1}$$

or $\cdot 374 \times {}_{n+1}V_x + \cdot 626 \times {}_nV_{x+1}.$

The following are specimens of the reserves brought out by this grouping. The comparative numbers indicate the relationship between the Actual Reserves and those based on an exact Valuation taken as 10,000.

Age of Office	WITHOUT ADJUSTMENTS		WITH PREMIUM ADJUSTMENTS ONLY		WITH CLAIM ADJUSTMENTS ONLY		WITH BOTH ADJUSTMENTS	
	Reserve	Com- parative Number	Reserve	Com- parative Number	Reserve	Com- parative Number	Reserve	Com- parative Number
5 years	25,959	9,219	27,776	9,864	26,348	9,357	28,193	10,012
25 "	348,873	9,674	355,040	9,845	354,106	9,816	360,366	9,993
50 "	695,363	9,735	703,235	9,845	705,793	9,881	713,784	9,993

This method practically gives the exact net premium reserves when both adjustments are made.

31. Now compare groupings (5) and (16).

No. of Group- ing	CHARACTERISTICS			Formula
	Age at Entry	Duration	Valuation Age	
(5)	N.B.D.	Curtate	{ Age n.b.d. at Entry assumed to be ex- actly attained at end of year of entry }	${}_nV_{x+1}$
(16)	...	Nearest	{ Age n.b.d. at Entry assumed to be ex- actly attained at end of year of entry }	$a_1 \times {}_{n+1}V_{x+1} + a_2 \times {}_nV_{x+1}$

Since $a_1 + a_2 = 1$, the difference between the two formulas is $a_1 \times ({}_{n+1}V_x - {}_nV_{x+1})$. Therefore (5) is obtained from (16) by increasing the age at entry by one year and diminishing the duration by one year in the case of $a_1 = .374$ of the policies; that is, it may be said to differ from an exact net premium valuation by adopting Dr. Sprague's modification of the net premium method for three-eighths of the policies.

VALUATION RESULTS.

32. Tables are given at the end of the paper showing the reserves of the Model Office by the various groupings, and the various methods of treating the adjustments already described, and giving a comparison of each with the result of an exact valuation.

There is a striking difference in the results brought out by the various valuations. To take two extreme cases, the comparison between grouping No. 5 with no adjustments, and grouping No. 7 with full adjustments is as follows:

	AT END OF		
	5 years	25 years	50 years
Comparative figure for grouping (5) without adjustments	8,197	9,444	9,600
Comparative figure for grouping (7) with both adjustments	12,093	10,675	10,465
Difference, expressed as a percentage of the liability by an exact valuation	38.96 %	12.31 %	8.65 %

33. The differences between the various valuations are due to different methods of treating the durations, the ages at entry, and the adjustments.

34. The following are examples of differences due to differences in duration only, the ages at entry in each case being taken as ages N.B.D., and full Adjustments being made:

No.	Method of obtaining Durations	Average Duration = $n+$	COMPARATIVE RESERVES AT END OF		
			5 Years	25 Years	50 Years
7	Curtate + 1 . .	1.000	12,093	10,675	10,465
8	Valuation Age = Age N.B.D.	.552	10,682	10,261	10,190
6	Curtate + $\frac{1}{2}$. .	.500	10,518	10,213	10,159
4	Nearest374	10,121	10,096	10,081
9	Valuation Age = Nearest Age	.059	9,134	9,806	9,888
10	Valuation Age = L.B.D. + $\frac{1}{2}$.	.052	9,117	9,801	9,885
5	Curtate000	8,942	9,751	9,852

The groupings in this list are arranged in descending order of durations. The differences in the reserves are proportional to the differences in the durations, and the variation in the liability produced by a variation of one year in the duration is at once obtained by comparing the reserves by groupings (7) and (5).

35. The effect of differences in the treatment of the ages at entry can be investigated in the same way. A comparison of groupings (2) and (3) shows the differences in the reserves corresponding to an average difference of .333 in the age at entry; and the differences at all periods due to a variation of one year in the age at entry are shown in the following Table, along with the differences due to a variation of one year in the duration. The differences are expressed as percentages of the liability by an exact valuation.

Age of Office	DIFFERENCES DUE TO A VARIATION OF ONE YEAR	
	In the Duration	In the Age at Entry
5 Years	31.51 %	3.33 %
10 "	18.95 %	3.24 %
15 "	13.79 %	3.15 %
20 "	10.98 %	3.03 %
25 "	9.24 %	2.91 %
30 "	8.09 %	2.79 %
35 "	7.29 %	2.70 %
40 "	6.74 %	2.61 %
45 "	6.37 %	2.52 %
50 "	6.13 %	2.49 %

36. A variation in the duration is, therefore, nearly ten times as important as an equal variation in the age at entry at the end of the first quinquennium, three times as important at the end of the sixth, and two and a half times as important at the end of the tenth.

37. The claim adjustment represents nearly $1\frac{1}{2}$ per-cent of the liability by an exact valuation at every period; and the premium adjustment represents nearly 6 per-cent of the exact liability at the end of the first quinquennium, $2\frac{1}{2}$ per-cent at the end of the fifth, and 1 per-cent at the end of the tenth.

The importance of the claim and premium adjustments cannot be questioned; but occasionally a valuation is made so stringent in other respects, as in the case of groupings (6), (7) and (8), that the omission of one or other of these adjustments does not bring the valuation below the standard of an exact net premium valuation.

I think, however, that it will be generally agreed that it is more satisfactory to deal with the adjustments as exactly as possible, particularly as there is no difficulty in so doing.

38. The valuations given in the present paper are on the common basis of H^M 3 per-cent. It may be assumed that on any other basis the relative results would be approximately the same, and therefore by using the Tables in this paper in conjunction with Mr. King's recent paper on the Comparative Reserves of Life Assurance Companies according to various Tables of Mortality, and at various rates of interest, it is possible to make a comparison between two valuations which differ, not only as regards grouping and adjustments, but also as regards the basis employed. For example, if office A, 30 years old, values on the basis of H^M and $H^{M(5)}$, 3 per-cent, using nearest ages at entry and valuation, and making both adjustments; and if office B, 30 years old, values on the basis of O^M $2\frac{1}{2}$ per-cent, using ages N.B.D. at entry and curtate durations, and making the claim adjustment only; then if the statistics of the present paper are appropriate to both offices the comparison between the valuations will be as follows:

Comparative figure for basis, from	Office A.	Office B.
Mr. King's paper	10,152	10,532
Comparative figure for grouping and adjustments (reduced to radix of 1.000)	1.0012	.9647
Product of comparative figures	10,164	10,160

In the case supposed, the 3 per-cent valuation is, therefore, more stringent than that at $2\frac{1}{2}$ per-cent.

The inclusion of an average amount of reversionary bonuses, however, would turn the balance in favour of the valuation at the lower rate of interest.

39. I have not thought it necessary to make any special investigation of the effect of including reversionary bonuses in the various valuations, but it is obvious that in the majority of cases, where the valuation ages are approximately equal to the true average ages, the effect would be to increase the reserves for a given age of the office by practically the same amount. In those cases, five in all, where the valuation ages are in excess of the true average ages the reserves would be increased to a greater extent, and as these are the cases of highest reserves the differences already existing would be accentuated.

40. Of all the groupings examined there are two which stand out as giving, when full adjustments are made, the closest agreement at all periods with the results of an exact net premium valuation. These groupings are No. (12) which uses nearest ages at entry, and No. (14) which uses nearest ages at valuation, the durations being in both cases nearest durations. The latter is Mr. King's well-known method of grouping.

The following Table shows the amounts by which valuations of the Model Office by these two groupings, with full adjustments, exceed or fall short of the results of an exact valuation.

Age of Office	Exact Valuation	EXCESS OR DEFICIENCY OF VALUATIONS BY	
		Grouping (12)	Grouping (14)
5 Years	28,158	29	19
10 "	86,310	32	39
15 "	166,268	23	59
20 "	260,298	5	78
25 "	360,632	-18	97
30 "	458,854	-40	115
35 "	517,486	-58	131
40 "	620,976	-71	145
45 "	676,766	-79	156
50 "	714,314	-84	161

The two groupings give results differing by such insignificant amounts from exact valuations, that a valuation by either method may be called exact.

Both groupings are convenient in use, and there is nothing to choose between them in this respect. As, however, grouping No. (12) gives the nearer approximation, and as, moreover, it leads to a distribution of the Policies, which corresponds almost exactly with that which gives the Exact Valuation, as can be seen by comparing the numerical coefficients in Table VII, it must, I think, be looked upon as the most satisfactory and most convenient method of grouping which can be used when it is desired to obtain the result of an exact Net Premium Valuation with the greatest possible degree of accuracy.

ADDENDUM.

41. The valuations of the Model Office were made on the arithmometer, and proofs were obtained by the following process:

Valuations by the simple formulas ${}_{n+1}V_{x+1}$, and ${}_{n-1}V_{x+1}$ were first made.

Valuations by the simple formulas ${}_{n+2}V_x$, ${}_{n+1}V_x$, and ${}_nV_x$, were deduced by the application of the identity

$${}_{n+1}V_x = {}_1V_x + (1 - {}_1V_x) \times {}_nV_{x+1}.$$

Valuations by the simple formulas ${}_{n+2}V_x$, ${}_{n+1}V_x$, and ${}_nV_x$, were then directly made, and the results compared.

The result of the valuation by the formula ${}_{n+1}V_x$ was also compared with Mr. King's valuation of the Model Office by the H^M 3 per-cent Table (see *J.I.A.*, xx, p. 273). There are trifling differences which appear to be due simply to the fact that one set of calculations was made by four-figure logarithms and the other set on the arithmometer.

TABLE NO. 8.—*Showing the subsidiary Valuations of the Model Office by the simple factors, with the corresponding Net Premiums.*

Age of Office	AGE AT ENTRY TAKEN AS AGE N.B.D. = $x+1$				AGE AT ENTRY TAKEN AS AGE L.B.D. = x		
	Valuation Factor			Net Premiums	Valuation Factor		Net Premiums
	${}_{n+1}V_{x+1}$	${}_nV_{x+1}$	${}_{n-1}V_{x+1}$	P_{x+1}	${}_{n+1}V_x$	${}_nV_x$	P_x
5 years	24,968	16,226	7,622	13,710	24,163	15,708	13,230
10 "	80,758	64,644	48,759	24,464	78,222	62,608	23,609
15 "	158,810	136,232	113,941	33,336	153,985	132,067	32,176
20 "	251,228	223,060	195,209	40,646	243,867	216,474	39,239
25 "	350,143	317,299	284,773	46,560	340,278	308,278	44,957
30 "	447,110	410,547	374,282	51,165	435,001	399,313	49,413
35 "	534,636	495,300	456,232	54,589	520,700	482,246	52,729
40 "	607,189	565,939	524,924	56,990	591,905	551,528	55,056
45 "	662,231	619,770	577,514	58,564	646,051	604,447	56,583
50 "	699,236	656,103	613,156	59,497	682,541	640,253	57,489

TABLE NO. 9.—Showing the Reserves of the Model Office by various Groupings, and a comparison of each with the Reserve by an Exact Net Premium Valuation. Basis of Valuation $H^M 3\frac{1}{2}\%$.

Description of Grouping	ACTUAL RESERVES				Age of Office	COMPARATIVE RESERVES			
	Without Adjustments	With Premium Adjustment only	With Claim Adjustment only	With both Adjustments		Without Adjustments	With Premium Adjustment only	With Claim Adjustment only	With both Adjustments
Age at Entry. Exact	28,158	Yrs. 5	10,000
	86,310	10	10,000
Duration. Exact	166,268	15	10,000
	260,298	20	10,000
Valuation Age. Exact	360,632	25	10,000
	458,854	30	10,000
	547,486	35	10,000
	620,976	40	10,000
	676,766	45	10,000
	714,314	50	10,000
Age at Entry. Nearest	26,133	27,731	26,525	28,147	5	9,281	9,849	9,420	9,996
	82,147	85,000	83,379	86,275	10	9,518	9,848	9,661	9,996
Duration. Exact	159,861	163,748	162,259	166,204	15	9,615	9,848	9,759	9,996
	251,618	256,358	255,392	260,203	20	9,667	9,849	9,812	9,996
	349,747	355,178	354,993	360,505	25	9,698	9,849	9,844	9,996
	445,953	451,921	452,642	458,699	30	9,719	9,849	9,865	9,997
	532,852	539,220	540,844	547,307	35	9,733	9,849	9,879	9,997
	604,959	611,607	614,033	620,780	40	9,742	9,849	9,888	9,997
	659,730	666,562	669,626	676,559	45	9,748	9,849	9,894	9,997
	696,607	703,548	707,056	714,100	50	9,752	9,849	9,898	9,997
Age at Entry. N.B.D.	26,421	28,039	26,817	28,460	5	9,383	9,958	9,524	10,107
	83,032	85,919	84,277	87,208	10	9,620	9,955	9,765	10,104
Duration. Exact	161,525	165,459	163,947	167,941	15	9,715	9,952	9,860	10,101
	254,144	258,941	257,955	262,825	20	9,764	9,948	9,910	10,097
	353,127	358,622	358,423	364,001	25	9,792	9,944	9,939	10,093
	450,097	456,136	456,848	462,978	30	9,809	9,941	9,956	10,090
	537,621	544,064	545,685	552,225	35	9,820	9,938	9,967	10,087
	610,192	616,918	619,345	626,172	40	9,826	9,935	9,974	10,084
	665,273	672,185	675,252	682,268	45	9,830	9,932	9,978	10,081
	702,329	709,352	712,864	719,993	50	9,832	9,931	9,980	10,080
Age at Entry. N.B.D.	26,350	28,077	26,745	28,498	5	9,358	9,971	9,498	10,121
	82,902	85,984	84,145	87,274	10	9,605	9,962	9,749	10,112
Duration. Nearest	161,343	165,543	163,763	168,026	15	9,704	9,956	9,849	10,106
	253,916	259,037	257,725	262,922	20	9,755	9,952	9,901	10,101
	352,861	358,727	358,154	364,107	25	9,785	9,947	9,931	10,096
	449,802	456,249	456,549	463,092	30	9,803	9,943	9,950	10,092
	537,304	544,182	545,364	552,344	35	9,814	9,940	9,961	10,089
	609,860	617,040	619,008	626,295	40	9,821	9,937	9,967	10,086
	664,931	672,309	674,905	682,393	45	9,825	9,934	9,972	10,083
	701,981	709,477	712,512	720,119	50	9,826	9,932	9,975	10,081
Age at Entry. N.B.D.	23,081	24,808	23,427	25,180	5	8,197	8,811	8,320	8,942
	76,876	79,958	78,029	81,157	10	8,906	9,264	9,041	9,403
Duration. Curtate	152,900	157,100	155,193	159,456	15	9,196	9,449	9,334	9,590
	243,383	248,504	247,033	252,231	20	9,350	9,547	9,490	9,690
	340,579	346,445	345,687	351,641	25	9,444	9,607	9,586	9,751
	436,129	442,576	442,670	449,214	30	9,505	9,645	9,647	9,790
	522,594	529,472	530,432	537,413	35	9,545	9,671	9,688	9,816
	594,434	601,614	603,350	610,639	40	9,573	9,688	9,716	9,834
	649,052	656,430	658,787	666,277	45	9,591	9,700	9,734	9,845
	685,851	693,347	696,138	703,748	50	9,600	9,706	9,746	9,852

TABLE No. 9—continued.

No. of Grouping	Description of Grouping	ACTUAL RESERVES				Age of Office	COMPARATIVE RESERVE			
		Without Adjustments	With Premium Adjustment only	With Claim Adjustment only	With both Adjustments		Without Adjustments	With Premium Adjustment only	With Claim Adjustment only	With both Adjustments
6	Age at Entry. N.B.D.	27,452	29,179	27,864	29,617	5	9,749	10,363	9,896	10,510
		84,933	88,015	86,207	89,336	10	9,840	10,198	9,938	10,296
	Duration. Curtate + $\frac{1}{2}$	164,189	168,389	166,652	170,916	15	9,875	10,128	10,023	10,176
		257,467	262,590	261,329	266,530	20	9,891	10,088	10,040	10,100
		357,001	362,869	362,356	368,313	25	9,899	10,062	10,048	10,100
		454,411	460,860	461,227	467,774	30	9,903	10,044	10,052	10,100
		542,262	549,142	550,396	557,380	35	9,905	10,030	10,053	10,100
		615,060	622,242	624,286	631,577	40	9,905	10,020	10,053	10,100
		670,283	677,663	680,337	687,829	45	9,904	10,013	10,053	10,100
		707,418	714,916	718,029	725,641	50	9,903	10,008	10,052	10,100
7	Age at Entry. N.B.D.	31,823	33,550	32,300	34,053	5	11,301	11,915	11,471	12,026
		92,990	96,072	94,385	97,513	10	10,775	11,131	10,936	11,292
	Duration. Curtate + 1	175,478	179,678	178,110	182,373	15	10,554	10,807	10,712	10,966
		271,551	276,672	275,624	280,822	20	10,432	10,629	10,589	10,833
		373,423	379,289	379,024	384,978	25	10,355	10,517	10,510	10,764
		472,692	479,139	479,782	486,326	30	10,302	10,442	10,456	10,710
		561,930	568,808	570,359	577,340	35	10,264	10,389	10,418	10,672
		635,684	642,864	645,219	652,507	40	10,237	10,352	10,390	10,646
		691,513	698,891	701,885	709,374	45	10,218	10,327	10,371	10,626
		728,984	736,480	739,918	747,527	50	10,205	10,310	10,358	10,616
8	Age at Entry. N.B.D.	27,906	29,633	28,325	30,077	5	9,910	10,524	10,059	10,623
		85,770	88,852	87,057	90,184	10	9,938	10,295	10,086	10,377
	Valuation Age. N.B.D.	165,362	169,562	167,843	172,105	15	9,946	10,198	10,095	10,341
		258,931	264,052	262,816	268,012	20	9,947	10,144	10,097	10,341
		358,708	364,574	364,090	370,042	25	9,947	10,109	10,096	10,341
		456,311	462,758	463,157	469,699	30	9,945	10,085	10,094	10,341
		544,306	551,184	552,472	559,451	35	9,942	10,068	10,091	10,341
		617,202	624,382	626,461	633,747	40	9,939	10,055	10,088	10,341
		672,488	679,866	682,576	690,063	45	9,937	10,046	10,086	10,341
		709,658	717,154	720,304	727,910	50	9,935	10,040	10,084	10,341
9	Age at Entry. N.B.D.	23,612	25,339	23,966	25,719	5	8,386	8,999	8,511	9,124
		77,852	80,934	79,020	82,148	10	9,020	9,377	9,156	9,513
	Valuation Age. Nearest	154,264	158,464	156,578	160,841	15	9,278	9,531	9,417	9,670
		245,080	250,201	248,756	253,954	20	9,415	9,612	9,557	9,810
		342,553	348,419	347,691	353,615	25	9,499	9,661	9,641	9,861
		438,319	444,766	444,893	451,437	30	9,555	9,693	9,696	9,916
		524,943	531,821	532,816	539,798	35	9,588	9,714	9,732	9,952
		596,892	604,072	605,844	613,133	40	9,614	9,728	9,756	9,978
		651,577	658,955	661,349	668,839	45	9,628	9,737	9,772	9,992
		688,413	695,909	698,738	706,347	50	9,637	9,742	9,782	9,997
10	Age at Entry. N.B.D.	23,566	25,293	23,919	25,672	5	8,369	8,982	8,494	9,107
		77,765	80,847	78,931	82,059	10	9,010	9,367	9,145	9,502
	Valuation Age. L.B.D. + $\frac{1}{2}$	154,138	158,338	156,450	160,712	15	9,271	9,523	9,410	9,663
		244,119	250,040	248,593	253,790	20	9,409	9,606	9,550	9,803
		342,358	348,224	347,494	353,447	25	9,493	9,656	9,636	9,889
		438,097	444,544	444,669	451,212	30	9,548	9,688	9,691	9,911
		524,699	531,577	532,570	539,550	35	9,584	9,709	9,728	9,948
		596,632	603,812	605,582	612,869	40	9,608	9,724	9,752	9,972
		651,305	658,683	661,075	668,563	45	9,624	9,733	9,768	9,992
		688,136	695,632	698,458	706,066	50	9,634	9,738	9,778	9,992

TABLE NO. 9—continued.

Grouping	Description of Grouping	ACTUAL RESERVES				Age of Office	COMPARATIVE RESERVES			
		Without Adjustments	With Premium Adjustment only	With Claim Adjustment only	With both Adjustments		Without Adjustments	With Premium Adjustment only	With Claim Adjustment only	With both Adjustments
	Age at Entry. Mean	27,049	28,749	27,455	29,180	Yrs	9,606	10,210	9,750	10,363
		83,718	86,752	84,974	88,053	5	9,700	10,051	9,815	10,202
	Duration. Curtate + $\frac{1}{2}$	161,915	166,051	164,344	168,541	10	9,738	9,987	9,884	10,137
		254,029	259,072	257,840	262,957	15	9,759	9,953	9,906	10,102
	Valuation Age. L.B.D. + $\frac{1}{2}$	352,412	358,189	357,699	363,561	20	9,772	9,932	9,919	10,081
		448,790	455,139	455,523	461,965	25	9,781	9,919	9,927	10,068
		535,801	542,575	543,839	550,713	30	9,787	9,911	9,933	10,059
		607,975	615,048	617,096	624,273	35	9,791	9,905	9,938	10,053
		662,782	670,052	672,725	680,102	40	9,793	9,901	9,940	10,048
		699,679	707,066	710,175	717,671	45	9,795	9,899	9,942	10,045
						50				
2	Age at Entry. Nearest	26,064	27,770	26,455	28,187	5	9,257	9,862	9,395	10,010
		82,021	85,066	83,251	86,342	10	9,503	9,856	9,646	10,004
	Duration. Nearest	159,684	163,833	162,079	166,291	15	9,604	9,854	9,748	10,001
		251,397	256,456	255,168	260,303	20	9,658	9,852	9,803	10,000
		349,490	355,285	354,732	360,614	25	9,691	9,852	9,836	10,000
		445,666	452,034	452,351	458,814	30	9,712	9,851	9,858	9,999
		532,543	539,338	540,531	547,428	35	9,727	9,851	9,873	9,999
		601,635	611,729	613,704	620,905	40	9,737	9,851	9,883	9,999
		659,397	666,687	669,287	676,687	45	9,743	9,851	9,889	9,999
		696,269	703,675	706,712	714,230	50	9,747	9,851	9,894	9,999
3	Age at Entry. Nearest	26,193	27,899	26,586	28,317	5	9,302	9,908	9,442	10,056
		82,262	85,307	83,496	86,586	10	9,531	9,884	9,674	10,032
	Valuation Age. Nearest	160,025	164,174	162,425	166,636	15	9,625	9,874	9,769	10,022
		251,827	256,886	255,604	260,739	20	9,675	9,869	9,820	10,017
		349,997	355,792	355,247	361,129	25	9,705	9,866	9,851	10,014
		446,237	452,605	452,931	459,394	30	9,725	9,864	9,871	10,012
		533,163	539,958	541,161	548,057	35	9,738	9,863	9,884	10,010
		605,291	612,385	614,371	621,570	40	9,747	9,862	9,894	10,010
		660,075	667,365	669,977	677,375	45	9,753	9,861	9,900	10,009
		696,961	704,367	707,416	714,932	50	9,757	9,861	9,905	10,009
	Duration. Nearest	26,053	27,762	26,444	28,178	5	9,253	9,859	9,391	10,007
		82,023	85,073	83,251	86,349	10	9,503	9,857	9,646	10,005
	Valuation Age. Nearest	159,714	163,869	162,110	166,327	15	9,606	9,856	9,750	10,004
		251,462	256,528	255,234	260,376	20	9,661	9,855	9,805	10,003
		349,595	355,398	354,839	360,729	25	9,694	9,855	9,841	10,003
		445,809	452,186	452,496	458,969	30	9,716	9,855	9,861	10,002
		532,720	539,524	540,711	547,617	35	9,730	9,855	9,876	10,002
		601,838	611,942	613,911	621,121	40	9,740	9,855	9,886	10,002
		659,618	666,918	669,513	676,922	45	9,747	9,854	9,894	10,002
		696,500	703,916	706,948	714,475	50	9,751	9,854	9,897	10,002
	Age at Entry. Nearest	27,152	28,858	27,559	29,291	5	9,643	10,249	9,787	10,402
		84,029	87,074	85,289	88,380	10	9,736	10,089	9,882	10,240
	Duration. Curtate + $\frac{1}{2}$	162,500	166,649	164,937	169,149	15	9,773	10,023	9,920	10,173
		254,911	259,970	258,734	263,870	20	9,793	9,987	9,940	10,137
		353,589	359,381	358,892	364,775	25	9,805	9,965	9,952	10,115
		450,231	456,599	456,984	463,448	30	9,812	9,951	9,959	10,100
		537,457	544,252	545,518	552,416	35	9,817	9,941	9,964	10,090
		609,792	616,886	618,938	626,140	40	9,820	9,934	9,967	10,083
		664,709	671,999	674,679	682,080	45	9,822	9,930	9,969	10,079
		701,667	709,073	712,191	719,710	50	9,823	9,927	9,970	10,076

ABSTRACT OF THE DISCUSSION.

Mr. H. T. ADLARD, in opening the discussion, said that the paper was interesting for two reasons—first, because it dealt with a very practical subject on which little, if any, information could be obtained from the *Journal of the Institute*; and secondly, because it introduced to their notice a further development and refinement in the construction of what was known as the “Model Office.” Several papers had been read before the Institute, describing the various general methods of classifying policies for valuation purposes, but none of them dealt with the different ways in which any one of those methods could be brought into practice. The paper naturally divided itself into three parts: (1) the construction of the Model Office; (2) the consideration of the various groupings, and the valuation formula for each; (3) a comparison of the results obtained by an H^M 3 per-cent valuation of the Model Office by means of the various groupings. With regard to the construction of the Model Office, as the relative results of valuations by means of the various methods of grouping employed in the paper depended entirely upon the way in which the dates of birth and entry were distributed over the year, it was necessary to make some modifications in the Model Office as constructed by Mr. King, who had assumed that all policies were effected on the 1 January, and at an exact age, and that the terminations in the first half of the calendar year of entry were equal to those in the second half. Mr. Fraser used the same data as Mr. King, namely, the Twenty Offices’ Experience; but in order to obtain the number existing at the end of the calendar year of entry he subtracted the actual terminations from the number of entrants, and then reduced the radix at each age of entry, so that all the figures, except the number of entrants at each age, were the same as Mr. King’s, but the column headed “Age at entry x ” in Mr. King’s table, was headed “Age at entry x to $x+1$ ” in Mr. Fraser’s table. In order to decide how the dates of birth and entry should be distributed over the year, Mr. Fraser had constructed Table 2, which showed out of 17,174 policies in an actual company the number effected in each month, subdivided according to the number of months to run until the next birthday. Mr. Fraser’s Model Office might therefore be described as a company in which (*a*) the flow of new business was constant; (*b*) the entrants were grouped as they were in the Twenty Offices, and were all assured for the same amount; (*c*) the dates of birth and entry were distributed as in the actual company selected; (*d*) no reversionary bonuses were declared. Before assuming that the relative results of valuing any particular office by the various groupings would be the same as those shown in the paper, it would be necessary to ascertain to what extent the Model Office represented the office in question. For example, the office on whose experience the figures in the paper were based, quoted premiums for every half-age, a fact which was reflected in Table 2 by the existence of two maximum points in the number of entrants, one six months before the next birthday, and the other just before the birthday, so that it was probable that the average time which elapsed between the date of the entry and the next birthday would be greater

than it would be in the case of an office quoting premiums only as "age next birthday." The effect of this would be that the difference between the results of valuations by grouping according to "age at entry next birthday" and "exact ages" respectively, would be less in the case of the latter office, than that shown in the paper. He did not suppose that in the majority of cases this difference would be of any practical importance, but still he thought it was a point which should not be lost sight of. Allowance could easily be made for reversionary bonuses, if thought necessary, by means of Table 10 in Mr. King's recent paper on "The Comparative Reserves of Life Assurance Companies" (*J.I.A.*, xxxvii, 474), in which he gave reversionary bonuses for his Model Office grouped according to ages attained. With regard to the groupings and valuation formulas, it appeared from Table 2 that the average date of entry was .382 of a year from the end of the calendar year, and the average interval between entry and next birthday was .321 of a year, and he would like to draw attention to the exceedingly neat way in which those figures were used to obtain the "Exact Valuation Formula" on p. 392. Table 4 was also very ingenious; it was the same as Table 2, except that it was thrown into such form that by means of a super-imposed diagram the policies were automatically divided into eight different classes according to (a) the dates of entry; (b) the intervals between entry and next birthday; (c) the dates of next birthday; and from these eight classes, and combinations of them, the policies could easily be grouped in any of the fifteen ways scheduled in Table 6, and the valuation formulas could easily be deduced. With regard to the valuation results, when they came to the question of comparing the results in Table 9, if the question was, "When valuing by the H^M Table, which method of grouping gave a result nearest to that given by Method 1"? he quite agreed with all the conclusions arrived at in the paper. But if the question was, "When valuing by the H^M Table, which method of grouping gave the most accurate result"? he thought it quite possible that the answer should be different, for to his mind it seemed probable that the figures in Table 9 against Method 1 were not the most strictly accurate reserves in the table. In the first place, he would like to point out that in order to get a perfectly accurate valuation as a standard, it was necessary, not only to make use of the "Exact Valuation Formula" given on p. 392, but also to employ values of ${}_nV_x$ and P_x derived from a mortality table constructed on the exact age and exact duration method, that was to say, ${}_nV_x$ must be the value of a policy taken out at exact age x , and which had been in force exactly n years, for the coefficients in the formula on p. 392 were only correct on that assumption. That was not the case in the H^M Table, the average duration in the year of entry being overstated by 30 per-cent, if Table 2 represented the Twenty Offices with any degree of accuracy, and, as pointed out by Mr. Whittall in his paper on "The Methods of Deducing the Rate of Mortality" (*J.I.A.*, xxxi, p. 161), the net result of the method being to entirely lose touch with the actual age, and with the year of assurance also. To explain what he meant further, let them suppose that a mortality table was constructed from the H^M data by the exact age and the exact duration method,

and the Model Office valued by it at 3 per-cent by Method 1 with both adjustments, and then let them suppose that the result, as compared with the results in the last column of Table 9, would be $10,000 \pm K$. Then it seemed to him, that when they were valuing by the H^M Table and 3 per-cent, the method of grouping which would give them the most accurate result was that method which in the last column of Table 9 gave the figure nearest to $10,000 \pm K$. Of course it would be impossible to construct a mortality table on the exact duration method from the H^M data, but he thought that Dr. Sprague's select tables gave them a very fair approximation to such a table. If the Model Office was valued by those tables and 3 per-cent by Method 1 the value of K would be found to vary from $+1,378$ for an office 5 years old to $+85$ for an office 50 years old. From an examination of the figures in the last column of Table 9, it would therefore appear that when valuing by the H^M Table at 3 per-cent Method 4 gave the most accurate result for an old office, and Method 7 for an office which had not been established more than about 25 years. When valuing by some other table than the H^M , in which different assumptions had been made, it was probable that one of the other methods of grouping would be found to give the most accurate result. For example, supposing they valued by Dr. Sprague's select tables, then on the assumption that the results according to the various groupings would bear the same proportions to one another as when valuing by the H^M Table, and that ${}_nV_x$ in Dr. Sprague's table represented the value of a policy taken out at exact age x and which had been in force exactly n years (an assumption which he thought was sufficiently accurate for all practical purposes) Methods 12 and 14 would seem to give the most accurate results. What he wanted to suggest was, that in order to determine which method of grouping would give the most accurate result, it was necessary to know what mortality table was used in the valuation.

Mr. G. J. LIDSTONE said that it was probable that those present who were responsible for the valuations of their offices must have considered from time to time the points discussed in the paper, but it was a very happy idea of Mr. Fraser's to reduce all these methods to the common denominator of the Model Office. To students in particular this placed the matter in a much clearer light than any merely theoretical considerations could do. As Mr. Adlard had stated, one of the first things that occurred to one in reading the paper was, that the author had been forced to use facts taken from an individual office, and it was important to consider how far the results agreed with those of offices in general. Since the paper had been written the figures contained in the last volume of the Institute and Faculty Mortality Experience had been published, which enabled one to test the point. In Mr. Ackland's contribution to the volume would be found a statement dealing with four blocks of ten thousand policies each, showing how the true age at entry compared with the age at entry derived by the nearest age method; and he found that the average true age was $\cdot610$ of a year in excess of the age last birthday, as compared with $\cdot679$ in the author's figures; also that the nearest age method gave an error of $\cdot020$ in excess, whereas the error in Mr. Fraser's figures was in the reverse direction, $\cdot012$ in defect. A

similar state of things was found in some figures which Mr. Whittall gave in a paper some years ago in reference to another individual office (*J.I.A.*, xxxi, 161). There the average true age at entry was $\cdot615$ in excess of the age last birthday, while the nearest age method gave $\cdot629$, showing an error, still in excess, of $\cdot014$. Dr. Sprague in a paper (*J.I.A.*, xxxi, 205) gave some similar figures, showing an error of $\cdot010$, still positive. The reason for the positive error was, he thought, fairly simple. If those policies were eliminated which were taken out practically at the close of the year of age (as to which no error was introduced by the nearest age method) there was left a series which was distributed more or less evenly over the year of age; but it was known that in point of fact there were more towards the end than towards the beginning. Those that occurred towards the end by the nearest age method were treated as having attained the full year of age, and it was therefore natural that the nearest age method should on the average overstate the true age at entry. That being so, they had to find the reason for the author's figures showing the deviation in the reverse direction. The reason was that given by Mr. Adlard—the practice of the office of accepting lives at half ages,—and if that were looked closely into, it would be found to have the effect of reversing the error. With regard to the effect of the nearest renewal date method, so far as he was aware there were no published figures dealing with offices in general. It might, therefore, be of interest to point to some which he had derived from his own office. He had occasion not long ago to examine the whole of the endowment assurance entrants in a given calendar year, comprising the considerable number of nearly nine hundred policies. He found that the average interval from the first January to the date of entry was $\cdot619$ of a year, which was almost exactly the same as the author's, $\cdot618$; also that the nearest renewal date method gave a positive error of $\cdot004$ as compared with the author's positive error of $\cdot008$, showing a very satisfactory agreement. The reason for that positive error was, of course, the same in principle as that which he had already given for the age at entry, substituting the year of entry for the year of birth. The author had given an analysis of the different methods of valuation, based on the combination of different policy values to obtain the final result. He (the speaker) found it gave rather a clearer view of the case to consider the facts which they more often dealt with in practice, namely, the age at entry, the proportion of net premium reserved, and the valuation age. He had therefore prepared the following table:

TABLE, showing the effect of the various methods of grouping as regards (1) the average assumed Age at Entry; (2) the average assumed Duration; (3) the average assumed Valuation Age; (4) the average reserved proportion of the net Annual Premiums.

Method	(1) AGE AT ENTRY	(2) DURATION	(3) VALUATION AGE	(4)
	(Average assumed Age) minus (Age l.b.d.)	(Average assumed) minus (curtate)	(Average assumed Age on Valuation Date) minus (Age l.b.d. at preceding anniversary)	Proportion of Net Annual Premiums reserved
1	·679	·382	1·061	·618
2	·667	·382	1·049	·618
3	1·000	·382	1·382	·618
4	1·000	·374	1·374	·626
5	1·000	·000	1·000	„
6	1·000	·500	1·500	„
7	1·000	1·000	2·000	„
8	1·000	·552	1·552	„
9	1·000	·059	1·059	„
10	1·000	·052	1·052	„
11	·552	·500	1·052	„
12	·667	·374	1·041	„
13	·667	·392	1·059	„
14	·685	·374	1·059	„
15	·667	·500	1·167	„

It would be seen that Methods 12, 13 and 14 agreed very closely in every respect with the "exact" valuation; they agreed also in reserving slightly too great a proportion of the net premium income, which was due to the operation of the nearest renewal date method. But in the case of Methods 12 and 14, the errors in the ages tended to counteract that error, while in the case of Method 13 both errors were in the same direction. If, however, the nearest age at entry gave, as they found it did more usually in practice, an age in *excess* of the true age at entry, then the two errors, instead of being cumulative, would tend to be counteractive. In those circumstances he believed that Method 13, which was certainly a very simple one, would be as accurate as Methods 12 or 14, in fact as accurate as anything one could possibly want. One advantage of Methods 13 and 14 over 12 was that the valuation age was a fixed quantity; and therefore if there were duplicate policies on one life they would all be found together in one group, and not distributed over two groups, as they might be with the other methods. That was a small point, but in practice he believed it to be a great convenience. Another smaller point of interest which might be mentioned was that Methods 12, 13 and 14 all agreed in reserving simply one year's full net premium for all policies effected exactly at the close of the valuation period. That was a result which might be useful in practice if it was desired to rapidly find the reserves on such business. The author had not touched in the paper on half-yearly or quarterly premiums, but it might be pointed out that the nearest renewal date method could be

employed very simply in such cases. If they were dealing with *true m-ly* premiums, and valued the *m-ly* net premiums ($P^{(m)}$), the annuity-value to be employed was $(a + \frac{m+1}{2m})$ —proportion to next renewal date). If the premiums were *instalment* premiums and the annual net premiums were valued, the annuity-value to be employed was $(a+1)$ —proportion to next renewal date). If, therefore, that proportion were treated as being 0 or $\frac{1}{m}$, whichever was the nearer to the exact proportion, they would get on the average exceedingly correct results, comparing well with those obtained for annual premiums by the nearest anniversary method.

MR. GEORGE KING thought he might be allowed to express not unpardonable satisfaction that Mr. Fraser had been able to make use of his (Mr. King's) work of twenty-seven years ago. He was very glad to find that, even at the present period, and for purposes altogether apart from those for which he did the work, it had proved useful. He wished, however, at the very outset to make a disclaimer. The author gave him more credit than was his due, because he spoke of him as having devised the Model Office, whereas it was Mr. Manly who did that work. He (Mr. King) improved upon it by bringing in actual facts as the material for building the Model Office, whereas Mr. Manly had, to a certain extent, gone by pure assumption. It was that change which had made the Model Office such a very useful instrument, but Mr. Manly was the first to conceive the idea. In various parts of the paper the author had referred to adjustments, but it was well known that he (Mr. King) did not make any adjustments in the valuations of his Model Office, for two reasons. In the first place, twenty-seven years ago adjustments were not so necessary as now, owing to the great change that had taken place during that period in life assurance business. In those former days they had not the great influx of policies at the end of the year, and had therefore very much more nearly a uniform distribution of entrants. At that time also it was not the custom to pay claims without discount immediately on proof of death, so that an adjustment for immediate payment of claims was not required. When, only the other day, he brought the Model Office up to date he again refrained from putting in any adjustments, for the reason that ratios only were what were required for the purpose in view, and the adjustments practically disappeared when ratios were used. He was very pleased with the investigation the author had given of the age incidence of new entrants, and Mr. Lidstone's additions to the enquiry were extremely valuable. He himself, at different times, had made investigations into that point, because it was an important one in dealing with valuations, and the results he had come to were very close indeed to those of the author. Passing to Mr. Fraser's various valuations, those that interested him most were Nos. 2, 12, 13 and 14. With regard to No. 2 and also No. 3, he would ask the author whether he was quite accurate in speaking of them as methods of grouping. No. 2—the nearest age at entry and duration exact—was the plan which he thought was universally followed in

America, but not by grouping. Every policy was valued separately by prepared tables, interpolated to the nearest month for the policy value, and policies of every kind were treated in that way, and the valuations run out in detail. They also took in America the age nearest birthday for the premium charged, and that was the age taken also in making the valuation, so that one might say that No. 2 was the American method, but not of grouping, rather of valuation in detail. It was satisfactory to find that this method gave results which were so very near exactitude, and one's confidence was confirmed in that method of valuation, although in this country it was not very much used. Nos. 12, 13 and 14, he thought, were particularly interesting. They gave the very nearest approximations to the truth, and were extremely easy to apply; but No. 13 was the one he had employed, because he thought it was much better to deal with ages attained rather than with durations. Moreover, as a matter of practice, it was much easier to group policies in that way now-a-days when the offices obtained the exact dates of birth; it was also less likely to lead to mistakes, and therefore was the better plan. No. 14 was peculiarly interesting to him, because it was founded on a formula which he had devised for an entirely different purpose, and it was satisfactory to see how a formula devised for one purpose could be used for another. It was also gratifying to him to find how very exact the results were, because that proved that, for the purpose for which the formula was intended, it was also exceedingly accurate. Mr. Ackland criticized it rather severely at one time—possibly theoretically it was open to his attacks, although he did not himself think so—but practically it gave most exact results, as he knew from long experience. It made an automatic adjustment for the incidence of policies coming in. It also made a corresponding automatic adjustment for exits, and gave an automatic adjustment for the days of grace, and for the maturing of endowment assurances, no matter how they might mature. Also, although it was first proposed for the purpose of mortality experience between two valuation epochs, it applied with equal certainty to an aggregate table formed from the whole experience of an unlimited time, and it even applied to select tables. He had used it for all those purposes, and had found it work with extreme simplicity and great accuracy, and a formula of that kind would get rid of the difficulties that were experienced in working out the British Offices' experience, where the explanations given of how the days of grace were dealt with certainly did not help to make matters clear. That explanation was confusing, and showed what an amount of labour was required when there was not an automatic formula. His own feeling was that a formula of that kind—there were others that did as well—would be better than the nearest duration method used by the Joint Committee; for instance, that which Dr. Sprague had called the final age method, as distinguished from duration. With regard to No. 5, the Institute method, it showed by no means a very close approximation; in fact, it understated the reserves, but that method when it was first adopted worked very much more accurately than at the present day. It fitted into the methods of business of that period, although it did not fit into the present methods of business, and therefore he did not think they

should pass a severe judgment on it merely because it gave results that were now found not to be very near the truth. At the time it was adopted it was a very good method, but it was one of the examples of how actuaries must keep up to date, watch the course of business, and not rely implicitly on a formula without from time to time examining it. In conclusion, he wished to say that there must have been an enormous amount of work bestowed on the paper. No one who had not undertaken similar labour knew what it involved, and he thought the members of the Institute should heartily thank Mr. Fraser for what he had done.

Mr. R. TODHUNTER said that the importance of the assumptions made in regard to the valuation age, and the age at entry in group valuations of whole-life policies, had, he supposed, always been recognized. Mr. King, in particular, had directed attention to it in his paper on the subject of legislation affecting life assurance companies (*J.I.A.*, xxix, 481). In that paper Mr. King urged that in the event of any changes being made in the requirements of the Board of Trade, one of the most important was that the companies should state in their answers under Schedule 5 the assumptions they had made with regard to those two points—the age at entry and the valuation age; and, as a matter of fact, this was asked for by some of the Colonial Acts. But although the importance of these assumptions had been recognized, he doubted whether the extent to which they affected the results had been generally realized. That, he thought, was perhaps the chief value of Mr. Fraser's paper, that it drew attention to the very considerable relative differences which the adoption of different assumptions as to the age at entry and the valuation age brought out, rather than that it indicated the most accurate method of obtaining a close approximation to the result of an individual net premium valuation. With reference to the method of procedure adopted in the paper, there was one question which he would like to put to the author, namely, whether it was strictly accurate to apply, as he had done, the two percentages .321 and .679 to the business of the two halves of the year separately. As a matter of fact, in regard to the business transacted in the first half of the year, the age next birthday was the most influential factor; in regard to the business effected in the second half of the year, the bonus had more influence. Consequently it would be found that the difference between the date of entry and the date of next birthday was less in the first half of the year than it was in the second, and therefore it seemed to him that different factors should be used to distribute the business of the two halves of the year, with the result that the two end terms in the author's formula for the valuation of the Model Office would be reduced in amount as compared with the middle term. Then there were the results of the paper, upon which he wished to say a word. He gathered that Mr. Lidstone and Mr. King were agreed in holding that Method 13 was to be preferred to Methods 12 and 14. In that view he would certainly concur. There seemed very little to choose between the three in point of results, but in point of convenience Method 13 was superior to Methods 12 and 14. There was another practical consideration besides that mentioned by

Mr. Lidstone, namely, that when the policies were classified it was not solely for valuation—there were other purposes for which the classification was used, such as the calculation of the expected death strain, and for that purpose it must certainly be better to have the policies scheduled at the nearest attained age. He noticed that while there had been some discussion as to the relative merits of Methods 12, 13 and 14, no one had ventured to put in a plea for No. 9, although, as a matter of fact, it was a method which was used by a very considerable number of offices. Notwithstanding the relatively weak reserves which, according to Mr. Fraser's table, it brought out, he thought there was something to be said in its defence. If the sole object were to attain the closest possible approximation to a net premium valuation, he was quite prepared to admit that No. 13 was much to be preferred, but there might be other considerations to be taken into account. After all, what the actuary had to do in a valuation was to value liabilities and assets, and it was surely incorrect from a practical point of view, when a man effected a policy at the beginning of a year at a premium of £3 per-cent, and effected another policy towards the end of the year at the same premium, to produce two different liabilities under the two policies, apart from the difference in the proportion of the current year's premium to be reserved. There was no great objection to doing so, but on the other hand he could see no special reason for the course. The net premium was not entirely unconnected, he thought, with the premium actually payable. The advantage he saw in Method 9 was that they took as their net premium the net premium corresponding to the premium actually payable at the age at which the assured was supposed to enter, and then, having taken that as the future premium receivable, proceeded to value the policy by a method which gave the best approximation. Probably Mr. Fraser's figures over-estimated to a certain extent the defect brought out by that method of valuation. It had been shown that, owing to the practice of the office from which the statistics were taken, the interval between the average date at which the policy was effected and the date of the next birthday was greater than it would be in many offices. If they turned to the author's comparison between Method 9 and Method 13, it would be found that Method 9 was derived from Method 13 by increasing the age at entry by one year and diminishing the duration by one year in the case of $\cdot333$ of the policies, this being the proportion of the policies for which the interval between the date at which the policy was effected and the date of the next birthday exceeded six months. He doubted whether in modern business that interval was so great as $\cdot333$. The practice of the author's office tended to increase the interval, and the fact that his experience was spread over a very long period would have a similar effect. The interval had probably for many years tended to become less: people were induced more and more to effect policies just before their birthdays: proposals were even dated back quite frequently a few days or a month, or it might be a couple of months, to the last birthday, so that his own impression was that the real difference in, at any rate, the majority of offices, between the results brought out by Method 9 and Method 13, would not be so great as it appeared in the tables. He did not wish

to advocate anything that was not strictly correct in the way of valuation, but he thought it was undesirable that they should be frightened away from a system of valuation which had great advantages, and certain practical merits, namely, that it did recognize some connection between the net premium and the premium actually payable, simply because it brought out a defect in reserve, which was possibly considerably over-estimated by the nature of the statistics upon which the paper was based.

Mr. KING asked Mr. Todhunter whether the effect of Method 9 was not to divide at the first distribution all the little gain from charging the premium at the next birthday, instead of letting it flow from time to time into the profit fund by instalments, as it was paid in?

Mr. TODHUNTER said he would like time to consider the question, but suggested that whether this were so or not would depend a good deal upon the proportion of the premium reserved for the current year.

Mr. RALPH PRICE HARDY remarked that, although it was many years since he ceased to concern himself with the minor differences arising from the various methods of grouping whole term assurances, having for long recognized that the results the most approved systems of classification brought out were merely relative, and rather indicatory than exact measures of the absolute truth, yet he could well understand that to many, whose opportunities of observation occurred only at distant intervals, such a series of numerical contrasts as were before them must prove of great interest. The analysis of the composing items of the groups, by means of the triangular arrangement, would be found peculiarly instructive to students. Before proceeding to discuss the results presented, he wished to clear the ground upon two points which had been incidentally raised. No statistics respecting the distribution of assurances, according to the months of entry and birth, had hitherto been available as a basis of comparison, and Mr. Fraser's were the first, to his knowledge, that had been published. He (Mr. Hardy) was in possession of a similar series, but based upon endowment assurances only. Again, while he was by no means sure that Mr. Fraser's suggested amendment upon the numbers brought to entrance in the Model Office was well founded, or would affect the ultimate results, he thought that all users of the test should always bear in mind that some assumptions were necessary in establishing a basis, and that Mr. King's broad intentions should be regarded, and his repeated cautions not forgotten. While gently demurring to Mr. Fraser's description of his standard as an "exact valuation"—such an epithet being always of doubtful validity in any matter involving probabilities and a good deal of unverified data, and one scarcely applicable to a case that was based upon a distribution of risks obtained from the experience of a single source—he was prepared to accept it as representing a defensible method, and as of some metrological value. But he thought that, if a standard for comparison had to be taken, such should rather be one resulting from the valuation of the contracts seriatim, according to the old-fashioned individual method. So far as the present figures were concerned (except those arising from Method 7, which were

evidently inserted for symmetry only), and speaking solely from the practical point of view, he did not see that any single set could specially claim preference, to the rigid exclusion of the others. Indeed, he went so far as to say that, if in any valuation any actuary got so close to the real liability as to be out only by the small differences shown in the paper, he might be sincerely congratulated upon his exceptional good fortune and skill, and might safely dismiss all hypercritical comments. Before he sat down he wished to revert to Dr. Karup's method of grouping.* That method, which but few in the room ever saw actually at work, though it was in general use when he entered life, had in its scope and results a great deal to recommend it. Karup's letter to the *Insurance Record* of the 26 May 1871, was far from clearly written, and hence it probably escaped general notice. Though he (Karup) had no claim to the merit of inventing the general method, yet he there showed, what was up to then unknown, that, by the help of two additional columns on the sheets, the advantages of a common age classification could be retained, while the results produced were identical with those of the ungrouped seriatim valuation. To his mind, that method was the only one by which they could make sure that a proper reserve had been made for the contracts effected just before the close of the books, and further, one that fully protected the reserve in the case of the half-yearly and quarterly premiums. It was known that some companies adopted the very questionable practice of taking credit as an asset for the uncollected instalments. That treatment was justifiable only where the account had been debited with the full premium, which, in some cases, to his knowledge, certainly had not been done. He could only add that he had personally tried Karup's method in the case of a large number of endowment assurances, and found that it worked with great facility and with most satisfactory results.

Mr. S. G. WARNER, in closing the discussion, said that one point upon which they were unanimous was that they cordially welcomed Mr. Fraser amongst them as a new contributor, who had taken up a practical subject which had not been worked out thoroughly before, and had devoted a large amount of laborious work to its elucidation. It was a subject that came before them in their daily experience, and therefore they could not give too careful study to an attempt to bring it before the Institute in a scientific and exhaustive way. The paper opened with a very clear statement of the problem, and an admirably ingenious investigation of formulas. Then followed what the author truly described as a serious difficulty. The problem could be easily and clearly stated, but when a statistical standard was required by which the various questions as to the distribution of policies throughout the policy year, the financial year, and the year of age could be determined, no truly representative statistics—no “universal” statistics, in the sense in which that term might be applied to the mortality experience of the combined offices—were available, and therefore all that could be done was to take an individual instance, and make that a standard. Then, of course,

* This method is explained in an actuarial note which follows this discussion.—Ed. *J.I.A.*

the question immediately arose how far the results of the whole paper were affected by such an inevitable circumstance, because they were dealing with large figures and comparatively small percentages of difference, and in doing so it was quite possible that comparatively slight variations in the standard would materially affect the results. Taking the figures employed, as exhibited in Mr. Fraser's Table 2, it was seen that they possessed certain marked and striking characteristics. They undoubtedly represented a progressive business, carried on under present-day conditions. Obviously, however, their distinctive features, being individual and not universal, might not occur in the experience of another office, and therefore the only way of arriving in any given case at similar exactitude would be to ascertain what the actual conditions were, make a single policy valuation, or an approximation thereto by the formulas given in the paper, and frame a standard accordingly. Until that was done they could not really tell how far in any individual case figures like those given could be implicitly relied upon. The only other suggestion with which he would trouble the meeting was in regard to the general summary of results in Table 9. There, importance was attached to two adjustments—that for premiums and that for claims—and the facts were set out in each case, without either of those adjustments, with one of them only, and with both. He did not know how far the labour expended in some of those directions was likely to justify itself in practical results. He should imagine, for instance, that the principle of claim adjustment, which stood quite apart from the main question of the paper, was now so fully admitted and generally practised, and in any case the result of dispensing with it was so easily ascertained, that the inclusion in the table of a column omitting that adjustment was perhaps its least necessary feature. Personally, he found it most useful to look at the column headed "With both adjustments", and to compare the results there with the standard results. When that was done, it was found that the variations, although they had truly been described as rather larger than were expected, and in some cases a little startling, were considerably diminished. He thought in only three cases—those of Methods 5, 9 and 10—did the result fall below what was set up as the standard. In all the other cases, as far as he could see, the reserve brought out was in advance of the standard. It was necessary to bear such considerations in mind when reading the paper and studying the figures, chiefly on the ground which he had already ventured to state, that they were dealing with rather minute, although very important, points, and with rather small percentages of difference shown on fairly large figures; in which case a little variation this way or that in the instrument of operation might make a considerable difference in the result.

The PRESIDENT (Mr. W. Hughes) said that reference had been made to what might have happened if the paper had been written twenty years ago. It was almost to be wished that it had been then written. Certainly its usefulness now was less than it would then have been, in consequence of the very great influx of endowment assurances, which threatened almost to submerge the whole-life classes. He asked the members to accord a hearty vote of thanks to Mr. Fraser for his interesting and instructive paper.

The resolution was carried with acclamation.

Mr. FRASER, in reply, thanked the members for the exceedingly kind manner in which they had received his paper, and for the friendly tone of their criticisms. In reply to one or two general points, he realized that the data which he was using for the purpose of comparing the groupings could not be universally applicable, but he had practically no choice in the matter; there were no general data available, and he was very fortunate, he thought, in being able to furnish data based on a sufficiently large number of policies. Mr. Adlard had made some interesting observations on the most accurate method of employing the H^M Experience in a Valuation. Such a question as that had been expressly excluded from consideration in the paper, but it might be pointed out that we used (say) the H^M Table, and not the H^M Experience, in our Valuations; the Table (whether deduced accurately or not from the original data) was a fixed datum, and its precise relation to the original experience could not in any way affect the methods and process of Valuation. It had been suggested that the paper would have been of greater use if it had been written twenty years ago, but he ventured, with all deference, to differ from that view. Twenty years ago actuaries were pre-occupied with the more important questions involved in the transition to a more stringent Mortality Table and a lower rate of interest. At the present time, when a very satisfactory position had been generally attained in regard to these points, and when the publication of the British Offices Tables was found not to involve any serious additions to their estimates of the liabilities under life assurance contracts, it was possible that in some cases actuaries might consider that the opportunity was favourable for strengthening the minor elements of the valuation, such as the method of grouping adopted, and he should be glad to think that his paper might be found to be of use in considering such questions. In reply to Mr. Todhunter's query as to whether it was strictly accurate to assume that the difference between the date of entry and the date of next birthday was the same for policies effected in the first half of the year as for those effected in the second, he mentioned that the difference expressed as a decimal of a year was identical to three decimal places for the two halves of the year. He was very much interested in Mr. Hardy's reference to Dr. Karup's letter which appeared in the *Insurance Record* in 1871. He thought that letter of Dr. Karup should certainly be resuscitated; and if it were put in a somewhat clearer form—because the letter had been dreadfully mangled by the printers—it would appeal to actuaries as a most interesting method, and one that might be put to practical use. Although authoritative opinions had been expressed in favour of his (the author's) Method 13 as the best method of grouping, he remained of the opinions expressed in the paper. He thought that duration was decidedly the most important element in the valuation, and Method 12, which dealt with the nearest durations, for that reason was the one that he still preferred in spite of the authorities against him.

ACTUARIAL NOTE.

DR. JOHANNES KARUP'S *Formula for valuing Policies in groups according to Exact Duration.*

THE readers of the *Journal* will, no doubt, welcome a reproduction of the formula of Dr. Karup, which was referred to, with deserved appreciation, by Mr. R. P. Hardy in the discussion on Mr. Fraser's paper.

The formula was given to the world in a letter in the *Insurance Record* of 26 May 1871, now long out of print, and, so far as we know, has never appeared elsewhere. The notation was somewhat difficult and obscure, and, the author apparently not having had an opportunity of revising the proofs, there were so many misprints as to render the argument not easy to follow. Moreover, in the numerical illustration included in the letter an unfortunate slip seems to have occurred, which vitiated the figures, and rendered the demonstration still more confused. On account of these accidents the formula attracted but little attention, and has been almost forgotten by the majority of actuaries, so that a useful end will be served if we re-demonstrate it in our own way, and in modern notation.

The value of a policy effected at age x , and which has been $n+t$ years in force, n being integral and t fractional, is

$${}_{n+t}V_x = (1-t)({}_nV_x + P_x) + t \times {}_{n+1}V_x$$

But ${}_nV_x + P_x = A_{x+n} - P_x \times a_{x+n}$

and ${}_{n+1}V_x = A_{x+n+1} - P_x(1 + a_{x+n+1})$

Therefore

$$\begin{aligned} {}_{n+t}V_x &= (1-t)(A_{x+n} - P_x \times a_{x+n}) + t\{A_{x+n+1} - P_x(1 + a_{x+n+1})\} \\ &= A_{x+n} - P_x \times a_{x+n} + t(A_{x+n+1} - A_{x+n}) \\ &\quad - P_x t(1 + a_{x+n+1} - a_{x+n}) \end{aligned}$$

and, if the sum assured be S , and the corresponding net premium be π , so that $\pi = S \times P_x$, then

$$\begin{aligned} S \times {}_{n+t}V_x &= SA_{x+n} - \pi a_{x+n} + St(A_{x+n+1} - A_{x+n}) \\ &\quad - \pi t(1 + a_{x+n+1} - a_{x+n}) \end{aligned}$$

Let there now be a group of policies for sums of S_1, S_2, S_3 , &c., on lives born in the same calendar year, effected at different

periods in different years, when the ages of the lives were x, y, z , &c. Also, let the valuation date be 31 December, and the integral durations of the respective policies n_1, n_2, n_3 , &c., and the fractional durations t_1, t_2, t_3 , &c. Then

$$x + n_1 = y + n_2 = z + n_3 = \&c. = m, \text{ say.}$$

If now ΣV be the sum of the values of all the policies, we shall have

$$\Sigma V = \left\{ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ \&c. \end{array} \right\} A_m - \left\{ \begin{array}{c} \pi_1 \\ \pi_2 \\ \pi_3 \\ \&c. \end{array} \right\} a_m + \left\{ \begin{array}{c} S_1 t_1 \\ S_2 t_2 \\ S_3 t_3 \\ \&c. \end{array} \right\} (A_{m+1} - A_m) - \left\{ \begin{array}{c} \pi_1 t_1 \\ \pi_2 t_2 \\ \pi_3 t_3 \\ \&c. \end{array} \right\} (1 + a_{m+1} -$$

To apply the formula we must therefore group the policies according to calendar years of birth. The first two columns of the above schedule are the usual columns of the Classification Register. The other two columns contain the functions of the form St and πt , which are constant for the whole duration of each policy, and which may therefore be prepared at the outset, and entered in additional columns in the Register. All four columns will then be summed in the usual way, and the valuation is easily effected.

The fraction t is the portion of the year from the day of issue of the policy to 31 December, so that for a policy granted on 7 September, t would be 115 days, or .315 of a year. In the original letter this was taken as .685 of a year, that is, $(1-t)$ was used instead of t , and confusion resulted.

In using the formula, Dr. Karup took for x, y, z , &c., the mean age at entry, that is, the calendar year of entry less the calendar year of birth; but obviously it would be at least equally suitable to take the nearest age at entry, or the ages at entry might be taken in any other way.

The formula is one for valuing in groups policies according to exact duration, and it avoids all necessity for adjustment on account of the distribution of premium income over the financial year. In Dr. Karup's letter, and in the above demonstration, annual premiums are assumed; but half-yearly and quarterly cases can be included if the proper allowance be made for the fractional premiums unpaid for the current policy year, which allowance, for any particular policy, would be the same at each

valuation. Also, by taking suitable assurance and annuity-values, the formula may be applied to Endowment Assurances, provided always that only those policies maturing at the same age are included in any one group.

In conclusion, we should like to thank the Editor of the *Insurance Record* for lending us his library copy of the volume for 1871, without which we could not have prepared this Note.

An Investigation into the Rates of Re-Marriage and Mortality amongst Widows in receipt of relief from the Patriotic (Russian War) Fund, 1854-1900. By J. BURN, F.I.A., and J. McDONALD, F.I.A., of the Prudential Assurance Company, Limited.

[Read before the Institute, 29 February 1904.]

READERS of the *Journal* will remember that in February 1901 our then President, Mr. Higham, addressed a letter on behalf of the Council to the Royal Commissioners of the Patriotic Fund, offering the services of the Council of the Institute in respect of any actuarial advice which might be necessary in connection with the various Funds administered by the Commissioners.

The offer of the Council was accepted with great cordiality, and complete statistical information as to the position of the Funds on 31 December 1900 was forwarded to the Council. A Committee of members of the Council was then formed to deal with the Valuations and Reports required. References to the correspondence at that date and on the further occasions when re-valuations were made will be found in the *Journal*, vol. xxxvi, pp. 85 and 400, and vol. xxxvii, p. 446.

In December 1900 the Transvaal War was still going on, subscriptions to the Fund were coming in well, the work of relief was continually in progress, and it was felt that an actuarial valuation at that date would serve little purpose.

The great majority of the widows in receipt of relief from the other Patriotic Funds were women whose husbands had died many years ago, and, it was concluded therefore, that they would not show any considerable rate of re-marriage, and that this element might be safely disregarded.

It was, however, foreseen that, when the Transvaal War Fund should come to be valued the rate of re-marriage would have a most important bearing on the liabilities of the Commissioners, and they were asked if it were possible to supply the Institute with information relative to the rate of re-marriage among widows on the older Fund. In response to the request of the Council, the Commissioners at once ordered complete returns of the Experience of the Widows of non-commissioned officers and men of the Patriotic (Russian War) Fund to be supplied, and schedules in the form recommended by the Committee of the Council were furnished with remarkable promptitude by Col. Young, the Secretary to the Royal Commissioners.

The present paper is an attempt to deduce from the data thus furnished rates of re-marriage and mortality in such a form as may be useful for the purpose of estimating the liabilities of similar funds, or for determining the benefits which could be safely offered on starting new funds with similar objects. We also believe that the statistics will be of some service in throwing additional light on the general question of re-marriage.

A brief account of the Patriotic Fund may here find a place.

The term "Patriotic Fund" was applied in 1803 to a fund raised by the subscribers to Lloyds' "to animate the efforts "of our defenders by sea and land, by providing a fund for "the relief of themselves when wounded, and of their widows "and orphans, and for granting pecuniary rewards and badges "of distinction for valour and merit."

What are now known as the Patriotic Funds had their origin at the commencement of the Crimean War. The public are frequently being reminded of points of resemblance between the recent Transvaal War and the Crimean War. In one respect there is an exact similarity. The Crimean War was at first extremely popular, and great sympathy was felt for the sufferers. Subscription lists were opened at local centres all over the country and large amounts were given. Unfortunately on both occasions the same defects in administration were observable; relief refused to some applicants was granted twice over to others, and actual fraud was not unknown. On 1 May 1854 an informal notice was published in *The Times* stating that a Commission was being formed, but it was not until 7 October 1854 that the Commission was actually appointed, the Prince Consort being President.

The Royal Commission commenced operations in November 1854, taking over most of the local funds subscribed by public

charity, and also the liability to provide for 75 widows and 114 orphans who were then in receipt of relief. Subscriptions came in rapidly, and in the course of three or four years reached a total of about a million and a half. Other Funds raised for the relief of sufferers by later wars and naval disasters have since been taken over by the Commissioners. From time to time Parliament has intervened in their administration, limiting the application of the monies subscribed and giving directions as to the distribution of the surpluses arising after the special trusts in each case had been satisfied. In 1886, they placed upon the Commissioners the responsibility "from time to time to ask for and receive contributions from the public for such purposes being for the benefit of the widows and children of officers and men of Her Majesty's naval and military forces."

The Funds have always been administered in a careful manner, the Commissioners having had from the outset the advice of Mr. A. G. Finlaison, and later of his distinguished son, Mr. A. J. Finlaison. The public, however, have looked with a jealous eye on the surpluses shown, and on the large amounts held in reserve for the purpose of meeting the accepted liabilities.

The conservative methods of the Commissioners have frequently been subjected to a considerable amount of hostile criticism, and the charge of actual maladministration has been preferred by the too zealous advocates of the claims of widows in need of relief which for the time being had to be refused or postponed. Such criticism has naturally been most acute when the Navy or Army has suffered losses in our various lesser wars, and when, consequently, more general interest has been taken in the administration of the Fund. For instance, after the Zulu War in 1880, and again in January 1881, the subject was brought before the House of Commons by the Baron de Worms. The recent agitation will be fresh in the memory of all, and no further reference need be made to the subject beyond noting the fact that when the offer of Actuarial advice by the Council of the Institute was accepted by the Commissioners, the Secretary, Col. Young, wrote: ". . . You may be ignorant of the allegation that has been made that Mr. Finlaison was required to make his valuations under conditions which were dictated by this Commission. Mr. Finlaison, in giving evidence before the Select Committee of the House of Commons in 1896, refuted this allegation, for which there has been no foundation, and I therefore emphasize the point now by making it clear to you

“and your Council that you have only to ask for such information as you may desire, and every available information will be promptly furnished.”

The following is a brief description of the Funds administered by the Commission:—

1st. Funds either subscribed directly by the public for administration by the Patriotic Fund Commissioners, or placed under their administration under certain conditions by bodies directly raising such Funds, the application of the Funds being for specific classes of sufferers in special wars, or in naval disasters, the application, therefore, necessitating the use of Capital as well as Income for payment of allowances to beneficiaries:—

1854. Patriotic Fund for widows whose husbands lost their lives in or by service in the War with Russia, 1854–6.

1870. Fund for widows, orphans, and other dependants of those losing their lives in H.M.S. “Captain.”

1878. Fund for widows, orphans, and other dependants of those losing their lives in H.M.S. “Eurydice.”

1879. Funds for widows of those losing their lives in the Zulu Campaign, 1879.

1880. Funds for widows and orphans of those losing their lives in H.M.S. “Atalanta.”

1888. Fund for widows of those losing their lives in the Ashanti War, 1873–4.

1890. Fund for most necessitous survivors of the Charge of the Light Brigade at the Battle of Balaclava, 1854.

1893. Fund for widows, orphans, and other dependants of those losing their lives in H.M.S. “Victoria.”

1901. Transvaal War Fund.

2nd. Funds not resulting directly from subscriptions by the Public for any special war or disaster, and therefore not applicable to special classes of widows, &c., from any particular war or disaster:—

Patriotic (General) Fund resulting from surpluses arising from Patriotic (Russian War, 1854) Fund, the “Captain”, “Eurydice”, and Zulu Funds under the provisions of Section IV Patriotic Fund Act 1881, and Supplementary Commissions of 3 April 1897 and 28 December 1899.

Patriotic (Army) Fund handed to this Commission in 1899 by the War Office, having been previously administered by the Quarter-Master-General of the Forces. This Fund is applicable to widows of deserving soldiers.

Rodriguez Fund originated in 1864 in a Bequest by Don Francisco Rodriguez and now applied for the benefit of widows and other dependants of deceased sailors and soldiers wounded in War.

Royal Naval Relief Fund founded by a contribution in 1878 from the Committee of the "Eurydice" Fund and constituted under a Royal Commission dated 7 June 1875. This Fund is applicable to widows, orphans and others of deceased sailors and marines.

Soldiers' Effects Funds placed under the administration of this Commission by the Secretary of State for War in 1884 to be applied as the Patriotic Fund Commissioners think fit in allowances and grants for the widows, orphans and other dependants of non-commissioned officers and men of His Majesty's land forces.

Zervudachi Fund created in 1883 by a donation from Sir Constantine Zervudachi, Greek Merchant, by gift of £1,300 without conditions, and now applied to widows, orphans and other dependants of officers and men of His Majesty's Navy and Army.

County of Forfar Fund created in 1886 by the County Authorities of Forfar paying to the Patriotic Fund Commissioners £674. 13s. 1d., and again in 1892 £171. 14s. 10d. to be applied for soldiers' widows nominated by the County of Forfar.

3rd. Funds for maintenance and education of orphans:—

Royal Victoria Patriotic Asylum Endowment Fund,
Thurlow Fund for helping girls in Royal Victoria Patriotic Asylum.

Roman Catholic Orphans' Fund for maintaining orphans in Roman Catholic Orphanages so far as income will allow.

The scale of allowances varies slightly among the various Funds, the following being the scale at present in force with regard to the "Russian War" Fund:—

Officers.

Widows of Colonels	} with corresponding ranks in the Navy	£66 per annum.	
Lieut.-Colonels		53	„
Majors		46	„
Captains		37	„
Lieutenants		30	„
Ensigns		27	„

With an increase of one quarter on attaining age 60.

Non-commissioned and Petty Officers, Sailors' and Soldiers' Weekly Allowances.

To Widows of	} with corres- ponding ranks in the Navy	Ordinary		After 60 years of age		After 70 years of age	
		s.	d.	s.	d.	s.	d.
Staff Non-com. Officers	}	9	0	11	0	12	0
Colour-Sergeants		8	6	10	6	11	6
Sergeants		8	0	10	0	11	0
Corporal or Bombardier		7	6	9	6	10	6
Drummer, Trumpeter, or Private		7	0	9	0	10	0

Special allowances are granted to some infirm orphans.

Re-married women receive about half the rate of allowance made to widows and are put on full allowance on again becoming widowed.

“A noteworthy feature in the administration of funds placed under the Patriotic Fund Commissioners is that in every way possible the widows are encouraged to maintain a proper spirit of independence in their mode of life, character and conduct, also in making earnings, and experience shows that this is contributed to in no slight degree by their receiving their allowances by remittances made to them direct from the Offices of the Commission punctually on the first day of every quarter instead of receiving these through local agents whose personal supervision is often disliked and sometimes has a tendency to pauperization.”

With regard to the management of special Funds, the practice has been to charge a fixed rate of 4 per-cent on the income of the Fund, to include all office services, printing, postage, stationery and audit of accounts, “but when a Fund subscribed by the public falls short of the amount required to give the same allowances as paid by the original Patriotic Fund, the highest allowances possible, according to the Actuarial Report, are paid, and no charge whatever is made for cost of management till the assets are valued by the Actuary to exceed the liabilities.”

The administration of the Patriotic Funds has been re-cast to a considerable extent by the Patriotic Fund Re-organization Act, 1903.

This Act creates as from the 1st January 1904, the "Royal Patriotic Fund Corporation", consisting of:—

- (a) Twelve members appointed by His Majesty the King.
- (b) The Lord Lieutenants of Counties.
- (c) The Chairmen of County Councils.
- (d) The Lord Mayors and Mayors of all County Boroughs in England, Wales and Scotland.
- (e) The Lord Provosts and Provost of every Royal, Parliamentary or Police Burgh in Scotland, with 50,000 inhabitants and upwards, and
- (f) Seven members by Co-optation.

In May 1901, a Joint Select Committee of the House of Lords and the House of Commons sat on the question of Charitable Agencies for Relief of Widows and Orphans of Soldiers and Sailors, and their Report was presented in July 1901.

Our past President, Mr. Higham, attended at the request of the Committee, and was closely questioned as to the reserves shown by the actuarial valuation to be necessary in respect of the various Funds. A full account of the proceedings will be found in the Parliamentary Report 289, 1901.

In the Forty-first Report of the Royal Commissioners to the Patriotic Fund a full statement as supplied by the Council of the Institute, is given of the amounts required to place the various classes of applicants for relief from the Transvaal War Fund, to December 1902, upon the same footing as those receiving relief from the older Funds.

A further sum of £231,389 was required, and it was hoped that a Parliamentary grant would be obtained to enable the Commissioners to deal with all the deserving cases presented to them. There was a particular desire to secure the reserves necessary for the restoration of full allowances to such re-married women as should become again widowed. The sum now required is, no doubt, much larger, as the number of applicants has greatly increased since the date of the above Report.

It is greatly to be desired that the necessary funds should be forthcoming, either through a direct grant from the Imperial exchequer or through the local bodies, who, it will be seen, are fully represented on the newly constituted Corporation.

Public interest in the sufferers from the late war has declined sadly, but the need for immediate relief in an immense number of cases is as acute as when the war was in progress. It is much to be regretted that the relief afforded should depend to so great an extent upon the applications being made at a propitious moment, and so little upon the measure of necessity.

It will be at once apparent that in the valuation of benefits such as those set out above, the rate of re-marriage is a most important factor. This was especially the case with the Transvaal War Fund, as the majority of the recipients of relief were young women who had only very recently become widowed. The allowances to widows in the last-mentioned Fund were granted on various conditions, as will be seen on reference to the Forty-first Report of the Royal Commissioners. Some received Post Office Annuities, which were inalienable and were to continue throughout life, irrespective of subsequent re-marriage. The greater number of the allowances, however, were made subject to discontinuance on re-marriage.

This fact must have a very great influence on the rate of re-marriage to be expected, and it is highly desirable that the experience of the Transvaal War Fund should be analyzed at an early date, so that an idea may be obtained of the trend of the difference in the re-marriage rates under the altered conditions.

In addition to statistics of re-marriage, it was hoped that data would be available for estimating the value to be placed upon the restitution to full allowances in the case of re-married women who again became widows.

A schedule in the following form was therefore suggested to the Royal Commissioners for supplying the necessary data.

Name of Pensioner	ENTRY ON FUND		RE-MARRIAGE		Calendar year of reverting to full Pension on death of Husband	Calendar year of Death
	Calendar year	Age	Calendar year	Age of Husband then		

It was impossible however to obtain the age of the husbands, and the schedules actually supplied were in the following form.

Schedule showing the Experience of Widows of Non-Commissioned Officers and Men provided for by the Royal Patriotic (Russian War) Fund.

Name of Pensioner	Age at Entry	Year of Entry	Year of Re-marriage	Year of further Widowhood	Year of forfeiture of Pension	Year in which Pension was re-instated	Year of Death
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No.

In this schedule only calendar years are noted and ages last birthday in the year of entry, as this was the only form in which the Commissioners could supply the information.

The most obvious method available for dealing with the statistics was the old "Institute" or "Calendar Year" method, by which the age at entry is assumed to be a half-year in excess of that stated, and all entries and retirements are assumed to occur in the middle of a calendar year.

The following Table (I) will show the extent of the data, and at the same time give in brief the history of the application of the Fund to the relief of widows. Table III gives the same items arranged according to years of age. The lists of widows of naval men were given separately from those of widows of men in the army, but no attempt has been made in the following investigation to distinguish between the experience of the two classes.

A Card was written for each case embodying the whole of the particulars supplied in the schedules, and those relating to Naval Widows were specially marked, although, as already stated, they were not treated separately. No doubt some interesting features would become apparent if the data were subjected to a more searching analysis than we have applied, and we shall be happy to place the cards at the disposal of any member wishing to pursue the subject further.

It is well known, at least amongst actuaries, that in the early years of widowhood the age of the widow is a much less important factor in determining the rate of re-marriage than the period which has elapsed since the death of her husband. We therefore felt that our first task must be the construction of tables in "select" form. For this purpose all widows were included who were placed upon the books of the Fund during the years 1854-1857, as it was considered a fair presumption

that only a short interval had elapsed since the death of their husbands. To these were added those who became widows after having re-married. The experience of those widows who entered subsequently to 1857 and did not re-marry appears only in the aggregate tables. A glance at Table I will show that a very small proportion of the total experience is thus excluded from the select tables.

The data were not sufficiently extensive to enable us to use the "select" form for individual ages at entry with good effect, and it was essential to group them in some way. The convenient arrangement of quinquennial groups of ages at entry was therefore adopted, and the following Tables comprise the whole of the facts brought out in "select" form (see Table II).

The greater portion of the data naturally relates to the first widowhood—i.e., to that which brought the widows on the Fund. There were, however, a great many cases of second and third widowhoods, and two of fourth widowhoods. The first widowhood alone gives very little experience with regard to women becoming widows comparatively late in life, and since there was no real evidence that the first observed widowhood was in all cases actually the *first* widowhood,—no distinction has been preserved in the tables here presented, although the experience of each widowhood was got out in select form, and may be examined in detail at some future time.

In order that the data should afford the fullest possible information with a minimum of labour to other enquirers, columns are included in Table II giving the rates and also the numbers "Exposed to risk" of mortality, forfeiture, re-marriage, re-marriage and mortality combined, and also of total decrement.

The true central age of each group does not exactly correspond with the middle age, as will be seen on referring to the numbers of "entrants" at the head of each table. No attempt has, however, been made in this series of tables to modify the original data, and the ages placed at the side of the table are merely approximate. This point is referred to later on.

On the assumptions adopted, since the age at entry is in all cases the age last birthday, and the first year of widowhood "0", is, as in the old Institute Experience, a half-year only, year "one" of widowhood commences on the average, at an attained age one year older than that stated as the age of "entrants."

We think there will be no real inconvenience from this circumstance, as the main use of the Tables will probably be in valuing allowances to widows, and it rarely happens that the actuary is supplied with more precise evidence as to exact ages than that given in the present experience. The age generally given is that last birthday in an inter-valuation period, which is usually the calendar year, and the assumption will consequently be that the next age is attained on the valuation date. If then, the time elapsed since widowhood be taken as some integral number of years plus a half, the form in which the present tables are presented will meet the necessities of the case.

The rates of re-marriage since widowhood are the most interesting, and considerable pains have been taken to graduate them by a graphic method. (See Table IV and Diagram I).

It was at first thought that one or more of Professor Karl Pearson's frequency curves might be made to represent the data, and we should have preferred the shifting of responsibility which such a method would have afforded. The data, however, when collated seemed to show in each group a tendency to a second maximum by a very marked arrest of the downward course of the rate of re-marriage after the fourth year from widowhood. Although the general form of the progression of rates could be fairly represented by a curve of Professor Pearson's Type I, we did not feel warranted in allowing such a marked and persistent feature to be sacrificed. It is hoped that the personal equation, unavoidably introduced in any application of the graphic method, does not at any rate misrepresent the actual facts in the graduations here given.

When dealing with the graduation of the re-marriage rates, we naturally turned to Dr. Sprague's papers in the *Journal* on rates of re-marriage, and first to the paper on Re-marriage of Widowers, vol. xxii, p. 77. Dr. Sprague's data were comparatively scanty, but the unadjusted rates of re-marriage figured on p. 88 (vol. xxii) certainly seem to indicate a second maximum, although in the graduated curves on the opposite page no such feature is apparent.

The maximum points in the curves for widowers are practically equal from ages 35 to 55, but in the widows' rates the maximum points show a rapid decline from age 25. (Diagram I.)

In his paper on Rates of Re-marriage among Widows and Spinsters, Dr. Sprague remarks that "the rate depends more

“upon the duration of widowerhood than upon the age of the
“widower, but I am inclined to think that in the case of widows
“the duration of widowhood will be a very much less important
“element, and that after a woman has been a widow for, say,
“two, or at the most three years, the probability of re-marriage
“would practically depend entirely upon her age, and not at all
“upon the length of time she had been a widow.”

This opinion is hardly borne out by the series of curves here shown. The rate of re-marriage is for many years after widowhood considerably in excess of the rates shown in the aggregate table, and also of the rates shown in the aggregate table formed after excluding the experience of the first ten (nine and a half) “years of widowhood.”

It will be seen that the maximum rate of re-marriage in each curve for the Patriotic Widows occurs at a later period than among Dr. Sprague’s widowers, and this is what one would expect, since there is often amongst the younger widows the possibility of a posthumous child, which must necessarily affect the probability of re-marrying early. This would also explain in part the earlier occurrence of the maximum rate among the older widows, though no doubt the explanation suggested by Dr. Sprague of a similar feature in the curves for widowers’ re-marriages at advanced ages has some force with regard to widows.

It is probable, also, that the slackening in the decline of the rate after five or six years may be due to the fact that the children, if any, of the previous marriage, would by that time be well beyond the period of infancy. There are other features which a careful examination of the curves will suggest, but we do not feel inclined to venture any explanation as to the causes underlying them. In reference to the general form of curve shown by the re-marriages of widows and widowers, it is curious to note that almost precisely the same form is shown by divorce statistics. Professor Karl Pearson gives an analysis of about 110,000 divorces in the United States in 1882-6, according to the duration of marriage, in his great paper on *Skew Variation*. The rough data show a maximum at a little short of 4 years from marriage, while the mode in Professor Pearson’s curve is just short of 3 years from marriage. Allowing for the time necessary to institute and complete divorce proceedings, the latter is probably about the true average period of the maximum rate of divorce. It would be interesting to know if any difference would be shown by

the rates of divorce according to duration of marriage if divorces with male and female respondents could be separated. It would appear almost that the problems suggested by the features of the curves are as much physiological as social.

It is highly probable that the progression of rates of re-marriage among widows in a higher sphere of life would be very different from those shown in this paper, whilst the absolute height of the rate would certainly be less amongst widows in better circumstances.

As already mentioned, marriage rates in Table IV have been deduced from the graduated curves, which were obtained by simply taking groups of five entrance ages, and following them through successive years of widowhood. The numbers of entrants at each age were not made equal, and it is therefore evident that the central age is not necessarily the true average age of the group.

Even if the number of entrants were made equal the results would not be completely satisfactory. The entrance age would be, of course, correct, but a very few years after entry it would be found that there was a considerable discrepancy between the true and the assumed average ages. In fact, a disturbance of the average age should be expected, since the younger widows in each group are eliminated by re-marriage at the greater rate, and the average age of the remaining widows tends to increase so long as the rate of re-marriage exceeds the death rate. In the present case, however, there is no doubt that the scantiness of the data is the chief cause of the irregularity in the average age in successive years since widowhood. It appeared that it would be of some interest to obtain the rates of re-marriage in such a manner as should enable us to use the true average ages throughout each group.

For this purpose we went back to the Select Tables for each age of entry. The tables were combined in groups of five ages at entry, and at each "year of widowhood" the mean of the rates of re-marriage for the five tables was taken as the average rate for the central age. This method of procedure has the effect of equalizing the "exposed to risk" at each duration passed through, and, consequently, the central ages are the true average ages throughout.

These rates were then graduated graphically, and the results are shown in Table V and Diagram II. No attempt was made to cross-graduate the rates, and, as the diagrams were originally drawn

on separate sheets, the rates at the old ages are not so consistent with each other as they undoubtedly should be. A reference to Table II will show that this point is relatively unimportant, since the facts, upon which the later rates were based, are very scanty.

The rates obtained reproduce the original number of marriages very closely, the accumulated error being never very large, and the number of changes of sign being fairly satisfactory.

The first group was taken with 22 as the central age because the method employed unduly exaggerated the effect of the comparatively few entrants below age 20. The curve shown is certainly not a reassuring one, and the decided maximum point which occurs in the tenth year of widowhood is quite contrary to what one would expect. The data, however, will not allow of this maximum being smoothed away, and we therefore reproduce it as it is, and will even venture to suggest a possible cause.

Is it not possible that at about age 30 there is a specially strong incentive for widows to re-marry while they are "still young"? The heaping up which is observable in each of the first three curves in Diagram II at about age 30 would seem to confirm this suggestion.

Further confirmation is afforded by the population statistics given in Table IX.

From the ungraduated rates of re-marriage and mortality combined in Table II, tables of annuities have been compiled, and are given in Table VI.

A very cursory examination of these will show that a slight amount of graduation is all that is necessary to make them serviceable for valuation purposes, but we have preferred to leave them untouched.

The annuity rates show a maximum in each group at 19, 17, 14, 11, 9 and 6 years respectively from the date of widowhood in the first six groups. In the remaining groups where the rate of re-marriage has very small effect, the maximum coincides with the first year of widowhood. The position of the maximum value of the annuity is not far removed from the point at which the re-marriage rate would merge into the rate depending upon age alone, as distinguished from duration of widowhood.

Before passing to the Aggregate Tables, a remark falls to be made as to the extent to which the completeness of the information may be relied upon.

Women who re-married forfeited only half their allowances as Widows, so that they were still on the books of the Fund, and their deaths would be notified in the usual way. There were, however, a few fraudulent cases in which the husband of a re-married woman did not report her death, but promptly substituted another woman to receive the half-allowance. The Secretary, Colonel Young, informs us that such cases were very rare, and were probably all eliminated by 1890 as a result of a searching enquiry. We think, therefore, that while it is proper to mention the fact, it has little bearing on the rates brought out. Colonel Young further informs us that the "usual reason for complete forfeiture of allowances were immorality and intemperance. There were no doubt other reasons, such as, becoming habitually chargeable as paupers either through living an idle and wasteful life, or becoming insane or senile; becoming possessed of means placing a woman beyond the need of assistance from a Charitable Fund; and emigrating to countries other than British possessions"; it appears, also, that there were two or three cases of forfeiture for concealment of re-marriage.

The whole of the experience was drawn upon to deduce aggregate rates of re-marriage. These are shown in the ungraduated and graduated forms in Table VII(a), and Diagram III shows the form which the graduated values assume. The use of such a table must necessarily be restricted, as has already been indicated by the insistence on the necessity for Select Tables in considering rates of re-marriage amongst widows.

Table VII(b) gives the aggregate rates of re-marriage amongst widows, excluding the first 10 (that is $9\frac{1}{2}$) years of widowhood. This table may possibly have a more extended application. Considerable care is always necessary when basing any estimates upon rates of re-marriage drawn from any particular class, since apart from the uncertainty due to the paucity of data, the rates will depend very largely upon the social position of the widows. In the case of Pension Funds, the reduction or stoppage of allowances to widows on re-marrying, forms a most important factor in determining the rate of re-marriage, and there will be observed differences depending on nationality or even locality. These remarks will be sufficiently obvious to all those who have had experience of Widows' Pension Funds, but notwithstanding the caution they enforce, we think that Table VII(b) may be found useful in a considerable number of cases.

Rates of re-marriage are generally employed in order to arrive at the value to be placed upon allowances to widows ceasing on re-marriage; it is therefore desirable not to over-estimate the rates of re-marriage expected. A reference to Tables IV and V will show that for the younger ages at widowhood, the rates of re-marriage shown in the Select Tables, do not merge into the rates shown in the truncated Table VII for a considerable period after widowhood. It follows, therefore, that in the valuation of a fund containing a large number of young widows of a class similar to that from which this experience has been drawn, and when the average duration of widowhood does not much exceed ten years, Table VII (*b*) can be used with reasonable confidence.

Messrs. Hewat and Chatham, in their admirable work on the Experience of the Widows' and Pension Funds of the Scottish Banks, give a table of central marriage rates of widows, and in Table VIII the Bankers' Widows' rates are placed side by side with those of the Patriotic Fund Widows.

The amount of difference is very remarkable, and is, we think, not in the main due to any difference in social position, but rather to the fact that widows on the Patriotic Fund continued to receive half the usual allowance when married, whereas the allowances to the Bankers' Widows ceased entirely on re-marriage.

The comparison serves to emphasize the caution usually advised when it is proposed to employ rates of re-marriage or any similar function, derived from a special experience, to an experience differing therefrom in any material degree.

In order to facilitate a comparison between the rates of re-marriage here brought out, and those among the general population, Table IX has been compiled from the Registrar-General's Reports and the Census Returns. The rate shown is in each case derived from the average of three years' re-marriages of widows, and the number of widows from the Census for the central year of the three. By the courtesy of the Registrar-General we were supplied with the marriage statistics for 1902, and these have been employed in obtaining the rates of re-marriage for 1901.

In the early periods considerable uncertainty arises from the fact that the age at marriage in numerous cases is not stated. In all such cases the numbers given as of unstated age have been distributed in the proportion of those whose ages are stated, but no correction has been applied to the Census figures to bring them to the middle of the Census year. The last is an

unimportant point, but the distribution of the unstated ages may involve a more or less serious departure from the true rates.

The Registrar-General's Office have made great efforts to reduce the number of unstated ages of marriage, and by persistently keeping the question in the foreground, they have succeeded in obtaining statements of age in nearly 99 per-cent of the total number married during 1902.

The marriage rate of spinsters for 1901 is appended in the last column of Table IX, as being of some interest.

The rates for widows on the Patriotic Fund, it will be noticed, lie between those for widows and spinsters of the general population. At the groups of ages 20-24, 25-34, 35-44, there is a fairly close correspondence with the rates among widows of the general population, but at the remaining groups, the spinsters' rates are more nearly approached.

One would expect the rates of re-marriage of the widows under observation to be less than the general rate of re-marriage, since they are kept above the pinch of poverty by the allowances received, and re-marriage is penalized by the withdrawal of half the pensions allowed during widowhood. There seems to be a steady decline in the rate of re-marriage of widows in the thirty years 1871-1901 in all the groups, with only two marked exceptions. For 1901, there is a rise in the rate among widows under 20, but as the numbers of such widows is always small, this is not a very significant fact. For ages 25-34 a small rise is apparent for 1901; the amount of the rise is comparatively small, and hardly affects the general trend.

The Registrar-General's Report shows a nearly continuous decline in the proportion of re-marriages to total marriages since 1874, but the total marriage rate, though lower in 1901 than in the early seventies, has risen since 1886, with the exception of a relapse in 1893. The total marriage rate depends, no doubt, on the general prosperity of the country, but it is not so easy to assign a reason for the decline in the proportion of re-marriages to total marriages.

In that most admirable compendium of statistics of the population, the recently issued Digest of the Results of the Census of England and Wales in 1901*, the progress of the marriage rate is shown as follows:

* A digest of the Results of the Census of England and Wales in 1901, compiled by William Sanders, and produced under the general supervision of Thomas G. Ackland. C. & E. Layton, 1903.

“The average marriage rate (persons married to 1000 living)
“ for the years—

1851– 60	was	16·9,
1861– 70	„	16·6,
1871– 80	„	16·2,
1881– 90	„	14·9,
1891–1900	„	15·6,
1901	„	15·9.”

These figures confirm the above remarks on Table IX.

The Mortality Table, founded on the aggregate experience, is set forth in Table X; and the graduated rates, with values of annuities and assurances derived therefrom, and commutation columns, are given in Table XI.

A table (XII) was also formed from the experience limited to the period of widowhood. In this table the mortality experience of re-married women was excluded, but those who became again widowed were treated as new entrants, and their experience until subsequent re-marriage or forfeiture included.

A comparison of the curves in Diagram IV will show that the rates of mortality amongst the widows is lower than that of the 3rd English Life Table until age 47, when it becomes higher, until it eventually coincides with the 3rd English at age 73.

The lower rates below age 47 are probably owing to the exclusion of the risks of maternity.

The rates of mortality in the Aggregate Table, which includes the experience of re-married women, are below those of the widows until age 34, after which they are higher until age 49. The lower rates until age 34 may possibly be due to the selection of re-marriage, and the higher rate observed until age 49 to maternity risks. Not much reliance can be placed on the rates shown at the higher ages, as the data upon which they were based were very scanty and irregular.

No table showing the rates of mortality peculiar to re-married women has been formed. It might, however, be expected that such a table would show the same differences from the experience of widows as are exhibited in the curves in Diagram IV between the total experience and the widows' experience, but in a more pronounced form.

The experience of re-married women has been used to obtain rates of widowhood, and these are given in rough and in adjusted

form in Table XIII. This is, of course, a table of death-rates of husbands, referred to the ages of the wives at the date of death of the husband. On comparing the rates with those of the English Life No 3 Males, it will be remarked that they progress very slowly with the ages of the wives, showing that, as the age of the wives increases, the disparity of age of husbands and wives on the average increases also.

This feature is probably mainly due to two causes. First, whatever the ages of husbands and wives at marriage may be, since the older husbands are subject to a higher rate of mortality, it follows that the age of the remaining husbands must become relatively less compared with that of the remaining wives. Secondly, widows who re-marry comparatively late in life have husbands younger than themselves, and these come into Table XIII as new entrants, exercising the same disturbing influence as do newly selected lives in an ordinary aggregate table of mortality of assured lives. The Registrar-General's Sixty-fourth Report contains particulars of the relative ages of husbands and wives in the marriages taking place during 1901. From these particulars Table XIV has been compiled, and the average differences of ages at marriage there shown seem to confirm the explanation offered in the foregoing paragraph with respect to the rates of widowhood.

In dealing with benefits which depend upon the rate of widowhood, it is usually necessary to make some assumption as to the average age of the husbands, and a further assumption as to the rate of mortality which they will experience. Table XIII enables us to dispense with both these assumptions for the particular class of wives here dealt with; and it is hoped, therefore, that it may be of some service to others who may have occasion to deal with pension or other funds where the rate of widowhood is of importance.

In concluding our remarks, we have to offer our grateful thanks to several gentlemen who have kindly assisted in working up the diagrams and tables. Though these are fairly voluminous, it will be readily understood that they do not show more than a small proportion of the labour expended in arriving at the results.

For particulars as to the history and working of the Patriotic Funds we are indebted mostly to the published Reports, and to the letters of Col. Young, the Secretary to the Royal Commission, who has very kindly helped us on many occasions by clearing up

difficulties and explaining many points which had a bearing on the methods to be employed in dealing with the statistics.

We are keenly alive to the many imperfections and shortcomings of this attempt to deal with a very interesting body of statistics, and we are in particular painfully conscious of the crude way in which many of the results are introduced.

Our only excuse is the very hurried conditions under which the paper had to be prepared, amidst the pressure of other duties.

We hope, however, that we have succeeded in putting the experience upon record in such a way that the principal features which it discloses may be readily recognized, and that our work may facilitate the efforts of any actuary who may desire to deduce from them further results.

TABLE I.

Number of Widows entering and leaving Patriotic (Russian War) Fund.

Year	Entered	Died	RE-MARRIED			RE-WIDOWED			Forfeited	Re-instated	Second Forfeiture	Existing according to year of entry
			First time	Second time	Third time	First time	Second time	Third time				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1854	133	...	1	1	39
1855	2,428	14	89	87	529
1856	466	22	251	1	68	1	...	97
1857	173	20	355	8	69	1	...	37
1858	123	24	207	2	...	12	35	6	...	27
1859	71	35	138	3	...	19	24	2	...	24
1860	38	29	161	4	...	17	26	11
1861	39	27	137	5	...	24	1	...	18	1	...	11
1862	34	28	97	4	...	18	16	3	...	10
1863	102	19	86	5	...	24	1	...	15	10	...	33
1864	40	41	61	14	...	22	2	...	21	16	...	8
1865	45	46	87	6	...	19	19	3	...	16
1866	33	33	60	11	...	27	27	2	1	6
1867	8	42	55	10	...	28	1	...	22	5	...	1
1868	12	49	35	7	...	21	2	...	24	6
1869	10	51	28	11	1	21	1	...	25	2	...	4
1870	13	49	21	6	...	27	1	...	22	2	...	7
1871	8	46	22	6	...	32	16	4	...	2
1872	8	49	14	5	...	23	1	...	11	5	...	1
1873	6	43	11	4	1	26	2	...	18	3	...	2
1874	6	41	7	3	2	20	11	2	...	2
1875	6	34	11	8	...	26	2	...	9	...	2	1
1876	2	48	1	6	...	22	2	...	7	2	...	1
1877	5	60	3	1	...	28	5	...	6	3
1878	...	50	7	4	...	25	1	...	7
1879	2	51	3	3	...	16	3	...	9	1
1880	1	48	5	3	...	33	4	...	10	...	1	1
1881	4	51	6	3	1	16	1	...	4	1
1882	1	60	3	1	...	16	4	...	6
1883	2	56	2	4	...	20	1	...	1	...	1	...
1884	6	61	1	3	...	17	2	4
1885	1	74	1	1	...	16	3	...	3	1
1886	...	62	...	2	...	25	1	...	2	1
1887	5	58	...	1	1	19	3	...	3	1
1888	3	65	...	1	...	31	1	1
1889	5	71	22	12	2
1890	5	71	...	1	...	21	3	1	8	3
1891	1	69	...	2	...	32	4	...	3
1892	4	89	10	...	1	3	3
1893	2	77	20	1	1	...	2
1894	2	63	24	1	2
1895	5	81	2	1	...	19	1	...	1	3
1896	8	63	11	1	...	1	6
1897	4	75	21	1	...	1	4
1898	...	68	13	1
1899	...	76	11
1900	...	72	20
...	3,870	2,361	1,968	151	6	923	54	2	675	78	5	907

Age last birthday at Widowhood, 18-22.

Average age of Entrants, 21.1.

Age attained	Year of Widow- hood	Married (m)	Died (d)	Forfeited (f)	Existing (ex.)	Total Decre- ment (dec.)	"EXPOSED"	
							Total Decrement E (dec.)	Marria and Dec E (m +
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
21½	0	19	4	15	...	38	334	326
22	1	38	2	10	...	50	296	291
23	2	42	1	6	...	49	246	243
24	3	22	2	3	...	27	197	195
25	4	23	2	2	...	27	170	169
26	5	8	...	2	...	10	143	142
27	6	17	17	133	133
28	7	8	1	2	...	11	116	115
29	8	10	1	11	105	105
30	9	5	...	1	...	6	94	93
31	10	8	1	2	...	11	88	87
32	11	5	5	77	77
33	12	3	3	1	...	7	72	71
34	13	2	...	2	...	4	65	64
35	14	3	...	1	...	4	61	60
36	15	1	...	2	...	3	57	56
37	16	3	1	1	...	5	54	53
38	17	1	1	2	49	49
39	18	2	...	1	...	3	47	46
40	19	...	1	1	...	2	44	43
41	20	1	...	1	...	2	42	41
42	21	40	40
43	22	1	...	1	40	39
44	23	1	1	39	39
45	24	1	...	1	...	2	38	37
46	25	36	36
47	26	1	1	36	36
48	27	...	2	2	35	35
49	28	1	1	33	33
50	29	...	1	1	32	32
51	30	1	1	2	31	31
52	31	29	29
53	32	29	29
54	33	...	1	1	29	29
55	34	...	1	1	28	28
56	35	...	1	1	27	27
57	36	26	26
58	37	26	26
59	38	...	1	1	26	26
60	39	...	1	1	25	25
61	40	...	2	2	24	24
62	41	22	22
63	42	...	1	1	22	22
64	43	...	1	...	1	2	21	21
65	44	4	4	19	19
66	45	14	14	15	15
67	46	1	1	1	1
...	...	226	33	55	20	334	3,219	3,191

Patriotic (Russian War) Fund.

No. of Entrants at age 18	10
" " " 19	17
" " " 20	56
" " " 21	99
" " " 22	152
Total of Group	334

Age at Entrance (<i>f</i>)	Marriage E (<i>m</i>)	Death E (<i>d</i>)	RATES OF					Year of Widow- hood
			Total Decrement <i>q</i> (<i>d</i> or <i>e</i>)	Marriage and Death <i>q</i> (<i>m</i> + <i>d</i>)	Forfeiture <i>q</i> (<i>f</i>)	Marriage <i>q</i> (<i>m</i>)	Death <i>q</i> (<i>d</i>)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	324.5	317.	.1138	.0704	.0465	.0586	.0126	0
1	290.	272.	.1689	.1375	.0362	.1310	.0074	1
2	242.5	222.	.1992	.1770	.0267	.1732	.0045	2
3	194.5	184.5	.1371	.1228	.0162	.1131	.0108	3
4	168.	157.5	.1588	.1479	.0127	.1369	.0127	4
5	142.	138.	.0699	.0563	.0144	.0563	...	5
6	133.	124.5	.1278	.12781278	...	6
7	114.5	111.	.0948	.0783	.0179	.0699	.0090	7
8	104.5	100.	.1048	.10480357	.0100	8
9	93.5	91.	.0638	.0535	.0109	.0535	...	9
10	86.5	83.	.1250	.1034	.0240	.0925	.0120	10
11	77.	74.5	.0649	.06490649	...	11
12	70.	70.	.0972	.0839	.0145	.0429	.0429	12
13	64.	63.	.0615	.0313	.0313	.0313	...	13
14	60.5	59.	.0656	.0496	.0168	.0496	...	14
15	56.	55.5	.0526	.0179	.0354	.0179	...	15
16	53.	52.	.0926	.0748	.0192	.0566	.0192	16
17	48.5	48.5	.0408	.04080206	.0206	17
18	46.5	45.5	.0638	.0430	.0217	.0430	...	18
19	43.	43.5	.0455	.0230	.02300230	19
20	41.5	41.	.0476	.0241	.0241	.0241	...	20
21	40.	40.	21
22	39.5	39.5	.0250250	22
23	39.	38.5	.0256	.02560256	...	23
24	37.5	37.	.0526	.0267	.0267	.0267	...	24
25	36.	36.	25
26	36.	35.5	.0278	.02780278	...	26
27	34.	35.	.0571	.05710571	27
28	33.	32.5	.0303	.03030303	...	28
29	31.5	32.	.0313	.03130313	29
30	30.5	30.5	.0645	.06450328	.0328	30
31	29.	29.	31
32	29.	29.	32
33	28.5	29.	.0345	.03450345	33
34	27.5	28.	.0357	.03570357	34
35	26.5	27.	.0370	.03700370	35
36	26.	26.	36
37	26.	26.	37
38	25.5	26.	.0355	.03550355	38
39	24.5	25.	.0400	.04000400	39
40	23.	24.	.0533	.05330533	40
41	22.	22.	41
42	21.5	22.	.0455	.04550455	42
43	20.5	21.	.0476	.04760476	43
44	19.	19.	44
45	15.	15.	45
46	1.	1.	46
5	3,175.	3,078.5

TABLE
Widows' Experience

Ages last birthday at Widowhood, 23-27.

Average age of Entrants, 24.9.

Age attained	Year of Widow- hood	Married (m)	Died (d)	Forfeited (f)	Existing (ex.)	Total Decre- ment (dec.)	" EXPOSE	
							Total Decrement E (dec.)	Mar- ried and E (m)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
25½	0	44	7	30	...	81	1,097	1,08
26	1	119	7	24	...	150	1,016	1,00
27	2	152	4	21	...	177	866	85
28	3	90	7	13	...	110	689	68
29	4	57	4	4	...	65	579	57
30	5	53	4	4	...	61	514	51
31	6	36	4	4	...	44	453	45
32	7	35	3	4	...	42	409	40
33	8	22	1	5	...	28	367	36
34	9	20	3	4	...	27	339	33
35	10	32	4	4	...	40	312	31
36	11	17	1	7	...	25	272	27
37	12	14	4	5	...	23	247	24
38	13	5	5	3	...	13	224	22
39	14	6	5	4	...	15	211	21
40	15	7	2	1	...	10	196	19
41	16	4	3	2	...	9	186	18
42	17	1	5	5	...	11	177	17
43	18	3	...	3	...	6	166	16
44	19	2	3	2	...	7	160	16
45	20	1	1	1	...	3	153	15
46	21	3	4	1	...	8	150	15
47	22	...	6	1	...	7	142	14
48	23	...	1	1	...	2	135	13
49	24	...	2	2	133	13
50	25	2	3	4	...	9	131	13
51	26	2	7	122	12
52	27	...	2	2	115	11
53	28	...	4	4	113	11
54	29	...	2	2	109	10
55	30	...	3	3	107	10
56	31	...	4	4	104	10
57	32	...	1	1	100	10
58	33	...	5	5	99	9
59	34	...	3	2	...	5	94	9
60	35	...	6	6	89	8
61	36	...	4	4	83	8
62	37	...	5	5	79	7
63	38	...	7	7	74	7
64	39	...	2	2	67	6
65	40	...	5	5	65	6
66	41	...	6	6	60	6
67	42	...	2	2	54	5
68	43	...	5	...	1	6	52	5
69	44	...	2	...	4	6	46	4
70	45	...	6	...	31	37	40	4
71	46	3	3	3	...
...	...	727	172	159	39	1,097	10,999	10,91

continued.

Patriotic (Russian War) Fund.

No of Entrants at age 23 . 191

" " " 24 . 231

" " " 25 . 252

" " " 26 . 215

" " " 27 . 208

Total of Group . 1,097

Age of			RATES OF					Year of Widow- hood
Entrance (f)	Marriage E (m)	Death E (d)	Total Decrement q (dec.)	Marriage and Death q (m+d)	Forfeiture q (f)	Marriage q (m)	Death q (d)	
(1)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0	1,078.5	1,060	.0738	.0471	.0280	.0408	.0066	0
1	1,000.5	944.5	.1476	.1255	.0252	.1189	.0074	1
2	853.5	779.5	.2044	.1823	.0266	.1782	.0051	2
3	679	637.5	.1597	.1421	.0203	.1325	.0110	3
4	575	548.5	.1123	.1057	.0073	.0991	.0073	4
5	510	485.5	.1187	.1113	.0082	.1039	.0082	5
6	449	433	.0971	.0887	.0092	.0802	.0092	6
7	405.5	389.5	.1027	.0934	.0103	.0863	.0077	7
8	364	353.5	.0763	.0631	.0141	.0604	.0028	8
9	335.5	327	.0796	.0682	.0122	.0596	.0092	9
10	308	294	.1282	.1161	.0136	.1039	.0136	10
11	268	260	.0919	.0670	.0266	.0634	.0038	11
12	242.5	237.5	.0931	.0736	.0210	.0577	.0168	12
13	220	220	.0580	.0449	.0137	.0227	.0227	13
14	206.5	206	.0711	.0526	.0195	.0291	.0243	14
15	194.5	192	.0510	.0460	.0052	.0360	.0104	15
16	183.5	183	.0484	.0378	.0110	.0218	.0164	16
17	172	174	.0621	.0344	.0287	.0058	.0287	17
18	164.5	163	.0361	.0182	.0182	.0182	...	18
19	157.5	158	.0438	.0314	.0127	.0127	.0190	19
20	152	152	.0196	.0131	.0066	.0066	.0066	20
21	147.5	148	.0533	.0468	.0068	.0203	.0270	21
22	138.5	141.5	.0493	.0424	.00720424	22
23	134	134.5	.0148	.0074	.00740074	23
24	132	133	.0150	.01500150	24
25	127.5	128	.0687	.0388	.0311	.0157	.0234	25
26	119.5	121	.0574	.05740167	.0413	26
27	114	115	.0174	.01740174	27
28	111	113	.0354	.03540354	28
29	108	109	.0183	.01830183	29
30	105.5	107	.0280	.02800280	30
31	102	104	.0385	.03850385	31
32	99.5	100	.0100	.01000100	32
33	96.5	99	.0505	.05050505	33
34	91.5	93	.0532	.0323	.02160323	34
35	86	89	.0674	.06740674	35
36	81	83	.0482	.04820482	36
37	76.5	79	.0633	.06330633	37
38	70.5	74	.0946	.09460946	38
39	66	67	.0299	.02990299	39
40	62.5	65	.0769	.07690769	40
41	57	60	.1000	.10001000	41
42	53	54	.0370	.03700370	42
43	49.5	52	.0962	.09620962	43
44	45	46	.0435	.04350435	44
45	37	40	.1500	.15001500	45
46	3	3	46
50	10,833.5	10,556

TABLE II
Widows' Experience

Ages last birthday at Widowhood, 28-32.

Average age of Entrants, 29·7.

Age attained	Year of Widow- hood	Married (<i>m</i>)	Died (<i>d</i>)	Forfeited (<i>f</i>)	Existing (<i>ec</i>)	Total Decre- ment (<i>dec.</i>)	" EXPOSED TO "	
							Total Decrement E (<i>dec.</i>)	Married and Dea E (<i>m+d</i>)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
30½	0	27	4	22	...	53	903	892
31	1	83	4	11	...	98	850	844
32	2	88	4	11	...	103	752	746
33	3	57	5	3	...	65	619	647
34	4	47	5	1	...	53	584	583
35	5	43	3	5	...	51	531	528
36	6	36	5	3	...	44	480	478
37	7	23	5	1	...	29	436	435
38	8	12	1	2	...	15	407	406
39	9	15	3	18	392	392
40	10	15	5	4	...	24	374	372
41	11	9	7	2	...	18	350	349
42	12	14	5	4	...	23	332	330
43	13	9	6	5	...	20	309	306
44	14	5	3	4	...	12	289	287
45	15	5	4	1	...	10	277	276
46	16	4	7	2	...	13	267	266
47	17	2	3	5	254	254
48	18	...	4	2	...	6	249	248
49	19	3	3	6	243	243
50	20	2	2	4	237	237
51	21	...	7	1	...	8	233	232
52	22	...	5	1	...	6	225	224
53	23	1	4	5	219	219
54	24	...	5	1	...	6	214	213
55	25	...	2	2	208	208
56	26	...	5	5	206	206
57	27	...	7	7	201	201
58	28	...	2	2	194	194
59	29	...	4	4	192	192
60	30	1	10	11	188	188
61	31	...	8	8	177	177
62	32	...	6	6	169	169
63	33	...	9	1	...	10	163	162
64	34	...	6	1	2	9	153	152
65	35	...	10	1	...	11	144	143
66	36	...	4	...	2	6	133	133
67	37	...	9	1	...	10	127	126
68	38	...	10	...	1	11	117	117
69	39	...	3	...	3	6	106	106
70	40	...	7	...	1	8	100	100
71	41	...	4	4	92	92
72	42	...	10	1	...	11	88	87
73	43	...	6	...	4	10	77	77
74	44	...	9	...	10	19	67	67
75	45	...	6	...	36	42	48	48
76	46	6	6	6	6
...	...	501	246	91	65	903	13,012	12,966

*continued.**Patriotic (Russian War) Fund.*

No. of Entrants at age 28	221
" " " 29	189
" " " 30	246
" " " 31	118
" " " 32	129
	<u>903</u>

RANK OF	RALES OF						Year of	
Forfeiture <i>q</i> (<i>f</i>)	Marriage <i>E</i> (<i>m</i>)	Death <i>E</i> (<i>d</i>)	Total Decrement <i>q</i> (<i>d-c</i>)	Marriage and Death <i>q</i> (<i>m+d</i>)	Forfeiture <i>q</i> (<i>f</i>)	Marriage <i>q</i> (<i>m</i>)	Death <i>q</i> (<i>d</i>)	Widow- hood
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
887.5	890.	878.5	.0587	.0348	.0248	.0303	.0046	0
806.5	842.5	803.	.1153	.1030	.0136	.0985	.0050	1
706.	744.5	702.5	.1370	.1232	.0156	.1182	.0057	2
618.	645.	619.	.1002	.0958	.0049	.0884	.0081	3
558.	581.	560.	.0908	.0891	.0018	.0809	.0089	4
508.	527.	507.	.0960	.0870	.0098	.0816	.0059	5
459.5	476.	460.5	.0917	.0857	.0065	.0756	.0109	6
422.	433.	424.	.0665	.0643	.0024	.0531	.0118	7
400.5	405.5	400.	.0369	.0320	.0050	.0296	.0025	8
83.	390.5	384.5	.0450	.04590384	.0078	9
64.	369.5	364.5	.0642	.0538	.0110	.0406	.0137	10
42.	345.5	344.5	.0514	.0458	.0058	.0260	.0203	11
22.5	327.5	323.	.0693	.0576	.0124	.0427	.0155	12
01.5	303.5	302.	.0647	.0489	.0166	.0297	.0199	13
85.	285.5	284.5	.0415	.0279	.0140	.0175	.0105	14
72.5	274.5	274.	.0361	.0325	.0037	.0182	.0146	15
61.5	262.5	264.	.0487	.0414	.0076	.0152	.0265	16
51.5	252.5	253.	.0197	.01970079	.0119	17
47.	246.	248.	.0241	.0161	.00810161	18
40.	241.5	241.5	.0247	.02470124	.0124	19
35.	236.	236.	.0169	.01690085	.0085	20
29.5	229.	232.5	.0343	.0301	.00440301	21
22.5	222.	224.5	.0267	.0223	.00450223	22
16.5	217.	218.5	.0228	.02280046	.0183	23
11.5	211.	213.5	.0280	.0234	.00470234	24
7.	207.	208.	.0096	.00960096	25
3.5	203.5	206.	.0243	.02430243	26
07.5	197.5	201.	.0348	.03180348	27
03.	193.	194.	.0103	.01030103	28
00.	190.	192.	.0208	.02080208	29
2.5	183.	187.5	.0585	.05850055	.0533	30
3.	173.	177.	.0452	.04520452	31
6.	166.	169.	.0355	.03550355	32
8.5	158.	162.5	.0613	.0554	.00630554	33
0.	149.5	152.5	.0458	.0393	.00670393	34
9.	138.5	143.5	.0764	.0697	.00720697	35
1.	131.	133.	.0301	.03010301	36
2.5	122.	126.5	.0787	.0711	.00820711	37
2.	112.	117.	.0855	.08550855	38
4.5	104.5	106.	.0283	.02830283	39
5.5	96.5	100.	.0700	.07000700	40
0.	90.	92.	.0435	.04350435	41
3.	82.5	87.5	.1250	.1143	.01201143	42
4.	74.	77.	.0779	.07790779	43
2.5	62.5	67.	.1343	.13431343	44
5.	45.	48.	.1250	.12501250	45
8.	6	6.	46
2,885	12,843.5	12,716.

TABLE I
Widows' Experience

Ages last birthday at Widowhood, 33-37.

Average age of Entrants, 34.9.

Age attained	Year of Widowhood	Married (m)	Died (d)	Forfeited (f)	Existing (ex.)	Total Decrement (dec.)	" EXPOSED	
							Total Decrement E (dec.)	Married and Dec E (m +)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
35½	0	17	2	11	...	30	569	563
36	1	36	5	4	...	45	539	537
37	2	48	6	5	...	59	494	491
38	3	21	7	2	...	30	435	434
39	4	16	4	2	...	22	405	404
40	5	19	2	21	383	383
41	6	9	4	13	362	362
42	7	12	3	15	349	349
43	8	5	1	2	...	8	334	333
44	9	8	9	3	...	20	326	324
45	10	7	5	1	...	13	306	305
46	11	5	2	7	293	293
47	12	6	5	3	...	14	286	284
48	13	3	5	1	...	9	272	271
49	14	3	5	8	263	263
50	15	1	3	2	...	6	255	254
51	16	...	2	2	...	4	249	248
52	17	2	5	1	...	8	245	244
53	18	...	8	1	...	9	237	236
54	19	...	6	1	...	7	228	227
55	20	2	1	3	221	220
56	21	...	5	2	...	7	218	217
57	22	...	3	1	...	4	211	210
58	23	...	10	2	...	12	207	206
59	24	...	2	2	195	194
60	25	1	9	10	193	192
61	26	...	6	1	...	7	183	182
62	27	2	2	4	176	176
63	28	...	9	1	...	10	172	172
64	29	...	9	1	1	11	162	162
65	30	...	8	...	1	9	151	151
66	31	...	3	...	3	6	142	142
67	32	...	5	5	136	136
68	33	...	6	...	2	8	131	131
69	34	...	9	...	4	13	123	123
70	35	...	7	7	110	110
71	36	...	6	6	103	103
72	37	...	5	...	1	6	97	97
73	38	...	6	...	1	7	91	91
74	39	...	9	...	1	10	84	84
75	40	...	6	...	1	7	74	74
76	41	...	9	...	1	10	67	67
77	42	...	5	5	57	57
78	43	...	4	...	1	5	52	52
79	44	...	4	...	10	14	47	47
80	45	...	3	...	29	32	33	33
81	46	1	1	1	1
...	...	223	240	49	57	569	10,267	10,241

*tinued.**Patriotic (Russian War) Fund.*

No. of Entrants at age 33	121
" " " 34	110
" " " 35	116
" " " 36	134
" " " 37	88
	<u>569</u>

AGE OF Entrant (f)	RATES OF						Year of Widow- hood
	Marriage E (m)	Death (d)	Total Decrement q (dec.)	Marriage and Death q (m+d)	Forfeiture q (f)	Marriage q (m)	
(1)	(11)	(12)	(13)	(14)	(15)	(16)	(18)
0	562.5	555.	.0527	.0337	.0197	.0302	0
1	534.5	519.	.0835	.0764	.0077	.0674	1
2	488.5	467.5	.1194	.1099	.0107	.0983	2
3	430.5	423.5	.0690	.0645	.0048	.0488	3
4	402.	396.	.0543	.0495	.0051	.0398	4
5	382.	373.5	.0548	.05480497	5
6	360.	357.5	.0359	.03590250	6
7	347.5	343.	.0430	.04300345	7
8	332.5	330.5	.0240	.0180	.0060	.0150	8
9	320.	320.5	.0613	.0524	.0094	.0250	9
10	303.	302.	.0425	.0393	.0033	.0231	10
11	292.	290.5	.0239	.02390171	11
12	282.	281.5	.0490	.0387	.0107	.0213	12
13	269.	270.	.0331	.0295	.0037	.0112	13
14	260.5	261.5	.0304	.03040115	14
15	252.5	253.5	.0235	.0157	.0079	.0040	15
16	247.	248.	.0161	.0081	.0081	...	16
17	242.	243.5	.0327	.0286	.0041	.0083	17
18	232.5	236.5	.0380	.0338	.0043	...	18
19	224.5	227.5	.0307	.0264	.0044	...	19
20	220.5	220.	.0136	.01360091	20
21	214.5	217.	.0321	.0230	.0093	...	21
22	209.	210.5	.0190	.0143	.0048	...	22
23	201.	206.	.0580	.0485	.0099	...	23
24	194.	195.	.0103	.0103	24
25	188.5	192.5	.0518	.05180053	25
26	179.5	182.5	.0383	.0329	.0056	...	26
27	175.	175.	.0227	.02270114	27
28	167.	171.5	.0581	.0525	.0060	...	28
29	157.	161.5	.0617	.0557	.0063	...	29
30	147.	151.	.0530	.0530	30
31	140.5	142.	.0211	.0211	31
32	133.5	136.	.0368	.0368	32
33	128.	131.	.0458	.0458	33
34	118.5	123.	.0732	.0732	34
35	106.5	110.	.0636	.0636	35
36	100.	103.	.0583	.0583	36
37	94.5	97.	.0515	.0515	37
38	88.	91.	.0659	.0659	38
39	79.5	84.	.1071	.1071	39
40	71.	74.	.0811	.0811	40
41	62.5	67.	.1343	.1343	41
42	54.5	57.	.0877	.0877	42
43	50.	52.	.0769	.0769	43
44	45.	47.	.0851	.0851	44
45	31.5	33.	.0909	.0909	45
46	1	1.	46
10,005	10,122.5	10,131.

TABLE II
Widows' Experience

Ages last birthday at Widowhood, 38-42.

Average age of Entrants, 39·8.

Age attained	Year of Widow- hood	Married (<i>m</i>)	Died (<i>d</i>)	Forfeited (<i>f</i>)	Existing (<i>ex.</i>)	Total Decre- ment (<i>dec.</i>)	"EXPOSED TO"	
							Total Decrement E (<i>dec.</i>)	Marriage and Dea E (<i>m+d</i>)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
40½	0	13	2	6	...	21	380	377
41	1	20	1	3	...	24	359	357
42	2	20	5	25	335	335
43	3	6	4	1	...	11	310	309
44	4	11	2	13	299	299
45	5	13	3	3	...	19	286	284
46	6	8	2	1	...	11	267	266
47	7	8	3	2	...	13	256	255
48	8	3	6	2	...	11	243	242
49	9	2	3	5	232	232
50	10	3	4	1	...	8	227	226
51	11	...	4	1	...	5	219	218
52	12	1	6	7	214	214
53	13	1	4	5	207	207
54	14	1	1	2	202	202
55	15	1	3	1	...	5	200	199
56	16	...	2	1	...	3	195	194
57	17	...	4	4	192	192
58	18	2	8	10	188	188
59	19	...	2	1	...	3	178	177
60	20	1	7	8	175	175
61	21	...	4	4	167	167
62	22	...	4	1	...	5	163	162
63	23	...	4	4	158	158
64	24	...	6	1	1	8	154	153
65	25	...	4	4	146	146
66	26	...	10	...	3	13	142	142
67	27	...	7	...	3	10	129	129
68	28	...	1	1	4	6	119	118
69	29	...	6	...	3	9	113	113
70	30	...	7	...	2	9	104	104
71	31	...	2	...	3	5	95	95
72	32	...	9	...	2	11	90	90
73	33	...	8	...	1	9	79	79
74	34	...	10	...	2	12	70	70
75	35	...	3	...	2	5	58	58
76	36	...	5	...	1	6	53	53
77	37	...	2	2	47	47
78	38	...	3	3	45	45
79	39	...	5	...	1	6	42	42
80	40	...	7	...	2	9	36	36
81	41	...	1	1	27	27
82	42	...	4	4	26	26
83	43	...	4	...	3	7	22	22
84	44	...	2	...	3	5	15	15
85	55	...	1	...	8	9	10	10
86	46	1	1	1	1
...	...	114	195	26	45	380	7,275	7,262

tinued.

Patriotic (Russian War) Fund.

No. of Entrants at age 38 . 92

" " " 39 . 62

" " " 40 . 110

" " " 41 . 59

" " " 42 . 57

Total of Group . 380

K" OF			RATES OF					Year of Widow- hood
Forfeiture (f)	Marriage E (m)	Death E (d)	Total Decrement q (dec.)	Marriage and Death q (m + d)	Forfeiture q (f)	Marriage q (m)	Death q (d)	
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
72·5	376·	370·5	·0553	·0398	·0161	·0346	·0054	0
48·5	357·	347·5	·0669	·0587	·0086	·0560	·0029	1
22·5	332·5	325·	·0746	·0746	...	·0602	·0154	2
05·	307·5	306·5	·0355	·0323	·0033	·0195	·0131	3
02·5	298·	293·5	·0435	·0435	...	·0369	·0068	4
78·	283·	278·	·0664	·0562	·0108	·0459	·0108	5
62·	265·5	262·5	·0412	·0375	·0038	·0301	·0076	6
50·5	253·5	251·	·0508	·0431	·0080	·0316	·0120	7
38·5	239·	240·5	·0453	·0372	·0084	·0126	·0249	8
29·5	230·5	231·	·0216	·0216	...	·0387	·0130	9
23·5	224·5	225·	·0352	·0309	·0045	·0134	·0178	10
17·	216·5	218·5	·0228	·0183	·0046	...	·0183	11
10·5	211·	213·5	·0327	·0327	...	·0047	·0281	12
04·5	205·	206·5	·0242	·0242	...	·0049	·0194	13
01·	201·5	201·5	·0099	·0099	...	·0050	·0050	14
98·	198·	199·	·0250	·0201	·0051	·0051	·0151	15
94·	193·5	194·5	·0154	·0103	·0052	...	·0103	16
90·	190·	192·	·0208	·0208	·0208	17
33·	184·	187·	·0532	·0532	...	·0109	·0428	18
77·	176·5	177·5	·0169	·0113	·0056	...	·0113	19
71·	171·5	174·5	·0457	·0457	...	·0058	·0401	20
65·	165·	167·	·0240	·0240	·0240	21
61·	160·5	162·5	·0307	·0246	·0062	...	·0246	22
56·	156·	158·	·0253	·0253	·0253	23
51·	150·5	153·5	·0455	·0391	·0066	...	·0391	24
44·	144·	146·	·0274	·0274	·0274	25
37·	137·	142·	·0704	·0704	·0704	26
35·5	125·5	129·	·0543	·0543	·0543	27
8·5	118·	118·5	·0168	·0084	·0084	...	·0084	28
0·	110·	113·	·0531	·0531	·0531	29
0·5	100·5	104·	·0673	·0673	·0673	30
4·	94·	95·	·0211	·0211	·0211	31
5·5	85·5	90·	·1000	·1000	·1000	32
5·	75·	79·	·1013	·1013	·1013	33
5·	65·	70·	·1429	·1429	·1429	34
6·5	56·5	58·	·0517	·0517	·0517	35
0·5	50·5	53·	·0943	·0943	·0943	36
6·	46·	47·	·0426	·0426	·0426	37
3·5	43·5	45·	·0667	·0667	·0667	38
9·5	39·5	42·	·1190	·1190	·1190	39
2·5	32·5	36·	·1944	·1944	·1944	40
6·5	26·5	27·	·0370	·0370	·0370	41
4·	24·	26·	·1538	·1538	·1538	42
0·	20·	22·	·1818	·1818	·1818	43
4·	14·	15·	·1333	·1333	·1333	44
9·5	9·5	10·	·1000	·1000	·1000	45
1·	1·	1·	46
10·5	7,164·5	7,205·

TABLE II
Widows' Experience

Ages last birthday at Widowhood, 43-47.

Average age of Entrants, 44·8

Age attained	Year of Widow- hood	Married (m)	Died (d)	Forfeited (f)	Existing (ex.)	Total Decre- ment (dec.)	" EXPOSED "	
							Total Decrement E (dec.)	Married and Dec E (m+d)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
45½	0	4	4	3	...	11	242	240
46	1	11	4	1	...	16	231	230
47	2	7	1	1	...	9	215	214
48	3	4	2	1	...	7	206	205
49	4	7	7	1	...	15	199	198
50	5	3	2	1	...	6	184	183
51	6	6	1	7	178	178
52	7	4	4	8	171	171
53	8	1	4	1	...	6	163	162
54	9	1	5	6	157	157
55	10	...	3	1	...	4	151	150
56	11	1	4	5	147	147
57	12	1	3	4	142	142
58	13	...	2	2	138	138
59	14	...	4	4	136	136
60	15	...	6	6	132	132
61	16	...	4	4	126	126
62	17	...	5	1	1	7	122	121
63	18	...	1	1	115	115
64	19	...	3	...	1	4	114	114
65	20	...	6	1	2	9	110	109
66	21	1	6	...	3	10	101	101
67	22	...	6	...	3	9	91	91
68	23	...	6	6	82	82
69	24	...	3	1	3	7	76	75
70	25	1	4	5	69	68
71	26	...	5	...	4	9	64	64
72	27	...	3	...	3	6	55	55
73	28	...	4	...	4	8	49	49
74	29	...	1	...	1	2	41	41
75	30	...	4	...	1	5	39	39
76	31	34	34
77	32	...	5	...	1	6	34	34
78	33	...	6	6	28	28
79	34	...	3	3	22	22
80	35	...	1	1	19	19
81	36	...	2	2	18	18
82	37	...	4	4	16	16
83	38	...	3	3	12	12
84	39	...	1	1	9	9
85	40	...	2	2	8	8
86	41	6	6
87	42	...	2	2	6	6
88	43	1	1	4	4
89	44	...	1	...	1	2	3	3
90	45	...	1	1	1	1
91	46
...	...	52	148	13	29	242	4,266	4,266

ferred.

riotic (Russian War) Fund.

No. of Entrants at age 43	57
" " " 44	46
" " " 45	63
" " " 46	34
" " " 47	42
Total of Group	242

OF	RATES OF							Year of Widow-hood
	Marriage E (m)	Death E (d)	Total Decrement q (dec.)	Marriage and Death q (m + d)	Forfeiture q (f)	Marriage q (m)	Death q (d)	
	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
	238.5	238.5	.0455	.0333	.0126	.0168	.0168	0
5	228.5	225.	.0693	.0651	.0045	.0481	.0178	1
	214.	211.	.0419	.0373	.0047	.0327	.0047	2
	204.5	203.5	.0340	.0292	.0049	.0196	.0098	3
	195.	195.	.0754	.0705	.0052	.0359	.0359	4
5	182.5	182.	.0326	.0272	.0055	.0164	.0110	5
5	177.5	175.	.0393	.03930338	.0057	6
	169.	169.	.0468	.04680237	.0237	7
5	160.5	162.	.0368	.0308	.0062	.0062	.0247	8
	154.5	156.5	.0382	.03820065	.0319	9
5	149.	150.5	.0265	.0199	.00670199	10
5	145.	146.5	.0340	.03400069	.0273	11
	140.5	141.5	.0282	.02820071	.0212	12
	137.	138.	.0145	.01450145	13
	134.	136.	.0294	.02940294	14
	129.	132.	.0455	.04550455	15
	124.	126.	.0317	.03170317	16
5	119.	121.5	.0492	.0412	.00840412	17
5	114.5	115.	.0087	.00870087	18
5	112.5	114.	.0263	.02630263	19
	106.5	109.5	.0636	.0548	.00930548	20
5	98.	100.5	.0693	.06930102	.0597	21
	88.	91.	.0659	.06590659	22
	79.	82.	.0732	.07320732	23
	74.	75.5	.0526	.0397	.01340397	24
6	67.	68.5	.0725	.07250149	.0584	25
1	61.5	64.	.0781	.07810781	26
3	53.5	55.	.0545	.05450545	27
0	47.	49.	.0816	.08160816	28
7	40.5	41.	.0244	.02440244	29
4	37.	39.	.1282	.12821026	30
1	34.	34.	31
5	31.5	34.	.1471	.14711471	32
5	25.	28.	.2143	.21432143	33
0	20.5	22.	.1364	.13641364	34
8	18.5	19.	.0526	.05260526	35
7	17.	18.	.1111	.11111111	36
4	14.	16.	.2500	.25002500	37
0	10.5	12.	.2500	.25002500	38
3	8.5	9.	.1111	.11111111	39
7	7.	8.	.2500	.25002500	40
6	6.	6.	41
5	5.	6.	.3333	.33333333	42
4	4.	4.	43
2	2.5	3.	.3333	.33333333	44
5	.5	1.	1.0000	1.0000	1.0000	45
	46
6	4,185.5	4,233.5

TABLE
Widows' Experience

Ages last birthday at Widowhood, 48-52.

Average age of Entrants, 50.

Age attained	Year of Widow- hood	Married (<i>m</i>)	Died (<i>d</i>)	Forfeited (<i>f</i>)	Existing (<i>ex</i>)	Total Decre- ment (<i>dec.</i>)	"EXPOSED"	
							Total Decrement E (<i>dec.</i>)	Married and I E (<i>m</i>)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
50½	0	1	2	2	...	5	171	17
51	1	3	5	8	166	16
52	2	4	2	1	...	7	158	15
53	3	2	2	4	151	15
54	4	...	2	2	147	14
55	5	3	1	4	145	14
56	6	2	3	5	141	14
57	7	1	4	5	136	13
58	8	2	2	1	...	5	131	13
59	9	1	1	1	...	3	126	13
60	10	1	2	3	123	11
61	11	1	3	4	120	11
62	12	1	1	116	11
63	13	...	3	3	115	11
64	14	...	8	...	1	9	112	11
65	15	...	3	...	1	4	103	11
66	16	...	5	...	3	8	99	
67	17	...	5	...	4	9	91	
68	18	...	5	...	4	9	82	
69	19	...	4	...	5	9	73	
70	20	...	1	...	9	10	64	
71	21	...	5	...	4	9	54	
72	22	...	2	...	3	5	45	
73	23	...	2	...	2	4	40	
74	24	...	3	...	3	6	36	
75	25	...	2	...	6	8	30	
76	26	...	2	...	1	3	22	
77	27	...	6	...	2	8	19	
78	28	...	3	3	11	
79	29	8	
80	30	...	1	1	8	
81	31	...	1	...	1	2	7	
82	32	...	1	1	5	
83	33	...	1	...	1	2	4	
84	34	2	
85	35	2	
86	36	2	
87	37	2	
88	38	2	
89	39	2	
90	40	2	
91	41	...	1	1	2	
92	42	1	
93	43	1	
94	44	1	1	1	
...	...	21	93	5	52	171	2,878	75

ued.

otic (Russian War) Fund.

No. of Entrants at age 48	.	37
" " " 49	.	25
" " " 50	.	47
" " " 51	.	32
" " " 52	.	30
Total of Group	.	<u>171</u>

RATES OF							Year of
Marriage E (m)	Death E (d)	Total Decrement q (dec.)	Marriage and Death q (m+d)	Forfeiture q (f)	Marriage q (m)	Death q (d)	Widow- hood.
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
169.	169.5	.0292	.0176	.0118	.0059	.0118	0
163.5	164.5	.0482	.04820183	.0304	1
156.5	155.5	.0443	.0381	.0065	.0256	.0129	2
150.	150.	.0265	.02650133	.0133	3
146.	147.	.0136	.01360136	4
144.5	143.5	.0276	.02760208	.007	5
139.5	140.	.0355	.03550143	.0214	6
134.	135.5	.0368	.03680075	.0295	7
129.5	129.5	.0382	.0397	.0078	.0154	.0154	8
125.	125.	.0238	.0159	.008	.008	.008	9
122.	122.5	.0244	.02440082	.0163	10
118.5	119.5	.0333	.03330084	.0251	11
116.	116.	12
113.5	115.	.0261	.02610261	13
103.	112.	.0714	.07140714	14
101.5	103.	.0291	.02910291	15
96.5	99.	.0505	.05050505	16
88.5	91.	.0549	.05490549	17
79.5	82.	.0610	.0610061	18
71.	73.	.0548	.05480548	19
63.5	64.	.0156	.01560156	20
51.5	54.	.0926	.09260926	21
44.	45.	.0444	.04440444	22
39.	40.	.05	.0505	23
34.5	36.	.0833	.08330833	24
29.	30.	.0667	.06670667	25
21.	22.	.0909	.09090909	26
16.	19.	.3158	.31583158	27
9.5	11.	.2727	.27272727	28
8.	8.	29
7.5	8.	.125	.125125	30
6.5	7.	.1429	.14291429	31
4.5	5.	.2	.22	32
3.5	4.	.25	.2525	33
2.	2.	34
2.	2.	35
2.	2.	36
2.	2.	37
2.	2.	38
2.	2.	39
2.	2.	40
1.5	2.	.5	.55	41
1.	1.	42
1.	1.	43
1.	1.	44
2,829.	2,865.

TABLE
Widows' Experiences

Ages last birthday at Widowhood, 53-57.

Average age of Entrants, 55.

Age attained	Year of Widow- hood	Married (<i>m</i>)	Died (<i>d</i>)	Forfeited (<i>f</i>)	Existing (<i>e.e.</i>)	Total Decre- ment (<i>dec.</i>)	" EXPOSURE "	
							Total Decrement E (<i>dec.</i>)	Mar- ried and E (<i>e.</i>)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
55½	0	1	1	136	1
56	1	...	2	2	135	1
57	2	1	1	1	...	3	133	1
58	3	...	5	5	130	1
59	4	1	4	5	125	1
60	5	1	4	...	1	6	120	1
61	6	1	4	5	114	1
62	7	1	2	1	1	5	109	1
63	8	...	4	...	1	5	104	1
64	9	...	3	...	5	8	99	
65	10	...	6	1	1	8	91	
66	11	...	4	...	1	5	83	
67	12	...	2	...	4	6	78	
68	13	...	3	...	6	9	72	
69	14	...	2	...	2	4	63	
70	15	...	4	...	4	8	59	
71	16	...	4	...	4	8	51	
72	17	...	1	...	2	3	43	
73	18	...	2	...	3	5	40	
74	19	...	2	...	4	6	35	
75	20	...	3	...	5	8	29	
76	21	...	3	3	21	
77	22	...	4	...	2	6	18	
78	23	...	1	...	3	4	12	
79	24	1	1	8	
80	25	...	1	...	1	2	7	
81	26	5	
82	27	5	
83	28	1	1	5	
84	29	...	1	1	4	
85	30	...	1	1	3	
86	31	2	
87	32	2	
88	33	2	
89	34	2	
90	35	...	2	2	2	
...	...	6	75	3	52	136	1,947	45

ued.

otic (Russian War) Fund.

No. of Entrants at age 53	.	28
" " " 54	.	29
" " " 55	.	30
" " " 56	.	24
" " " 57	.	25
Total of Group	.	136

			RATES OF					Year of Widow- hood.
Marriage E (m)	Death E (d)	Total Decrement q (dec.)	Marriage and Death q (m + d)	Forfeiture q (f)	Marriage q (m)	Death q (d)		
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	
136·	135·5	·0074	·0074	...	·0074	...	0	
134·	135·	·0148	·0148	·0148	1	
132·	132·	·0226	·0151	·0076	·0076	·0076	2	
127·5	130·	·0385	·0385	·0385	3	
123·	124·5	·0400	·0400	...	·0081	·0321	4	
118·	119·5	·0417	·0417	...	·0085	·0335	5	
112·	113·5	·0439	·0439	...	·0089	·0352	6	
107·5	108·	·0367	·0276	·0093	·0093	·0185	7	
102·	104·	·0385	·0385	·0385	8	
97·5	99·	·0303	·0303	·0303	9	
87·5	90·5	·0769	·0663	·0113	...	·0663	10	
81·	83·	·0482	·0482	·0482	11	
77·	78·	·0256	·0256	·0256	12	
70·5	72·	·0417	·0417	·0417	13	
62·	63·	·0317	·0317	·0317	14	
57·	59·	·0678	·0678	·0678	15	
49·	51·	·0784	·0784	·0784	16	
42·5	43·	·0233	·0233	·0233	17	
39·	40·	·0500	·0500	·0500	18	
34·	35·	·0571	·0571	·0571	19	
27·5	29·	·1034	·1034	·1034	20	
19·5	21·	·1429	·1429	·1429	21	
16·	18·	·2222	·2222	·2222	22	
11·5	12·	·0833	·0833	·0833	23	
8·	8·	24	
6·5	7·	·1429	·1429	·1429	25	
5·	5·	26	
5·	5·	27	
5·	5·	28	
3·5	4·	·2500	·2500	·2500	29	
2·5	3·	·3333	·3333	·3333	30	
2·	2·	31	
2·	2·	32	
2·	2·	33	
2·	2·	34	
1·	2·	...	1·0000	1·0000	35	
1,908·	1,942·5	

Age last birthday at Widowhood, 58-62.

Average age of Entrants, 60.

Age attained	Year of Widow- hood	Married (<i>m</i>)	Died (<i>d</i>)	Forfeited (<i>f</i>)	Existing (<i>ex.</i>)	Total Decre- ment (<i>dec.</i>)	"EXPOS"	
							Total Decrement E (<i>dec.</i>)	Ma and E
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
60½	0	1	1	146	1
61	1	2	4	3	1	10	145	1
62	2	...	3	...	1	4	135	1
63	3	...	2	...	2	4	131	1
64	4	...	3	...	2	5	127	1
65	5	...	5	...	3	8	122	1
66	6	...	3	...	5	8	114	1
67	7	...	6	1	5	12	106	1
68	8	...	2	...	3	5	94	1
69	9	...	9	...	6	15	89	1
70	10	...	2	...	7	9	74	1
71	11	...	7	...	5	12	65	1
72	12	...	1	...	8	9	53	1
73	13	...	3	...	6	9	44	1
74	14	...	3	...	5	8	35	1
75	15	...	1	...	1	2	27	1
76	16	...	1	...	5	6	25	1
77	17	...	2	...	3	5	19	1
78	18	...	1	...	1	2	14	1
79	19	...	1	...	1	2	12	1
80	20	...	2	2	10	1
81	21	8	1
82	22	...	1	...	1	2	8	1
83	23	1	1	6	1
84	24	1	1	5	1
85	25	4	1
86	26	4	1
87	27	4	1
88	28	4	1
89	29	...	1	...	1	2	4	1
90	30	...	1	1	2	1
91	31	...	1	1	1	1
...	...	2	65	4	75	146	1,637	...

*inued.**patriotic (Russian War) Fund.*

No. of Entrants at age 58 .	31
„ „ „ 59 .	30
„ „ „ 60 .	22
„ „ „ 61 .	36
„ „ „ 62 .	27
Total of Group .	<u>146</u>

" OF			RATES OF					Year of Widow- hood
ture <i>f</i>)	Marriage E (<i>m</i>)	Death E (<i>d</i>)	Total Decrement <i>q</i> (<i>dcc.</i>)	Marriage and Death <i>q</i> (<i>m+d</i>)	Forfeiture <i>q</i> (<i>f</i>)	Marriage <i>q</i> (<i>m</i>)	Death <i>q</i> (<i>d</i>)	
(0)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0	146	146	0
1	141.5	142.5	.0621	.0418	.0211	.0111	.0281	1
2	133.5	135	.0222	.02220222	2
3	130	131	.0153	.01530153	3
4	125.5	127	.0236	.02360236	4
5	119.5	122	.0410	.04100410	5
6	112.5	114	.0263	.02630263	6
7	102.5	105.5	.0660	.0569	.00970569	7
8	93	94	.0213	.02130213	8
9	84.5	89	.1011	.10111010	9
10	73	74	.0270	.02700270	10
11	61.5	65	.1077	.10771077	11
12	52.5	53	.0189	.01890189	12
13	42.5	44	.0682	.06820682	13
14	33.5	35	.0857	.08570857	14
15	26.5	27	.0370	.03700370	15
16	24.5	25	.0400	.04000400	16
17	18	19	.1053	.10531053	17
18	13.5	14	.0714	.07140714	18
19	11.5	12	.0833	.08330833	19
20	9	10	.2000	.20002000	20
21	8	8	21
22	7.5	8	.1250	.12501250	22
23	6	6	23
24	5	5	24
25	4	4	25
26	4	4	26
27	4	4	27
28	4	4	28
29	3.5	4	.2500	.25002500	29
30	1.5	2	.5000	.50005000	30
31	.5	1	1	1	1	31
05	1,602.5	1,634

TABLE II—continued.

Widows' Experience. Patriotic (Russian War) Fund.

	No. of Entrants at age 63 .	26
	" " " 64 .	23
Ages last birthday at Widowhood, 63-67.	" " " 65 .	29
	" " " 66 .	13
Average age of Entrants, 64·8.	" " " 67 .	20
	Total of Group .	<u>111</u>

Age attained	Year of Widowhood	Died (d)	Existing (ex.)	Total Decrement (dec.)	" EXPOSED TO RISK " OF		Rates of Death q (d)
					Marriage E (m)	Death E (d)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
65½	0	2	6	8	110·	111	·0180
66	1	4	4	8	101·	103	·0388
67	2	10	3	13	90·	95	·1053
68	3	6	7	13	79·	82	·0732
69	4	2	3	5	68·	69	·0290
70	5	3	4	7	62·5	64	·0469
71	6	1	10	11	56·5	57	·0175
72	7	5	7	12	43·5	46	·1087
73	8	1	2	3	33·5	34	·0294
74	9	2	8	10	30·	31	·0645
75	10	2	3	5	20·	21	·0952
76	11	...	1	1	16·	16	...
77	12	1	4	5	14·5	15	·0667
78	13	10·	10	...
79	14	...	2	2	10·	10	...
80	15	4	2	6	6·	8	·5000
81	16	...	1	1	2·	2	...
82	17	1·	1	...
83	18	1·	1	...
84	19	1·	1	...
85	20	1·	1	...
86	21	1·	1	...
87	22	1·	1	...
88	23	1·	1	...
89	24	1·	1	...
90	25	1·	1	...
91	26	1	...	1	·5	1	1·0000
...	...	44	67	111	762·	784	...

TABLE II—*continued.**Widows' Experience. Patriotic (Russian War) Fund.*

	No. of Entrants at age	68	. 19
	"	"	69 . 14
Ages last birthday at Widowhood, 68-72.	"	"	70 . 12
	"	"	71 . 6
Average age of Entrants, 69.	"	"	72 . 4
	Total of Group		<u>55</u>

Age attained	Year of Widowhood	Died (d)	Forfeited (f)	Existing (ex.)	Total Decrement (dec.)	"EXPOSED TO RISK" OF			RATES OF		
						Total Decrement E (dec.)	Forfeiture E (f)	Death E (d)	Total Decrement q (dec.)	Forfeiture q (f)	Death q (d)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
69½	0	3	...	7	10	55	53·5	55·	·0545	...	·0545
70	1	3	1	3	7	45	43·5	44·5	·0889	·0230	·0674
71	2	4	...	5	9	38	36·	38·	·1053	...	·1053
72	3	6	6	29	29·	29·
73	4	2	...	3	5	23	22·	23·	·0870	...	·0870
74	5	2	2	18	18·	18·
75	6	1	...	4	5	16	15·5	16·	·0625	...	·0625
76	7	1	1	11	11·	11·
77	8	10	10·	10·
78	9	1	...	3	4	10	9·5	10·	·1000	...	·1000
79	10	3	3	6	6·	6·
80	11	1	1	3	2·5	3·	·3333	...	·3333
81	12	1	1	2	2·	2·
82	13	1	1·	1·
83	14	1	1·	1·
84	15	1	1·	1·
85	16	1	1	1	·5	1·	1·0000	...	1·0000
...	...	16	1	38	55	270	262·	269·5

TABLE II—continued.

Widows' Experience. Patriotic (Russian War) Fund.

	No. of Entrants at age 73 .	5
	" " " 74 .	7
Ages last birthday at Widowhood, 73-77.	" " " 75 .	4
	" " " 76 .	4
Average age of Entrants, 74·7.	" " " 77 .	3
	Total of Group .	23

Age attained.	Year of Widowhood.	Died (d)	Existing (ex.)	Total Decrement (dec.)	" EXPOSED TO RISK " OF		Rates of Death q (d)
					Total Decrement E (dec.)	Marriage E (m)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
75½	0	1	4	5	23	22·5	·0435
76	1	1	3	4	18	17·5	·0556
77	2	1	2	3	14	13·5	·0714
78	3	...	1	1	11	11·	...
79	4	1	2	3	10	9·5	·1
80	5	...	1	1	7	7·	...
81	6	6	6·	...
82	7	1	2	3	6	5·5	·1667
83	8	1	...	1	3	2·5	·3333
84	9	1	...	1	2	1·5	·5
85	10	1	1·	...
86	11	...	1	1	1	1·	...
...	...	7	16	23	102	98·5	...

	No. of Entrants at age 78 .	4
	" " " 79 .	1
Ages last birthday at Widowhood, 78-82.	" " " 80 .	1
	" " " 81 .	1
Average age of Entrants, 79·2.	" " " 82 .	1
	Total of Group .	8

Age attained	Year of Widowhood	Died (d)	Existing (f)	Total Decrement (dec.)	" Exposed to Risk " of Total Decrement E (dec.)	Rates of Death q (d)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
79½	0	...	1	1	8	...
80	1	1	...	1	7	·1429
81	2	2	...	2	6	·3333
82	3	1	...	1	4	·25
83	4	3	...
84	5	2	1	3	3	·6667
...	...	6	2	8	31	...

TABLE III.

*Patriotic (Russian War) Fund.**Numbers entering and leaving Fund during the period 1854-1900.*

x	Entered	FORFEITED			RE-IN-STATED			Re-married	Re-married women again Widowed	DIED			EXISTING		
		Married	Widows	Total	Married	Widows	Total			Married	Widows	Total	Married	Widows	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
17	1
18	10	2	1	1
19	17
20	56	...	4	4	8	1
21	101	1	7	8	20	2	2
22	158	1	15	16	27	1	1
23	196	...	6	6	49	2	...	5	5
24	249	...	25	25	1	1	2	64	1	1	5	6
25	264	3	24	27	101	3	3	4	7
26	234	3	31	34	2	...	2	94	4	2	4	6
27	228	2	14	16	107	7	...	7	7
28	245	4	23	27	1	...	1	124	8	4	6	10
29	204	4	24	28	1	...	1	108	9	4	11	15
30	250	3	24	27	...	1	1	112	17	4	9	13
31	133	4	19	23	121	10	6	7	13
32	147	8	14	22	1	2	3	126	8	9	13	22
33	137	7	11	18	2	2	4	107	16	8	12	20
34	124	6	18	24	4	2	6	117	18	11	10	21
35	126	8	21	29	1	1	2	80	17	9	8	17
36	137	3	21	24	2	4	6	86	22	15	14	29
37	94	3	19	22	1	2	3	79	20	13	14	27
38	89	4	10	14	1	5	6	94	18	12	15	27
39	65	7	10	17	1	2	3	54	15	17	17	34
40	121	6	14	20	1	3	4	59	29	16	20	36
41	33	5	20	25	1	1	2	53	35	31	7	38
42	57	11	9	20	1	4	5	52	23	21	20	41
43	46	7	12	19	1	1	2	29	30	23	19	42
44	33	2	11	13	1	4	5	49	23	16	14	30
45	48	3	13	16	...	5	5	28	28	17	26	43
46	26	1	11	12	...	1	1	29	20	18	21	39
47	29	7	6	13	1	1	2	17	20	19	16	35
48	23	7	9	16	21	23	19	27	46
49	16	5	8	13	2	...	2	20	19	16	24	40
50	23	3	5	8	...	1	1	12	36	9	24	33
51	14	5	8	13	1	...	1	13	28	22	20	42
52	16	6	5	11	6	21	23	31	54
53	8	2	1	3	1	...	1	11	23	19	22	41
54	11	3	1	4	...	1	1	7	24	15	31	46
55	15	2	4	6	6	22	22	23	45
Carried forward	3,784	146	477	623	28	44	72	2,092	600	424	510	934

TABLE III—continued.

æ	Entered	FORFEITED			RE-INSTATED			Re-married	Re-married women again Widowed	DIED			EXISTING		
		Married	Widows	Total	Married	Widows	Total			Married	Widows	Total	Married	Widows	Total
Brought forward	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	3,784	146	477	623	28	44	72	2,092	600	424	510	934
56	8	1	7	8	3	22	21	31	52
57	3	2	4	6	1	1	2	4	24	19	32	51
58	5	2	2	4	7	31	18	47	65
59	6	2	2	4	4	30	22	32	54
60	11	2	4	6	...	1	1	3	17	25	34	59
61	11	1	2	3	...	1	1	4	33	21	47	68	1	5	6
62	3	2	3	5	4	27	21	41	62	3	4	7
63	8	...	2	2	2	26	27	53	80	4	8	12
64	4	...	2	2	1	...	1	...	22	13	49	62	7	12	19
65	4	...	3	3	29	21	48	69	10	23	33
66	2	1	4	5	1	13	19	64	83	13	32	45
67	4	...	1	1	20	20	62	82	21	51	72
68	2	...	3	3	1	19	17	46	63	30	50	80
69	1	1	14	12	41	53	19	43	62
70	4	...	1	1	1	...	1	...	11	7	66	73	24	57	81
71	4	6	8	45	53	27	44	71
72	2	4	8	45	53	14	39	53
73	1	...	1	1	5	8	50	58	13	48	61
74	7	7	38	45	12	53	65
75	2	4	10	32	42	8	35	43
76	1	1	4	...	35	35	6	25	31
77	3	4	31	35	8	31	39
78	1	4	2	23	25	3	18	21
79	1	1	1	1	12	13	3	19	22
80	1	4	13	17	4	16	20
81	1	15	15	...	23	23
82	1	12	13	3	12	15
83	1	1	10	11	...	9	9
84	2	10	12	...	3	3
85	6	6	1	3	4
86	1	1	3	4	...	4	4
87	1	1	...	2	2
88	2	2
89	3	3	...	2	2
90	6	6	...	1	1
91	1	1
92
93
94	1	...	1	...	1	1
95
	3,870	159	521	680	31	47	78	2,125	979	765	1,596	2,361	234	673	907

AGES 18-22										AGES 23-27						
Year of Widowhood	Exposed to Risk	Re-Marrriages (m)	Rate of Re-marriage Un-graduated	Rate of Re-marriage Graduated	Expected Re-marrriages (m')	Difference (m'-m)	Σ Difference	Year of Widowhood	Exposed to Risk	Re-Marrriages (m)	Rate of Re-marriage Un-graduated	Rate of Re-marriage Graduated	Expected Re-marrriages (m')	Difference (m'-m)	Σ Difference	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
0	324.5	(38)19	.1170	.1125	36.5	-1.5	-1.5	0	1078.5	(88)44	.0816	.0835	90.1	+2.1	+2.1	
1	290.	38	.1310	.1340	38.9	+9	-6	1	1000.5	119	.1189	.1208	120.9	+1.9	+4.	
2	242.5	42	.1732	.1700	41.2	-8	-1.4	2	853.5	152	.1781	.1730	147.7	-4.3	-3	
3	194.5	22	.1131	.1200	23.3	+13	-1	3	679.	90	.1325	.1325	90.	...	-3	
4	168.	23	.1369	.1063	17.9	-5.1	-5.2	4	575.	57	.0991	.1061	61.	+4.	+3.7	
5	112.	8	.0563	.1005	14.3	+6.3	+1.1	5	510.	53	.1039	.0926	47.2	-5.8	-2.1	
6	133.	17	.1278	.0963	12.8	-4.2	-3.1	6	449.	36	.0802	.0862	38.7	+2.7	+6	
7	114.5	8	.0699	.0927	10.6	+2.6	-5	7	405.5	35	.0863	.0824	33.4	-1.6	-1.	
8	104.5	10	.0957	.0880	9.2	-8	-1.3	8	364.	22	.0604	.0781	28.4	+6.4	+5.4	
9	93.5	5	.0535	.0806	7.5	+2.5	+1.2	9	335.5	20	.0596	.0725	24.3	+4.3	+9.7	
10	86.5	8	.0925	.0718	6.2	-1.8	-4	10	308.	32	.1039	.0618	20.	-12.	-2.3	
11	77.	5	.0619	.0630	4.9	-1	-7	11	268.	17	.0634	.0572	15.3	-1.7	-4.	
12	70.	3	.0429	.0550	3.9	+9	+2	12	242.5	14	.0577	.0500	12.	-2.	-6.	
13	64.	2	.0313	.0478	3.0	+1	+1.2	13	220.	5	.0227	.0429	9.4	+4.4	-1.6	
14	60.5	3	.0496	.0417	2.5	-5	+7	14	206.5	6	.0291	.0353	7.3	+1.3	-3	
15	56.	1	.0179	.0369	2.1	+1.1	+8	15	194.5	7	.0360	.0282	5.5	-1.5	-18	
16	53.	3	.0566	.0328	1.7	-1.3	+5	16	183.5	4	.0218	.0231	4.2	+2	-1.6	
17	48.5	1	.0206	.0292	1.4	+4	+9	17	172.	1	.0058	.0196	3.4	+2.1	+8	
18	46.5	2	.0130	.0262	1.3	-7	+2	18	161.5	3	.0182	.0162	2.7	-3	+5	
19	43.0235	1.0	+1	+1.2	19	157.5	2	.0127	.0133	2.1	+1	+6	
20	41.5	1	.0241	.0209	.9	-1	+1.1	20	152.	1	.0066	.0111	1.7	+7	+1.3	
21	40.0186	.7	+7	+1.8	21	147.5	3	.0263	.0097	1.4	-1.6	-3	
22	39.50164	.6	+6	+2.4	22	138.50079	1.1	+1.1	+8	
23	39.	1	.0256	.0143	.6	-4	+2	23	134.0070	.9	+9	+1.7	
24	37.5	1	.0267	.0125	.5	-5	+1.5	24	132.0056	.7	+7	+2.4	
25	36.0109	.4	+4	+1.9	25	127.5	2	.0157	.0018	.6	-1.4	+1.	
26	36.	1	.0278	.0095	.3	-7	+1.2	26	119.5	2	.0167	.0039	.5	-1.5	-5	
27	34.0082	.3	+3	+1.5	27	114.0036	.4	+4	-1	
28	33.	1	.0303	.0070	.2	-8	+7	28	111.0032	.4	+4	+3	
29	31.50060	.2	+2	+9	29	108.0029	.3	+3	+6	
30	30.5	1	.0328	.0050	.2	-8	+1	30	105.50026	.3	+3	+9	
...	...	245	245.1	+1	771	771.9	+9	...	

TABLE IV—continued.

AGES 28-32								AGES 33-37							
Year of Widow- hood.	Exposed to Risk	Re- Marriages (m)	Rate of Re- Marriage Un- graduated	Rate of Re- Marriage Graduated	Expected Re- Marriages (m')	Difference (m'-m)	Σ Difference	Year of Widow- hood.	Exposed to Risk	Re- Marriages (m)	Rate of Re- Marriage Un- graduated	Rate of Re- Marriage Graduated	Expected Re- Marriages (m')	Difference (m'-m)	Σ Difference
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	890	27	.0606	.0662	58.9	+4.9	+4.9	0	562.5	17	.0604	.0552	31.1	-2.9	-2.9
1	842.5	83	.0985	.0933	78.6	-4.4	+5	1	534.5	36	.0674	.0725	38.8	+2.8	-1
2	744.5	88	.1182	.1168	87	-1.0	-7	2	488.5	48	.0983	.0962	47.0	-1.0	-1.1
3	645	57	.0884	.0935	60.3	+3.3	+2.6	3	430.5	21	.0488	.0574	24.7	+3.7	+2.6
4	581	47	.0809	.0807	46.9	-1	+2.5	4	402	16	.0398	.0445	17.9	+1.9	+4.5
5	527	43	.0816	.0739	38.9	-4.1	-1.6	5	382	19	.0497	.0384	14.7	-4.3	+2
6	476	36	.0756	.0665	31.7	-4.3	-5.9	6	360	9	.0250	.0340	12.2	+3.2	+3.4
7	433	23	.0531	.0590	25.5	+2.5	-3.4	7	347.5	12	.0345	.0304	10.6	-1.4	+2.0
8	405.5	12	.0296	.0515	20.9	+8.9	+5.5	8	332.5	5	.0150	.0267	8.9	+3.9	+5.9
9	390.5	15	.0384	.0442	17.3	+2.3	+7.8	9	320	8	.0250	.0225	7.2	-8	+5.1
10	369.5	15	.0406	.0379	14	-1.0	+6.8	10	303	7	.0231	.0187	5.7	-1.3	+3.8
11	345.5	9	.0260	.0320	11.1	+2.1	+8.9	11	292	5	.0171	.0150	4.4	-6	+3.2
12	327.5	14	.0428	.0269	8.8	-5.2	+3.7	12	282	6	.0213	.0121	3.4	-2.6	+6
13	303.5	9	.0297	.0223	6.8	-2.2	+1.5	13	269	3	.0112	.0094	2.5	-5	+1
14	285.5	5	.0175	.0177	5.1	+1	+1.6	14	260.5	3	.0115	.0079	2.1	-9	-8
15	274.5	5	.0182	.0140	3.8	-1.2	+4	15	252.5	1	.0040	.0063	1.6	+6	-2
16	262.5	4	.0152	.0112	2.9	-1.1	-7	16	2470050	1.2	+1.2	+1.0
17	252.5	2	.0079	.0086	2.2	+2	-5	17	242	2	.0083	.0042	1.0	-1.0	...
18	2460070	1.7	+1.7	+1.2	18	232.50035	.8	+8	...
19	241.5	3	.0124	.0050	1.2	-1.8	-6	19	224.50030	.7	+7	+1.5
20	236	2	.0085	.0045	1.1	-9	-1.5	20	220.5	2	.0091	.0025	.6	-1.4	+1
21	2290035	.8	+8	-7	21	214.50024	.5	+5	+6
22	2220030	.7	+7	...	22	208.50022	.5	+5	+1.1
23	217	1	.0046	.0024	.5	-5	-5	23	2010021	.4	+4	+1.5
24	2110017	.4	+4	+1	24	1940020	.4	+4	+1.9
25	2070010	.2	+2	+3	25	188	1	.0053	.0019	.4	-6	+1.3
...	26	179.50018	.0018	.3	+3	+1.6
...	27	175	2	.0114	.0017	.3	-1.7	-1
...	...	527	527.3	28	166.50016	.2	+2	+1
...	240	...	240.1

TABLE IV—continued.

AGES 38-42							AGES 43-47								
Year of Widow-hood.	Exposed to Risk	Re-Marrriages (m)	Rate of Re-Marrriage Un-graduated	Rate of Re-Marrriage Graduated	Expected Re-Marrriages (m')	Difference (m' - m)	Σ Difference	Year of Widow-hood.	Exposed to Risk	Re-Marrriages (m)	Rate of Re-Marrriage Un-graduated	Rate of Re-Marrriage Graduated	Expected Re-Marrriages (m')	Difference (m' - m)	Σ Difference
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	376	(26) 13	.0692	.0625	23.5	-2.5	-2.5	0	238.5	4	.0336	.0383	8.7	+ .7	+
1	357	20	.0560	.0633	22.6	+2.6	+ .1	1	228.5	11	.0481	.0470	10.7	- .3	+
2	332.5	20	.0602	.0500	16.6	-3.4	-3.3	2	214	7	.0327	.0349	7.5	+ .5	+
3	307.5	6	.0195	.0409	12.6	+6.6	+3.3	3	201.5	4	.0136	.0275	5.6	+1.6	+2.5
4	298	11	.0369	.0367	10.9	- .1	+3.2	4	195	7	.0359	.0233	4.5	-2.5	...
5	283	13	.0459	.0340	9.6	-3.4	- .2	5	182.5	3	.0164	.0225	4.1	+1.1	+1.1
6	265.5	8	.0301	.0308	8.2	+ .2	...	6	177.5	6	.0338	.0221	3.9	-2.1	-1.1
7	253.5	8	.0316	.0246	6.2	-1.8	-1.8	7	169	4	.0237	.0195	3.3	- .7	-1.7
8	239	3	.0126	.0179	4.3	+1.3	- .5	8	160.5	1	.0062	.0148	2.4	+1.4	- .3
9	230.5	2	.0087	.0125	2.9	+ .9	+ .4	9	154.5	1	.0065	.0100	1.5	+ .5	+ .2
10	224.5	3	.0134	.0090	2.0	-1.0	- .6	10	1490031	.9	+ .9	+1.1
11	216.50072	1.6	+1.6	+1.0	11	145	1	.0039	.0037	.5	- .5	+ .6
12	211	1	.0017	.0062	1.3	+ .3	+1.3	12	140.5	1	.0071	.0028	.4	- .6	...
13	205	1	.0049	.0053	1.1	+ .1	+1.4	13	1370025	.3	+ .3	+ .3
14	201.5	1	.0050	.0014	.9	- .1	+1.3	14	1340023	.3	+ .3	+ .6
15	198	1	.0051	.0036	.7	- .3	+1.0	15	1290022	.3	+ .3	+ .9
16	193.50028	.5	+ .5	+1.5	16	1240020	.2	+ .2	+1.1
17	1900024	.5	+ .5	+2.0	17	1190015	.2	+ .2	+1.3
18	184	2	.0109	.0020	.4	-1.6	+ .4	18	114.50014	.2	+ .2	+1.5
19	176.50016	.3	+ .3	+ .7	19	112.50013	.1	+ .1	+1.6
20	171.5	1	.0058	.0014	.2	- .8	- .1	20	106.50012	.1	- .1	+1.7
21	165	21	98	1	.0102	.0011	.1	- .9	+ .8
22	160.5	22	88.50010	.1	+ .1	+ .9
23	156	23	78.50009	.1	+ .1	+1.0
24	150.5	24	740008	.1	+ .1	+1.1
25	144	25	67	1	.0149	.0007	...	-1.0	+ .1
...	...	127	126.9	56

TABLE V.

*Widows. Patriotic (Russian War) Fund.**Rates of Re-marriage.**Exposed to Risk made equal at each age passed through.*

AGE LAST BIRTHDAY, 20-24 TRUE CENTRAL AGE, 22½			AGE LAST BIRTHDAY, 23-27 TRUE CENTRAL AGE, 25½			AGE LAST BIRTHDAY, 28-32 TRUE CENTRAL AGE, 30½		
Year of Widow- hood.	Un- graduated $q(m)$	Graduated $q(m)$	Year of Widow- hood.	Un- graduated $q(m)$	Graduated $q(m)$	Year of Widow- hood.	Un- graduated $q(m)$	Graduated $q(m)$
0	·1144	·1144	0	·0810	·0810	0	·0608	·0608
1	·1514	·1514	1	·1195	·1195	1	·0996	·0996
2	·1843	·1843	2	·1803	·1803	2	·1168	·1168
3	·1216	·1216	3	·1316	·1316	3	·0880	·0880
4	·1036	·0950	4	·0980	·1033	4	·0784	·0790
5	·0659	·0813	5	·1031	·0925	5	·0757	·0773
6	·0874	·0800	6	·0818	·0855	6	·0742	·0725
7	·0660	·0828	7	·0853	·0800	7	·0508	·0613
8	·0981	·0865	8	·0612	·0760	8	·0262	·0465
9	·0650	·0875	9	·0603	·0720	9	·0403	·0363
10	·1219	·0845	10	·1060	·0668	10	·0383	·0290
11	·0798	·0760	11	·0630	·0619	11	·0269	·0240
12	·0457	·0565	12	·0590	·0537	12	·0397	·0205
13	·0153	·0425	13	·0207	·0462	13	·0307	·0175
14	·0429	·0340	14	·0320	·0377	14	·0165	·0150
15	·0423	·0290	15	·0443	·0304	15	·0160	·0135
16	·0258	·0246	16	·0200	·0237	16	·0153	·0118
17	·0180	·0212	17	·0103	·0188	17	·0076	·0110
18	·0325	·0182	18	·0154	·0150	18	...	·0098
19	...	·0155	19	...	·0125	19	...	·0088
20	...	·0133	20	...	·0101	20	...	·0078
21	...	·0114	21	...	·0085	21	...	·0075
22	...	·0098	22	...	·0070	22	...	·0063
23	...	·0080	23	...	·0052	23	...	·0050
24	...	·0069	24	...	·0047	24	...	·0042
25	...	·0062	25	...	·0037	25	...	·0033

TABLE V—*continued.*

AGE LAST BIRTHDAY, 33-37 TRUE CENTRAL AGE, 35½			AGE LAST BIRTHDAY, 38-42 TRUE CENTRAL AGE, 40½			AGE LAST BIRTHDAY, 43-47 TRUE CENTRAL AGE, 45½		
Year of Widow- hood.	Ungraduated $q(m)$	Graduated $q(m)$	Year of Widow- hood.	Ungraduated $q(m)$	Graduated $q(m)$	Year of Widow- hood.	Ungraduated $q(m)$	Graduated $q(m)$
0	·0652	·0560	0	·0606	·0562	0	·0320	·0320
1	·0670	·0800	1	·0558	·0577	1	·0137	·0437
2	·0091	·0091	2	·0602	·0540	2	·0330	·0327
3	·0455	·0455	3	·0214	·0125	3	·0218	·0290
4	·0381	·0381	4	·0391	·0342	4	·0358	·0290
5	·0496	·0370	5	·0467	·0335	5	·0185	·0280
6	·0249	·0362	6	·0278	·0325	6	·0305	·0248
7	·0341	·0340	7	·0332	·0298	7	·0253	·0203
8	·0148	·0280	8	·0143	·0230	8	·0062	·0147
9	·0232	·0205	9	·0097	·0150	9	·0065	·0085
10	·0232	·0162	10	·0146	·0097	10	...	·0065
11	·0166	·0135	11	...	·0070	11	·0056	·0050
12	·0214	·0125	12	·0063	·0050	12	·0060	·0038
13	·0112	·0112	13	·0067	·0043	13	...	·0025
14	·0120	·0100	14	·0061	·0040	14	...	·0012
15	·0032	·0092	15	·0043	·0037	15	...	·0005
16	...	·0078	16	...	·0034
17	·0085	·0070	17	...	·0030
18	...	·0063	18	...	·0026
19	...	·0053	19	...	·0022
20	...	·0049
21	...	·0034
22	...	·0025
23	...	·0020
24	...	·0010
25

TABLE VI.

*Widows. Patriotic (Russian War) Fund.**Value of £1 per annum to cease at Death, or Re-marriage
(from Ungraduated Rates).*

INTEREST 3 PER-CENT.

Year of Widow- hood.	AGES AT WIDOWHOOD									
	18-22	23-27	28-32	33-37	38-42	43-47	48-52	53-57	58-62	63-67
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	7.138	7.398	9.765	11.865	12.615	12.182	12.875	12.408	12.111	8.101
1	7.557	7.412	9.811	12.105	13.117	12.443	12.745	11.972	11.474	8.366
2	8.024	7.731	10.265	12.499	13.353	12.708	12.792	11.516	11.334	8.244
3	9.043	8.738	11.058	13.463	13.865	12.596	12.698	11.044	10.939	8.480
4	9.619	9.491	11.596	13.824	13.755	12.364	12.435	10.831	10.443	8.443
5	10.627	9.931	12.112	13.980	13.813	12.701	11.985	10.621	10.015	7.944
6	10.599	10.511	12.664	14.234	14.075	12.448	11.694	10.415	9.757	7.588
7	11.516	10.879	13.266	14.207	14.061	12.346	11.488	10.221	9.321	6.955
8	11.869	11.360	13.603	14.292	14.136	12.341	11.286	9.826	9.180	7.038
9	12.657	11.489	13.475	13.990	14.122	12.115	10.992	9.526	8.661	6.466
10	12.774	11.699	13.547	14.207	13.867	11.974	10.505	9.118	8.925	6.111
11	13.674	12.632	13.747	14.232	13.738	11.584	10.091	9.059	8.447	5.966
12	14.061	12.946	13.810	14.017	13.414	11.351	9.752	8.803	8.751	5.144
13	14.809	13.394	14.127	14.019	13.283	11.031	9.044	8.305	8.187	4.671
14	14.746	13.444	14.299	13.879	13.021	10.529	8.566	7.926	8.050	...
15	14.981	13.616	14.150	13.743	12.545	10.174	8.502	7.431	8.068	...
16	14.712	13.701	14.065	13.382	12.186	9.978	8.019	7.211	7.630	...
17	15.379	13.666	14.112	12.896	11.682	9.614	7.700	7.059	7.187	...
18	15.513	13.577	13.828	12.674	11.288	9.328	7.391	6.445	7.274	...
19	15.697	13.243	13.477	12.510	11.280	8.692	7.108	5.988	7.069	...
20	15.549	13.083	13.233	12.235	10.751	8.195	6.745	5.541	6.942	...
21	15.411	12.653	12.863	11.776	10.604	7.930	6.058	5.366	7.938	...
22	14.873	12.673	12.661	11.415	10.190	7.776	5.876	5.448	7.176	...
23	14.319	12.632	12.339	10.928	9.761	7.574	5.334	6.215	7.448	...
24	14.136	12.108	12.006	10.829	9.315	7.418	4.783	5.983	6.671	...
25	13.960	11.661	11.662	10.270	8.985	6.956	4.374	5.163	5.871	...
26	13.379	11.496	11.129	10.156	8.515	6.725	3.827	5.204	5.047	...
27	13.174	11.562	10.748	9.817	8.434	6.514	3.336	4.360	4.198	...
28	13.392	11.120	10.469	9.346	8.186	6.096	4.022	3.491	3.324	...
29	13.224	10.874	9.895	9.160	7.503	5.837	4.697	2.596	2.423	...
30	13.061	10.409	9.409	8.992	7.162	5.162	3.837	2.565	2.328	...
31	13.381	10.030	9.293	8.780	6.909	5.099	3.517	2.962
32	12.782	9.744	9.024	8.238	6.270	4.252	3.227
33	12.166	9.137	8.637	7.810	6.175	4.135	3.155
34	11.978	8.913	8.418	7.430	6.078	4.420	3.332
35	11.794	8.486	8.026	7.258	6.304	4.273
36	11.615	8.373	7.886	6.983	5.847	3.645
37	10.964	8.061	7.375	6.638	5.649	3.224
38	10.293	7.864	7.178	6.208	5.078	3.427
39	10.026	7.947	7.084	5.846	4.604	3.706
40	9.758	7.438	6.509	5.743	4.383	3.294
41	9.964	7.299	6.209	5.438	4.604	3.524
42	9.263	7.353	5.687	5.470	3.925	2.630
43	8.995	6.865	5.613	5.175	3.777	3.062
44	8.728	6.824	5.270	4.775	3.755	2.154
45	...	6.348	5.270	4.376	3.462	2.328
46	...	6.693	5.204	3.958	2.962

TABLE VII.—*Aggregate Marriage-rates among Widows on the Patriotic (Russian War) Fund from 1854-1900.*

Age	TOTAL EXPERIENCE (a)				EXPERIENCE EXCLUDING FIRST TEN YEARS OF WIDOWHOOD (b)				Age
	Exposed to Risk	Marriages	Rate of Re-marriage		Exposed to Risk.	Marriages	Rate of Re-marriage		
			Un-graduated	Graduated			Un-graduated	Graduated	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
17	5	17
18	5.5	2	.3636	.1425	18
19	16.51411	19
20	51.5	8	.1553	.1395	20
21	116	20	.1724	.1379	21
22	213.5	27	.1265	.1362	22
23	352.5	49	.1390	.1338	23
24	509	64	.1257	.1313	24
25	676	101	.1494	.1251	25
26	798.5	94	.1177	.1245	26
27	915	107	.1169	.1197	27
28	1029.5	124	.1204	.1141	1	28
29	1112	108	.0971	.1086	4	29
30	1215.5	112	.0921	.1026	17	1	.0586	.0973	30
31	1283.5	121	.0943	.0962	44	7	.1591	.0917	31
32	1288.5	126	.0978	.0898	77	4	.0519	.0860	32
33	1293	107	.0828	.0835	119	12	.1008	.0800	33
34	1308.5	117	.0894	.0767	169.5	15	.0885	.0732	34
35	1306.5	80	.0612	.0708	217.5	10	.0460	.0662	35
36	1348.5	86	.0638	.0648	257.5	15	.0582	.0593	36
37	1370.5	79	.0576	.0593	301	17	.0565	.0530	37
38	1377.5	94	.0682	.0538	366	22	.0601	.0465	38
39	1354	54	.0399	.0487	409	10	.0244	.0405	39
40	1388	59	.0425	.0434	481	16	.0333	.0346	40
41	1410.5	53	.0376	.0384	504	12	.0238	.0295	41
42	1405	52	.0370	.0340	533.5	15	.0281	.0254	42
43	1402.5	29	.0207	.0298	564	13	.0231	.0217	43
44	1412.5	49	.0347	.0261	601.5	11	.0183	.0185	44
45	1397.5	28	.0200	.0225	639.5	10	.0156	.0157	45
46	1395.5	29	.0208	.0192	682.5	10	.0147	.0132	46
47	1387.5	17	.0123	.0163	707	6	.0085	.0109	47
48	1389	21	.0151	.0138	737	9	.0122	.0090	48
49	1374.5	20	.0146	.0118	748	7	.0094	.0075	49
50	1371.5	12	.0087	.0103	798	3	.0038	.0063	50
51	1382	13	.0094	.0087	811.5	3	.0037	.0052	51
52	1376.5	6	.0044	.0074	822.5	1	.0012	.0044	52
53	1375	11	.0080	.0065	841.5	6	.0071	.0036	53
54	1369.5	7	.0051	.0056	851	2	.0023	.0030	54
55	1369	6	.0044	.0049	859	2	.0023	.0025	55
56	1364.5	3	.0022	.0041	856	2	.0023	.0021	56
57	1354	4	.0030	.0035	860.50018	57
58	1339.5	7	.0052	.0029	860	2	.0023	.0016	58
59	1327	4	.0030	.0023	851	3	.0035	.0013	59
60	1319.5	3	.0023	.0016	856	1	.0012	.0011	60
61	1309.5	4	.0031	.0012	851.5	1	.0012	.0009	61
62	1291	4	.0031	.0010	8400007	62
63	1265.5	2	.0016	.0008	826.5	1	.0012	.0006	63
64	1232.50006	804.50005	64
65	1199	7790004	65
66	1140.5	1	.0009	...	745	1	.0013	.0003	66
67	1061.5	6960002	67
68	977	1	.0010	...	649.5	1	.0015	...	68
69	898	615.5	69
70	815	558.5	70
...	59042	2125	24813.5	251

TABLE VIII.

*Central Marriage Rates of Widows of Scottish Bankers
compared with Widows on Patriotic Fund.*

Age	WIDOWS ON PATRIOTIC FUND		Widows of Scottish Bankers (Hewat)
	All Widows	Excluding first ten years of Widowhood	
(1)	(2)	(3)	(4)
18	·1534
19	·1518
20	·1500
21	·1481	...	·0001
22	·1462	...	·0015
23	·1434	...	·0032
24	·1405	...	·0051
25	·1369	...	·0072
26	·1328	...	·0095
27	·1273	...	·0120
28	·1210	...	·0145
29	·1148	...	·0170
30	·1081	·1023	·0195
31	·1011	·0961	·0220
32	·0940	·0899	·0245
33	·0871	·0833	·0270
34	·0798	·0760	·0270
35	·0734	·0685	·0230
36	·0670	·0613	·0195
37	·0611	·0544	·0165
38	·0553	·0476	·0140
39	·0499	·0413	·0115
40	·0444	·0352	·0090
41	·0392	·0299	·0070
42	·0346	·0257	·0050
43	·0303	·0219	·0035
44	·0264	·0187	·0025
45	·0228	·0158	·0018
46	·0194	·0133	·0014
47	·0164	·0110	·0012
48	·0139	·0090	·0011
49	·0119	·0075	·0010
50	·0104	·0063	·0009
51	·0087	·0052	·0008
52	·0074	·0044	·0007
53	·0065	·0036	·0006
54	·0056	·0030	·0005
55	·0049	·0025	·0004
56	·0041	·0021	·0003
57	·0035	·0018	·0002
58	·0029	·0016	·0002
59	·0023	·0013	·0001
60	·0016	·0011	·0001
61	·0012	·0009	...
62	·0010	·0007	...
63	·0008	·0006	...
64	·0006	·0005	...

TABLE X.

*Widows. Patriotic (Russian War) Fund.**Aggregate Mortality Table, including Experience of Re-married Women.
(Ungraduated.)*

Age at Entry x	$\frac{1}{2}(n_x + r_x - f_x)$	Col. 2 Age $x-1$ + age x	Exist- ing ex_x	Died d_x	Col. 3 - Cols. 4 & 5 Age $x-1$	Exposed to Risk E_x	q_x	p_x	e_x	Age at Entry x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
17	+ .5	+ .5	+ .5	.5	17
18	+ 5.	+ 5.5	...	1	+ 5.5	6.	.1667	.8333	35.83	18
19	+ 8.5	+ 13.5	+ 12.5	18.5	...	1.0000	42.00	19
20	+ 26.	+ 34.5	+ 34.5	53.	...	1.0000	41.00	20
21	+ 46.5	+ 72.5	...	2	+ 72.5	125.5	.0159	.9841	40.00	21
22	+ 71.	+ 117.5	...	1	+ 115.5	241.	.0041	.9959	39.65	22
23	+ 95.	+ 166.	...	5	+ 165.	406.	.0123	.9877	33.81	23
24	+ 113.	+ 208.	...	6	+ 203.	609.	.0099	.9901	38.29	24
25	+ 118.5	+ 231.5	...	7	+ 225.5	834.5	.0084	.9916	37.67	25
26	+ 101.	+ 219.5	...	6	+ 212.5	1,047.	.0057	.9943	36.99	26
27	+ 106.	+ 207.	...	7	+ 201.	1,248.	.0056	.9944	36.20	27
28	+ 109.5	+ 215.5	...	10	+ 208.5	1,456.5	.0069	.9931	35.40	28
29	+ 88.5	+ 198.	...	15	+ 188.	1,644.5	.0091	.9909	34.65	29
30	+ 112.	+ 200.5	...	13	+ 185.5	1,830.	.0071	.9929	33.97	30
31	+ 55.	+ 167.	...	13	+ 154.	1,984.	.0066	.9934	33.21	31
32	+ 64.	+ 119.	...	22	+ 106.	2,090.	.0105	.9895	32.43	32
33	+ 61.5	+ 125.5	...	20	+ 103.5	2,193.5	.0091	.9909	31.77	33
34	+ 53.	+ 114.5	...	21	+ 94.5	2,288.	.0092	.9908	31.06	34
35	+ 49.5	+ 102.5	...	17	+ 81.5	2,369.5	.0072	.9928	30.35	35
36	+ 59.5	+ 109.	...	29	+ 92.	2,461.5	.0118	.9882	29.57	36
37	+ 37.5	+ 97.	...	27	+ 68.	2,529.5	.0107	.9893	28.92	37
38	+ 40.5	+ 78.	...	27	+ 51.	2,580.5	.0105	.9895	28.23	38
39	+ 25.5	+ 66.	...	34	+ 39.	2,619.5	.0130	.9870	27.53	39
40	+ 52.5	+ 78.	...	36	+ 44.	2,663.5	.0135	.9865	26.89	40
41	+ 5.	+ 57.5	...	38	+ 21.5	2,685.	.0142	.9858	26.26	41
42	+ 21.	+ 26.	...	41	- 12.	2,673.	.0153	.9847	25.64	42
43	+ 14.5	+ 35.5	...	42	- 5.5	2,667.5	.0157	.9843	25.04	43
44	+ 12.5	+ 27.	...	30	- 15.	2,652.5	.0113	.9887	24.44	44
45	+ 18.5	+ 31.	...	43	+ 1.	2,653.5	.0162	.9838	23.72	45
46	+ 7.5	+ 26.	...	39	- 17.	2,636.5	.0148	.9852	23.11	46
47	+ 9.	+ 16.5	...	35	- 22.5	2,614.	.0134	.9866	22.46	47
48	+ 3.5	+ 12.5	...	46	- 22.5	2,591.5	.0178	.9822	21.77	48
49	+ 2.5	+ 6.	...	40	- 40.	2,551.5	.0157	.9843	21.16	49
50	+ 8.	+ 10.5	...	33	- 29.5	2,522.	.0131	.9869	20.50	50
51	+ 1.	+ 9.	...	42	- 24.	2,498.	.0168	.9832	19.77	51
52	+ 2.5	+ 3.5	...	54	- 38.5	2,459.5	.0220	.9780	19.11	52
53	+ 3.	+ 5.5	...	41	- 48.5	2,411.	.0170	.9830	18.54	53
54	+ 4.	+ 7.	...	46	- 34.	2,377.	.0194	.9806	17.86	54
55	+ 4.5	+ 8.5	...	45	- 37.5	2,339.5	.0192	.9808	17.21	55
Carried forward	934	...	71,631.5

TABLE X—continued.

Age at Entry x	$\frac{1}{2}(n_x + r_x - f_x)$ *	Col. 2 Age $x-1$ + age x	Exist- ing ex_x	Died dx	Col. 3 - Cols. 4 & 5 Age $x-1$	Exposed to Risk E_x	q_x	p_x	e_x	Age at Entry x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Brought forward	934	...	71,631.5
56	...	+ 4.5	...	52	- 40.5	2,299.	.0220	.9774	16.55	56
57	- .5	- .5	...	51	- 52.5	2,246.5	.0227	.9773	15.93	57
58	+ .5	65	- 51.	2,195.5	.0296	.9704	15.30	58
59	+ 1.	+ 1.5	...	54	- 63.5	2,132.	.0253	.9747	14.77	59
60	+ 3.	+ 4.	...	59	- 50.	2,082.	.0283	.9717	14.15	60
61	+ 4.5	+ 7.5	6	68	- 51.5	2,030.5	.0335	.9665	13.56	61
62	- 1.	+ 3.5	7	62	- 70.5	1,960.	.0316	.9684	13.03	62
63	+ 3.	+ 2.	12	80	- 67.	1,893.	.0423	.9577	12.46	63
64	+ 1.5	- 4.5	19	62	- 87.5	1,805.5	.0343	.9657	12.01	64
65	+ .5	+ 2.	33	69	- 79.	1,726.5	.0400	.9600	11.44	65
66	- 1.5	- 1.	45	83	- 103.	1,623.5	.0511	.9489	10.92	66
67	+ 1.5	...	72	82	- 128.	1,495.5	.0548	.9452	10.51	67
68	- .5	+ 1.	80	63	- 153.	1,342.5	.0469	.9531	10.12	68
69	- .5	- 1.	62	53	- 144.	1,198.5	.0442	.9558	9.62	69
70	+ 2.	+ 1.5	81	73	- 113.5	1,085.	.0673	.9327	9.06	70
71	+ 2.	+ 4.	71	53	- 150.	935.	.0567	.9433	8.71	71
72	+ 1.	+ 3.	53	53	- 121.	814.	.0651	.9349	8.23	72
73	...	+ 1.	61	58	- 105.	709.	.0818	.9182	7.80	73
74	65	45	- 119.	590.	.0763	.9237	7.49	74
75	+ 1.	+ 1.	43	42	- 109.	481.	.0873	.9127	7.11	75
76	- .5	+ .5	31	35	- 84.5	396.5	.0883	.9117	6.79	76
77	...	- .5	39	35	- 66.5	330.	.1061	.8939	6.45	77
78	+ .5	+ .5	21	25	- 73.5	256.5	.0975	.9025	6.21	78
79	- .5	...	22	13	- 46.	210.5	.0618	.9382	5.88	79
80	...	- .5	20	17	- 35.5	175.	.0971	.9029	5.27	80
81	+ .5	+ .5	23	15	- 36.5	138.5	.1083	.8917	4.84	81
82	...	+ .5	15	13	- 37.5	101.	.1287	.8713	4.43	82
83	9	11	- 28.	73.	.1507	.8493	4.09	83
84	3	12	- 20.	53.	.2264	.7736	3.81	84
85	4	6	- 15.	38.	.1579	.8421	3.92	85
86	4	4	- 10.	28.	.1429	.8571	3.66	86
87	2	1	- 8.	20.	.0500	.9500	3.27	87
88	2	- 3.	17.	.1176	.8824	2.44	88
89	2	3	- 2.	15.	.2000	.8000	1.77	89
90	1	6	- 5.	10.	.6000	.4000	1.21	90
91	1	- 7.	3.	.3333	.6667	2.03	91
92	- 1.	2.	2.05	92
93	93
94	1	1	94
95	- 2.	95
...	+1,634.	+3,268.	907	2,361	...	104,143.

* See Table III, Cols. 1, 7, and 4.

TABLE XI.

*Widows. Patriotic (Russian War) Fund.**Aggregate Mortality Table (including Experience of Re-married Women).**(Graduated.)*

Age	l_x	d_x	q_x	p_x	INTEREST 3 PER-CENT				Age
					D_x	N_x	a_x	A_x	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)
17	10,000	65	·0065	·9935	6,050·2	134,560·5	22·241	·32308	17
18	9,935	68	·0068	·9932	5,835·8	128,724·7	22·058	·32840	18
19	9,867	70	·0071	·9929	5,627·	123,097·7	21·876	·33371	19
20	9,797	73	·0074	·9926	5,424·4	117,673·3	21·693	·33904	20
21	9,724	75	·0077	·9923	5,227·1	112,446·2	21·512	·34431	21
22	9,649	75	·0078	·9922	5,035·7	107,410·5	21·330	·34961	22
23	9,574	76	·0079	·9921	4,851·1	102,559·4	21·141	·35512	23
24	9,498	75	·0079	·9921	4,672·4	97,887·	20·950	·36068	24
25	9,423	73	·0078	·9922	4,500·5	93,386·5	20·750	·36650	25
26	9,350	72	·0077	·9923	4,335·5	89,051·	20·540	·37262	26
27	9,278	71	·0077	·9923	4,176·9	84,874·1	20·320	·37903	27
28	9,207	72	·0078	·9922	4,024·2	80,849·9	20·091	·38570	28
29	9,135	72	·0079	·9921	3,876·4	76,973·5	19·857	·39252	29
30	9,063	73	·0081	·9919	3,733·8	73,239·7	19·615	·39956	30
31	8,990	75	·0083	·9917	3,595·9	69,613·8	19·368	·40676	31
32	8,915	77	·0086	·9914	3,462·	66,181·8	19·117	·41407	32
33	8,838	79	·0088	·9912	3,332·2	62,849·6	18·861	·42152	33
34	8,759	81	·0092	·9908	3,206·2	59,643·4	18·603	·42904	34
35	8,678	84	·0097	·9903	3,084·	56,559·4	18·340	·43670	35
36	8,594	88	·0102	·9898	2,965·2	53,594·2	18·075	·44441	36
37	8,506	92	·0108	·9892	2,849·4	50,744·8	17·808	·45220	37
38	8,414	96	·0114	·9886	2,736·5	48,008·3	17·544	·45988	38
39	8,318	100	·0120	·9880	2,626·4	45,381·9	17·279	·46760	39
40	8,218	104	·0126	·9874	2,519·3	42,862·6	17·014	·47532	40
41	8,114	107	·0132	·9868	2,415·	40,447·6	16·749	·48304	41
42	8,007	110	·0137	·9863	2,313·7	38,133·9	16·482	·49081	42
43	7,897	111	·0141	·9859	2,215·4	35,918·5	16·213	·49865	43
44	7,786	113	·0145	·9855	2,120·7	33,797·8	15·937	·50669	44
45	7,673	115	·0150	·9850	2,029·	31,768·8	15·657	·51485	45
46	7,558	116	·0154	·9846	1,940·4	29,828·4	15·372	·52314	46
47	7,442	118	·0158	·9842	1,855·	27,973·4	15·080	·53165	47
48	7,324	119	·0162	·9838	1,772·4	26,201·	14·783	·54030	48
49	7,205	120	·0166	·9834	1,692·8	24,508·2	14·478	·54919	49
50	7,085	121	·0170	·9830	1,616·1	22,892·1	14·165	·55830	50
51	6,964	122	·0175	·9825	1,542·3	21,349·8	13·843	·56768	51
52	6,842	124	·0181	·9819	1,471·1	19,878·7	13·513	·57729	52
53	6,718	126	·0188	·9812	1,402·4	18,476·3	13·175	·58713	53
54	6,592	130	·0197	·9803	1,336·	17,140·3	12·830	·59718	54
55	6,462	134	·0208	·9792	1,271·5	15,868·8	12·480	·60738	55

TABLE XI—*continued.*

Age	l_x	d_x	q_x	p_x	INTEREST 3 PER-CENT				Age
					D_x	N_x	a_x	A_x	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
56	6,328	140	·0221	·9779	1,208·9	14,659·9	12·127	·61766	56
57	6,188	145	·0235	·9765	1,147·7	13,512·2	11·773	·62767	57
58	6,043	152	·0252	·9748	1,088·2	12,424·	11·417	·63834	58
59	5,891	159	·0270	·9730	1,029·9	11,394·1	11·063	·64865	59
60	5,732	166	·0289	·9711	972·91	10,421·17	10·711	·65890	60
61	5,566	173	·0311	·9689	917·22	9,503·95	10·362	·66907	61
62	5,393	182	·0337	·9663	862·82	8,641·13	10·015	·67917	62
63	5,211	189	·0363	·9637	809·42	7,831·71	9·676	·68905	63
64	5,022	195	·0389	·9611	757·35	7,074·36	9·341	·69880	64
65	4,827	201	·0417	·9583	706·74	6,367·62	9·010	·70845	65
66	4,626	208	·0449	·9551	657·58	5,710·04	8·684	·71794	66
67	4,418	214	·0484	·9516	609·72	5,100·32	8·365	·72723	67
68	4,204	219	·0522	·9478	563·29	4,537·03	8·055	·73626	68
69	3,985	224	·0562	·9438	518·39	4,018·64	7·752	·74509	69
70	3,761	226	·0602	·9398	475·00	3,543·64	7·461	·75356	70
71	3,535	227	·0642	·9358	433·46	3,110·28	7·175	·76189	71
72	3,308	226	·0682	·9318	393·81	2,716·37	6·898	·76996	72
73	3,082	223	·0724	·9276	356·22	2,360·15	6·626	·77789	73
74	2,859	220	·0768	·9232	320·82	2,039·33	6·357	·78572	74
75	2,639	215	·0815	·9185	287·51	1,751·82	6·093	·79341	75
76	2,424	210	·0865	·9135	256·39	1,495·43	5·833	·80098	76
77	2,214	203	·0917	·9083	227·36	1,268·07	5·578	·80841	77
78	2,011	195	·0971	·9029	200·50	1,067·57	5·325	·81577	78
79	1,816	187	·1030	·8970	175·78	891·79	5·074	·82308	79
80	1,629	178	·1094	·8906	153·09	738·70	4·826	·83032	80
81	1,451	168	·1160	·8840	132·39	606·31	4·580	·83748	81
82	1,283	158	·1230	·8770	113·65	492·663	4·336	·84459	82
83	1,125	148	·1315	·8685	96·753	395·910	4·093	·85166	83
84	977	137	·1405	·8595	81·577	314·333	3·854	·85862	84
85	840	126	·1505	·8495	68·095	246·238	3·617	·86553	85
86	714	116	·1620	·8380	56·195	190·075	3·384	·87231	86
87	598	104	·1740	·8260	45·694	144·381	3·160	·87883	87
88	494	93	·1880	·8120	36·648	107·738	2·940	·88524	88
89	401	82	·2034	·7966	28·882	78·849	2·729	·89139	89
90	319	70	·2200	·7800	22·307	56·507	2·529	·89722	90
91	249	59	·2371	·7629	16·905	39·589	2·340	·90272	91
92	190	48	·2548	·7452	12·524	27·058	2·159	·90799	92
93	142	39	·2733	·7267	9·087	17·990	1·984	·91308	93
94	103	30	·2938	·7062	6·399	11·591	1·812	·91810	94
95	73	23	·3151	·6849	4·403	7·206	1·643	·92303	95
96	50	17	·3391	·6609	2·928	4·290	1·471	·92803	96
97	33	12	·3651	·6349	1·876	2·419	1·292	·93324	97
98	21	8	·3931	·6069	1·159	1·265	1·096	·93896	98
99	13	5	·4221	·5779	·697	·584	·860	·94583	99
100	8	4	·4521	·5479	·416	·203	·532	·95538	100
101	4	4	1·	...	·202	101

TABLE XII.

*Patriotic (Russian War) Fund.**Aggregate Mortality Table.**Widows (excluding Re-married Women).*

Age	Exposed to Risk	Deaths.	RATE OF MORTALITY OF WIDOWS <i>q_x</i>		Experience of Widows and Re-married Women combined <i>q_x</i>	English Life Table No. 3 (Females) <i>q_x</i>	Age
			Un-graduated	Graduated			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
17	5	·0065	·0066	17
18	5·	1	·2000	...	·0068	·0072	18
19	16·5	·0071	·0079	19
20	47·5	·0074	·0086	20
21	107·	2	·0187	...	·0077	·0088	21
22	200·5	1	·0049	...	·0078	·0090	22
23	330·5	5	·0151	...	·0079	·0092	23
24	479·5	5	·0104	·0079	·0079	·0094	24
25	627·5	4	·0064	·0080	·0078	·0096	25
26	753·5	4	·0053	·0081	·0077	·0098	26
27	865·	7	·0089	·0082	·0077	·0100	27
28	970·5	6	·0062	·0084	·0078	·0102	28
29	1063·5	11	·0103	·0085	·0079	·0104	29
30	1164·	9	·0077	·0086	·0081	·0106	30
31	1226·5	7	·0057	·0087	·0083	·0108	31
32	1232·	13	·0106	·0088	·0086	·0110	32
33	1245·5	12	·0096	·0089	·0088	·0112	33
34	1255·	10	·0080	·0090	·0092	·0114	34
35	1270·5	8	·0063	·0091	·0097	·0116	35
36	1312·5	14	·0107	·0093	·0102	·0118	36
37	1338·	14	·0105	·0095	·0108	·0120	37
38	1338·	15	·0112	·0098	·0114	·0123	38
39	1335·5	17	·0127	·0101	·0120	·0125	39
40	1368·5	20	·0146	·0104	·0126	·0128	40
41	1387·5	7	·0050	·0109	·0132	·0130	41
42	1389·	20	·0144	·0115	·0137	·0133	42
43	1397·5	19	·0136	·0122	·0141	·0136	43
44	1395·	14	·0100	·0130	·0145	·0139	44
45	1396·5	26	·0186	·0137	·0150	·0143	45
46	1391·5	21	·0151	·0144	·0154	·0146	46
47	1387·	16	·0115	·0151	·0158	·0150	47
48	1392·	27	·0194	·0158	·0162	·0154	48
49	1376·5	24	·0174	·0166	·0166	·0158	49
50	1377·5	24	·0174	·0174	·0170	·0162	50
51	1385·5	20	·0144	·0182	·0175	·0167	51
52	1389·	31	·0223	·0190	·0181	·0171	52
53	1380·5	22	·0166	·0199	·0188	·0176	53
54	1381·5	31	·0224	·0209	·0197	·0197	54
55	1377·5	23	·0167	·0220	·0208	·0210	55
56	1378·5	31	·0225	·0232	·0221	·0223	56
Carried forward	41,735·5	541

TABLE XII—*continued.*

Age	Exposed to Risk	Deaths	RATES OF MORTALITY OF WIDOWS q_x		Experience of Widows and Re-married women combined q_x graduated	English Life Table No. 3 (Females) q_x	Age
			Un-graduated	Graduated			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Brought forward	41735·5	541
57	1368·	32	·0234	·0246	·0235	·0238	57
58	1359·5	47	·0346	·0261	·0252	·0253	58
59	1341·	32	·0239	·0281	·0270	·0270	59
60	1335·	34	·0255	·0302	·0289	·0289	60
61	1331·	47	·0353	·0324	·0311	·0309	61
62	1309·5	41	·0313	·0347	·0337	·0331	62
63	1291·	53	·0411	·0372	·0363	·0355	63
64	1257·	49	·0390	·0400	·0389	·0382	64
65	1223·	48	·0392	·0429	·0417	·0411	65
66	1172·	64	·0546	·0460	·0449	·0444	66
67	1092·5	62	·0568	·0493	·0484	·0479	67
68	999·5	46	·0460	·0529	·0522	·0518	68
69	918·5	41	·0446	·0568	·0562	·0560	69
70	848·	66	·0778	·0611	·0602	·0606	70
71	737·	45	·0611	·0658	·0642	·0656	71
72	656·	45	·0686	·0710	·0682	·0709	72
73	577·5	50	·0866	·0767	·0724	·0767	73
74	485·5	38	·0783	·0830	·0768	·0830	74
75	401·	32	·0798	·0896	·0815	·0897	75
76	338·5	35	·1034	·0968	·0865	·0968	76
77	281·5	31	·1101	·1044	·0917	·1044	77
78	223·5	23	·1029	·1125	·0971	·1125	78
79	185·	12	·0649	·1211	·1030	·1211	79
80	154·5	13	·0841	·1302	·1094	·1302	80
81	126·5	15	·1186	·1398	·1160	·1398	81
82	89·	12	·1348	·1499	·1230	·1499	82
83	65·5	10	·1527	·1604	·1315	·1604	83
84	47·	10	·2128	·1715	·1405	·1715	84
85	34·	6	·1765	·1831	·1505	·1831	85
86	25·5	3	·1176	·1952	·1620	·1952	86
87	19·	1	·0526	·2078	·1740	·2078	87
88	16·	2	·1250	·2208	·1880	·2208	88
89	14·	3	·2143	·2343	·2034	·2343	89
90	9·	6	·6665	·2483	·2200	·2483	90
91	2·	1	·5000	·2627	·2371	·2627	91
92	1·	92
93	1·	93
94	1·	94
95	95
...	63071·5	1596

TABLE XIII.

*Patriotic (Russian War) Fund.**Rate of Widowhood among Re-married Women
(i.e., Death Rate of Husbands).*

Age of Wife	Half of Re-Married less forfeited and Died Married	Col. (2) Age $x-1$ + Age x	Existing Married	Widowed	Col. (3) less Cols. (4 and 5) Age $x-1$	Exposed to Risk	q_x Un-graduated	q_x Graduated	Age of Wife
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)
17	17
18	+ 1'	+ 1'	+ 1'	1'	18
19	...	+ 1'	+ 1'	2'	19
20	+ 4'	+ 4'	...	1	+ 4'	6'	·1667	...	20
21	+ 10'	+ 14'	+ 13'	19'	21
22	+ 13'	+ 23'	+ 23'	42'	22
23	+ 24·5	+ 37·5	...	2	+ 37·5	79·5	·0252	·0146	23
24	+ 31·5	+ 56'	...	1	+ 54'	133·5	·0075	·0148	24
25	+ 47·5	+ 79'	...	3	+ 78'	211·5	·0142	·0150	25
26	+ 45'	+ 92·5	...	4	+ 89·5	301'	·0133	·0151	26
27	+ 52·5	+ 97·5	...	7	+ 93·5	394·5	·0177	·0152	27
28	+ 59'	+ 111·5	...	8	+ 104·5	499'	·0160	·0153	28
29	+ 50'	+ 109'	...	9	+ 101'	600'	·0150	·0154	29
30	+ 53'	+ 103'	...	17	+ 94'	694'	·0245	·0155	30
31	+ 55·5	+ 108·5	...	10	+ 91·5	785·5	·0127	·0157	31
32	+ 56'	+ 111·5	...	8	+ 101·5	887'	·0090	·0159	32
33	+ 46'	+ 102'	...	16	+ 94'	981'	·0163	·0161	33
34	+ 50'	+ 96'	...	18	+ 80'	1061'	·0170	·0163	34
35	+ 31·5	+ 81·5	...	17	+ 63·5	1124·5	·0151	·0165	35
36	+ 34·5	+ 66'	...	22	+ 49'	1173·5	·0187	·0167	36
37	+ 31·5	+ 66'	...	20	+ 44'	1217·5	·0164	·0170	37
38	+ 39'	+ 70·5	...	18	+ 50·5	1268'	·0142	·0172	38
39	+ 15'	+ 54'	...	15	+ 36'	1304'	·0115	·0175	39
40	+ 18·5	+ 33·5	...	29	+ 18·5	1322·5	·0219	·0178	40
41	+ 9'	+ 27·5	...	35	- 1·5	1321'	·0265	·0182	41
42	+ 10'	+ 19'	...	23	- 16'	1305'	·0176	·0186	42
43	- 5	+ 9·5	...	30	- 13·5	1291·5	·0232	·0191	43
44	+ 15·5	+ 15'	...	23	- 15'	1276·5	·0180	·0195	44
45	+ 4'	+ 19·5	...	28	- 3·5	1273'	·0220	·0201	45
46	+ 5·5	+ 9·5	...	20	- 18·5	1254·5	·0159	·0206	46
47	- 4·5	+ 1'	...	20	- 19'	1235·5	·0162	·0212	47
48	- 2'	- 6·5	...	23	- 26·5	1209'	·0190	·0218	48
49	- 5	- 2·5	...	19	- 25·5	1183·5	·0161	·0221	49
50	...	- 5	...	36	- 19·5	1164'	·0309	·0231	50
51	- 7'	- 7'	...	28	- 43'	1121'	·0250	·0238	51
52	- 11·5	- 18·5	...	21	- 46·5	1074·5	·0195	·0245	52
53	- 5'	- 16·5	...	23	- 37·5	1037'	·0222	·0253	53
54	- 5·5	- 10·5	...	24	- 33·5	1003·5	·0239	·0262	54
55	- 9'	- 14·5	...	22	- 38·5	965'	·0228	·0271	55
56	- 9·5	- 18·5	...	22	- 40·5	924·5	·0238	·0280	56
Carried forward	+ 757·5	1524·5	...	622	924·5	32746·5

TABLE XIII—*continued.*

Age of Wife	Half of Re-Married less forfeited and Died Married *	Col. (2) Age $x-1$ + Age x	Existing Married	Widowed	Col. (3) less Cols. (4 and 5) Age $x-1$	Exposed to Risk	q_x Un-graduated	q_x Graduated	Age of Wife
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(1)
Brought forward	757·5	1524·5	...	622	924·5	32746·5
57	— 8·5	— 18·	...	24	— 40·	884·5	·0271	·0290	57
58	— 6·5	— 15·	...	31	— 39·	845·5	·0367	·0300	58
59	— 10·	— 16·5	...	30	— 47·5	798·	·0376	·0312	59
60	— 12·	— 22·	...	17	— 52·	746·	·0228	·0324	60
61	— 9·	— 21·	1	33	— 38·	708·	·0466	·0337	61
62	— 9·5	— 18·5	3	27	— 52·5	655·5	·0412	·0350	62
63	— 12·5	— 22·	4	26	— 52·	603·5	·0431	·0363	63
64	— 6·5	— 19·	7	22	— 49·	554·5	·0397	·0376	64
65	— 10·5	— 17·	10	29	— 46·	508·5	·0570	·0389	65
66	— 9·5	— 20·	13	13	— 59·	449·5	·0289	·0403	66
67	— 10·	— 19·5	21	20	— 45·5	404·	·0495	·0417	67
68	— 8·	— 18·	30	19	— 59·	345·	·0551	·0432	68
69	— 6·	— 14·	19	14	— 63·	282·	·0496	·0449	69
70	— 3·5	— 9·5	24	11	— 42·5	239·5	·0459	·0466	70
71	— 4·	— 7·5	27	6	— 42·5	197·	·0305	·0484	71
72	— 4·	— 8·	14	4	— 41·	156·	·0256	·0504	72
73	— 4·	— 8·	13	5	— 26·	130·	·0385	·0526	73
74	— 3·5	— 7·5	12	7	— 25·5	104·	·0670	·0548	74
75	— 5·	— 8·5	8	4	— 27·5	77·	·0519	·0571	75
76	...	— 5·	6	4	— 17·	60·	·0667	·0597	76
77	— 2·	— 2·	8	3	— 12·	48·	·0625	·0623	77
78	— 1·	— 3·	3	4	— 14·	34·	·1176	·0652	78
79	— ·5	— 1·5	3	1	— 8·5	25·5	·0392	·0686	79
80	— 2·	— 2·5	4	1	— 6·5	19·	·0526	·0724	80
81	...	— 2·	— 7·	12·	...	·0768	81
82	— ·5	— ·5	3	...	— ·5	11·5	...	·0818	82
83	— ·5	— 1·	...	1	— 4·	7·5	·1333	·0873	83
84	— 1·	— 1·5	— 2·5	5·	...	·0930	84
85	...	— 1·	1	...	— 1·	4·	...	·0990	85
86	— ·5	— ·5	...	1	— 1·5	2·5	·4000	·1055	86
87	...	— ·5	— 1·5	1·	...	·1125	87
88	1·	88
89	1·	89
90	1·	90
91	1·	91
92	1·	92
93	1·	93
94	— ·5	— ·5	— ·5	·5	94
95	..	— ·5	— ·5	95
...	+ 812·5	+ 1,619·5	234	979	+ 1,322·5	41,671·5
...	- 206·	- 406·5	- 1,322·5
...	606·5	1,213·

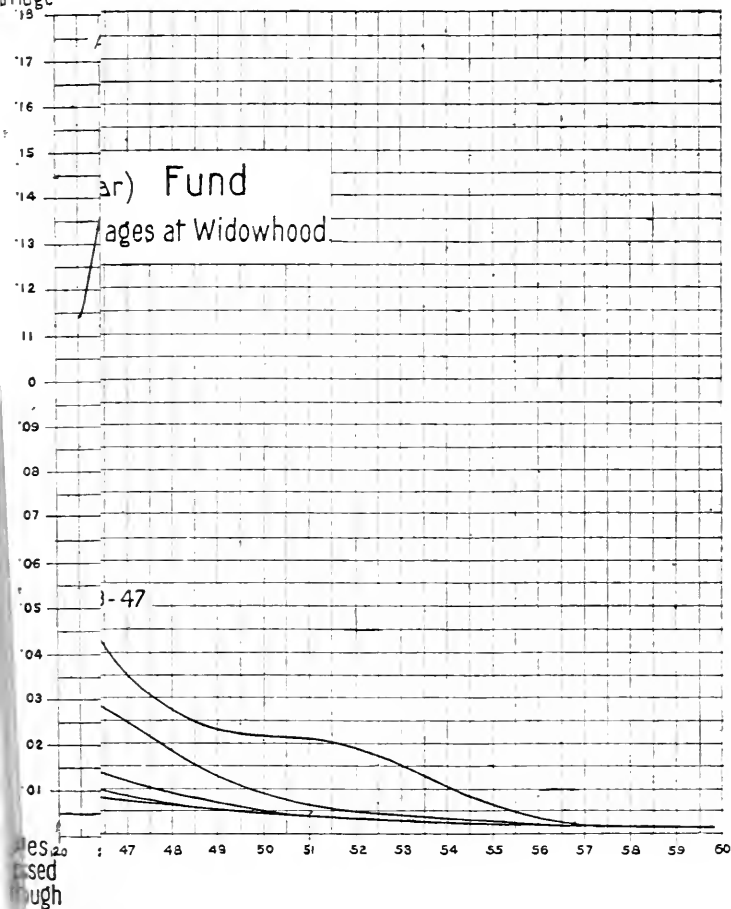
* See Table III, Cols. 8, 2, and 10.

TABLE XIV.

Differences in Age at Marriage of Husbands and Wives Married during 1901.

WIDOWS MARRY WIDOWERS				WIDOWS MARRY BACHELORS			
No. of Marriages	Average Age of Widow	Average Age of Widower	Difference in Age	No. of Marriages	Average Age of Widow	Average Age of Bachelor	Difference in Age
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
76	23·	33·9	10·9	505	23·	26·1	3·1
444	27·5	36·9	9·4	1,777	27·5	28·2	·7
1,005	32·5	40·4	7·9	2,303	32·5	31·4	- 1·1
1,552	37·5	43·5	6·	1,941	37·5	34·9	- 2·6
1,716	42·5	47·7	5·2	1,219	42·5	38·5	- 4·
1,624	47·5	51·8	4·3	623	47·5	42·3	- 5·2
1,194	52·5	55·8	3·3	282	52·5	47·6	- 4·9
817	57·5	59·6	2·1	115	57·5	51·2	- 6·3
481	62·5	62·2	- ·3	56	62·5	52·2	- 10·3
188	67·5	66·4	- 1·1	9	67·5	62·5	- 5·
44	72·5	69·5	- 3·	3	72·5	59·2	- 13·3
13	77·5	76·	- 1·5
3	80·	75·8	- 4·2
and upwards							
9,157	8,833
Widows marry Widowers who are on the average 4·73 years older than themselves.				Widows marry Bachelors who are on the average 1·77 years younger than themselves.			
WIDOWS MARRY BACHELORS OR WIDOWERS				SPINSTERS OR WIDOWS MARRY BACHELORS OR WIDOWERS			
No. of Marriages	Average Age of Widow	Average Age of Bachelor or Widower	Difference in Age	No. of Marriages	Average Age of Spinster or Widow	Average Age of Bachelor or Widower	Difference in Age
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
581	23·	27·1	4·1	103,544	23·	25·5	2·5
2,221	27·5	30·	2·5	65,643	27·5	28·6	1·1
3,308	32·5	34·1	1·6	21,899	32·5	33·4	·9
3,493	37·5	38·7	1·2	10,220	37·5	39·2	1·7
2,935	42·5	43·9	1·4	5,878	42·5	43·6	1·1
2,247	47·5	49·2	1·7	3,523	47·5	50·0	2·5
1,476	52·5	54·2	1·7	2,034	52·5	54·9	2·4
932	57·5	58·6	1·1	1,149	57·5	58·8	1·3
537	62·5	61·2	- 1·3	624	62·5	61·6	- ·9
197	67·5	66·2	- 1·3	224	67·5	66·4	- 1·1
47	72·5	68·8	- 3·7	59	72·5	70·	- 2·5
13	77·5	76·	- 1·5	15	77·5	76·4	- 1·1
3	80·	75·8	- 4·2	3	80·	75·8	- 4·2
and upwards				and upwards			
17,990	214,815
Widows marry Bachelors or Widowers who are on the average 1·54 years older than themselves.				Spinsters or Widows marry Bachelors or Widowers who are on the average 1·81 years older than themselves.			

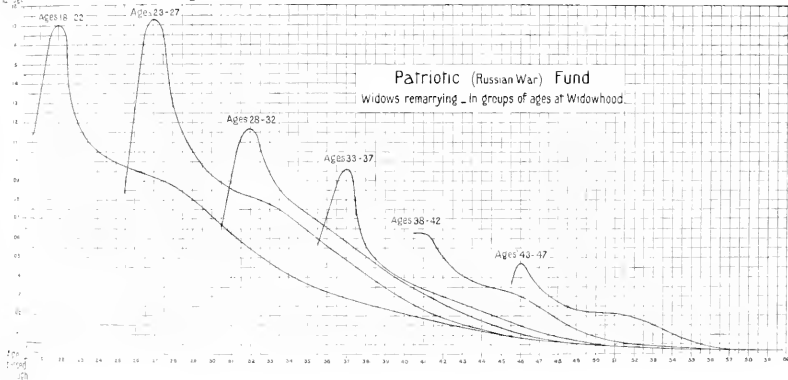
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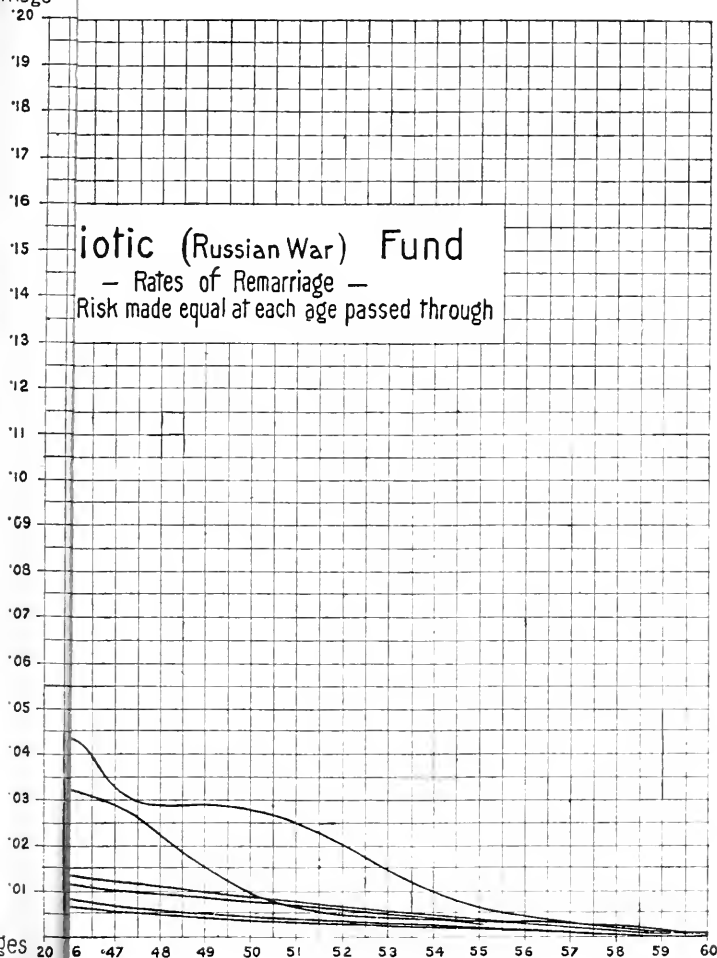
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Age of
remarriage

Diagram I



Rate of
marriage

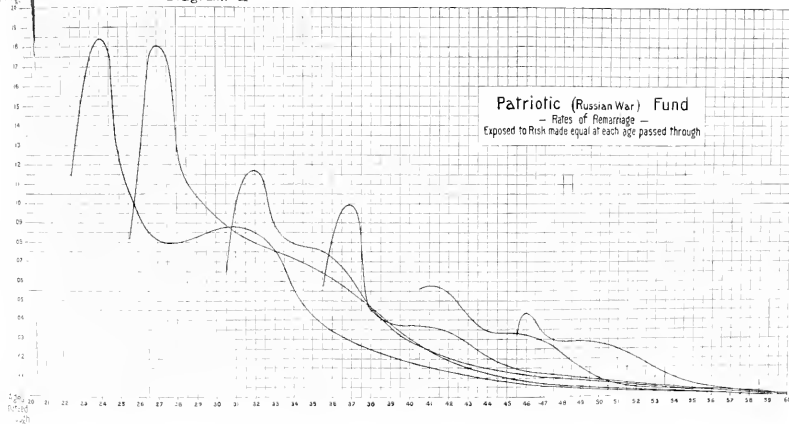


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Rate of
Survival

Diagram II

Patriotic (Russian War) Fund
— Rates of Remarriage —
Exposed to Risk made equal at each age passed through



Rare
re-

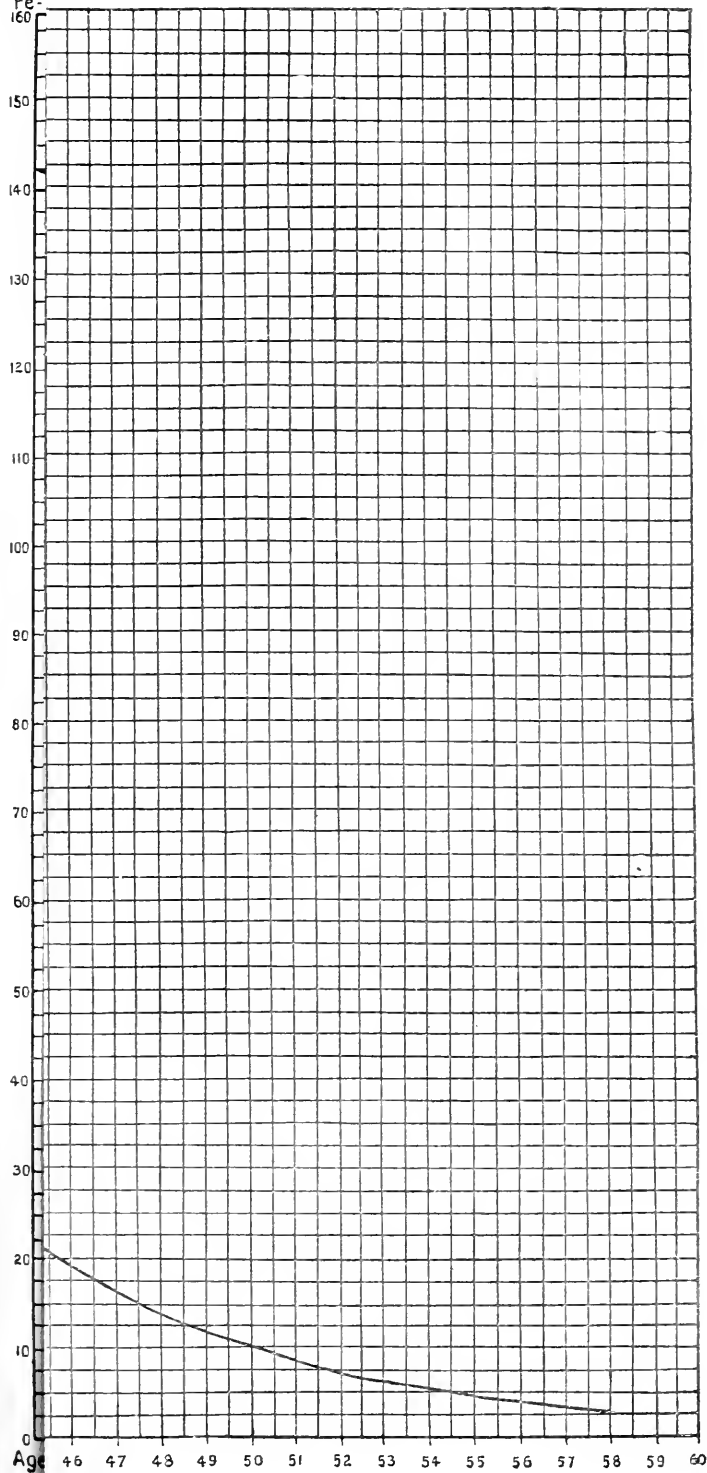
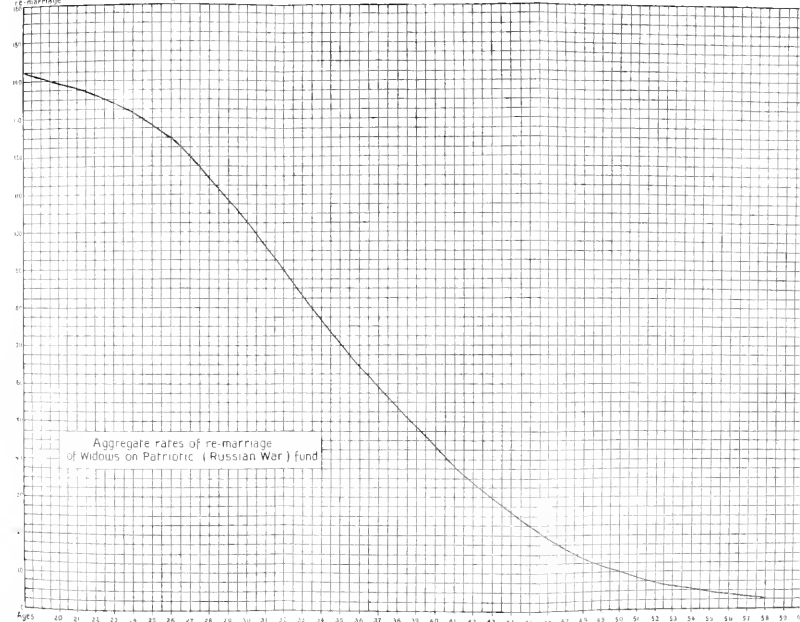


Diagram III

RATE OF
RE-MARRIAGE



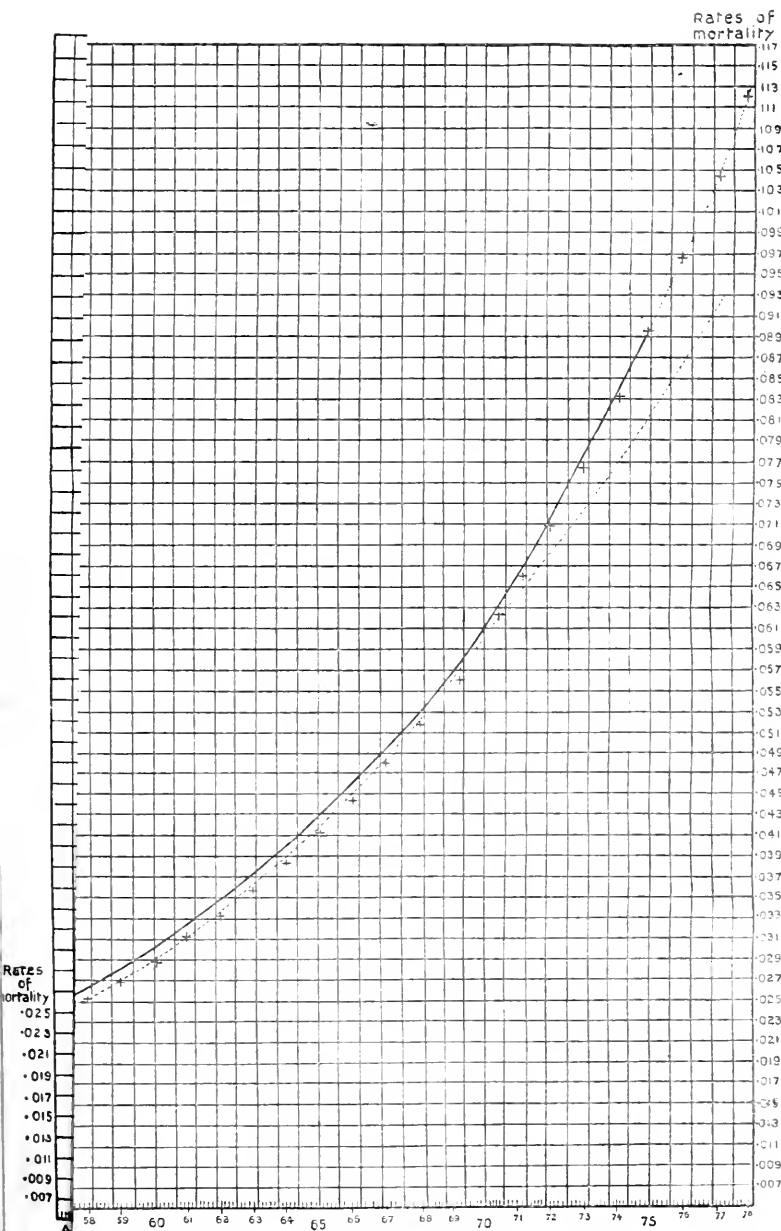


Diagram IV.

PATRIOTIC (Russian War) FUND.

Aggregate death rate - Widows only

** Including re married women

English life No.3 Females + + + + +

Rate

of

mortality

100

90

80

70

60

50

40

30

20

10

0

Age 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Rate of

mortality

100

90

80

70

60

50

40

30

20

10

0

100

90

80

70

60

50

40

30

20

10

0

100

90

80

70

60

50

40

30

20

10

0

100

90

80

70

60

50

40

30

20

10

0

100

90

80

70

60

50

40

30

20

10

0

ABSTRACT OF THE DISCUSSION.

Mr. F. L. COLLINS, in opening the discussion, said that a glance at the numerous tables appended to the paper enabled one to appreciate the exhaustive manner in which the facts, so kindly supplied by the Patriotic Commissioners, had been treated. He considered that the efforts of the authors were worthy of more precise data. Turning to the numbers entering in Table 2 at each age of entry, he noticed the large number of widows who were entrants on the fund at the decennial ages, such as 30, 40, and 50. This was most marked at the age of 40, where there were 110 entered as compared with 62 at 39 and 59 at 41. This preference for round figures was also slightly apparent at the more advanced quinquennial ages, and it gave rise to the supposition that the ages were merely supplied by the applicants for pension and were not in any way authenticated. In fact, as all the widows must have been born prior to the Registration Act, this was what one would anticipate. He thought that it would be correct to designate the ages as "reputed ages." It seemed to him, therefore, that the trouble to which the authors had gone in so adjusting the rates of re-marriage as to attach equal weight to each age at entry in the quinquennial groups was unnecessary. The durations were arrived at by subtracting the calendar year of entry from that of exit. As these calendar years were all that could be supplied by the Commissioners the use of the mean duration thus arrived at was, he supposed, unavoidable. The authors said that the results brought out in this manner were more suitable for valuation purposes, and that no real inconvenience would be caused. He submitted that it was a fundamental principle that all tables should be so constructed that the results might be brought to bear on the exact integral age, and, in the case of select tables, on the exact integral duration. The particular circumstances of a valuation could easily be met by suitable adjustments. In case of a similar investigation being made into the Transvaal War Fund experience, he wished to enter a plea for exact particulars with regard to the date of widowhood and the date of re-marriage. However great the difficulty of obtaining a truthful answer to a question with regard to age, there should be, he thought, none in obtaining from the widows themselves the exact date of widowhood, and the exact date of their second marriage. With regard to graduation, he thought it a great advantage that the authors had adopted the graphic method, which was probably the most suitable that could be used in a case like this, where the rates were so rapidly changing. At the point where the earlier rates of re-marriage were at their maximum it appeared that the curves were drawn through only three points. Here, again, the need of more exact particulars became noticeable. If these were supplied they would be enabled to follow the rates of re-marriage not only through years of duration, but even through quarters of years, and almost through months, so as to see the exact form that the curves really did take in the early years of widowhood.

Mr. F. SCHOOLING wished to call attention to the paragraph on page 434 which said that the paper might be useful "for the purpose of estimating the liabilities of similar funds or for determining the

benefits which could be safely offered on starting new funds with similar objects." Although the authors had very properly afterwards stated that the tables, and so on, must be used with very great caution, yet he thought that that point needed to be emphasized in the paragraph to which he had referred, as in the case of valuing friendly societies it was very necessary that the experience of the society about to be valued should be investigated before using any tables. So here he thought that it was doubly important, before these tables were used in connection with any fund for the purpose of estimating the rates of re-marriage, that an investigation into the particular fund under consideration should be most carefully undertaken. The authors had noted that point afterwards, but the statement which they had made at this particular place might be a little misleading. He thought, with regard to the point which Mr. Collins had mentioned about the ages, when they remembered that the chief purpose of the paper was to obtain the rates of re-marriage, it would be apparent that the age question did not become of so much importance as it would in an ordinary mortality table; and in many of the important tables here the ages were grouped so that the fact of widows stating that they were aged 40 or 35 really did not matter very much if they came within their right groups.

Mr. T. G. ACKLAND said the subject was in some senses a difficult one, and was one with which perhaps not many of the members had had a very large practical dealing. There were one or two points which were not altogether clear to him. The writers referred, on page 441, to the exclusion from the select tables of widows who came on the fund as such after the year 1857. He had not been able to gather why those cases were excluded. The writers stated that it was an unimportant and small matter, but as far as he (Mr. Ackland) could gather, it appeared that those who entered the fund in 1858 and subsequently, and re-married, were included in the data on their second widowhood, but those who did not re-marry were excluded from the data. It did look as if this exclusion was a little arbitrary, and as if, indeed, it might be in the nature of excluding something from the denominator, and not from the numerator. He thought that it was pretty clear that in this investigation they were dealing with a very special class of widows. The widows of non-commissioned officers and men of the army, and of the corresponding class in the navy, could be compared with but very few other classes which would have to be dealt with, and no doubt that ought to be borne in mind as a caution with regard to the use of the tables. There would be a great temptation, in having before them the rate of re-marriage in so complete a form, to employ such a table when they required that function; but undoubtedly a great deal of caution should be employed on account of the special class that was here dealt with. As regarded the quinary grouping of entry ages, it was clear that what Mr. Collins had called the "reputed ages" were given; but, at the same time, they were grouped in quinary groups, and he did not know that the true ages were of so much importance in this case as if they were dealing with the matter of mortality. It was undoubtedly the case that the grouping of ages, apart from the question of the accuracy of the ages, was open to the objection that, as duration proceeded, the

average age was disturbed, and the central age no longer became the true age, because, whether mortality or re-marriage was in question, there was a tendency on one side or other of the central line for the rate to vary unequally from the average. He believed that Dr. Sprague was the first, in the application of his Graphic method, to use the expedient (employed also in this paper) of equal exposures, by taking the mean of the rates for the five ages. The differences from the rate deduced by the authors, as compared with the Scottish Bankers' rate of re-marriage, were indeed remarkable; and this was another illustration of how cautious they should be in making use of the tables. The rate of the army widows was three times, four times, and at some points ten and twenty times, as high as the rate among the bankers' widows; and no doubt there were special reasons for that, some of which were referred to in the paper. The reason would be, partly, he supposed, that, on re-marriage, in one case the pension was absolutely extinguished, whilst in the other case it was only reduced, but it would lie perhaps more particularly in the difference between classes in life from which the widows dealt with came. The graphic graduation, he thought, had been carried out with very great success, considering the paucity of the data. When they compared the expected with the actual re-marriages throughout Table 4, they would find a remarkably close identity, and the differences and the accumulated differences were both well within what one would have anticipated. The writers referred to the maximum rate of re-marriage at the age of 30, but he thought that there was not very much in that. He should like to emphasize what had been said by Mr. Collins as to the desirability of getting from the Patriotic Fund Commissioners some more precise facts as to ages and durations in tabulating the experience of the Transvaal War Fund. As regarded the ages of the widows, surely they could be required to supply a certificate of birth, and, as they would in all cases come under the Registration Acts, there would be no difficulty in obtaining the precise ages: and the same remarks applied as to the true ages at the death of husbands.

Mr. A. W. WATSON, in closing the discussion, said that the authors of the paper were fortunate in having what may be termed completed experience in the sense that the lives at risk had been followed through from the beginning to a period at which the probability of re-marriage vanished, and the further continuance of the observations would have been of little practical value. This, he thought, was in strong contrast with the usual conditions of practice, in which they found that the "existing" at every duration of the contract loomed very largely in the schedules, and induced a degree of uncertainty as to the results brought out. The authors pointed out that the data afforded little experience as to first widowhoods in the later years of life, and that their experience as to the later ages was drawn from second and third widowhoods. Whether this would afford a reliable guide to the probability of re-marriage on a first widowhood was of course problematical, but the authors went far to disarm criticism by stopping their graduated rates of re-marriage at the initial central age of 45. It might be of interest in this connection to observe that he had consulted an experience including 563 widows of a fund for providing annuities to the widows of railway servants of the working class, and had found that of 563 widows

coming on the fund there were 283 cases of widows becoming such at ages over 47, and there was not a single case of re-marriage recorded amongst them. In that case the pension ceased on re-marriage. The rate of pay was similar to that of the Patriotic Fund—namely, 6s. a week. For his part he should prefer to rule out the element of re-marriage entirely in valuations after the age of 50 years had been passed, whether the duration of widowhood was long or short; but he understood that in Table 8 the authors had carried on the ultimate rates to the age of 64. As Mr. Ackland had said, a reader of the paper could not fail to be struck by the difference between the re-marriage rates of the widows of the Patriotic Fund and the Scottish Bankers' Widows. If the tables could be regarded as having a general application they would be extremely useful to what he might term the general practitioner of the profession, who sometimes was puzzled to know where to turn for data to suit some special case which he had in hand; but he shared with Mr. Ackland the doubt as to whether this was not a special experience, and one which they could not fully rely upon. In order to test it in some little degree, he had calculated from the rates of re-marriage given in Table 5 the expected re-marriages amongst the widows of the railway fund to which he had referred, and he found that against an expected number of re-marriages of 98 the actual number recorded was only 57; so that, as far as that little experience went, it seemed to corroborate the view that the Patriotic Fund widows were a special class.

Mr. SCHOOLING.—In the railway fund did the pension cease on re-marriage?

Mr. WATSON said that the pension did cease in that case on re-marriage. He should like to suggest in respect to Table 9 that if there was one point which the authors had more clearly established than another it was that these data must be collected in the select form, and that such comparisons as those set up in Table 9 were apt to be misleading. The widows' experience of the general population is that of a class which is constantly being recruited by fresh widowhoods, and the re-marriages amongst the widows of the general population would undoubtedly come largely from those who had been widowed in the few years previous to the year of observation. But in the special class that they were dealing with in the present discussion at least one-half of the experience came from widows who were placed on the fund in the years 1855 and 1856; and he should expect in such circumstances that the rates of re-marriage amongst them, especially at the later ages, would be very much lower than those of the general population. On the general subject of variation of marriage rates he agreed with the authors that such elements as locality must be allowed considerable weight. A reference to the Digest of the Census Returns of 1901 would show that there was an enormous difference in the distribution of the population, and that in the mining districts the male population was generally in excess of the female population; whilst in the agricultural districts the reverse was the case. To take these as examples, he should expect to find that the re-marriages amongst widows were much more numerous in the mining districts than in the agricultural districts; and in valuing any particular fund, he should deem it necessary to consider

the locality in which the beneficiaries of the fund resided before proceeding to apply such general tables as those in the paper. The rates of widowhood shown in Table 13 deserved attention; these seemed to be really rates of second widowhood. He should like the authors to tell what in their view was the reason for the extraordinarily high rates of widowhood at the ages 23 to 43 in Table 13. Viewing those rates of widowhood rather as rates of mortality amongst husbands, they found that the widows of the age of 23 showed a rate of widowhood of .0146, which at such age was extremely high, and he should be glad if the authors could suggest the reason for that feature. If it was suggested that this rate of widowhood was to be of much practical value, he did not altogether agree with the suggestion. In a fund of this character the mass of the benefits was generally in reversion, and what was required was not the value of the interest of a given woman after the death of her husband, but rather the value of an annuity payable at the death of the male member to any wife that he might leave behind him. Therefore, what they required was not to make some assumption as to the average age of the husband, but rather to take the male member as the existing life, and to make assumptions as to the probable average ages of the women who would be left widowed at each successive year of the husbands' age. It appeared to him, viewing the problem in that form, that this function was not likely to be of much assistance to them. The mortality rates in Table 10 must, he thought, be viewed with some reserve. At page 447 were given a number of causes of forfeiture, including insanity, senility, and intemperance, and it seemed to him that by the exclusion of lives subject to those disqualifications the probable death rate would be largely reduced. Consequently, he should somewhat hesitate to use tables obtained in that way. He thought that the criticism offered by Mr. Collins as to the method of tabulation, *i.e.*, referring to the question of "duration", was somewhat undeserved. The date of the widowhood was not recorded in the schedules, but only the year of entry, and the year of entry, or the precise date of entry, upon the fund was not a matter of any significance, and therefore the anniversary of being placed upon the fund would equally be a matter of insignificance. It seemed to him that unless the actual date of the beginning of the widowhood could be ascertained the method of tabulation adopted was quite as good as any they could expect, or hope to get.

The PRESIDENT (Mr. W. Hughes) said he should like to emphasize one or two things that had been said, especially as to the matter of accuracy of the data, on which some doubt had been expressed. He should think that it was very probable that 45 years ago or thereabouts, when the Russian War ceased, there was so much enthusiasm about providing for the wounded soldiers and sailors, that some essential details were overlooked, and there was not sufficient care taken to secure accurate data, and that ages and other statistical matters were entered a little haphazard. They had reason to believe that at the conclusion of the Transvaal War the data obtained were more accurate, and that in any future investigation they would have some surer ground to go upon. A remark had been made as to this being a very particular investigation, and one which, if used hereafter for the valuation of military funds of any kind,

must be used with the greatest possible caution. That would be especially apparent when they came to consider that the army which was engaged in the Crimea in 1855 consisted of a very different class of men from those of which the army now consisted, and in consequence the wives of those soldiers were of a very different class from the widows that were left by the late Transvaal War, or that were likely to be left in any future war. At the time of the Crimean War they were all soldiers of the regular army, but on the recent occasion there were, and probably on any future war in which the British army was concerned there would be, very large contingents of Yeomanry and Volunteers, as well as a much better class of soldiers than the men who served under Lord Raglan in the Crimea. That fact would make it additionally necessary that there should be caution in using the old experience and attempting to apply it to new facts.

Mr. J. BURN, in reply, hoped that in the figures connected with the Transvaal War fuller particulars would be given, and that they would be able to deal with the data in a more scientific manner. Mr. Schooling had mentioned that very great care should be exercised in dealing with the rate of re-marriage in any particular class of widows. With that, of course, they agreed. They had hoped that they had sufficiently emphasized that point. Mr. Ackland had asked why the widows who came on the fund after 1857 were excluded from the select tables. The reason of their exclusion was that they considered that the widows who came on up to 1856 were probably beneficiaries who were included among the members of the fund almost immediately upon the death of their husbands; but that those who came on after 1857 and after the termination of the war were probably, or very possibly, widows whose husbands had died some time previously, and that, therefore, they could not really be considered as "select" widows—that is to say, the date of widowhood was, perhaps, in those cases uncertain, but that really did not affect the actual rate of re-marriage. Mr. Ackland had thought that probably it had the effect of lessening the denominator of the fraction, but that was not so. Mr. Ackland had also said that he was struck with the difference between the rate of re-marriage shown amongst the Scottish Bankers' widows and the Patriotic Fund widows. Possibly there were several reasons which might be given, but the only one which occurred to him at the moment was that it was generally known that the rate of re-marriage in Scotland was rather less than the rate of re-marriage in England. Of course, that would not fully account for the very great difference shown. Mr. Collins had mentioned a point with regard to the central age, and other speakers had also referred to it. He had hoped that by grouping quinquennially they would be able to avoid any difficulty as to the decennial ages 30, 40, and so on. As to the widows of the railway workers and the rate of re-marriage appearing to cease almost entirely after age 50, Mr. Watson had reminded them of the fact that the benefit being entirely discontinued in the event of re-marriage would greatly alter the rate of re-marriage shown, but he (Mr. Burn) believed that in any investigation where the rate of re-marriage involved an extra benefit, in the same way as in the valuation of this fund, it would be advisable to entirely neglect the rate of re-marriage after age 50. With regard to the special class of

annuitants dealt with, and the remark which Mr. Watson had made in reference thereto, he might say that one of the chief reasons of the paper was to give a complete record of the information which the Commissioners had placed at their disposal, and that they were aware that the facts set out there would seldom be suitable for use in their present form, but he hoped that they might provide some guide to other investigations of a similar kind. He should not think of adopting those rates of re-marriage in any other investigation, even if he was dealing with a class of life which might be very similar to the Patriotic Fund widows, without first making a careful investigation of the facts. Speaking with regard to Table 13, dealing with the death rates of husbands, he failed to understand why the rate was so very high at the lower ages. Of course it must be understood that it was the mortality amongst the husbands, and it referred to the age of the widows. It was quite possible that the widows, re-marrying at the age of 23, married men who were so considerably older than themselves that they would show the high rate of mortality indicated in that column, and it was for the purpose of getting some explanations of that extraordinary result that Table 14 was got out. Taking the average age of widows who re-married either bachelors or widowers at 23, the average age of the husband was over four years in excess. In connection with Table 10, Mr. Watson referred to the forfeitures on account of insanity and several other causes, and he intimated that the rates of mortality shown in the table were perhaps rather doubtful owing to the exclusion of what were technically called the very bad lives. But he would point out that there were really very few of those cases, and he doubted whether the effect of them was very important.

The British Offices Life Tables, 1893. Memorandum on the Graduation of the Whole-Life Without-Profit Mortality Table—Male Lives. By GEORGE FRANCIS HARDY, Fellow of the Institute of Actuaries.

[This Memorandum was prepared for the private information of the Joint Committee of the Institute of Actuaries and the Faculty of Actuaries on Mortality Investigation, but, in view of its importance and general interest, the Committee have kindly agreed that it should be published in the *Journal* of the Institute and the *Transactions* of the Faculty. Since the Memorandum was submitted to the Committee, its value has been much enhanced by the calculation and inclusion of a table of annuity-values and single and annual premiums at 3 per-cent interest forming Table No. XVIII of this reprint.—ED. J.I.A.]

1. **AT** the request of the Committee, I undertook to consider the graduation of the Whole-Life Without-Profit Experience, and I now have the pleasure to submit the result of my investigation.

2. In the graduation of this experience, it has been borne in mind that the main practical purpose to which the table will be

put will be the calculation of premiums, rather than the determination of reserves. It is most important, therefore, that the graduation should not under-estimate the mortality, and, consequently, the premiums.

3. A comparison of the ungraduated curtate expectations of life led to the conclusion that a junction of the select tables for individual ages at entry with the truncated aggregate table could be satisfactorily effected after five years from entry. In fact, the mortality of the fifth year of assurance is already higher than that of the aggregate table, excluding the first five years from entry.

4. It seemed desirable that the graduation should be, as far as possible, on similar lines to those adopted for the $O^{M(5)}$ and $O^{[M]}$ Tables. A trial graduation was therefore made by Makeham's formula, employing the same value ($\cdot 039$) for $\log_{10} c$ as in those tables; and, as the results appear sufficiently close to the original facts for all practical purposes, it is suggested for adoption.

5. Dealing first with the graduation of the ultimate table (given on pages 504-5 of the "Whole-Life Assurance Experience—Unadjusted Data"), the constants A and B in $\mu_x = A + Bc^x$ were found in the following manner:

From the unadjusted data, the number of lives exposed to risk in the middle of each year of age was ascertained, and then multiplied by the proper value of $c^{x+\frac{1}{2}}$. On the assumption that $\mu_{x+\frac{1}{2}} = m_x$, values of A and B were then found from the equations:

$$A \Sigma E_{x+\frac{1}{2}} + B \Sigma E_{x+\frac{1}{2}} c^{x+\frac{1}{2}} = \Sigma \theta_x \quad . \quad . \quad . \quad . \quad (1)$$

$$A \Sigma^2 E_{x+\frac{1}{2}} + B \Sigma^2 E_{x+\frac{1}{2}} c^{x+\frac{1}{2}} = \Sigma^2 \theta_x \quad . \quad . \quad . \quad . \quad (2)$$

The ages included ranged from 20-89 inclusive, the data beyond age 89 being excluded upon the same grounds as in the case of the With-Profit Experience.

6. From these values of A and B ($\cdot 006\,619\,293$ and $\cdot 000\,108\,883$ respectively) $\text{colog } p_x$, and hence q_x , were obtained, and also the value of the ratio $\mu_{x+\frac{1}{2}} \div m_x$ (where $m_x = q_x \div \overline{1 - \frac{1}{2}q_x}$). By means of this ratio, the error in the total number of expected deaths caused by the assumption in (1) and (2) that $\mu_{x+\frac{1}{2}} = m_x$ was ascertained. The values of $\Sigma \theta_x$ and $\Sigma^2 \theta_x$ in (1) and (2) were then increased, respectively, by the first and second summations of the difference between the expected deaths deduced by means of $\mu_{x+\frac{1}{2}}$ and m_x , and final values of A and B were determined from these modified equations.

7. The following tables show that the agreement between the graduated and ungraduated figures may be considered sufficiently close.

TABLE I.

AGGREGATE TABLE, EXCLUDING FIRST FIVE YEARS' EXPERIENCE.

Combined "Old" and "New" Assurances.

$$A = .006\ 607\ 555$$

$$B = .000\ 108\ 987$$

$$\mu_x = A + Bc^x$$

$$\alpha = .002\ 869\ 625$$

$$\beta = .000\ 049\ 523$$

$$\text{Col } p_x = a + \beta c^x$$

Group of Ages	Expected Deaths	Actual Deaths	Expected Deviations ±	DEVIATIONS		ACCUMULATED DEVIATIONS	
				+	-	+	-
20-4	10.6	9	3	1.6	...	1.6	...
25-9	65.0	69	6	...	4.0	...	2.4
30-4	206.3	205	11	1.3	1.1
35-9	373.8	369	15	4.8	...	3.7	...
40-4	561.7	588	19	...	26.3	...	22.6
45-9	797.5	801	22	...	3.5	...	26.1
50-4	1,060.8	1,064	26	...	3.2	...	29.3
55-9	1,402.5	1,399	30	3.5	25.8
60-4	1,804.9	1,752	33	52.9	...	27.1	...
65-9	2,169.1	2,164	36	5.1	...	32.2	...
70-4	2,260.7	2,216	37	44.7	...	76.9	...
75-9	1,892.1	1,965	33	...	72.9	4.0	...
80-4	1,211.3	1,237	25	...	25.7	...	21.7
85-9	516.1	494	16	22.14	...
Totals	14,332.4	14,332	±312	136.0	135.6	145.9	129.0
				±271.6		±274.9	

TABLE II.

Comparison of Expectations of Life (Mean of Five Values).

Group of Ages	QNM ⁽⁵⁾		Deviation (G-U)		QNM ⁽¹⁰⁾	QM ⁽⁵⁾	HM ⁽⁵⁾
	Un-graduated	Graduated	+	-	Un-graduated		
25-9	35.72	35.80	.08	...	36.08	36.77	35.39
30-4	32.19	32.21	.02	...	32.29	33.07	32.04
35-9	28.55	28.60	.05	...	28.52	29.38	28.53
40-4	25.03	25.03	25.06	25.71	25.02
45-9	21.57	21.5205	21.58	22.12	21.55
50-4	18.17	18.1403	18.18	18.67	18.20
55-9	14.99	14.9405	14.97	15.40	15.02
60-4	12.03	12.0003	12.00	12.38	12.05
65-9	9.35	9.35	9.31	9.68	9.41
70-4	7.01	7.06	.05	...	7.00	7.32	7.05
75-9	5.07	5.14	.07	...	5.08	5.35	5.15

Average deviation, irrespective of sign ±.039, or, with signs +.01.

8. The graduation of the mortality tables for the first five years after entry was effected in the following manner: Values of the constants A_t and B_t were determined separately for each year of assurance from the unadjusted data by means of equations similar to (1) and (2) given above. The expected deaths were then found by multiplying the exposed to risk in the middle of each year of age by the proper value of $\mu_{[x]+t+\frac{1}{2}}$, the deaths due to the A_t constant being separated from those derived from B_t , *e.g.*, total expected deaths = $\sum E_{[x]+t+\frac{1}{2}} A_t + \sum E_{[x]+t+\frac{1}{2}} B_t c^{x+t+\frac{1}{2}}$. These may be termed the "Expected A Deaths" and the "Expected B Deaths." It was then ascertained what the expected A deaths and B deaths would have been had the constants A and B of the ultimate table $O^{NM(5)}$ been adopted. The results are shown in the following table, where the difference between the A and B deaths "expected" by the ultimate and by the select constants have been termed for short, "A Deaths Saved" and "B Deaths Saved."

TABLE III.

Year of Assurance	EXPECTED A DEATHS		A Deaths Saved (3)-(2)	EXPECTED B DEATHS		B Deaths Saved (6)-(5)
	Select Constant A_t	Ultimate Constant A		Select Constant B_t	Ultimate Constant B	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	156.6	290.8	134.2	205.4	291.6	86.2
1	222.1	255.8	33.7	213.9	284.2	70.3
2	186.7	232.8	46.1	230.3	285.3	55.0
3	195.6	214.0	18.4	267.4	289.6	22.2
4	237.3	201.3	-36.0	284.7	294.2	9.5
			196.4			243.2

9. The following diagrams represent in graphic form the figures in columns (4) and (7) of Table III, and also the curves which were ultimately adopted for the purpose of obtaining the necessary deductions to be made from the values of A and B applicable to the ultimate table in order to obtain the values of A_t and B_t in $\mu_{[x]+t}$.

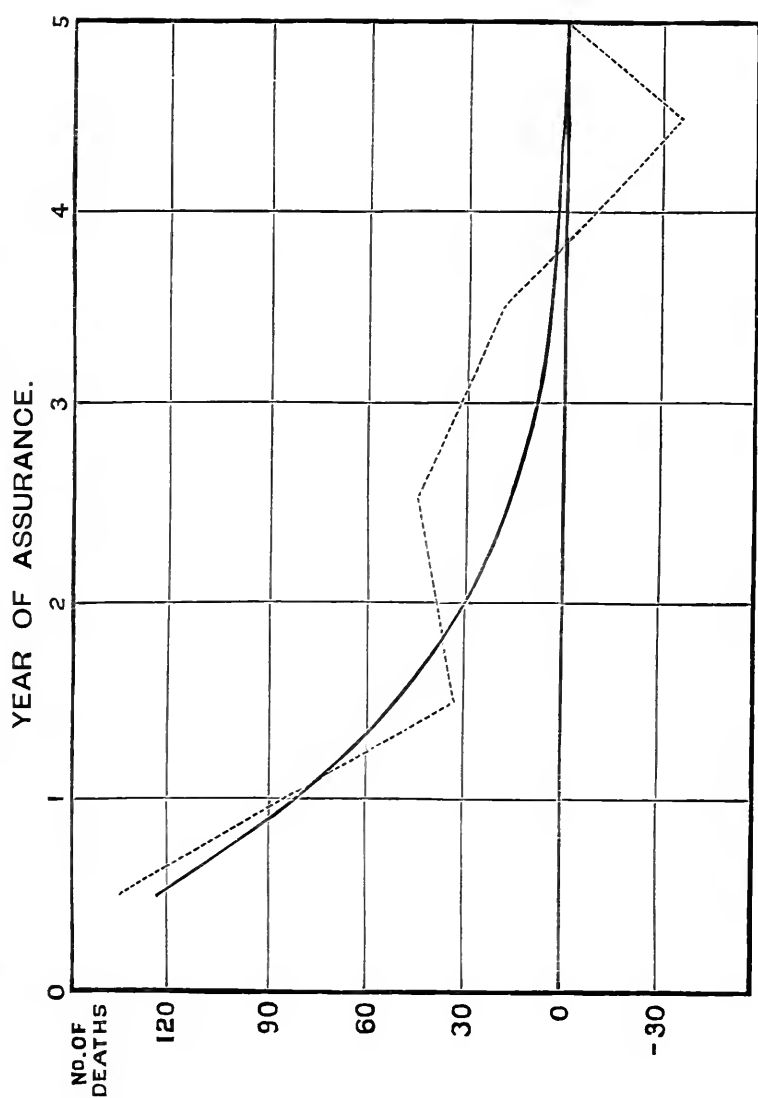
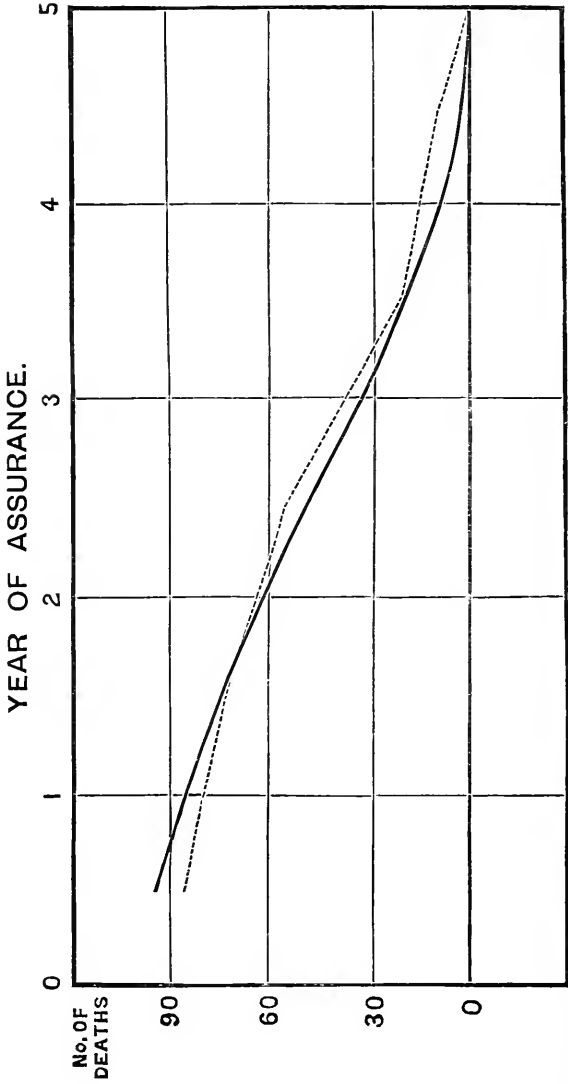


Diagram showing difference between Expected B_t Deaths in the first 5 years of Assurance and Expected B Deaths by the Ultimate ($ONM^{(5)}$) Table.



10. The extremely irregular curve representing the A deaths was not very amenable to graduation, and obviously could only be dealt with by throwing into the earlier years of assurance some of the excessive mortality of the fifth year. This proceeding will not practically affect the values of Whole-Life Premiums, and will not affect materially the Term Premiums for 1, 3, or 5 years, or longer. The formula adopted for the graduation of the A deaths saved (col. (4) in Table III), considered as a function of the duration t , was $(4\frac{1}{2}-t)^3 \cdot m \cdot E_{t+\frac{1}{2}}$, $E_{t+\frac{1}{2}}$ representing the sum of the exposed to risk $(t+\frac{1}{2})$ years after entry, for all ages at entry from 15-75 inclusive, the value of m being determined from the equation:

$$m \sum_{t=0}^{t=4} [(4\frac{1}{2}-t)^3 E_{t+\frac{1}{2}}] = 196 \cdot 4 \quad . \quad . \quad . \quad (3)$$

Inserting the values for $t=0, 1, 2, 3$ and 4 in (3) the graduated A deaths saved were found (see Table IV).

11. A rough graduation of the B deaths saved showed that these could be represented by a curve (having a single point of inflexion) of the form $(ay^2 + by^3) \cdot B \cdot E_{t+\frac{1}{2}}$, or

$$[a(4\frac{1}{2}-t)^2 + b(4\frac{1}{2}-t)^3] \cdot B \cdot E_{t+\frac{1}{2}} \quad . \quad . \quad . \quad (4)$$

To avoid an anomalous progression in the rates of mortality, the value of (4) should not be increasing at the moment of entry (when $t=-\frac{1}{2}$), and it should also vanish when $t=4\frac{1}{2}$. As the nature of the curve requires as high a commencing numerical value of the B deaths saved as is permissible, $[a(4\frac{1}{2}-t)^2 + b(4\frac{1}{2}-t)^3]$ was made a maximum when $t=-\frac{1}{2}$; i.e.:

$$\frac{d}{dt} [a(4\frac{1}{2}-t)^2 + b(4\frac{1}{2}-t)^3] = 0$$

$$\text{or} \quad -2(4\frac{1}{2}-t)a - 3(4\frac{1}{2}-t)^2b = 0$$

whence (putting $t=-\frac{1}{2}$), $a = -7\frac{1}{2}b$.

We then have (see Table III) the following equation for determining b :

$$\begin{aligned} -b[\{291 \cdot 6 \times 3 \times (4\frac{1}{2})^2\} + \{284 \cdot 2 \times 4 \times (3\frac{1}{2})^2\} \\ + \dots + \{294 \cdot 2 \times 7 \times (\frac{1}{2})^2\}] = 243 \cdot 2 \quad . \quad . \quad (5) \end{aligned}$$

whence $b = -\cdot 005408$ and $a = \cdot 04056$.

TABLE V.
Graduation Functions.

t	A_t	B_t	a_t	β_t	$\log_{10} B_t$	$\log_{10} \beta_t$	$\log_{10} a_t$	$\log_{10} q_t$
0	·0027201	·000072150	·0016236	·000033393	5·8582344	5·5236554	5·06308466	— ·000573064
1	·0046172	·000075981	·0022787	·000036503	5·8807039	5·5622286	5·0620915	— ·000554370
2	·0057679	·000085117	·0026501	·000041183	5·9300141	5·6147180	5·0626825	— ·000540126
3	·0063588	·000093621	·0028190	·000045873	5·9823642	5·6615571	5·0629019	— ·000531381
4	·0065765	·000105156	·0028662	·000048933	4·0218353	5·6896018	5·0629525	— ·000527074
5 (or more)	·0066076	·000108987	·0028696	·000049523	4·0373764	5·6948069	5·0629560	— ·000527087

ABSOLUTE CONSTANTS.

$$c = 1·0639564; \log_{10} c = ·039; \log_e c = ·0898008; \log_{10} s - a_5 = -·0028696.$$

13. The expected deaths for the first five years of assurance, computed from the values of q given in Table XIV, are as follows :

TABLE VI.

Expected Deaths (Ordinary Type), Actual Deaths (Black Type).

Grouped Ages at Date of Assurance	YEARS ELAPSED SINCE DATE OF ASSURANCE					Total for each Group of Ages at Date of Assurance
	0	1	2	3	4	
15-9	2.8 5	3.4 6	3.6 6	3.5 4	3.4 11	16.7 32
20-4	19.4 25	22.5 19	23.3 24	22.5 14	21.6 20	109.3 102
25-9	34.1 31	40.6 44	42.6 33	42.3 32	41.3 30	200.9 170
30-4	35.7 26	42.2 46	45.1 34	45.8 45	46.2 57	215.0 208
35-9	37.8 34	44.1 47	48.1 40	49.5 54	50.1 62	229.6 237
40-4	38.0 37	44.2 54	48.3 49	51.5 62	52.8 61	234.8 263
45-9	38.3 41	43.9 48	48.5 53	52.2 52	54.0 57	236.9 251
50-4	40.3 49	45.8 35	51.5 42	56.0 52	59.3 65	252.9 243
55-9	39.6 43	44.2 49	50.1 45	55.9 60	59.2 65	249.0 262
60-4	36.9 26	41.1 49	45.9 56	50.2 48	53.3 50	227.4 229
65-9	24.3 25	26.1 27	29.4 19	32.8 24	33.8 30	146.4 125
70-4	13.2 20	13.7 9	15.1 15	15.6 13	16.3 13	73.9 70
75	1.1 ...	1.2 3	1.4 1	1.7 3	1.7 1	7.1 8
Total for each year of Assurance	361.5 362	413.0 436	452.9 417	479.5 463	493.0 522	2199.9 2200

14. The deviations from the expected and actual deaths are shown below :

TABLE VII.

Expected Deaths less Actual Deaths.(+ *Errors in Ordinary Type.*) (− *Errors in Black Type.*)

Grouped Ages at Date of Assurance	YEARS ELAPSED SINCE DATE OF ASSURANCE					Total for each Group of Ages at Date of Assurance
	0	1	2	3	4	
15-9	− 2·2	− 2·6	− 2·4	− ·5	− 7·6	− 15·3
20-4	− 5·6	+ 3·5	− ·7	+ 8·5	+ 1·6	+ 7·3
25-9	+ 3·1	− 3·4	+ 9·6	+ 10·3	+ 11·3	+ 30·9
30-4	+ 9·7	− 3·8	+ 11·1	+ ·8	− 10·8	+ 7·0
35-9	+ 3·8	− 2·9	+ 8·1	− 4·5	− 11·9	− 7·4
40-4	+ 1·0	− 9·8	− ·7	− 10·5	− 8·2	− 28·2
45-9	− 2·7	− 4·1	− 4·5	+ ·2	− 3·0	− 14·1
50-4	− 8·7	+ 10·8	+ 9·5	+ 4·0	− 5·7	+ 9·9
55-9	− 3·4	− 4·8	+ 5·1	− 4·1	− 5·8	− 13·0
60-4	+ 10·9	− 7·9	− 10·1	+ 2·2	+ 3·3	− 1·6
65-9	− ·7	− ·9	+ 10·4	+ 8·8	+ 3·8	+ 21·4
70-4	− 6·8	+ 4·7	+ ·1	+ 2·6	+ 3·3	+ 3·9
75	+ 1·1	− 1·8	+ ·4	− 1·3	+ ·7	− ·9
Total	− ·5	− 23·0	+ 35·9	+ 16·5	− 29·0	− ·1

15. It will be noticed that the actual deaths for the group of ages 15-19 at date of assurance are greatly in excess of the expected deaths for each of the first five years of assurance, and it seems probable, therefore, that Makeham's Law is not applicable to the graduation of this Experience for ages less than, say, 20 at date of assurance. Having regard to this fact, and to the paucity of the data at the younger entry ages, it has been thought desirable to start the select table at age 20, and the ultimate table at age 25. Strictly speaking, the constants should perhaps be redetermined from the data as thus curtailed, but as the difference would be insignificant (very slightly reducing the graduated rates of mortality at the younger ages), it was not considered necessary to make this redetermination. The radix of the final mortality table has therefore been made $l_{[20]}=100,000$, the values of the various functions being given in Tables VIII-XVII.

16. I have, in conclusion, to express my thanks to Mr. H. J. Baker, who has kindly been responsible for the numerical work connected with the graduation, and has also assisted me in preparing this Memorandum for the use of the Committee.

March 1904.

G. F. HARDY.

TABLE VIII.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	l_x	l_{x+1}	l_{x+2}	l_{x+3}	l_{x+4}	l_{x+5}	
20	100,000	99,580	99,003	98,333	97,616	96,879	25
21	99,264	98,844	98,267	97,596	96,877	96,137	26
22	98,530	98,109	97,530	96,857	96,135	95,392	27
23	97,794	97,369	96,790	96,115	95,389	94,641	28
24	97,055	96,630	96,048	95,370	94,639	93,886	29
25	96,316	95,887	95,302	94,619	93,884	93,124	30
26	95,567	95,135	94,547	93,862	93,122	92,355	31
27	94,818	94,382	93,791	93,100	92,353	91,578	32
28	94,059	93,618	93,023	92,327	91,574	90,790	33
29	93,300	92,854	92,253	91,550	90,788	89,995	34
30	92,529	92,079	91,472	90,763	89,991	89,187	35
31	91,749	91,294	90,682	89,964	89,183	88,365	36
32	90,962	90,500	89,879	89,154	88,361	87,529	37
33	90,163	89,693	89,066	88,330	87,525	86,678	38
34	89,355	88,877	88,241	87,494	86,674	85,810	39
35	88,536	88,048	87,402	86,642	85,806	84,924	40
36	87,700	87,203	86,545	85,771	84,918	84,016	41
37	86,850	86,342	85,670	84,881	84,010	83,085	42
38	85,981	85,459	84,775	83,971	83,079	82,130	43
39	85,092	84,559	83,861	83,039	82,124	81,148	44
40	84,188	83,639	82,924	82,081	81,141	80,136	45
41	83,257	82,693	81,962	81,096	80,131	79,095	46
42	82,300	81,720	80,969	80,081	79,086	78,017	47
43	81,319	80,720	79,948	79,035	78,010	76,904	48
44	80,310	79,691	78,897	77,956	76,897	75,753	49
45	79,267	78,627	77,807	76,837	75,743	74,557	50
46	78,192	77,528	76,683	75,682	74,549	73,318	51
47	77,083	76,394	75,520	74,483	73,308	72,029	52
48	75,931	75,216	74,312	73,239	72,019	70,690	53
49	74,736	73,993	73,057	71,945	70,679	69,296	54
50	73,499	72,724	71,753	70,599	69,284	67,845	55
51	72,212	71,405	70,398	69,199	67,831	66,333	56
52	70,878	70,036	68,989	67,744	66,319	64,759	57
53	69,483	68,606	67,518	66,225	64,743	63,118	58
54	68,036	67,120	65,989	64,644	63,102	61,409	59
55	66,527	65,571	64,395	62,997	61,392	59,629	60
56	64,952	63,954	62,732	61,279	59,610	57,776	61
57	63,315	62,274	61,004	59,494	57,758	55,851	62
58	61,610	60,523	59,202	57,634	55,832	53,852	63
59	59,830	58,698	57,327	55,699	53,829	51,777	64

TABLE VIII—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x + 5$
	0	1	2	3	4	5 or more	
	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$	$l_{[x]+4}$	l_{x+5}	
60	57,982	56,802	55,380	53,693	51,756	49,632	65
61	56,060	54,831	53,357	51,611	49,608	47,414	66
62	54,059	52,784	51,259	49,456	47,389	45,130	67
63	51,991	50,669	49,093	47,233	45,106	42,785	68
64	49,815	48,476	46,853	44,942	42,757	40,382	69
65	47,630	46,218	44,549	42,589	40,355	37,932	70
66	45,348	43,891	42,185	40,182	37,904	35,445	71
67	43,002	41,512	39,766	37,726	35,415	32,929	72
68	40,603	39,080	37,304	35,235	32,901	30,401	73
69	38,152	36,602	34,803	32,717	30,372	27,874	74
70	35,662	34,092	32,280	30,186	27,845	25,365	75
71	33,146	31,564	29,747	27,659	25,336	22,893	76
72	30,615	29,030	27,221	25,152	22,864	20,476	77
73	28,083	26,506	24,717	22,682	20,418	18,135	78
74	25,570	24,012	22,255	20,271	18,108	15,889	79
75	23,091	21,564	19,854	17,936	15,863	13,758	80
						11,760	81
						9,910·8	82
						8,224·5	83
						6,710·7	84
						5,375·3	85
						4,219·4	86
						3,239·5	87
						2,427·8	88
						1,771·9	89
						1,256·2	90
						862·88	91
						572·49	92
						365·70	93
						224·10	94
						131·24	95
						73·134	96
						38·599	97
						19·197	98
						8·947	99
						3·883	100
						1·559	101
						·575	102
						·193	103
						·059	104
						·016	105

TABLE IX.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\log l_{[x]}$	$\log l_{[x]+1}$	$\log l_{[x]+2}$	$\log l_{[x]+3}$	$\log l_{[x]+4}$	$\log l_{x+5}$	
20	5.00000	4.99817	4.99565	4.99270	4.98952	4.98623	25
21	4.99679	.99495	.99241	.98943	.98622	.98289	26
22	.99357	.99171	.98914	.98613	.98288	.97951	27
23	.99031	.98842	.98583	.98279	.97950	.97608	28
24	.98702	.98511	.98249	.97941	.97607	.97260	29
25	.98370	.98176	.97910	.97598	.97259	.96906	30
26	.98031	.97834	.97565	.97249	.96905	.96546	31
27	.97689	.97489	.97216	.96895	.96545	.96179	32
28	.97340	.97136	.96859	.96533	.96177	.95804	33
29	.96988	.96780	.96498	.96166	.95803	.95422	34
30	.96623	.96416	.96129	.95791	.95420	.95030	35
31	.96260	.96044	.95752	.95407	.95028	.94628	36
32	.95886	.95665	.95366	.95014	.94626	.94215	37
33	.95503	.95276	.94971	.94611	.94213	.93791	38
34	.95112	.94879	.94567	.94198	.93789	.93354	39
35	.94712	.94472	.94152	.93773	.93352	.92903	40
36	.94300	.94053	.93724	.93334	.92900	.92436	41
37	.93877	.93622	.93283	.92881	.92433	.91952	42
38	.93440	.93176	.92827	.92413	.91949	.91450	43
39	.92989	.92716	.92356	.91928	.91447	.90928	44
40	.92525	.92241	.91868	.91424	.90924	.90383	45
41	.92042	.91747	.91361	.90900	.90380	.89815	46
42	.91540	.91233	.90832	.90353	.89810	.89219	47
43	.91019	.90698	.90281	.89782	.89215	.88595	48
44	.90477	.90141	.89706	.89185	.88591	.87940	49
45	.89909	.89557	.89102	.88557	.87934	.87249	50
46	.89316	.88946	.88470	.87899	.87244	.86521	51
47	.88696	.88306	.87806	.87206	.86515	.85751	52
48	.88042	.87631	.87106	.86474	.85745	.84936	53
49	.87353	.86919	.86366	.85700	.84929	.84071	54
50	.86628	.86168	.85584	.84880	.84063	.83152	55
51	.85861	.85373	.84756	.84010	.83143	.82173	56
52	.85051	.84532	.83878	.83087	.82164	.81130	57
53	.84188	.83636	.82942	.82102	.81119	.80015	58
54	.83274	.82685	.81947	.81053	.80004	.78823	59
55	.82300	.81671	.80885	.79932	.78811	.77546	60
56	.81259	.80587	.79749	.78731	.77532	.76175	61
57	.80151	.79431	.78536	.77447	.76161	.74703	62
58	.78965	.78192	.77234	.76068	.74688	.73120	63
59	.77692	.76862	.75836	.74585	.73102	.71414	64

TABLE IX—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\log l_{[x]}$	$\log l_{[x]+1}$	$\log l_{[x]+2}$	$\log l_{[x]+3}$	$\log l_{[x]+4}$	$\log l_{x+5}$	
60	4.76329	4.75436	4.74335	4.72992	4.71396	4.69576	65
61	4.74865	4.73903	4.72719	4.71274	4.69555	4.67591	66
62	4.73287	4.72250	4.70977	4.69422	4.67568	4.65447	67
63	4.71593	4.70474	4.69102	4.67425	4.65423	4.63129	68
64	4.69762	4.68553	4.67074	4.65265	4.63101	4.60619	69
65	4.67788	4.66481	4.64884	4.62930	4.60590	4.57901	70
66	4.65656	4.64241	4.62516	4.60403	4.57869	4.54955	71
67	4.63349	4.61817	4.59951	4.57664	4.54919	4.51758	72
68	4.60856	4.59195	4.57175	4.54698	4.51721	4.48289	73
69	4.58152	4.56350	4.54162	4.51477	4.48247	4.44520	74
70	4.55221	4.53265	4.50893	4.47981	4.44474	4.40424	75
71	4.52043	4.49919	4.47345	4.44184	4.40374	4.35970	76
72	4.48594	4.46285	4.43491	4.40058	4.35916	4.31125	77
73	4.44844	4.42334	4.39299	4.35569	4.31065	4.25851	78
74	4.40773	4.38042	4.34743	4.30687	4.25786	4.20109	79
75	4.36345	4.33373	4.29785	4.25373	4.20038	4.13855	80
						4.07039	81
						3.99611	82
						4.91511	83
						4.82677	84
						4.73040	85
						4.62525	86
						4.51048	87
						4.38521	88
						4.24843	89
						4.09907	90
						2.93595	91
						4.75777	92
						4.56312	93
						4.35045	94
						4.11806	95
						1.86412	96
						4.58658	97
						4.28324	98
						0.95166	99
						4.58921	100
						4.19296	101
						1.75976	102
						4.28612	103
						2.76825	104
						4.20199	105

TABLE X.
WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.
Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x + 5$
	0	1	2	3	4	5 or more	
	$d_{[x]}$	$d_{[x]+1}$	$d_{[x]+2}$	$d_{[x]+3}$	$d_{[x]+4}$	d_{x+5}	
20	420	577	670	717	737	742	25
21	420	577	671	719	740	745	26
22	421	579	673	722	743	751	27
23	425	579	675	726	748	755	28
24	425	582	678	731	753	762	29
25	429	585	683	735	760	769	30
26	432	588	685	740	767	777	31
27	436	591	691	747	775	788	32
28	441	595	696	753	784	795	33
29	446	601	703	762	793	808	34
30	450	607	709	772	804	822	35
31	455	612	718	781	818	836	36
32	462	621	725	793	832	851	37
33	470	627	736	805	847	868	38
34	478	636	747	820	864	886	39
35	488	646	760	836	882	908	40
36	497	658	774	853	902	931	41
37	508	672	789	871	925	955	42
38	522	684	804	892	949	982	43
39	533	698	822	915	976	1,012	44
40	549	715	843	940	1,005	1,041	45
41	564	731	866	965	1,036	1,078	46
42	580	751	888	995	1,069	1,113	47
43	599	772	913	1,025	1,106	1,151	48
44	619	794	941	1,059	1,144	1,196	49
45	640	820	970	1,094	1,186	1,239	50
46	664	845	1,001	1,133	1,231	1,289	51
47	689	874	1,037	1,175	1,279	1,339	52
48	715	904	1,073	1,220	1,329	1,394	53
49	743	936	1,112	1,266	1,383	1,451	54
50	775	971	1,154	1,315	1,439	1,512	55
51	807	1,007	1,199	1,368	1,498	1,574	56
52	842	1,047	1,245	1,425	1,560	1,641	57
53	877	1,088	1,293	1,482	1,625	1,709	58
54	916	1,131	1,345	1,542	1,693	1,780	59
55	956	1,176	1,398	1,605	1,763	1,853	60
56	998	1,222	1,453	1,669	1,834	1,925	61
57	1,041	1,270	1,510	1,736	1,907	1,999	62
58	1,087	1,321	1,568	1,802	1,980	2,075	63
59	1,132	1,371	1,628	1,870	2,052	2,145	64

TABLE X—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1893-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$d_{[x]}$	$d_{[x]+1}$	$d_{[x]+2}$	$d_{[x]+3}$	$d_{[x]+4}$	d_{x+5}	
60	1,180	1,422	1,687	1,937	2,124	2,218	65
61	1,229	1,474	1,746	2,003	2,194	2,284	66
62	1,275	1,525	1,803	2,067	2,259	2,345	67
63	1,322	1,576	1,860	2,127	2,321	2,403	68
64	1,369	1,623	1,911	2,185	2,375	2,450	69
65	1,412	1,669	1,960	2,234	2,423	2,487	70
66	1,454	1,709	2,003	2,278	2,459	2,516	71
67	1,490	1,746	2,040	2,311	2,486	2,528	72
68	1,523	1,776	2,069	2,334	2,500	2,527	73
69	1,550	1,799	2,086	2,345	2,498	2,509	74
70	1,570	1,812	2,094	2,341	2,480	2,472	75
71	1,582	1,817	2,088	2,323	2,443	2,417	76
72	1,585	1,809	2,069	2,288	2,388	2,341	77
73	1,577	1,789	2,035	2,234	2,313	2,246	78
74	1,558	1,757	1,984	2,163	2,219	2,131	79
75	1,527	1,710	1,918	2,073	2,105	1,998	80
						1,849·2	81
						1,686·3	82
						1,513·8	83
						1,335·4	84
						1,155·9	85
						979·9	86
						811·7	87
						655·9	88
						515·7	89
						393·32	90
						290·39	91
						206·79	92
						141·60	93
						92·86	94
						58·106	95
						34·535	96
						19·402	97
						10·250	98
						5·064	99
						2·324	100
						·934	101
						·382	102
						·134	103
						·043	104
						·016	105

TABLE XI.
WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.
Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\log d_{[x]}$	$\log d_{[x]+1}$	$\log d_{[x]+2}$	$\log d_{[x]+3}$	$\log d_{[x]+4}$	$\log d_{x+5}$	
20	2·62325	2·76118	2·82607	2·85552	2·86747	2·87040	25
21	·62325	·76118	·82672	·85673	·86923	·87216	26
22	·62428	·76268	·82802	·85854	·87099	·87564	27
23	·62839	·76268	·82930	·86094	·87390	·87795	28
24	·62839	·76492	·83123	·86392	·87680	·88196	29
25	·63246	·76716	·83142	·86629	·88081	·88593	30
26	·63548	·76938	·83569	·86923	·88480	·89042	31
27	·63949	·77159	·83948	·87332	·88930	·89653	32
28	·64444	·77452	·84261	·87680	·89432	·90037	33
29	·64933	·77887	·84696	·88196	·89927	·90741	34
30	·65321	·78319	·85065	·88762	·90526	·91487	35
31	·65801	·78675	·85612	·89265	·91275	·92221	36
32	·66464	·79309	·86034	·89927	·92012	·92993	37
33	·67210	·79727	·86688	·90580	·92788	·93852	38
34	·67943	·80346	·87332	·91381	·93651	·94743	39
35	·68842	·81023	·88081	·92221	·94547	·95809	40
36	·69636	·81823	·88874	·93095	·95521	·96895	41
37	·70586	·82737	·89708	·94002	·96614	·98000	42
38	·71767	·83506	·90526	·95036	·97727	·99211	43
39	·72673	·84386	·91487	·96142	·98945	3·00518	44
40	·73957	·85431	·92583	·97313	3·00217	·01745	45
41	·75128	·86392	·93752	·98453	·01536	·03262	46
42	·76343	·87564	·94841	·99782	·02898	·04650	47
43	·77743	·88762	·96047	3·01072	·04376	·06108	48
44	·79169	·89982	·97359	·02490	·05843	·07773	49
45	·80618	·91381	·98677	·03902	·07408	·09307	50
46	·82217	·92686	3·00043	·05423	·09026	·11025	51
47	·83822	·94151	·01578	·07004	·10687	·12678	52
48	·85431	·95617	·03060	·08636	·12353	·14426	53
49	·87099	·97128	·04610	·10243	·14082	·16167	54
50	·88930	·98722	·06221	·11893	·15806	·17955	55
51	·90687	3·00303	·07882	·13609	·17551	·19700	56
52	·92531	·01995	·09517	·15381	·19312	·21511	57
53	·94300	·03663	·11160	·17085	·21085	·23274	58
54	·96190	·05346	·12872	·18808	·22866	·25042	59
55	·98046	·07041	·14551	·20548	·24625	·26788	60
56	·99913	·08707	·16227	·22246	·26340	·28443	61
57	3·01745	·10380	·17898	·23955	·28035	·30081	62
58	·03623	·12090	·19535	·25575	·29667	·31702	63
59	·05385	·13704	·21165	·27184	·31218	·33143	64

TABLE XI—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\log d_{[x]}$	$\log d_{[x]+1}$	$\log d_{[x]+2}$	$\log d_{[x]+3}$	$\log d_{[x]+4}$	$\log d_{x+5}$	
60	3.07188	3.15290	3.22712	3.28713	3.32715	3.34596	65
61	.08955	.16850	.24204	.30168	.34124	.35870	66
62	.10551	.18327	.25600	.31534	.35392	.37014	67
63	.12123	.19756	.26951	.32777	.36568	.38075	68
64	.13640	.21032	.28126	.33945	.37566	.38917	69
65	.14983	.22246	.29226	.34908	.38435	.39568	70
66	.16256	.23274	.30168	.35755	.39076	.40071	71
67	.17319	.24204	.30963	.36380	.39550	.40278	72
68	.18270	.24944	.31576	.36810	.39794	.40261	73
69	.19033	.25503	.31931	.37014	.39759	.39950	74
70	.19590	.25816	.32098	.36940	.39445	.39305	75
71	.19921	.25935	.31973	.36605	.38792	.38328	76
72	.20003	.25744	.31576	.35946	.37803	.36940	77
73	.19783	.25261	.30856	.34908	.36418	.35141	78
74	.19257	.24477	.29754	.33506	.34616	.32858	79
75	.18384	.23300	.28285	.31660	.32325	.30060	80
						.26698	81
						.22693	82
						.18007	83
						.12561	84
						.06292	85
						2.99118	86
						.90940	87
						.81684	88
						.71240	89
						.59475	90
						.46298	91
						.31553	92
						.15106	93
						1.96783	94
						.76422	95
						.53826	96
						.28785	97
						.01072	98
						0.70449	99
						.36624	100
						1.99300	101
						.58206	102
						.12710	103
						2.63347	104
						.20412	105

TABLE XII.
WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.
Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x + 5$
	0	1	2	3	4	5 or more	
	$P[x]$	$P[x]+1$	$P[x]+2$	$P[x]+3$	$P[x]+4$	$P[x]+5$	
20	·99580	·99421	·99323	·99270	·99245	·99234	25
21	·99577	·99417	·99316	·99264	·99236	·99225	26
22	·99573	·99410	·99309	·99254	·99227	·99213	27
23	·99566	·99405	·99302	·99245	·99216	·99202	28
24	·99561	·99399	·99293	·99234	·99204	·99188	29
25	·99554	·99389	·99284	·99222	·99190	·99174	30
26	·99547	·99383	·99275	·99211	·99177	·99159	31
27	·99541	·99373	·99264	·99197	·99161	·99140	32
28	·99531	·99364	·99252	·99184	·99145	·99124	33
29	·99522	·99353	·99238	·99168	·99127	·99101	34
30	·99513	·99341	·99225	·99149	·99106	·99079	35
31	·99504	·99330	·99209	·99131	·99083	·99054	36
32	·99492	·99314	·99193	·99111	·99058	·99028	37
33	·99479	·99300	·99175	·99088	·99033	·98999	38
34	·99465	·99284	·99154	·99063	·99003	·98967	39
35	·99449	·99266	·99131	·99035	·98971	·98930	40
36	·99433	·99245	·99106	·99006	·98937	·98892	41
37	·99415	·99222	·99079	·98974	·98899	·98851	42
38	·99394	·99200	·99051	·98937	·98858	·98805	43
39	·99373	·99174	·99019	·98899	·98812	·98753	44
40	·99348	·99145	·98983	·98855	·98762	·98701	45
41	·99323	·99115	·98944	·98810	·98707	·98637	46
42	·99296	·99081	·98903	·98757	·98648	·98573	47
43	·99264	·99044	·98858	·98703	·98583	·98503	48
44	·99229	·99003	·98808	·98642	·98512	·98422	49
45	·99193	·98958	·98753	·98576	·98435	·98338	50
46	·99152	·98910	·98694	·98503	·98349	·98243	51
47	·99106	·98855	·98628	·98422	·98256	·98141	52
48	·99058	·98798	·98555	·98335	·98154	·98028	53
49	·99006	·98735	·98478	·98240	·98044	·97906	54
50	·98946	·98664	·98392	·98136	·97924	·97771	55
51	·98883	·98589	·98297	·98023	·97791	·97627	56
52	·98812	·98505	·98195	·97897	·97647	·97465	57
53	·98737	·98415	·98084	·97762	·97490	·97293	58
54	·98653	·98315	·97963	·97614	·97317	·97102	59
55	·98562	·98206	·97830	·97452	·97129	·96892	60
56	·98465	·98089	·97683	·97277	·96924	·96667	61
57	·98356	·97960	·97524	·97082	·96699	·96421	62
58	·98236	·97818	·97351	·96872	·96454	·96148	63
59	·98107	·97665	·97161	·96643	·96188	·95856	64

TABLE XII—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x + 5$
	0	1	2	3	4	5 or more	
	$P[x]$	$P[x]+1$	$P[x]+2$	$P[x]+3$	$P[x]+4$	$P[x]+5$	
60	·97965	·97497	·96955	·96392	·95896	·95532	65
61	·97809	·97311	·96728	·96119	·95578	·95183	66
62	·97641	·97111	·96483	·95821	·95234	·94803	67
63	·97456	·96890	·96212	·95495	·94855	·94384	68
64	·97255	·96652	·95920	·95139	·94445	·93933	69
65	·97035	·96390	·95600	·94755	·93996	·93442	70
66	·96794	·96106	·95251	·94332	·93510	·92903	71
67	·96534	·95794	·94870	·93875	·92980	·92323	72
68	·96248	·95455	·94456	·93375	·92402	·91687	73
69	·95936	·95087	·94005	·92832	·91776	·91000	74
70	·95596	·94685	·93515	·92242	·91096	·90253	75
71	·95227	·94245	·92980	·91601	·90357	·89444	76
72	·94822	·93769	·92400	·90903	·89555	·88565	77
73	·94384	·93250	·91770	·90149	·88687	·87615	78
74	·93905	·92685	·91084	·89328	·87747	·86588	79
75	·93386	·92070	·90340	·88440	·86730	·85475	80
						·84279	81
						·82985	82
						·81594	83
						·80100	84
						·78496	85
						·76777	86
						·74943	87
						·72983	88
						·70899	89
						·68688	90
						·66347	91
						·63878	92
						·61282	93
						·58561	94
						·55726	95
						·52779	96
						·49735	97
						·46604	98
						·43406	99
						·40156	100
						·36881	101
						·33602	102
						·30348	103
						·27148	104
						·00000	105

TABLE XIII.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x + 5$
	0	1	2	3	4	5 or more	
	$\log p_{[x]}$	$\log p_{[x]+1}$	$\log p_{[x]+2}$	$\log p_{[x]+3}$	$\log p_{[x]+4}$	$\log p_{x+5}$	
20	1.99817	1.99748	1.99705	1.99682	1.99671	1.99666	25
21	.99816	.99746	.99702	.99679	.99667	.99662	26
22	.99814	.99743	.99699	.99675	.99663	.99657	27
23	.99811	.99741	.99696	.99671	.99658	.99652	28
24	.97809	.99738	.99692	.99666	.99653	.99646	29
25	.99806	.99734	.99688	.99661	.99647	.99640	30
26	.99803	.99731	.99684	.99656	.99641	.99633	31
27	.99800	.99727	.99679	.99650	.99634	.99625	32
28	.99796	.99723	.99674	.99644	.99627	.99618	33
29	.99792	.99718	.99668	.99637	.99619	.99608	34
30	.99788	.99713	.99662	.99629	.99610	.99598	35
31	.99784	.99708	.99655	.99621	.99600	.99587	36
32	.99779	.99701	.99648	.99612	.99589	.99576	37
33	.99773	.99695	.99640	.99602	.99578	.99563	38
34	.99767	.99688	.99631	.99591	.99565	.99549	39
35	.99760	.99680	.99621	.99579	.99551	.99533	40
36	.99753	.99671	.99610	.99566	.99536	.99516	41
37	.99745	.99661	.99598	.99552	.99519	.99498	42
38	.99736	.99651	.99586	.99536	.99501	.99478	43
39	.99727	.99640	.99572	.99519	.99481	.99455	44
40	.99716	.99627	.99556	.99500	.99459	.99432	45
41	.99705	.99614	.99539	.99480	.99435	.99404	46
42	.99693	.99599	.99521	.99457	.99409	.99376	47
43	.99679	.99583	.99501	.99433	.99380	.99345	48
44	.99664	.99565	.99479	.99406	.99349	.99309	49
45	.99648	.99545	.99455	.99377	.99315	.99272	50
46	.99630	.99524	.99429	.99345	.99277	.99230	51
47	.99610	.99500	.99400	.99309	.99236	.99185	52
48	.99589	.99475	.99368	.99271	.99191	.99135	53
49	.99566	.99447	.99334	.99229	.99142	.99081	54
50	.99540	.99416	.99296	.99183	.99089	.99021	55
51	.99512	.99383	.99254	.99133	.99030	.98957	56
52	.99481	.99346	.99209	.99077	.98966	.98885	57
53	.99448	.99306	.99160	.99017	.98896	.98808	58
54	.99411	.99262	.99106	.98951	.98819	.98723	59
55	.99371	.99214	.99047	.98879	.98735	.98629	60
56	.99328	.99162	.98982	.98801	.98643	.98528	61
57	.99280	.99105	.98911	.98714	.98542	.98417	62
58	.99227	.99042	.98834	.98620	.98432	.98294	63
59	.99170	.98974	.98749	.98517	.98312	.98162	64

TABLE XIII—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [<i>x</i>]	DURATION						Age attained <i>x</i> + 5
	0	1	2	3	4	5 or more	
	$\log p_{[x]}$	$\log p_{[x]+1}$	$\log p_{[x]+2}$	$\log p_{[x]+3}$	$\log p_{[x]+4}$	$\log p_{x+5}$	
60	·199107	·198899	·198657	·198404	·198180	·198015	65
61	·99038	·98816	·98555	·98281	·98036	·97856	66
62	·98963	·98727	·98445	·98146	·97879	·97682	67
63	·98881	·98628	·98323	·97998	·97706	·97490	68
64	·98791	·98521	·98191	·97836	·97518	·97282	69
65	·98693	·98403	·98046	·97660	·97311	·97054	70
66	·98585	·98275	·97887	·97466	·97086	·96803	71
67	·98468	·98134	·97713	·97255	·96839	·96531	72
68	·98339	·97980	·97523	·97023	·96568	·96231	73
69	·98198	·97812	·97315	·96770	·96273	·95904	74
70	·98044	·97628	·97088	·96493	·95950	·95546	75
71	·97876	·97426	·96839	·96190	·95596	·95155	76
72	·97691	·97206	·96567	·95858	·95209	·94726	77
73	·97490	·96965	·96270	·95496	·94786	·94258	78
74	·97269	·96701	·95944	·95099	·94323	·93746	79
75	·97028	·96412	·95588	·94665	·93817	·93184	80
						·92572	81
						·91900	82
						·91166	83
						·90363	84
						·89485	85
						·88523	86
						·87473	87
						·86322	88
						·85064	89
						·83688	90
						·82182	91
						·80535	92
						·78733	93
						·76761	94
						·74606	95
						·72246	96
						·69666	97
						·66842	98
						·63755	99
						·60375	100
						·56680	101
						·52636	102
						·48213	103
						·43374	104
						...	105

TABLE XIV.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$q[x]$	$q[x]+1$	$q[x]+2$	$q[x]+3$	$q[x]+4$	$q[x]+5$	
20	·00420	·00579	·00677	·00730	·00755	·00766	25
21	·00423	·00583	·00684	·00736	·00764	·00775	26
22	·00427	·00590	·00691	·00746	·00773	·00787	27
23	·00434	·00595	·00698	·00755	·00784	·00798	28
24	·00439	·00601	·00707	·00766	·00796	·00812	29
25	·00446	·00611	·00716	·00778	·00810	·00826	30
26	·00453	·00617	·00725	·00789	·00823	·00841	31
27	·00459	·00627	·00736	·00803	·00839	·00860	32
28	·00469	·00636	·00748	·00816	·00855	·00876	33
29	·00478	·00647	·00762	·00832	·00873	·00899	34
30	·00487	·00659	·00775	·00851	·00894	·00921	35
31	·00496	·00670	·00791	·00869	·00917	·00946	36
32	·00508	·00686	·00807	·00889	·00942	·00972	37
33	·00521	·00700	·00825	·00912	·00967	·01001	38
34	·00535	·00716	·00846	·00937	·00997	·01033	39
35	·00551	·00734	·00869	·00965	·01029	·01070	40
36	·00567	·00755	·00894	·00994	·01063	·01108	41
37	·00585	·00778	·00921	·01026	·01101	·01149	42
38	·00606	·00800	·00949	·01063	·01142	·01195	43
39	·00627	·00826	·00981	·01101	·01188	·01247	44
40	·00652	·00855	·01017	·01145	·01238	·01299	45
41	·00677	·00885	·01056	·01190	·01293	·01363	46
42	·00704	·00919	·01097	·01243	·01352	·01427	47
43	·00736	·00956	·01142	·01297	·01417	·01497	48
44	·00771	·00997	·01192	·01358	·01488	·01578	49
45	·00807	·01042	·01247	·01424	·01565	·01662	50
46	·00848	·01090	·01306	·01497	·01651	·01757	51
47	·00894	·01145	·01372	·01578	·01744	·01859	52
48	·00942	·01202	·01445	·01665	·01846	·01972	53
49	·00994	·01265	·01522	·01760	·01956	·02094	54
50	·01054	·01336	·01608	·01864	·02076	·02229	55
51	·01117	·01411	·01703	·01977	·02209	·02373	56
52	·01188	·01495	·01805	·02103	·02353	·02535	57
53	·01263	·01585	·01916	·02238	·02510	·02707	58
54	·01347	·01685	·02037	·02386	·02683	·02898	59
55	·01438	·01794	·02170	·02548	·02871	·03108	60
56	·01535	·01911	·02317	·02723	·03076	·03333	61
57	·01644	·02040	·02476	·02918	·03301	·03579	62
58	·01764	·02182	·02649	·03128	·03546	·03852	63
59	·01893	·02335	·02839	·03357	·03812	·04144	64

TABLE XIV—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$q[x]$	$q[x]+1$	$q[x]+2$	$q[x]+3$	$q[x]+4$	q_{x+5}	
60	·02035	·02503	·03045	·03608	·04104	·04468	65
61	·02191	·02689	·03272	·03881	·04422	·04817	66
62	·02359	·02889	·03517	·04179	·04766	·05197	67
63	·02544	·03110	·03788	·04505	·05145	·05616	68
64	·02745	·03348	·04080	·04861	·05555	·06067	69
65	·02965	·03610	·04400	·05245	·06004	·06558	70
66	·03206	·03894	·04749	·05668	·06490	·07097	71
67	·03466	·04206	·05130	·06125	·07020	·07677	72
68	·03752	·04545	·05544	·06625	·07598	·08313	73
69	·04064	·04913	·05995	·07168	·08224	·09000	74
70	·04404	·05315	·06485	·07758	·08904	·09747	75
71	·04773	·05755	·07020	·08399	·09643	·10556	76
72	·05178	·06231	·07600	·09097	·10445	·11435	77
73	·05616	·06750	·08230	·09851	·11313	·12385	78
74	·06095	·07315	·08916	·10672	·12253	·13412	79
75	·06614	·07930	·09660	·11560	·13270	·14525	80
						·15721	81
						·17015	82
						·18406	83
						·19900	84
						·21504	85
						·23223	86
						·25057	87
						·27017	88
						·29101	89
						·31312	90
						·33653	91
						·36122	92
						·38718	93
						·41439	94
						·44274	95
						·47221	96
						·50265	97
						·53396	98
						·56594	99
						·59844	100
						·63119	101
						·66398	102
						·69652	103
						·72852	104
						1·00000	105

TABLE XV.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x + 5$
	0	1	2	3	4	5 or more	
	$\log q_{[x]}$	$\log q_{[x]+1}$	$\log q_{[x]+2}$	$\log q_{[x]+3}$	$\log q_{[x]+4}$	$\log q_{x+5}$	
20	$\bar{3}\cdot62325$	$\bar{3}\cdot76268$	$\bar{3}\cdot83059$	$\bar{3}\cdot86332$	$\bar{3}\cdot87795$	$\bar{3}\cdot88423$	25
21	$\cdot62634$	$\cdot76567$	$\cdot83506$	$\cdot86688$	$\cdot88309$	$\cdot88930$	26
22	$\cdot63043$	$\cdot77085$	$\cdot83948$	$\cdot87274$	$\cdot88818$	$\cdot89597$	27
23	$\cdot63749$	$\cdot77452$	$\cdot84386$	$\cdot87795$	$\cdot89432$	$\cdot90200$	28
24	$\cdot64246$	$\cdot77887$	$\cdot84942$	$\cdot88423$	$\cdot90091$	$\cdot90956$	29
25	$\cdot64933$	$\cdot78604$	$\cdot85491$	$\cdot89098$	$\cdot90849$	$\cdot91698$	30
26	$\cdot65610$	$\cdot79029$	$\cdot86034$	$\cdot89708$	$\cdot91540$	$\cdot92480$	31
27	$\cdot66181$	$\cdot79727$	$\cdot86688$	$\cdot90472$	$\cdot92376$	$\cdot93450$	32
28	$\cdot67117$	$\cdot80346$	$\cdot87390$	$\cdot91169$	$\cdot93197$	$\cdot94250$	33
29	$\cdot67943$	$\cdot81090$	$\cdot88196$	$\cdot92012$	$\cdot94101$	$\cdot95376$	34
30	$\cdot68753$	$\cdot81889$	$\cdot88930$	$\cdot92993$	$\cdot95134$	$\cdot96426$	35
31	$\cdot69548$	$\cdot82607$	$\cdot89818$	$\cdot93902$	$\cdot96237$	$\cdot97589$	36
32	$\cdot70586$	$\cdot83632$	$\cdot90687$	$\cdot94890$	$\cdot97405$	$\cdot98767$	37
33	$\cdot71684$	$\cdot84510$	$\cdot91645$	$\cdot95999$	$\cdot98543$	$\cdot2\cdot00043$	38
34	$\cdot72835$	$\cdot85491$	$\cdot92737$	$\cdot97174$	$\cdot99870$	$\cdot01410$	39
35	$\cdot74115$	$\cdot86570$	$\cdot93902$	$\cdot98453$	$\bar{2}\cdot01242$	$\cdot02938$	40
36	$\cdot75358$	$\cdot87795$	$\cdot95134$	$\cdot99739$	$\cdot02653$	$\cdot04454$	41
37	$\cdot76716$	$\cdot89098$	$\cdot96426$	$\bar{2}\cdot01115$	$\cdot04179$	$\cdot06032$	42
38	$\cdot78247$	$\cdot90309$	$\cdot97727$	$\cdot02653$	$\cdot05767$	$\cdot07737$	43
39	$\cdot79727$	$\cdot91698$	$\cdot99167$	$\cdot04179$	$\cdot07482$	$\cdot09587$	44
40	$\cdot81425$	$\cdot93197$	$\bar{2}\cdot00732$	$\cdot05581$	$\cdot09272$	$\cdot11361$	45
41	$\cdot83059$	$\cdot94694$	$\cdot02366$	$\cdot07555$	$\cdot11160$	$\cdot13450$	46
42	$\cdot84757$	$\cdot96332$	$\cdot04021$	$\cdot09447$	$\cdot13098$	$\cdot15442$	47
43	$\cdot86688$	$\cdot98046$	$\cdot05767$	$\cdot11294$	$\cdot15137$	$\cdot17522$	48
44	$\cdot88705$	$\cdot99870$	$\cdot07628$	$\cdot13290$	$\cdot17260$	$\cdot19811$	49
45	$\cdot90687$	$\bar{2}\cdot01787$	$\cdot09587$	$\cdot15351$	$\cdot19451$	$\cdot22063$	50
46	$\cdot92840$	$\cdot03743$	$\cdot11594$	$\cdot17522$	$\cdot21775$	$\cdot24477$	51
47	$\cdot95134$	$\cdot05881$	$\cdot13735$	$\cdot19811$	$\cdot24155$	$\cdot26928$	52
48	$\cdot97405$	$\cdot07990$	$\cdot15987$	$\cdot22141$	$\cdot26623$	$\cdot29491$	53
49	$\cdot99739$	$\cdot10209$	$\cdot18241$	$\cdot24551$	$\cdot29137$	$\cdot32098$	54
50	$\bar{2}\cdot02284$	$\cdot12581$	$\cdot20629$	$\cdot27045$	$\cdot31723$	$\cdot34811$	55
51	$\cdot04805$	$\cdot14953$	$\cdot23121$	$\cdot29601$	$\cdot34420$	$\cdot37530$	56
52	$\cdot07482$	$\cdot17464$	$\cdot25648$	$\cdot32284$	$\cdot37162$	$\cdot40398$	57
53	$\cdot10140$	$\cdot20003$	$\cdot28240$	$\cdot34986$	$\cdot39967$	$\cdot43249$	58
54	$\cdot12937$	$\cdot22660$	$\cdot30899$	$\cdot37767$	$\cdot42862$	$\cdot46210$	59
55	$\cdot15776$	$\cdot25382$	$\cdot33646$	$\cdot40620$	$\cdot45803$	$\cdot49248$	60
56	$\cdot18611$	$\cdot28126$	$\cdot36493$	$\cdot43505$	$\cdot48799$	$\cdot52284$	61
57	$\cdot21590$	$\cdot30963$	$\cdot39375$	$\cdot46509$	$\cdot51865$	$\cdot55376$	62
58	$\cdot24650$	$\cdot33885$	$\cdot42308$	$\cdot49527$	$\cdot54974$	$\cdot58569$	63
59	$\cdot27715$	$\cdot36829$	$\cdot45317$	$\cdot52595$	$\cdot58115$	$\cdot61742$	64

TABLE XV—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

[illegible]

TABLE XVI.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [<i>x</i>]	DURATION						Age attained <i>x</i> + 5
	0	1	2	3	4	5 or more	
	μ_x	μ_{x+1}	μ_{x+2}	μ_{x+3}	μ_{x+4}	μ_{x+5}	
20	·00316	·00512	·00638	·00712	·00748	·00764	25
21	·00320	·00517	·00644	·00719	·00757	·00773	26
22	·00324	·00522	·00650	·00727	·00766	·00784	27
23	·00329	·00527	·00657	·00735	·00776	·00796	28
24	·00334	·00533	·00665	·00744	·00788	·00808	29
25	·00340	·00540	·00673	·00755	·00800	·00822	30
26	·00347	·00548	·00682	·00766	·00813	·00837	31
27	·00354	·00556	·00692	·00778	·00828	·00854	32
28	·00361	·00564	·00703	·00791	·00844	·00872	33
29	·00370	·00574	·00715	·00806	·00861	·00892	34
30	·00379	·00585	·00728	·00822	·00880	·00913	35
31	·00389	·00596	·00742	·00839	·00901	·00937	36
32	·00400	·00609	·00757	·00858	·00924	·00963	37
33	·00412	·00623	·00774	·00879	·00949	·00992	38
34	·00425	·00638	·00793	·00902	·00977	·01023	39
35	·00439	·00654	·00813	·00927	·01007	·01057	40
36	·00455	·00672	·00835	·00955	·01039	·01094	41
37	·00472	·00692	·00859	·00985	·01075	·01134	42
38	·00491	·00714	·00886	·01017	·01114	·01179	43
39	·00512	·00738	·00915	·01053	·01157	·01228	44
40	·00534	·00764	·00947	·01092	·01204	·01281	45
41	·00559	·00792	·00981	·01135	·01256	·01339	46
42	·00586	·00823	·01019	·01182	·01312	·01403	47
43	·00615	·00857	·01061	·01233	·01373	·01473	48
44	·00647	·00894	·01107	·01290	·01441	·01549	49
45	·00682	·00935	·01156	·01351	·01514	·01632	50
46	·00721	·00979	·01211	·01418	·01595	·01723	51
47	·00763	·01028	·01270	·01492	·01683	·01823	52
48	·00809	·01081	·01335	·01572	·01779	·01933	53
49	·00860	·01139	·01407	·01660	·01885	·02052	54
50	·00915	·01203	·01485	·01756	·02000	·02183	55
51	·00975	·01272	·01570	·01862	·02126	·02326	56
52	·01042	·01348	·01663	·01977	·02264	·02482	57
53	·01114	·01432	·01765	·02103	·02415	·02653	58
54	·01193	·01523	·01877	·02241	·02580	·02840	59
55	·01280	·01622	·01999	·02391	·02761	·03045	60
56	·01374	·01731	·02133	·02556	·02958	·03269	61
57	·01478	·01851	·02279	·02737	·03174	·03514	62
58	·01591	·01981	·02439	·02934	·03411	·03782	63
59	·01715	·02124	·02614	·03150	·03670	·04076	64

TABLE XVI—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\mu[x]$	$\mu[x]+1$	$\mu[x]+2$	$\mu[x]+3$	$\mu[x]+4$	$\mu[x]+5$	
60	·01850	·02280	·02805	·03386	·03953	·04397	65
61	·01999	·02451	·03015	·03645	·04262	·04748	66
62	·02161	·02638	·03244	·03927	·04601	·05132	67
63	·02339	·02842	·03494	·04236	·04971	·05552	68
64	·02533	·03066	·03768	·04575	·05377	·06011	69
65	·02745	·03311	·04068	·04945	·05820	·06511	70
66	·02977	·03578	·04396	·05350	·06305	·07064	71
67	·03232	·03871	·04755	·05793	·06836	·07665	72
68	·03510	·04192	·05148	·06277	·07416	·08324	73
69	·03814	·04542	·05577	·06807	·08051	·09043	74
70	·04147	·04926	·06047	·07387	·08746	·09831	75
71	·04511	·05345	·06561	·08021	·09506	·10693	76
72	·04909	·05804	·07123	·08715	·10337	·11635	77
73	·05345	·06306	·07738	·09474	·11247	·12666	78
74	·05821	·06855	·08411	·10304	·12242	·13794	79
75	·06343	·07455	·09147	·11213	·13330	·15028	80
						·16378	81
						·17855	82
						·19470	83
						·21238	84
						·23171	85
						·25286	86
						·27599	87
						·30131	88
						·32890	89
						·35928	90
						·39242	91
						·42867	92
						·46833	93
						·51171	94
						·55916	95
						·61108	96
						·66787	97
						·73000	98
						·79797	99
						·89233	100
						·95367	101
						1·04265	102
						1·13999	103
						1·24648	104
						1·36297	105

TABLE XVII.
WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.
Male Lives. Without Participation in Profits.

Age at Entry [x]	DURATION						Age attained $x + 5$
	0	1	2	3	4	5 or more	
	$\log \mu_{[x]}$	$\log \mu_{[x]+1}$	$\log \mu_{[x]+2}$	$\log \mu_{[x]+3}$	$\log \mu_{[x]+4}$	$\log \mu_{x+5}$	
20	$\bar{3}\cdot49969$	$\bar{3}\cdot70927$	$\bar{3}\cdot80482$	$\bar{3}\cdot85248$	$\bar{3}\cdot87390$	$\bar{3}\cdot88309$	25
21	$\cdot50515$	$\cdot71349$	$\cdot80889$	$\cdot85673$	$\cdot87910$	$\cdot88818$	26
22	$\cdot51055$	$\cdot71767$	$\cdot81291$	$\cdot86153$	$\cdot88423$	$\cdot89432$	27
23	$\cdot51720$	$\cdot72181$	$\cdot81757$	$\cdot86629$	$\cdot88986$	$\cdot90091$	28
24	$\cdot52375$	$\cdot72673$	$\cdot82282$	$\cdot87157$	$\cdot89653$	$\cdot90741$	29
25	$\cdot53148$	$\cdot73239$	$\cdot82802$	$\cdot87795$	$\cdot90309$	$\cdot91487$	30
26	$\cdot54033$	$\cdot73878$	$\cdot83378$	$\cdot88423$	$\cdot91009$	$\cdot92273$	31
27	$\cdot54900$	$\cdot74507$	$\cdot84011$	$\cdot89098$	$\cdot91803$	$\cdot93146$	32
28	$\cdot55751$	$\cdot75128$	$\cdot84696$	$\cdot89818$	$\cdot92634$	$\cdot94052$	33
29	$\cdot56820$	$\cdot75891$	$\cdot85431$	$\cdot90634$	$\cdot93500$	$\cdot95036$	34
30	$\cdot57864$	$\cdot76716$	$\cdot86213$	$\cdot91487$	$\cdot94448$	$\cdot96047$	35
31	$\cdot58995$	$\cdot77525$	$\cdot87040$	$\cdot92376$	$\cdot95472$	$\cdot97174$	36
32	$\cdot60206$	$\cdot78462$	$\cdot87910$	$\cdot93349$	$\cdot96567$	$\cdot98363$	37
33	$\cdot61490$	$\cdot79449$	$\cdot88874$	$\cdot94399$	$\cdot97727$	$\cdot99651$	38
34	$\cdot62839$	$\cdot80482$	$\cdot89927$	$\cdot95521$	$\cdot98989$	$\bar{2}\cdot00988$	39
35	$\cdot64246$	$\cdot81558$	$\cdot91009$	$\cdot96708$	$\bar{2}\cdot00303$	$\cdot02408$	40
36	$\cdot65801$	$\cdot82737$	$\cdot92169$	$\cdot98000$	$\cdot01662$	$\cdot03902$	41
37	$\cdot67394$	$\cdot84011$	$\cdot93399$	$\cdot99344$	$\cdot03141$	$\cdot05461$	42
38	$\cdot69108$	$\cdot85370$	$\cdot94743$	$\bar{2}\cdot00732$	$\cdot04689$	$\cdot07151$	43
39	$\cdot70927$	$\cdot86806$	$\cdot96142$	$\cdot02243$	$\cdot06333$	$\cdot08920$	44
40	$\cdot72754$	$\cdot88309$	$\cdot97635$	$\cdot03822$	$\cdot08063$	$\cdot10755$	45
41	$\cdot74741$	$\cdot89873$	$\cdot99167$	$\cdot05500$	$\cdot09899$	$\cdot12678$	46
42	$\cdot76790$	$\cdot91540$	$\bar{2}\cdot00817$	$\cdot07262$	$\cdot11793$	$\cdot14706$	47
43	$\cdot78888$	$\cdot93298$	$\cdot02572$	$\cdot09096$	$\cdot13767$	$\cdot16820$	48
44	$\cdot81090$	$\cdot95134$	$\cdot04415$	$\cdot11059$	$\cdot15866$	$\cdot19005$	49
45	$\cdot83378$	$\cdot97081$	$\cdot06296$	$\cdot13066$	$\cdot18013$	$\cdot21272$	50
46	$\cdot85794$	$\cdot99078$	$\cdot08314$	$\cdot15168$	$\cdot20276$	$\cdot23629$	51
47	$\cdot88252$	$\bar{2}\cdot01199$	$\cdot10380$	$\cdot17377$	$\cdot22608$	$\cdot26079$	52
48	$\cdot90795$	$\cdot03383$	$\cdot12548$	$\cdot19645$	$\cdot25018$	$\cdot28623$	53
49	$\cdot93450$	$\cdot05652$	$\cdot14829$	$\cdot22011$	$\cdot27531$	$\cdot31218$	54
50	$\cdot96142$	$\cdot08027$	$\cdot17173$	$\cdot24452$	$\cdot30103$	$\cdot33905$	55
51	$\cdot98900$	$\cdot10449$	$\cdot19590$	$\cdot26998$	$\cdot32756$	$\cdot36661$	56
52	$\bar{2}\cdot01787$	$\cdot12969$	$\cdot22089$	$\cdot29601$	$\cdot35488$	$\cdot39480$	57
53	$\cdot04689$	$\cdot15594$	$\cdot24674$	$\cdot32284$	$\cdot38292$	$\cdot42374$	58
54	$\cdot07664$	$\cdot18270$	$\cdot27346$	$\cdot35044$	$\cdot41162$	$\cdot45332$	59
55	$\cdot10721$	$\cdot21005$	$\cdot30081$	$\cdot37858$	$\cdot44107$	$\cdot48359$	60
56	$\cdot13799$	$\cdot23830$	$\cdot32899$	$\cdot40756$	$\cdot47100$	$\cdot51441$	61
57	$\cdot16967$	$\cdot26741$	$\cdot35774$	$\cdot43727$	$\cdot50161$	$\cdot54580$	62
58	$\cdot20167$	$\cdot29688$	$\cdot38721$	$\cdot46746$	$\cdot53288$	$\cdot57772$	63
59	$\cdot23426$	$\cdot32715$	$\cdot41731$	$\cdot49831$	$\cdot56467$	$\cdot61023$	64

TABLE XVII—continued.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Age at Entry [<i>x</i>]	DURATION						Age attained <i>x</i> + 5
	0	1	2	3	4	5 or more	
	$\log \mu_{[x]}$	$\log \mu_{[x]+1}$	$\log \mu_{[x]+2}$	$\log \mu_{[x]+3}$	$\log \mu_{[x]+4}$	$\log \mu_{x+5}$	
60	2.26717	2.35793	2.44793	2.52969	2.59693	2.64316	65
61	.30081	.38934	.47929	.56170	.62961	.67651	66
62	.33465	.42127	.51108	.59406	.66285	.71029	67
63	.36903	.45362	.54332	.62696	.69644	.74445	68
64	.40364	.48657	.57611	.66039	.73054	.77895	69
65	.43854	.51906	.60938	.69417	.76492	.81385	70
66	.47378	.55364	.64306	.72835	.79969	.84905	71
67	.50947	.58782	.67715	.76290	.83480	.88451	72
68	.54531	.62242	.71164	.79775	.87017	.92033	73
69	.58138	.65725	.74640	.83296	.90585	.95631	74
70	.61773	.69249	.78154	.86847	.94181	.99260	75
71	.65427	.72795	.81697	.90423	.97800	1.02910	76
72	.69099	.76373	.85266	.94027	1.01489	.06577	77
73	.72795	.79975	.88863	.97653	.05104	.10264	78
74	.76500	.83601	.92485	1.01301	.08785	.13969	79
75	.80229	.87245	.96128	.04972	.12483	.17690	80
						.21426	81
						.25176	82
						.28937	83
						.32711	84
						.36494	85
						.40288	86
						.44089	87
						.47901	88
						.51718	89
						.55543	90
						.59375	91
						.63212	92
						.67055	93
						.70902	94
						.74754	95
						.78610	96
						.82469	97
						.86332	98
						.90199	99
						.95053	100
						.97940	101
						0.01814	102
						.05690	103
						.09568	104
						.13449	105

TABLE XVIII.

WHOLE-LIFE ASSURANCE EXPERIENCE, 1863-1893.

Male Lives. Without Participation in Profits.

Q[NM]

3 PER-CENT.

Age at Entry [<i>x</i>]	<i>a</i> _[<i>x</i>]	<i>A</i> _[<i>x</i>]	<i>P</i> _[<i>x</i>]	Age at Entry [<i>x</i>]	<i>a</i> _[<i>x</i>]	<i>A</i> _[<i>x</i>]	<i>P</i> _[<i>x</i>]
20	21·862	·33411	·01461	50	13·887	·56641	·03805
21	21·680	·33944	·01497	51	13·543	·57642	·03964
22	21·491	·34493	·01534	52	13·195	·58656	·04132
23	21·297	·35060	·01573	53	12·846	·59672	·04310
24	21·096	·35641	·01613	54	12·494	·60697	·04498
25	20·890	·36242	·01656	55	12·141	·61728	·04698
26	20·680	·36855	·01700	56	11·786	·62760	·04908
27	20·463	·37489	·01747	57	11·430	·63794	·05132
28	20·240	·38136	·01796	58	11·074	·64834	·05370
29	20·011	·38804	·01847	59	10·719	·65867	·05621
30	19·777	·39487	·01901	60	10·363	·66902	·05883
31	19·536	·40186	·01957	61	10·009	·67936	·06171
32	19·289	·40904	·02016	62	9·657	·68959	·06471
33	19·038	·41639	·02078	63	9·307	·69981	·06790
34	18·779	·42393	·02143	64	8·959	·70992	·07128
35	18·514	·43164	·02212	65	8·615	·71995	·07488
36	18·243	·43952	·02284	66	8·275	·72988	·07870
37	17·966	·44758	·02360	67	7·938	·73966	·08275
38	17·684	·45580	·02440	68	7·606	·74934	·08707
39	17·396	·46417	·02523	69	7·280	·75884	·09165
40	17·102	·47276	·02612	70	6·959	·76819	·09652
41	16·803	·48147	·02704	71	6·644	·77736	·10170
42	16·498	·49033	·02802	72	6·335	·78636	·10721
43	16·189	·49937	·02905	73	6·034	·79513	·11304
44	15·873	·50856	·03014	74	5·739	·80371	·11926
45	15·553	·51788	·03129	75	5·452	·81206	·12586
46	15·229	·52733	·03250				
47	14·899	·53693	·03377				
48	14·565	·54665	·03512				
49	14·228	·55645	·03654				

CORRESPONDENCE.

MANCHESTER UNITY EXPERIENCE 1893-7.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The author of the review of the "Manchester Unity Experience 1893-7" has devoted part of his most interesting article to a brief criticism of the method of obtaining the "Exposed to Risk of Sickness", which, as he points out, differs from that usually followed. In order that my purpose in departing from the common plan may be fully apprehended by students of the subject, I venture to ask that space may be found in the *Journal* for the following explanation.

The Sickness rates of the Manchester Unity Investigation may, by analogy with ordinary practice, be called "central sickness rates." Symbolizing such rate at age x by s'_x the following identity is established in cases in which the *same body of data* have supplied both the sickness and mortality rates.

$$s_x = \frac{1}{2} p_x s'_x$$

The expression $\frac{1}{2} p_x s'_x$ shows that in the ordinary rate of Sickness (s_x) a certain rate of mortality at the age x is involved, and leads to the conclusion that, although the sickness per unit of actual exposure may not change, the introduction of varying death rates will produce varying rates of sickness. A good example of this is found in Sutton's Tables, page 1171, wherein it is shown that the adjusted sickness rate decreases from 45 weeks per member per annum at age 90 to 26 weeks at age 100. Premising that nearly all the lives at risk at these advanced ages were constantly sick (which appears to have been the case), it is clear that the diminishing "rate of sickness" can only arise from an increasing rate of mortality, and consequent increase in the number of fractions of exposure counted as whole units in the denominator.

Now, without discussing whether this is a satisfactory "rate of sickness", or not, I may point out that for the purpose of the new Manchester Unity Experience it was necessary to separate the two factors s'_x and $\frac{1}{2} p_x$, which make up the ordinary s_x . The rate of sickness was obtained from data grouped by occupations, but the rates of mortality came from the geographical areas, and thus with *one* value of s'_x were combined *three* values of $\frac{1}{2} p_x$, so yielding (what could not have been obtained by any other ready process of calculation) three distinct values of s_x , responding to the variations in the mortality of the several bodies at risk.

EXAMPLES.

At age 80, $s'_x = 29.777$ weeks (p. 183)—

Area 1, $\frac{1}{2} p_x = .93085$

„ 2, $\frac{1}{2} p_x = .90504$

„ 3, $\frac{1}{2} p_x = .92064$

$\frac{1}{2} p_x s'_x = s_x$ (the ordinary sickness rate)—

Area 1, $29.777 \times .93085 = 27.72$ weeks.

„ 2, $\times .90504 = 26.95$ „

„ 3, $\times .92064 = 27.41$ „

It will be seen that these differences are not altogether insignificant. Without the complete separation of the mortality and sickness factors it would have been impossible to avoid scientific incongruities in the various combinations of the data.

I am, Sir,

Yours faithfully,

ALFRED W. WATSON.

Nottingham, 19 May 1904.

To the Editor of the Journal of the Institute of Actuaries.

DEAR SIR,—My attention has been drawn to the fact that the formulæ I gave on pages 354 and 355 of vol. xxxviii (April 1904) of this *Journal*, have already been given by Mr. G. F. Hardy in a slightly different form on page 137 of the "British Offices Life Tables, Account of Principles and Methods", which was published in December last year. My result was arrived at quite independently; the part of my note referring to the H^m Table was written before the publication of the British Offices Life Annuity Tables in Feb. 1903, and on their appearance the remainder was immediately added and the manuscript sent to you in the latter half of Feb. 1903. Unfortunately, at the time of reading over the proofs, it was overlooked that Mr. G. F. Hardy had, in the interim, called attention to the property of the tables mentioned in my note.

I am,

Yours faithfully,

Liverpool, 3 June 1904.

H. W. CURJEL.

[It is due to Mr. Curjel that we should insert the above letter, with the additional explanation that his communication was originally received by us on 27 Feb. 1903, and was unavoidably held over on account of the crowded state of the *Journal*.—ED. J.I.A.]

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1903, published in 1904.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
Balance at the beginning of the Year.	£ 257,800,325	£ 23,302,992	£ 281,103,317
Adjustments, for balances transferred from Ordinary to Industrial Branch (£16,606); to other accounts (£343,513), and for one Industrial Return discontinued (£22,831)	— 360,119	— 6,225	— 366,344
	257,440,206	23,296,767	280,736,973
Premiums	22,998,588	10,265,896	33,264,484
Consideration for Annuities	2,012,131	3,420	2,015,551
Interest and Dividends (less Tax)	9,452,697	776,549	10,229,246
Increase in value of Investments	66,291	322	66,613
Fines, Fees, &c.	14,197	1,385	15,582
Capital Paid-up	25,000	10,461	35,461
Customs Timber Measuring, &c.	3,426	—	3,426
Transfers from other Accounts	12,116	71,747	83,863
Miscellaneous	11,712	11,992	23,704
	292,036,364	34,438,539	326,474,903
OUTGO	Ordinary Companies	Industrial Companies	TOTAL
Claims	£ 16,344,325	£ 3,916,391	£ 20,260,716
Cash Bonuses and Reduction of Premiums	1,056,684	...	1,056,684
Surrenders	1,392,743	50,430	1,443,173
Annuities	1,941,147	8,645	1,949,792
Commission	1,190,644	2,553,294	3,743,938
Expenses of Management	1,951,935	1,880,101	3,832,036
Bad Debts	1,354	480	1,834
Decrease in value of Investments	197,268	2,596	199,864
Interest on Capital and Dividends and Bonuses to Shareholders	323,362	534,560	857,922
Transfers to other Accounts	263,385	150,886	414,271
Miscellaneous	15,371	...	15,371
Balance* at the end of the Year	267,358,146	25,341,156	292,699,302
	292,036,364	34,438,539	326,474,903

* This Balance includes the whole of the Life and Annuity Funds (£289,136,132), and, in addition, the Capital, &c., of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1903).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . . .	11,402,841	1,827,272	13,230,113
Life and Annuity Funds . . .	264,985,953	24,150,179	289,136,132
Fire Funds of Companies trans-acting Life Business . . .	12,862,827	...	12,862,827
Marine Funds of Companies trans-acting Life Business . . .	851,537	...	851,537
Reserve Funds . . .	4,354,990	1,056,818	5,411,838
Other Funds . . .	2,675,537	240,447	2,915,984
Profit and Loss Balances . . .	4,168,389	8,335	4,176,724
Depreciation and Investment Balances . . .	1,376,284	1,805	1,378,089
Globe Annuitants (Liverpool and London) . . .	1,654,200	...	1,654,200
Outstanding Claims . . .	4,188,754	9,089	4,197,843
Outstanding Accounts . . .	719,413	49,100	768,513
Temporary Loans . . .	250,813	8,754	259,567
	309,491,538	27,351,820	336,843,367

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages . . .	86,132,193	2,691,017	88,823,210
Loans on Policies . . .	14,882,758	68,990	14,951,748
„ Rates . . .	27,096,979	10,065,991	37,162,970
British Government Securities . . .	7,708,393	2,080,034	9,788,427
Indian and Colonial Government Securities . . .	19,352,529	463,722	19,816,251
Foreign Government Securities . . .	10,435,709	376,313	10,812,022
Debentures . . .	55,746,557	2,391,425	58,137,982
Shares and Stocks . . .	37,975,099	293,273	38,268,372
Companies' own Shares . . .	622,018	...	622,018
Land and House Property and Ground Rents . . .	24,473,414	7,160,739	31,634,153
Life Interests and Reversions . . .	8,388,761	2,350	8,391,111
Loans on Personal Security . . .	1,536,794	2,687	1,539,481
Agents' Balances and Outstanding Premiums . . .	5,889,988	677,078	6,567,066
Outstanding Interest . . .	2,869,809	240,495	3,110,304
Cash, Deposits, Stamps, &c. . .	6,048,920	244,416	6,293,336
Customs Timber Measuring Balances, &c. . .	2,305	...	2,305
Deficiencies, Establishment Expenses, &c. . .	329,312	593,299	922,611
	309,491,538	27,351,829	336,843,367

INCREASE (+) or DECREASE (—) in the Chief Items of this Year's
SUMMARY as compared with the corresponding Items for the
previous Year.

	Ordinary Companies	Industrial Companies
INCOME.		
	£	£
Premiums	+ 540,166	+ 651,767
Consideration for Annuities	— 30,526	— *80,629
Interest and Dividends (less Tax)	— 332,110	+ 63,401
Net Result of Realization and Re-valuation of Investments	— 61,332	— 975
OUTGO.		
Claims	— 236,572	+ 177,965
Annuities	— 78,312	— *67,719
Surrenders	+ 100,733	— 9,322
Commission	— 4,264	+ 143,928
Expenses of Management	+ 54,738	+ 154,596
LIABILITIES.		
Paid-up Capital (including sundry Share- holders' Balances)	+ 90,731	+ 49,653
Life and Annuity Funds	+ 9,670,508	+ 2,099,702
ASSETS.		
Mortgages (including Loans on Rates)	+ 3,375,642	+ 933,097
Life Interests and Reversions	+ 271,322	+ 334
Loans on Policies	+ 878,931	+ 23,625
British Government Securities	+ 610,678	+ 28,302
Indian and Colonial Government Securities	+ 216,488	+ 197,226
Foreign Government Securities	+ 121,900	+ 37,869
Debentures	+ 3,743,713	+ 282,371
Shares and Stocks	+ 2,225,299	+ 146,611
Companies' own Shares	— 4,321	...
Land and House Property and Ground Rents	+ 782,937	+ 609,857
Loans on Personal Security	— 110,075	— 6,711

* Excluding the figures of "Nelson & Co." as returns are not yet due.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 93, of which 73 are classed as Ordinary, 11 as Industrial, and 9 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately. The accounts of the Empire Guarantee, and Profits and Income are included for the first time.

During the year five names have been removed from the Official List of Companies, namely, Bells Stores; the deposit being returned as the Memorandum of Association was altered to exclude Life Business; and the Imperial; Imperial Life; National Guardian; Universal; their business being transferred. And two names have been added, namely, Consolidated Assurance Company, Limited; and the Hearts of Oak Life and General Assurance Company, Limited; in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Companies.*
ORDINARY BUSINESS.

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances — Amount	Net Amount
	No.	Amount	No.	Amount	No.	Amount		
ASSURANCES.		£		£		£	£	£
Whole Term of Life Limited number of Premiums . . .	798,648	381,003,959	165,645	72,787,502	964,293	453,791,461	28,363,438	425,428,023
	50,897	32,253,711	11,194	4,866,095	62,091	37,119,806	2,093,359	35,026,447
	849,545	413,257,670	176,839	77,653,597	1,026,384	490,911,267	30,456,797	460,454,470
Endowments . . .	1,684	430,925	20,311	4,945,797	21,995	5,376,722	18,500	5,358,222
Endowment Assur- ances . . .	1,005,547	166,567,167	66,391	19,882,215	1,071,938	186,449,382	2,920,297	183,529,085
Joint Lives . . .	16,730	3,327,061	2,751	973,986	19,481	4,301,047	246,231	4,054,816
Last Survivor . . .	831	683,530	1,094	1,481,188	1,925	2,164,718	385,061	1,779,657
Contingent . . .	36	69,363	4,256	6,309,104	4,292	6,378,467	1,451,399	4,927,068
Issue . . .	11	25,250	1,517	5,378,359	1,528	5,403,609	1,630,734	3,772,875
Miscellaneous . . .	2,660	1,617,918	9,754	13,190,561	12,414	14,808,479	2,011,038	12,797,441
	1,877,044	585,978,884	282,913	129,814,807	2,159,957	715,793,691	39,120,057	676,673,634
ANNUITIES.								
Immediate	36,350	1,890,991	52,971	1,838,020
Deferred	14,702	403,600	22,131	381,469
	51,052	2,294,591	75,102	2,219,489

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances — Amount	Net Amount
	No.	Amount	No.	Amount	No.	Amount		
ASSURANCES.						£	£	£
Whole Term of Life Limited number of Premiums . . .	252	16,381	20,116,979	195,807,615	20,117,231	195,823,996	1,180	195,822,816
	1,336	7,999	1,336	7,999	...	7,999
	252	16,381	20,118,315	195,815,614	20,118,567	195,831,995	1,180	195,830,815
Endowments . . .	140	42,475	1,521,292	14,160,193	1,521,432	14,202,668	...	14,202,668
Endowment Assur- ances . . .	45	2,599	486,339	4,822,527	486,384	4,825,126	88	4,824,742
Joint Lives	391,657	6,276,510	391,657	6,276,510	...	6,276,510
Contingent	4	1,910	4	1,910	400	1,910
Miscellaneous	2	2,100	2	2,100	1,000	2,100
	437	61,455	22,517,609	221,078,854	22,518,046	221,140,309	2,668	221,137,641
ANNUITIES.								
Immediate	65	2,034	...	34
Deferred	14	310	...	10
	79	2,344	...	44

The above figures are based on Returns deposited, for the most part, during the last five years and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of one Company, namely, the Northern, the amount of business at a more recent date has been included. The figures of the Colonial and Foreign Companies have been excluded, as their Returns separately show the extent of business in the United Kingdom.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

No. CCXVIII.—OCTOBER 1904.



"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: 5, RUE LAMARTINE, 5.
BERLIN: CARLSTRASSE 11. MELBOURNE: MCCARRON, BIRD & Co.
NEW YORK: THE SPECTATOR COMPANY.

PRINTED BY C. & E. LAYTON, FARRINGDON STREET. E.C.
TO WHOM ALL COMMUNICATIONS FOR THE EDITOR SHOULD BE SENT, POST PAID.

Price 2s. 6d.

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VOL. XXXVIII. — PART VI.

OCTOBER 1904.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

PARIS: 8, RUE LAMARTINE 8.

BERLIN: CARLSTRASSE 11.

MELBOURNE: MCCARRON, BIRD & CO.

NEW YORK: THE SPECTATOR COMPANY.

[The Council of the Institute of Actuaries wish it to be understood, that while they consider it their duty to give, from time to time, publicity to certain of the papers presented to the Institute, and to abstracts of the discussions at the Sessional Meetings, they are not responsible for the opinions put forward therein.]

LONDON;
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGTON STREET.

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NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Valuation of Whole-Life Industrial Assurances, with allowance for Lapses. By THOMAS G. ACKLAND, F.I.A., and JAMES BACON, F.I.A.

[Read before the Institute, 28 March 1904.]

INTRODUCTION.

THE subject of lapses or secessions has received comparatively little attention from writers in the *Journal of the Institute*. The question of secessions in connection with the valuation of Friendly Societies has been incidentally dealt with by Mr. G. F. Hardy (vol. xxvii, p. 312), and Mr. A. W. Watson (vol. xxxv, p. 313). Mr. A. Hunter, in his paper on "The effect of using the Lapse Element in calculating Premiums and Reserves" (vol. xxxvi, p. 51), makes some interesting comparisons of policy reserves, with and without allowance for discontinuances, but limits his investigations to the first 10 years of assurance, with a rate of exit over that period deduced from the tabulated H^M data. One of the present writers has also contributed to the *Journal* (vol. xxxiii, p. 164) an account of the methods followed and results arrived at in the valuation, with and without allowance for lapses, of Clerks' Associations granting benefits to their members during unemployment, but the results arrived at are, of course, only applicable to those particular Societies.

There must be a rich storehouse of information in the notebooks and reports of those Actuaries who have, from time to time, made investigations of the experience, as to secessions, of Friendly Societies, Workmen's Benefit Societies, or the Superannuation Funds of large Institutions, such as Banks and Railway Companies, and who have conducted valuations of such

Societies or Funds with allowance for secessions. A notable instance was the investigation by Dr. Sprague of the Royal Liver Friendly Society, with allowance for secessions, the scale adopted being given in *J.I.A.*, vol. xxvii, p. 317. The results of these investigations are, however, seldom published, and, being confidential in their nature, are not usually available for the benefit of the profession generally, and, when so available, it is frequently found that the experience of any particular Benefit Society is quite *sui generis*, and not applicable to the case of another Society differently constituted, or even necessarily to one whose benefits and membership seem to be similar in character.

As regards the rate of lapse experienced by Life Assurance Companies, Ordinary or Industrial, the pages of the *Journal* contain, apart from the paper by Mr. A. Hunter above referred to, little or nothing to reward the search of students. The element of lapse must, however, be an important factor in some of these Institutions. In the case of many of the old-established and conservative Companies, the rate of lapse is no doubt inconsiderable. With the more progressive Companies, however, where a large new business is to some extent obtained under pressure, with a correspondingly heavy list of "non-renewals", the rate of lapse must be an important factor, demanding due consideration in investigations as to mortality and expenditure, and also, as we would submit, not altogether to be ignored in the determination of valuation reserves. In the case of the Industrial Companies, granting, without medical examination, small sums assured, mainly to working men and women, forming the general mass of the population, in consideration of weekly premiums of a few pence only, the rate of lapse is known to be very high, especially in the early years of assurance. This will perhaps be sufficiently demonstrated by reference to the Table given by Mr. C. H. E. Rea, in his paper entitled "Some Observations on Industrial Assurance" (*J.I.A.*, vol. xxxiv, p. 107). The summary statement as to Industrial Assurance Companies given in the Board of Trade Returns for 1902 also shows that, whilst the total premium income was £9,611,129, the increase for the year was only £315,082, or slightly over $3\frac{1}{4}$ per-cent on the premium income. This indicates a heavy rate of secession, having regard to the considerable amount of new business known to be transacted.

Confining our attention, in the present paper, to the case of an Industrial Assurance Office, we would submit that it is unscientific for the Actuary to ignore, in his valuation of the

prospective assets and liabilities, an element so important as that of lapse, the operation of which cannot but materially affect the actual value of those assets and liabilities. If, for example, it is practically a known fact that, in a particular Company, out of 100 existing policies, not more than 30 will be regularly renewed and mature for payment at death, whilst, in respect of the other 70 policies, the premiums will cease by lapse after different durations, and the liabilities will be reduced to a comparatively small surrender allowance on lapse (where such allowance is applicable); then it appears to us that the calculation of valuation reserves, altogether without allowance for lapses, cannot be regarded as satisfactory, or as accurately representing, in any real sense, the liabilities of the Office. It will be seen, on reference to the tables appended to this paper, that the rate of lapse experienced, even on the most favourable assumptions, is, in the early years of assurance, largely in excess of the rate of mortality; and we would suggest that a valuation which, whilst making scientific and exact allowance for the minor element of decrement (that of mortality), ignores the major element of decrement (that of lapse), cannot be considered as a true measure of the liability of a Company, especially where a heavy rate of lapse is experienced.

It is no doubt the case that the rate of lapse is more or less fluctuating, and liable to sudden and incalculable changes, both in incidence and amount; whilst the rate of mortality is, on the whole, likely to be more steady, and with less material departures from the rate assumed. But we consider that this fact does not supply a justification for the exclusion of the one element, and the inclusion of the other, in the calculation of valuation reserves; and that, in an Industrial Assurance Company whose operations are widespread, and extending over a number of years, it is quite possible to deduce from the experience of the Company rates of lapse at different ages and durations, which shall form a trustworthy basis as to the rates which may confidently be expected in the future experience of the Company, and may therefore properly be included in the valuation bases and estimates. ¶

It does not by any means necessarily follow that such inclusion of the element of lapse will, in a particular case, increase or diminish the valuation reserves; and we know of no method by which it can be mathematically proved that, even on a net premium valuation basis, the effect will on the whole be to

increase or decrease the net liability,* the variation in which will be materially affected by the incidence of mortality and lapse, and the scale of surrender allowances adopted. But, whether the reserves be increased or diminished by the introduction of a lapse rate, we would submit that only upon these lines can the true liability of the Company be determined. In certain cases, also, it appears to us most desirable to adopt this (as we think) more scientific basis, as, for example, where it is proposed to transfer the business of an existing Company to another Company; or, where, from the acceptance of insufficient premium-rates, or from depreciation of assets, wasteful expenditure, or losses arising from other causes, it is desirable to review the position of the Company from the point of view of solvency, and to apply a true measure of its liabilities.

In this connection there is also to be considered the great diversity of practice obtaining in the actual valuations of the whole life assurances of the leading Industrial Offices, as shown in the following statement, the particulars included in which are taken from the Blue Books up to and including 1902 :—

Office No.	Valuation Basis.
(1)	Net premium reserves.
+ (2)	Policies under 5 years' duration—no reserves.
	„ over „ „ —net premium at age $(x+5)$.
(3)	The premiums valued were the office premiums, less 30 per-cent.
(4)	Policies over 30 years' duration—net reserves at true entry ages.
	Policies between 15 and 30 years' duration—net premium at age $(x+5)$.
	Policies under 15 years' duration—35 per cent. off office premiums.
° (5)	Net premium reserves.
+ (6)	Policies under 5 years' duration—no reserves.
	„ over „ „ —net premium at age $(x+5)$.

This statement appears to indicate the apparent unsuitability, to the case of Industrial Offices, of the methods adopted in the

* The difficulty of a mathematical proof is, as will be readily seen, increased by the fact that the usual relation does not hold between the annuity-value ${}^w d_{\overline{x}|i}$, with allowance for lapse and mortality, and the single premium for the assurance payable on death (with allowance for lapses) ${}^{dw} \overline{A}^x$.

valuation of Companies doing ordinary business, and the desirability of some generally approved method, which shall have a close regard to the practical conditions upon which industrial business is conducted.✓

No doubt the introduction of an assumed lapse rate, based upon the Company's experience, is a delicate task, and one requiring the exercise of due judgment and caution on the part of the Actuary, especially where a valuation is made with a view to the ascertainment of surplus. And this is more particularly the case, where it is proposed to recommend a scale of office premiums for adoption, with allowance for lapses, as it is then evidently of the first importance that the rates should be such as shall not involve the Company in loss. The difficulty, and the need of caution, are still further accentuated, where it is desired to compute a scale of office premiums for an Industrial Office newly established, and where there can, in the nature of things, be no available experience as to lapses of the particular Office.

The object of the present contribution, is, then (a) to deduce, from the available experience of an Industrial Office, rates of lapse, at different entry ages and durations; (b) to show the methods by which valuation factors, including the element of lapse, may be computed, to be employed in the valuation of the assets and liabilities; (c) to apply these factors in the valuation of certain illustrative office data, upon different assumptions as to (i) the rate of interest, (ii) the rate of lapse, (iii) the provision of a margin for future expenses; (d) to compare the results of such valuations, and discuss their general bearing on the subject.

INVESTIGATION OF THE RATE OF LAPSE.

There is no great difficulty in deducing the rates of lapse experienced by an Industrial Assurance Company in a particular year, or over any convenient period, with discrimination of the entry ages of the lives, and the durations of the assurances. The data required would include: (a) cases existing at the commencement of the period, scheduled according to entry ages and durations; (b) cases entering during the period, scheduled according to entry ages, and dates of entry; (c) cases passing out of observation, whether by death, surrender, or lapse, during the period, scheduled according to entry ages, durations, and modes of exit. In some cases, cards are doubtless already employed by the Offices for recording the policy data for

valuation, or for investigations of the mortality experienced by the lives; and these cards would furnish the necessary material for the investigation of the lapse-rates experienced. Where such records are not available, the valuation schedules will usually conveniently supply the data under (a) above; but, in that case, consideration must be given to the fact that the cases are scheduled at a fixed date (usually the end of a calendar year) and not according to policy years. Some of the Industrial Offices, no doubt, make an investigation of their lapse-rate experienced, as part of the regular office routine, either periodically, or by a continuous record of the data necessary for deducing the desired results.

In the case of the experience here dealt with, there were no ready means of independently deducing, without considerable labour, and disturbance of the work of the Office, the rates of lapse experienced; and the method most readily available was found to be by comparison of the preliminary valuation schedules. Annual valuations had been made for some years past, the data for which had been recorded upon sheets of a uniform pattern. By comparison of the data entered on these sheets for successive annual valuations, as explained below, *rates of exit* were deduced, appropriate to particular entry ages and durations. The process will be clearly followed by setting out the type of schedule employed in the valuations:

WHOLE-LIFE ASSURANCES.

Valuation 1900. Present Age 38.

Year of Entry	Age at Entry	Number of Policies	Sums Assured	Office Premiums	Net Premiums
...
...
...
...
...
...
1897	35	462
1898	36
1899	37
1900	38
Totals

Valuation 1901.

Present Age 39.

Year of Entry	Age at Entry	Number of Policies	Sums Assured	Office Premiums	Net Premiums
...
...
...
...
...
...
...
1897	35	413
1898	36
1899	37
1900	38
1901	39
<hr/>					
Totals

Here it is evident that the cases entered in the 1901 schedules, in respect of any specified year of entry and age at entry, are the survivors of those entered, under the same year and entry age, in the 1900 schedules, after allowing for the operation of mortality and lapse (including surrender) during the year 1901. Thus there were, at attained age 38, at the 1900 valuation, 462 policies effected in the year 1897, at entry age 35. At the 1901 valuation, these identical cases had been reduced, by the operation of mortality and lapse during the year 1901, to 413 policies. The annual probability of surviving death and lapse at attained age 38, in respect of lives aged 35 at entry, was thus equal to $\frac{413}{462} = .894$; and the combined rate of exit would

be the complement of this value, or .106. Comparing, similarly, the valuation schedules at present ages 38 and 39, in respect of the valuations of 1897 and 1898, 1898 and 1899, and 1899 and 1900, values were deduced, for the rate of exit, of cases entering at age 35, of .112, .103, and .100 respectively. The average of the four values thus deduced, or .105, was then adopted as the unadjusted rate of exit from mortality and lapse combined, at attained age 38, in respect of lives entering at age 35.

These processes were gone through in respect of cases entering at ages 1, 5, 10, 15 70, 75, and the

combined rates of exit were deduced, at each of these entry ages, in respect of each separate year of duration passed through. It will be observed that these rates are in respect of *calendar years* of duration. Thus, for cases entering at age 35, the rates are in respect of the first, second, third, &c., years of duration, *reckoned from the end of the calendar year of entry*. These rates, not being deduced from observations according to policy-years, are thus not applicable for the calculation of net premiums; but it is submitted that, as the data from which they are taken are precisely under the conditions, as to entry and duration, usually obtaining in a valuation, the rates thus obtained may appropriately and correctly be employed in the calculation of the factors (at ages attained) to be employed in the valuation of the assurances, as at the expiration of a calendar year.

The rates deduced as above, including as they do the combined operation of mortality and secession, are appropriate (subject to graduation) for the calculation of the annuity, payable until death or lapse, to be employed in the valuation of the office or net premiums. In the calculation of the factors for the valuation of the sum assured (payable at death only), and of the surrender value (payable at lapse only), it is, however, necessary to separate the combined rates into their constituent parts, so as to ascertain the lapse rate, and the mortality rate, separately. This was effected by dividing the combined probability of surviving death and lapse at age $(x+t)$, by p_{x+t} , the probability of surviving death only, in order to obtain separately the probability of surviving lapse only. The mortality table employed was the English Life Table No. 3 (Males), and, where the mortality shown by the table used does not greatly differ from that experienced amongst the industrial assured lives, the method indicated will, we think, give a sufficiently correct indication of the ungraduated rate of lapse experienced at different entry ages and durations; although it would no doubt be more satisfactory to investigate the rate of lapse separately, rather than the rate of exit.

By the above method, the rates of lapse appropriate for the calculation of the valuation factors were deduced for every fifth age at entry, and for each successive year of duration, reckoned from the end of the calendar year of entry. The rates were then plotted out graphically, and roughly adjusted so as to exhibit, without material departure from the rates

experienced, a fairly smooth progression, when traced from a given entry age through successive years of duration, and also when compared, in respect of different entry ages, at the same point of duration. The resulting graduated rates are shown in detail in the columns marked β in Table I, and are graphically set out in the Diagram, page 569.

The next step was to deduce the graduated rates of lapse, *as arising in successive policy years*, which were required for the calculation of the net premiums to be employed in the valuation. These rates were obtained from those tabulated for calendar years as above, by observing, in the Diagram (which was originally on a much larger scale), where the curve, in respect of a given entry age, crossed the central ordinate in each calendar year of duration. This was of course equivalent to a rough method of interpolation, and after the first few years of duration did not differ greatly from the arithmetic mean of the values in respect of calendar years.

For the first year of assurance (dating from entry) the available data furnished no information as to the rate of lapse experienced in the calendar year of entry; and the method adopted was to trace the curve, showing the rate, back to the assumed date of entry, in such a way as to preserve the smoothness, and to follow the general contour, of the curve. The transition from the end of the calendar year of entry to the assumed date of entry is shown by the dotted lines in the Diagram. In thus tracing the curve back to an assumed origin, and also in determining the rate of lapse in the first two or three policy years, some allowance was made for the known tendency of the cases entering to preponderate in the second half of the calendar year of entry. The rates for durations exceeding 35, at all entry ages, and, in respect of entry age 80, for all durations, were arbitrarily deduced, so as to be generally consistent with those observed, at similar ages attained, at other entry ages and durations.

The graduated rates of lapse, as thus determined in policy years, are set out in the columns marked α , Table I, for the selected central entry ages, and successive years of duration. At each entry age above 1, the rates of lapse in columns α and β are separately tabulated for durations varying from 9 to 25 years, after which they merge in the ultimate table given, under ages attained (up to age 89), in respect of age at entry 1. The points of junction were deduced by observing the attained ages

In respect of all ages up to $(x+n-1)$ inclusive:

$$\mathbf{D}_{x+n-1} = \frac{wd}{\tau p_{x+n-1}} \mathbf{D}_{x+n}$$

$${}^{wd}\mathbf{D}_{x+n-1} = \frac{{}^{wd}\mathbf{D}_{x+n}}{{}^{wp}_{x+n-1}} \times \frac{{}^{wl}_{x+n-1}}{{}^{wl}_{x+n}} = \mathbf{D}_{x+n-1}, \quad \frac{{}^{wl}_{x+n-1}}{{}^{wl}_{x+n}}$$

$${}^{wd}D_{x+n-2} = \frac{{}^{wd}D_{x+n-1}}{vp_{x+n-2}} \times \frac{w|_{x+n-2}}{w|_{x+n-1}} = D_{x+n-2} \cdot \frac{w|_{x+n-2}}{w|_{x+n}}$$

and, generally,

$${}^{wd}D_x = D_x \times \frac{{}^w l_x}{{}^w l_{x+n}} . \quad (2)$$

$${}^{wd}\mathbf{N}_x = {}^{wd}\mathbf{D}_{x+1} + {}^{wd}\mathbf{D}_{x+2} + \dots \quad (3)$$

[illegible]

$$\begin{aligned} {}^{wd}\overline{a}_x &= {}^{wd}a_x + \frac{1}{2} - \frac{{}^{wd}\mu_x + \delta}{12} \\ &= {}^{wd}a_x + \frac{1}{2} - \frac{{}^{wd}D_{x-1} - {}^{wd}D_{x+1}}{24{}^{wd}D_x} \text{ approximately ; (5)} \end{aligned}$$

where ${}^{wd}\mu_x$ represents the *force of exit* at age x , which is equal to the sum of the forces of mortality and withdrawal.

Mortality only, with allowance for Lapse—

Similarly,

$${}^d w C_x = C_x \left(1 - \frac{w q_x}{2} \right)^{wl_{x-}} (6)$$

$${}^d w \overline{C}_x = {}^d w C_x (1 + i)^{\frac{1}{2}} \text{ approximately ; } \quad . \quad . \quad . \quad (7)$$

$${}^{dw}\overline{\mathbf{M}}_x = {}^{dw}\mathbf{C}_x + {}^{dw}\mathbf{C}_{x+1} + \dots \quad (8)$$

$${}^{dw}\bar{\mathbf{A}}_{\mathbf{x}} = \frac{{}^d w \overline{\mathbf{M}}_{\mathbf{x}}}{{}^{wd}\mathbf{D}_r} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (9)$$

Lapse only, with allowance for Mortality—

$$w^d \bar{\mathbf{M}}_x = w^d \bar{\mathbf{M}}_x - {}^d w \bar{\mathbf{M}}_x. \quad (10)$$

where ${}^{wd}\overline{\mathbf{M}}_x = (v \cdot {}^{wd}\mathbf{N}_{x-1} - {}^{wd}\mathbf{N}_x)(1+i)^{\frac{1}{2}}$ approximately; (11)

$${}^{w,d)}\overline{\mathbf{R}}_x = {}^{w,d)}\overline{\mathbf{M}}_x + {}^{w,d)}\overline{\mathbf{M}}_{x+1} + \dots \quad (12)$$

Then we have, for the value, at entry age x , of an allowance on lapse of 1 per annum (with proportionate allowance to date of lapse) reckoned from the commencement, but not to take effect until after the expiration of 5 years from entry :

$${}^{w'd}(\bar{IA})_{[x]} = \frac{{}^{w'd}\bar{R}_{[x]+5} + 4 \cdot 5 {}^{w'd}\bar{M}_{[x]+5}}{{}^{wd}D_{[x]}} \quad . \quad . \quad . \quad (13)$$

For the value, at attained age $(x+t)$, of such a future allowance, it being assumed that $(t+\frac{1}{2})$ years have elapsed since entry, we have, where t is not >5 ,

$${}^{w'd}(\bar{IA})_{[x]+t} = \frac{{}^{w'd}\bar{R}_{[x]+5} + 5 {}^{w'd}\bar{M}_{[x]+5}}{{}^{wd}D_{[x]+t}} \text{ approximately. } (14)$$

and where t is not <5 ,

$${}^{w'd}(\bar{IA})_{[x]+t} = \frac{{}^{w'd}\bar{R}_{[x]+t} + t {}^{w'd}\bar{M}_{[x]+t}}{{}^{wd}D_{[x]+t}} \quad . \quad . \quad . \quad (15)$$

Net Premiums—

$${}^{wd}\pi_{[x]} = \frac{{}^d w \bar{M}_{[x]} + ({}^{w'd}\bar{R}_{[x]+5} + 4 \cdot 5 {}^{w'd}\bar{M}_{[x]+5}) \times .25 P_{[x]}}{{}^{wd}N_{[x]}} \quad (16)$$

where P = the office annual premium for 1 assured at death, and the allowance on lapse is equal to 25 per-cent of the premiums paid.

The valuation factors computed comprise (a) the single premium, ${}^d w \bar{A}_{[x]+t}$, for the valuation, by formula (9), of the sums payable immediately on death; (b) the single premium, ${}^{w'd}(\bar{IA})_{[x]+t}$, for the valuation, by formulas (14) and (15), of the allowance payable immediately on lapse; (c) the continuous annuity, ${}^{wd}\bar{a}_{[x]+t}$, for the valuation, by formula (5), of the office and net premiums. These were each computed, at 3 per-cent interest, for entry ages $[x] = 1, 5, 10, 15 \dots 70, 75, 80$, and, in respect of each entry age, for each curtate duration (t), until merged in the ultimate Table; the rates of lapse employed being those headed β in Table I. Specimen values of the valuation factors (a) and (c) above are given, in Table II, for each year of duration, at the selected entry ages 1, 5, 15, 25, 35, 45, 55, 65 and 75, those at intermediate ages being omitted for economy of space. In Table IV are given, at the above entry ages, and for all durations over which the data for valuation extended, the values of factor (b).

In Tables III and V are given similar valuation factors, computed at 3 per-cent interest, and based throughout upon rates of lapse *one-half* of those specified in Tables II and IV.

Net Premiums were computed, by formula (16), upon the

basis of the full rates of lapse marked a in Table I, and also upon the basis of one-half of those rates throughout, for entry ages 1, 5, 10 . . . 70, 75 and 80. The net premiums are set out in Table XVII, computed at 3 per-cent on the basis of the full and half lapse rates; and also those computed at 3 and 4 per-cent, without allowance for lapse.

In computing the continuous annuities at each entry age for duration 0, the approximation to the value of $({}^{red}\mu_x + \delta)$, as given in formula (5), could not be directly arrived at; but was deduced by fourth differences from the values as computed at higher durations. This approximation was felt to be only a rough one; but time did not permit of any extended study of the question, with a view to obtaining a more satisfactory solution.

The net premiums were calculated at the office ages at entry, which, although not theoretically correct, is in accordance with the usual practice. The age at entry is thus slightly over-estimated, but the resulting effect on the premium rate must be insignificant, excepting, perhaps, at office age 1, where the mortality is exceptionally heavy, although it must be borne in mind that at this particular age, the sum assured is at a minimum.

In computing the net premiums at central entry ages 1, 5, and 10, formula (16) was suitably modified, so as to provide for the reduced sum assured paid on the death of children under 10 years of age; and formula (9) was similarly modified, in computing the single premiums for valuation purposes. The scale actually adopted was that employed with practical uniformity by all the Industrial Offices, and was as follows at entry ages 1, 5, and 10:

Duration of Assurance	AGE AT ENTRY		
	1	5	10
First 3 months	£ s. nil	£ s. nil	£ nil
3-6 "	1 10	2 10	5
6-12 "	2 10	4 10	5
1-2 years	3 0	5 0	10
2-3 "	3 10	6 0	10
3-4 "	4 0	7 0	10
4-5 "	4 10	8 0	10
5-6 "	5 0	9 0	10
6-7 "	6 0	10 0	10
7-8 "	7 0	10 0	10
8-9 "	8 0	10 0	10
9-10 "	9 0	10 0	10
10 years and after	10 0	10 0	10

In the practical application of formulas (9) and (16), no allowance was made for the fact that the full sum assured is not paid (except in cases of fatal accident) where death occurs during the first year. The usual practice appears to be: one-fourth of the sum assured, if death occur in the first six months; one-half, between six and twelve months; and the full sum assured, after twelve months.

VALUATION DATA.

It was thought desirable to test the effect of a valuation with allowance for lapse, by practical application to a body of illustrative data, in which the distribution of the business at different entry ages and durations should not greatly differ from that obtaining in some of the existing Industrial Assurance Companies. For the purpose of constructing the data, we had the advantage of access to the detailed valuation sheets of two Industrial Offices, each established in Great Britain more than 30 years ago, whose combined whole-life business, existing at the last valuation date, comprised sums assured of £12,260,681, at aggregate office premiums of £695,351·5 per annum. The sums assured, and the office annual premiums, in the whole-life class, were entered on valuation sheets of the form already referred to on pp. 544, 545, so that the facts, in respect of each entry age and curtate duration, were at once available for combination from the valuation sheets of the two Offices. The combined data were scheduled so as to bring together cases existing, after the same curtate duration, in respect of consecutive entry ages. Thus, the combined sums assured and office premiums for "curtate duration 0" were set out on one sheet, in respect of entry ages 1, 2, 3, &c., up to the highest entry age; the combined data for curtate duration 1 were similarly entered on another sheet, at successive entry ages; and so on to the highest observed duration, of 36 years. The sums assured and office premiums, thus entered, were then summed on each sheet so as to bring together the data, for a given duration, in respect of the consecutive entry ages (1, 2), (3, 4, 5, 6, 7), (8, 9, 10, 11, 12) . . . (78, 79, 80, 81, 82). The cases of infancy entering at office ages 1 and 2, were separately grouped, and the data at all other entry ages were thus combined in quinary groups, of which the central age was always a multiple of 5. In Table VI appended, are given at each group of entry ages the combined data as thus scheduled, the details being given in respect of each of the first 15 years of duration, after

which they are combined in quinary groups of duration for economy of space.

In order to eliminate any material error arising from the valuation of cases, having a given curtate duration, at every fifth entry age, instead of at the individual entry ages, a modification of the grouping of the data was adopted, which should now be explained. Let the sums assured, scheduled for valuation as above, in respect of a particular curtate duration, at the successive entry ages $(x-2)$, $(x-1)$, x , $(x+1)$, and $(x+2)$, be represented by $(S+A)$, $(S+B)$, S , $(S+B+C)$, and $(S+A+D)$ respectively. Omitting for the moment the excess quantities C and D (which may be positive or negative in sign) arising at ages $(x+1)$ and $(x+2)$, it is evident that the remaining sums $(S+A)$, $(S+B)$, S , $(S+B)$ and $(S+A)$ may be grouped, and valued by the factor appropriate to the central age x , without material error; the assumption made in such grouped valuation being that the factors at the successive ages $(x-2)$, $(x-1)$, x , $(x+1)$ and $(x+2)$ vary by constant first differences. As regards the excess sum C , arising at age $(x+1)$, the appropriate factor, upon the same assumption of constant first differences, is

$$\bar{A}_x + \frac{1}{5}(\bar{A}_{x+5} - \bar{A}_x) = \frac{1}{5}(\bar{A}_{x+5} + 4\bar{A}_x);$$

whilst for the excess sum D , arising at age $(x+2)$, the appropriate factor would similarly be

$$\bar{A}_x + \frac{2}{5}(\bar{A}_{x+5} - \bar{A}_x) = \frac{1}{5}(2\bar{A}_{x+5} + 3\bar{A}_x).$$

If, therefore, one-fifth of the excess quantity C , and two-fifths of the excess quantity D , be transferred, from the group with central age x , to the group with central age $(x+5)$, these amounts will also be valued correctly, upon the assumption made.

If the signs of the quantities C and D are negative, that is, if the quantities at ages $(x-1)$ and $(x-2)$ are in excess of those at ages $(x+1)$ and $(x+2)$, respectively, precisely the same argument holds good; but the proportions $\frac{1}{5}C$ and $\frac{2}{5}D$ must then be transferred to the next *preceding* group, with central age $(x-5)$. If, in a given group, the excess quantities C and D arise on different sides of the central age x ,—as, for example, where the excess C arises at age $(x-1)$ and the excess D at age $(x+2)$,—the work may be simplified, without affecting the accuracy of the result, if the *difference* between the proportions $\frac{1}{5}C$ and $\frac{2}{5}D$ be transferred to the group nearest to the larger proportion, instead of transferring the two amounts to their nearest adjacent groups.

The premiums were dealt with in precisely the same manner as the sums assured. In practice, the modifications of the grouped amounts are simply and rapidly effected; and the rules may be expressed as follows: Ascertain the difference (D) between the sums assured (or the premiums) at ages $(x-2)$ and $(x+2)$, where x is the central age of the group, and enter such difference opposite the larger amount. Proceed similarly with the difference (C) at ages $(x-1)$ and $(x+1)$. Compute $\frac{2}{5}D (= \cdot 4D)$ and $\frac{1}{5}C (= \cdot 2C)$. If D and C are entered on *different* sides of the central age x , deduct the *difference* between the above proportions from the group with central age x , and add it to the group nearest to the larger proportion. If D and C are entered on the *same* side of the central age x , deduct the *sum* of the above proportions from the group with central age x , and add it to the nearest group.

A practical illustration, taken from the data for curtate duration 0, in respect of entry ages 8-22 inclusive, will, it is

Ages at Entry	Sums Assured	
8	(S + A)	44,655
9	(S + B)	41,115
10	S	41,360
11	(S + B + C)	49,062
12	(S + A + D)	43,289
		(D) $1,366 \times \cdot 4 = 546$
		(C) $7,947 \times \cdot 2 = 1,589$
		Difference = 1,043
	(Original Total)	219,481
		- 1,043
		+ 1,871
	(Modified Total)	220,309
13		41,424
14		43,750
15		42,764
16		40,317
17		38,464
		(D) $2,960 \times \cdot 4 = 1,184$
		(C) $3,437 \times \cdot 2 = 687$
		Sum = 1,871
		206,719
		+ 1,043
		- 1,871
		205,891
18		36,586
19		36,387
20		40,221
21		44,165
22		47,304
		(C) $7,778 \times \cdot 2 = 1,556$
		(D) $10,718 \times \cdot 4 = 4,287$
		Sum = 5,843
		204,663
		- 5,843

hoped, makes these operations perfectly clear. In practice, the differences (C) and (D) would alone be entered on the schedules, the multiplications by $\cdot 2$ and $\cdot 4$ being performed on rough sheets, and the sum or difference of these proportions entered under the total sums assured of each group.

For the entry ages 1-7 inclusive, the above modification was not applied during the first 7 durations, as the sum assured over the whole of these ages and durations was not uniform, but increased from birth up to age 10. The modification was therefore applied from duration 8 inclusive, at which point the sums assured in respect of the grouped entry ages 3-7 had throughout reached their maximum. The tabulated sums assured are throughout the maximum amounts. As the large preponderance of the business in the first group arose at entry age 1, no modifications arose in practice by transfer from the group with entry ages 1, 2 to that with ages 3 to 7.

The method above explained (which was first suggested to us by Mr. W. S. Anderson, F.I.A.) would be applicable in any case of grouped valuation, and it has occurred to us that it would materially reduce the labour, where it is desired to apply extended or Select Tables (as for example those deduced for the first 10 years of assurance on the $O^{[M]}$ basis) in the valuation of the policies of an Assurance Company. The method is of course only a first approximation to the true results, but, in the case of a smoothly graduated Table, such as the $O^{[M]}$, where the second differences of the factors (and therefore of the reserves) are not very considerable, we believe that the figures arrived at would not differ very much from the true reserves.

In Table VII appended are given the sums assured and premiums, as thus adjusted for valuation, classified similarly to the unadjusted facts in Table VI. The data thus adjusted were then valued at the entry ages and durations shown in Table VII; for entry ages 3-82 inclusive, the central age of a quinary group was always taken; whilst the fact that, on the whole, $86\frac{1}{2}$ per-cent of the sums assured in the grouped entry ages 1, 2, arose at office age 1, entitled us, without material error, to assume that the cases in that group were all effected at age 1. We observed, on the schedules of original sums assured, a rather marked tendency to a maximum at the central entry ages 40, 50, and 60. This was observed at almost all durations, and is illustrated by the following extract from the schedules, in respect of curtate duration 0:—

Entry Age	Original Sum Assured
	£
38	34,007
39	32,139
40	40,047
41	31,525
42	34,474
...	...
48	48,414
49	48,470
50	66,082
51	49,520
52	52,088
...	...
58	45,484
59	42,675
60	62,819
61	41,208
62	47,627
...	...

This tendency of the cases to aggregate at the entry ages which were multiples of 10, whilst casting an interesting light on the question of the accuracy of the ages given in the proposals, fitted in very well with our assumption that the cases in each group entered at the central age.

SURRENDER ALLOWANCES.

We have not been able to obtain very full information as to the practice of the Industrial Offices in the grant of allowances on lapse, to policies which have attained a certain duration. The particulars given in the Returns to the Board of Trade, in reply to Q. 10 of Schedule 6 of the Life Assurance Companies Act, 1870, are not in all cases very definite and clear; but it would appear from these replies, and from what we can learn as to the practice of the Companies, that it is usual to give free or paid-up policies, rather than cash surrender values, after payment of premiums over a certain number of years (usually five years). One of the Companies gives cash surrender values varying from 15 per-cent of the premiums paid (after 10 years) to 25 per-cent (after 20 years and upwards); and free policies, varying, according to present age and duration, from 20 to 60 per-cent of the premiums paid; whilst the scale of free policies given by two other Offices does not appear to differ very much from a cash equivalent of one-third of the premiums paid (on the basis of the English Life Table No. 3, at 3 per-cent).

For the purposes of the present paper, it appears to be immaterial whether the allowance on lapse is made by way of a

cash surrender value, or a reversionary payment at death. Having regard to the probability that a certain proportion of the free policies granted for small amounts do not actually become claims upon the death of the assured, it seems likely that on the whole a fair cash equivalent for the paid-up policy would be somewhat below one-third of the premiums paid; and, in view of this consideration, and of the terms of cash surrender value which have come under our notice, we decided to proceed on the assumption that a cash allowance, equal to 25 per-cent upon the premiums paid from the commencement, would be granted on lapse to all policies having five years' duration and upwards. No doubt a certain proportion of the lives assured do not claim, upon lapse, the surrender value or free policy to which they are entitled. We find, however, from an investigation made in the case of a single Industrial Office, that not less than 80 per-cent of such policyholders claimed the surrender value. We have, therefore, throughout the present paper, assumed that the allowance will be made in every case of lapse arising after 5 years' duration.

In the calculation of the net premiums, it was necessary to assume a scale of office premiums, in connection with the allowance on surrender (see Formula 16). The scale actually adopted was based upon the prospectus rates of five leading Industrial Offices, the maximum sum assured payable at each age being taken as the basis, in cases where the Office had an increasing scale. The rates at the central entry ages were as follows :

Entry Age	Maximum Sum Assured for 1 <i>l.</i> weekly	Equivalent Annual Premium (P) for £1 Assured
	£ s.	£
1	10 0	·02167
5	10 0	·02167
10	10 0	·02167
15	9 12	·02257
20	8 8	·02579
25	7 6	·02968
30	6 5	·03467
35	5 7	·04050
40	4 11	·04762
45	3 16	·05702
50	3 2	·06989
55	2 10	·08667
60	1 19	·11111
65	1 10	·14445
70	1 3	·18841
75	0 17	·25491
80	0 12	·36111

VALUATION RESULTS.

The selected data, as set out in Table VII, were valued by the English Life Table No. 3 (Males) at 3 per-cent interest (i) with allowance for the *full* rate of lapse, as given in Table I; (ii) with allowance throughout for *half* that rate of lapse, (iii) *without* allowance for lapse; the latter valuation being made by the normal net premium method, but the data and results at each age attained, in respect of each central age at entry, being kept separate, for purposes of comparison. A further valuation (iv) without allowance for lapses was similarly made at 4 per-cent interest, this being considered to be the maximum rate obtainable, with due regard to safety. It will be remembered that 4 per-cent is the rate of interest prescribed, under the First Schedule of the Life Assurance Companies Act, 1872, for a valuation as a test of solvency. We also thought it would be interesting to compare the results brought out on a normal basis, at two different rates of interest, with those brought out at the lower rate with allowance for lapses.

On each of the four bases specified above, we have also computed the net liability on the assumption that a reduced premium, equivalent to two-thirds of the office premium, would be available to meet the liabilities. We adopted this assumption, after examination of the actual margins reserved for expenses in the valuations of the leading Industrial Offices, as set out in reply to Q. 5 of the Returns under the Fifth Schedule of the Life Assurance Companies Act of 1870. These margins practically varied between 30 and 40 per-cent in individual cases, but on the average would not greatly differ from $33\frac{1}{3}$ per-cent.

The valuation results on these several bases are shown in the appended Tables VIII to XVI. In Tables VIII to XI are set out, for *all ages at entry combined*, in respect of each curtate duration, the present values of (1) sums assured, (2) surrender allowances, (3) office premiums, (4) $\frac{2}{3}$ office premiums, (5) net premiums; also the net liability, on the basis of (6) $\frac{2}{3}$ office premiums, (7) net premiums. In Tables XII to XV these results are set out for *all durations combined*, in respect of each central age at entry. For purposes of convenient comparison, the net liabilities on each valuation basis are brought together in Table XVI for grouped entry ages, and for each of the first fifteen years of duration, subsequent durations being tabulated in quinary groups. The grouped central entry ages are 1, 5-15, 20-40, 45-60, and 65-80; and, in determining these groups, we gave due regard to

the general similarity of the rate of lapse at the individual entry ages included in each group.

Having regard to the importance of the amount of the surrender allowance granted, as affecting the liabilities of an office, we have also thought it well to show the financial effect of the assumption that no surrender allowance is granted on lapse. For this purpose, in the net premium valuations with allowance for lapse, it was necessary to eliminate, from the value of the net premiums, that portion which was a provision for the surrender allowance. In Table XVII, the net premiums are set out on the different valuation bases assumed; and, in the cases of full and half lapse rate, the net premiums with and without provision for surrender allowance are separately stated. Columns are also included in the table, showing separately the provision for surrender allowance included in the net premiums (columns 8 and 10), and, also, the proportions of such provision to the whole net premiums (columns 9 and 11), which proportions are, it will be seen, somewhat considerable at the early entry ages, but rapidly diminish as the entry age increases.

Turning now to the actual valuation results, it will be convenient, in the first instance, to set out the aggregate net liability on each valuation basis adopted. These are as follows:

STATEMENT A.

Valuation Basis		Office Premiums valued	Net Premiums valued
Full Lapse rate	3 per-cent	£	£
		+ 697,708·9 - 200,701·5 <hr/> 497,007·4	1,112,171·3
Half Lapse rate.	3 per-cent	+ 720,517·2 - 274,316·1 <hr/> 446,201·1	1,040,875·1
Excluding Lapse rate.	3 per-cent	+ 754,234·3 - 346,125·8 <hr/> 408,108·5	977,202·2
Excluding Lapse rate.	4 per-cent	+ 576,616·9 - 525,225·5 <hr/> 51,391·4	876,296·8

It will be observed that, upon a net premium basis, the liability at 3 per-cent is *increased* by £63,672·9 by inclusion of the half lapse, and by a further £71,296·2, or £134,969·1 in all, by inclusion of the full lapse rate. The difference, upon a net premium basis without allowance for lapse, between the 3 per-cent and 4 per-cent results, is £100,905·4; so that the introduction of the full lapse rate on a 3 per-cent basis makes a greater increase in the reserves than the reduction of the rate of interest from 4 to 3 per-cent.

Taking now the valuations on an office premium basis, we have throughout the tables shown separately the positive and negative reserves, the latter being printed in italic figures. Assuming, for the moment, that the usual course is adopted of eliminating negative values throughout, the above results indicate that the assumption of a lapse rate has the effect of *reducing* the policy liabilities; but the inclusion of the full lapse rate only reduces the liabilities by about one-third, and the half lapse rate by about one-fifth, of the difference between the reserves by valuations at 3 and 4 per-cent, without allowance for lapses.

It does not appear to us to be demonstrable that, in the case of a net premium valuation, the introduction of the element of lapse will always tend to increase reserves, nor that, where the valuation is made on an office premium basis, with elimination of negative values, the tendency will be to decrease reserves; and it may be interesting to compare the reserves, in the case of the particular experience here investigated, at different entry ages and durations, on the several valuation bases employed.

Taking first the results at grouped entry ages, the following summary statement shows the liability on each basis, the figures being extracted from the total lines in Table XVI at the different grouped entry ages. The results on a net and office premium basis are separately stated:—

STATEMENT B.
Net Premium Valuations.

Valuation Basis	GROUPED ENTRY AGES				
	(1)	(5)–(15)	(20)–(40)	(45)–(60)	(65)–(80)
	£	£	£	£	£
Full Lapse rate. 3 %	59,639	149,283	319,281	449,643	134,321
Half Lapse rate. „	65,397	133,768	296,703	420,019	124,994
Excludg. Lapse rate. „	71,035	123,442	277,334	390,745	114,657
„ „ „ 4 %	53,385	102,759	243,792	366,128	110,242

Office Premium Valuations.

	(1)	(5)-(15)	(20)-(40)	(45)-(60)	(65)-(80)
	£	£	£	£	£
Full Lapse rate. 3 %	(+ 26,431 - 30,187)	+ 68,497 - 16,668	+ 224,872 - 27,527	+ 309,103 - 68,880	+ 68,807 - 57,439
Half Lapse rate. "	(+ 27,381 - 49,182)	+ 74,971 - 30,179	+ 238,188 - 36,376	+ 313,046 - 87,850	+ 66,926 - 70,726
Excl'dg. Lapse rate. "	(+ 29,688 - 76,308)	+ 86,399 - 45,056	+ 255,985 - 34,742	+ 317,292 - 103,877	+ 64,870 - 86,148
" " " 4 %	(+ 15,753 - 100,804)	+ 48,543 - 83,992	+ 184,450 - 98,665	+ 269,219 - 148,565	+ 58,652 - 93,199

From the above comparison, it appears that the introduction of a lapse rate, on a net premium valuation basis, *diminishes* the aggregate reserves at entry age 1, whilst *increasing* them at all other entry ages; also that, on an office premium basis (ignoring negative values) the reserves are *diminished* by the introduction of a lapse rate at each group of entry ages, excepting that for ages (65) to (80) inclusive, where the reserves are *increased*. It therefore appears that no general law can be laid down, applicable to all entry ages and durations, as to the effect of the introduction of a lapse rate upon the resulting reserves, whether deduced upon a net premium or an office premium basis.

Turning now to the results according to grouped policy durations, the following statement shows the comparative reserves, the figures being taken from the totals of the groups in Tables VIII to XI:—

STATEMENT C.
Net Premium Valuations.

Valuation Basis	GROUPED DURATIONS						
	0-4	5-9	10-14	15-19	20-24	25-29	30-36
	£	£	£	£	£	£	£
Lapse rate. 3 %	239,861	277,723	209,960	192,656	129,992	46,388	15,591
Lapse rate. „	210,103	254,442	200,208	187,257	127,698	45,813	15,355
g.Lapse rate. „	175,817	234,456	193,612	184,875	127,301	45,825	15,316
„ „ 4 %	158,775	210,051	173,907	165,175	113,687	40,952	13,719

Office Premium Valuations.

	£	£	£	£	£	£	£
Lapse rate. 3 %	(+ 21,054 - 190,507)	+ 137,197 - 9,475	+ 168,025 - 719	+ 184,260 ...	+ 128,480 ...	+ 44,626 ...	+ 14,067 ...
Lapse rate. "	(+ 21,596 - 257,300)	+ 139,286 - 15,212	+ 173,554 - 1,805	+ 191,536 ...	+ 133,741 ...	+ 46,307 ...	+ 14,498 ...
g. Lapse rate. "	(+ 24,393 - 321,154)	+ 144,895 - 21,860	+ 181,003 - 3,112	+ 200,562 ...	+ 140,114 ...	+ 48,282 ...	+ 14,986 ...
" " 4 %	(+ 9,598 - 478,721)	+ 89,702 - 35,879	+ 145,558 - 9,700	+ 163,118 - 926	+ 115,657 ...	+ 40,335 ...	+ 12,650 ...

Here it will be seen that the reserves with introduction of lapse-rate are throughout greater, on a net premium basis (excepting at grouped durations 25-29 on the half lapse-rate), and less, on an office premium basis, than the reserves computed without lapse. The differences shown in Statement B are, however, concealed by the grouping of the results in Statement C, but are more clearly seen on reference to the following tables, which show the differences arising from the introduction of the full lapse-rate, for grouped entry ages and grouped durations, both on a net premium and office premium basis.

STATEMENT D.

DIFFERENCES IN RESERVES.

NET PREMIUM *Reserves on the assumption of FULL LAPSE-RATE,*
less those EXCLUDING LAPSE-RATE. 3 PER-CENT.

Grouped Durations	GROUPED ENTRY AGES					Totals
	(1)	(5)-(15)	(20)-(40)	(45)-(60)	(65)-(80)	
	£	£	£	£	£	£
0- 4	- 1,609	+ 7,779	+ 17,216	+ 23,086	+ 12,561	+ 61,033
5- 9	- 1,031	+ 7,786	+ 12,523	+ 18,091	+ 5,894	+ 43,263
10-14	- 1,912	+ 4,060	+ 5,868	+ 7,347	+ 987	+ 16,350
15-19	- 3,200	+ 3,389	+ 3,788	+ 3,599	+ 204	+ 7,780
20-24	- 2,634	+ 1,931	+ 1,895	+ 1,481	+ 17	+ 2,690
25-29	- 860	+ 628	+ 527	+ 267	+ 1	+ 563
30-36	- 150	+ 268	+ 130	+ 27	...	+ 275
Totals	-11,396	+25,841	+41,947	+58,898	+19,664	+134,954

DIFFERENCES IN RESERVES, *as above, on an OFFICE PREMIUM basis (ignoring negative values).* 3 PER-CENT.

Grouped Durations	GROUPED ENTRY AGES					Totals
	(1)	(5)-(15)	(20)-(40)	(45)-(60)	(65)-(80)	
	£	£	£	£	£	£
0- 4	...	- 2,229	- 2,614	{ - 228 } { + 185 }	+ 1,548	- 3,338
5- 9	...	- 2,970	- 6,708	{ - 558 } { + 137 }	+ 2,401	- 7,698
10-14	+ 777	- 2,525	- 7,476	- 3,764	{ - 68 } { + 79 }	- 12,977
15-19	- 1,011	- 4,398	- 7,993	- 2,877	- 23	- 16,302
20-24	- 2,169	- 3,858	- 4,654	- 952	...	- 11,633
25-29	- 751	- 1,429	- 1,352	- 124	...	- 3,656
30-36	- 103	- 493	- 316	- 8	...	- 920
Totals	- 3,257	-17,902	-31,113	- 8,189	+ 3,937	- 56,524

These comparative results would seem to confirm the view that no general law can be stated as to the effect upon reserves of introducing the element of lapse.

We now turn to the question of *margins* on the above valuation bases. The proportions of the office premiums reserved for future expenses are, on a net premium 3 per-cent basis,

With allowance for lapses . . .	45·83 per-cent,
Without „ „ „ . . .	39·92 „

whilst, on an office premium basis, on the assumption made that one-third of the premiums are reserved, and negative values eliminated, the proportions become

With allowance for lapses . . .	37·41 per-cent,
Without „ „ „ . . .	37·33 „

It appears from these results that, assuming that the future expenses of the office will not exceed one-third of the gross premiums, a net premium valuation, whether with or without allowance for lapses, provides margins in excess of the requirements of the case. If, as there is some reason to think, the office premiums included in our valuation data (especially in the case of the older policies) are below the average rates, the disproportion between the rate of expense assumed and that provided for in the valuation would be accentuated.

Considering now the margins provided for expenses on an office premium valuation basis, it will be seen that the elimination of negative values has the effect of increasing the margin by about 4 per-cent of the office premiums, whether the valuation be made with or without allowance for lapses. There appears to us, however, to be an important distinction between the two cases; for, where the valuation is made without allowance for lapses, the inclusion of negative values assumes that the premiums under all policies will be continued throughout life, that is, that there will be no lapses; and as this condition does not obtain in practice, the assumption is inadmissible, and the negative values must be excluded. This exclusion is necessary, to the extent to which lapses may reasonably be expected to arise. It is, however, impossible to determine, without a detailed valuation which shall allow for the anticipated lapse rate, to what extent the negative values in question will arise as the result of such valuation; and, therefore, as a measure of precaution, the only available course is to eliminate such values altogether.

The effect of a lapse rate, whilst negative values are in operation, is to extinguish an estimated asset, and at the same time (in respect of policies of upwards of five years' duration) to introduce an immediate liability for a cash surrender allowance, or equivalent free policy; whilst, if positive values are in operation, the effect is to extinguish an estimated liability, which is replaced (in the case of policies of upwards of five years' duration) by the immediate liability for surrender, which may or may not be greater. The ultimate effect upon the whole valuation can only be determined, as stated above, by a detailed valuation with full allowance for lapses. In the case, however, of a valuation with allowance for full lapse rate, due weight has been given to all the circumstances, and, as the operation of that lapse rate can therefore have no effect upon the valuation results that has not been foreseen, and fully provided for, there is no occasion to exclude negative values.

It might be supposed that the assumption of a rate of lapse lower than that actually experienced would have the effect of providing, by increased reserves, a margin of safety; but it will be seen, on reference to the comparative tables, that this is not necessarily so; as, in the present experience, the assumption of a half lapse rate involves *smaller* aggregate reserves than the assumption of a full lapse rate, both in the case of a net premium valuation, and in that of an office premium valuation including negative values; although, if negative values be excluded in the latter case, the reserves are increased. The arguments as to the inclusion of negative values, as above adduced in the case of an office premium valuation without allowance for lapse rate, apply also to the case of a valuation with only partial allowance for lapses.

It appears to us, therefore, that the appropriate course for the Actuary to adopt is to make a valuation with full allowance for lapses, throwing off such a proportion of the office premiums as shall provide for the anticipated expenses; and, further, that where negative values arise on such a basis, they may safely be included.

In the particular experience here valued, the effect of such inclusion of negative values is to reduce the reserves by £200,702, from £697,709 to £497,007, a reduction of nearly 30 per-cent. This reduced liability is only about two-thirds of that brought out by an office premium valuation without allowance for lapses, namely, £754,234; and about one-half of that brought out by a net premium valuation, without allowance for lapses, namely, £977,202.

(See Statement E.) We are, however, of opinion, as above stated, that the smaller amount is a truer measure of the actual liabilities, assuming that the lapse rate adopted is likely to obtain in the future experience of the Company. Any unfavourable departure from the rate assumed in the valuation may be provided for by a special precautionary reserve, rather than by a different assumption as to the rate of lapse.

Having regard to the considerable amount of the liability for surrender allowance, in relation to the total reserves, for full and half lapse rate respectively, we have thought that it would be of interest to show the effect of a variation in the scale of surrender allowances. The effect of the exclusion of the surrender allowance on the reserves, whether on an office or net premium basis, is shown in Tables XII and XIII, also at foot of Tables VIII and IX, for full and half lapse rate respectively; and the results are set out in the following statement, together with the reserves excluding lapse rate, as shown in Tables X and XIV :

STATEMENT E.

RESERVES.

With, and without, Provision for Surrender Allowance.

Valuation Basis	FULL LAPSE RATE		HALF LAPSE RATE		EXCLUDING LAPSE RATE	
	Office Premium basis (including Negative values)	Net Premium basis	Office Premium basis (including Negative values)	Net Premium basis	Office Premium basis (excluding Negative values)	Net Premium basis
	£	£	£	£	£	£
With Surrender Allowance	497,007	1,112,171	446,201	1,040,875
Without „ „	186,811	1,005,296	238,823	1,000,068	754,234	977,202

These results are, we think, worthy of attention. The reserves on a net premium basis, without surrender allowance, do not very materially vary, whether the full or half lapse rate be included, or whether lapses be excluded altogether; but the characteristics previously observed still obtain, of a steady increase in the liability, as the magnitude of the lapse rate increases. Upon an office premium basis, however, it will be observed that whilst the liability, including surrender allowance, on a full lapse basis exceeds that on a half lapse basis, the contrary is the case where no surrender allowance is granted; as, in the former case, the reserves are increased by £50,806; whilst, in the latter case, they are reduced by £52,012. These

results would, of course, be varied according to the scale of surrender allowance assumed, and the margin provided for expenses; and the effect of proportionate variations can readily be seen, by suitably modifying the totals in Tables XII and XIII of the columns headed "Present Value of Surrender Allowances" and "Present Value of Office Premiums." It seems clear that, in basing a valuation upon a rate of lapse other than that experienced by an office, the effect of introducing a particular scale of surrender allowance must be carefully considered; and our view is confirmed that the best course to adopt is to adhere as closely as practicable to the actual experience of the Company as regards lapse rate, surrender allowance, and expenses.

INFANTILE RESERVES.

It appears to be the usual practice of the Industrial Offices to assume that no reserves, or merely nominal reserves, are required in the case of policies on lives under 10 years of age, presumably upon the ground that the rate of mortality is diminishing from birth up to age 13, according to the English Life Tables No. 3 (Males). It is, however, to be borne in mind that the sum assured is increasing in amount up to age 11, the scale usually adopted not differing very much from that given on page 551. It will be seen, on reference to Table XVI, that at central age at entry 1, where the net premium has been calculated with due allowance for the increasing sums assured, the reserves are throughout positive; and, although comparatively small at the infantile ages, we find that the reserves required (without allowance for lapses) vary from one-third to nearly four times the net premium. At entry age 5, the reserves for policies on lives under 11 years of age vary from one-third to three times the net premium. It would thus appear that, on a true net premium basis, the reserves at these early entry ages can hardly be considered as negligible.

The introduction of a lapse rate has the effect, at age at entry 1, of diminishing the reserves on a net premium basis; but, at the early ages attained, the proportions of the net premiums do not differ very much from those given above.

We have already shown that the net premium reserves, with allowance for lapse, in respect of policies effected at age 1, are throughout less than those effected without such allowance, the opposite being the case at older entry ages. This is no doubt

due to the sum assured being at a minimum, when the rates of lapse and mortality are both at a maximum.

It is also to be remarked that, as will be seen from Table XVII, the net premium at entry age 1, on the half lapse basis, does not, as might be anticipated, lie between those computed on the full lapse basis, and without allowance for lapse. This exception probably arises from the incidence of the lapse and mortality rates, and the variation in the sum assured, at this early entry age.

EXPENSE RATIOS.

The valuation factors, calculated with allowance for lapse, may be applied in questions arising as to the distribution of the expenditure of an office incurred in the early years of assurance, mainly in the procurement of new business, which distribution can only be properly ascertained with due allowance for lapses. Table XVIII shows, upon the different assumptions of $33\frac{1}{3}$, 40, and 45 per-cent as the uniform rate of expenditure upon all premiums, the equivalent outlay in terms of the first t years' premiums (where t is not > 7), and a reduced percentage on those of later years' premiums. Provision has been made for the death claims arising in the first t years, and also for the surrender allowances (on the scale adopted throughout this paper), estimated as payable in cash over the same period. The reduced provision has been computed on the basis of the full and half lapse rate, with interest at 3 per-cent, and also, excluding lapses, at 3 and 4 per-cent interest.

It may be convenient to deduce the formula adopted in the calculations of these percentages. Let ϕ be the uniform rate of expenditure upon all premiums, and ψ_t the rate of reduced outlay after t years, assuming that the first t years' premiums are spent in meeting expenses and current claims arising on policies of less than t years' duration; also, let the annual premium be unity, and the sum assured at death $= \frac{1}{P_x}$, where P_x is the average rate of office premium set out on page 557. When allowance is made for lapses, we have, where t is not > 5 ,

$$\phi {}^w d \bar{N}_x = ({}^w d \bar{N}_x - {}^w d \bar{N}_{x+t}) - ({}^d w \bar{M}_x - {}^d w \bar{M}_{x+t}) \frac{1}{P_x} + \psi_t {}^w d \bar{N}_{x+t}$$

and

$$\psi_t = 1 - \frac{(1 - \phi) {}^w d \bar{N}_x - ({}^d w \bar{M}_x - {}^d w \bar{M}_{x+t}) \frac{1}{P_x}}{{}^w d \bar{N}_{x+t}}$$

whilst, where t is > 5 ,

$$\begin{aligned} \phi^{wd} \bar{N}_x &= ({}^{wd} \bar{N}_x - {}^{wd} \bar{N}_{x+t}) - ({}^{d(w)} \bar{M}_x - {}^{d(w)} \bar{M}_{x+t}) \frac{1}{P_x} \\ &- \cdot 25 \{ {}^{w(d)} \bar{R}_{x+5} + 4 \cdot 5 {}^{w(d)} \bar{M}_{x+5} - ({}^{w(d)} \bar{R}_{x+t} + t - \frac{1}{2} {}^{w(d)} \bar{M}_{x+t}) \} + \psi_t {}^{wd} \bar{N}_{x+t} \\ \text{and} \\ \psi_t &= 1 - \frac{(1 - \phi) {}^{wd} \bar{N}_x - ({}^{d(w)} \bar{M}_x - {}^{d(w)} \bar{M}_{x+t}) \frac{1}{P_x}}{{}^{wd} \bar{N}_{x+t}} \\ &- \frac{\cdot 25 \{ {}^{w(d)} \bar{R}_{x+5} - {}^{w(d)} \bar{R}_{x+t} + 4 \cdot 5 {}^{w(d)} \bar{M}_{x+5} - (t - \frac{1}{2}) {}^{w(d)} \bar{M}_{x+t} \}}{{}^{wd} \bar{N}_{x+t}} \end{aligned}$$

If the lapse element is not involved, the formula becomes

$$\psi_t = 1 - \frac{(1 - \phi) \bar{N}_x - (\bar{M}_x - \bar{M}_{x+t}) \frac{1}{P_x}}{\bar{N}_{x+t}}$$

for all values of t .

A table constructed on the lines of Table XVIII may be useful in connection with questions as to the relative charges arising in respect of new and old business, as, for instance, where a valuation is made on the basis of net premiums computed at ages t years older than the true entry ages.

CONCLUSION.

In conclusion, we desire to state that we are conscious that the subject of this paper is a difficult one, and that our deductions, being drawn from a particular experience, may not be generally applicable; but, whilst there is little in the work of previous investigators bearing on the subject which tends either to confirm or refute those deductions, we believe that further research will, in the main, tend to establish our views. The field of research is a wide one, and it would be interesting to extend the investigation to classes of assurance other than whole life, and into the effect of differing assumptions as to the rate and incidence of the lapses; as, for instance, that the lapse rate ceases after a certain duration, irrespective of age attained; or, at a certain age (other than that assumed by us), irrespective of duration. We hope, however, that our present contribution may be useful at least as illustrating the need of further consideration of the subject; and that other investigations may be forthcoming which shall tend, by further elucidation of the question, to more settled convictions thereon amongst the profession generally.

Graphic Illustration of the Full Rate of Lapse observed at central Ages at entry and in years of duration, as adjusted for use in the valuation.

The ordinates represent the rates of lapse, and the abscissæ the ages:— The dotted lines show the assumed movement in the rate of lapse between the date of entry and the end of the calendar year of entry.

The curve for central entryage 80 was arbitrarily inserted.

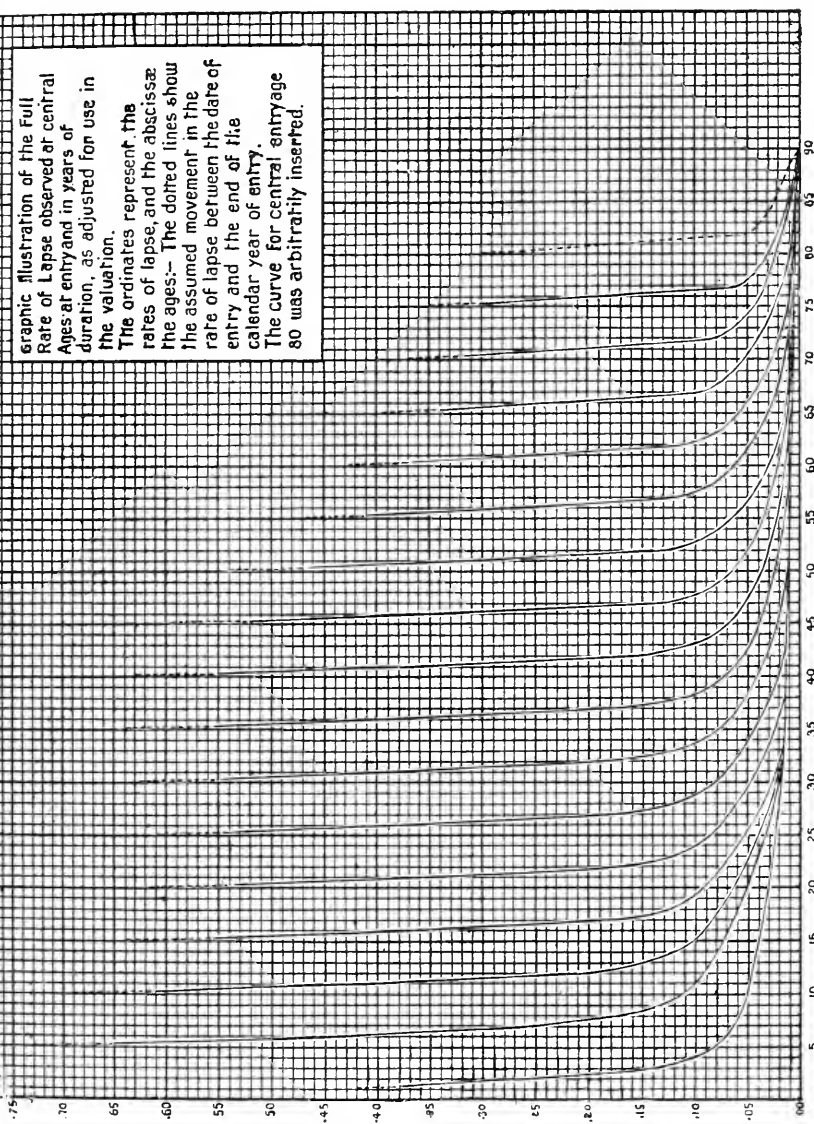


TABLE I—continued.
 FULL RATE OF LAPSE, as observed at the specified ages at entry, and durations reckoned
 (a) from entry, (β) from the end of the calendar year of entry.

attained ($x+n$)	HIGHER ENTRY AGES		Dura- tion (n)	$[x]=45$		$[x]=50$		$[x]=55$		$[x]=60$		$[x]=65$		$[x]=70$		$[x]=75$		$[x]=80$		Dura- tion (n)
	α	β		α	β	α	β	α	β	α	β	α	β	α	β	α	β	α	β	
46	0130	0129	0	5950	5200	5140	4660	4650	4100	4250	3660	3950	3100	3700	3180	3500	3020	3000	2590	0
47	0127	0126	1	3300	1950	3000	1725	2650	1530	2400	1370	2175	1190	1975	0990	1880	0730	1610	0626	1
48	0124	0123	2	1350	1160	1230	1075	1130	0990	1050	0910	0950	0810	0800	0675	0550	0470	0470	0410	2
49	0121	0120	3	0960	0830	0900	0800	0850	0770	0810	0725	0730	0660	0610	0550	0440	0390	0380	0334	3
50	0118	0117	4	0750	0680	0730	0660	0700	0610	0660	0600	0610	0540	0500	0460	0350	0325	0290	0270	4
51	0115	0114	5	0610	0535	0630	0580	0600	0560	0570	0530	0525	0483	0430	0400	0300	0280	0250	0233	5
52	0112	0111	6	0550	0515	0530	0500	0520	0490	0490	0455	0460	0430	0370	0345	0260	0240	0210	0190	6
53	0109	0108	7	0480	0450	0470	0435	0460	0425	0420	0400	0400	0370	0320	0300	0220	0200	0170	0150	7
54	0106	0105	8	0420	0400	0400	0375	0390	0370	0380	0350	0340	0315	0280	0265	0180	0160	0120	0110	8
55	0103	0102	9	0380	0360	0350	0325	0350	0320	0320	0295	0290	0270	0250	0235	0140	0120	0070	0060	9
56	0099	0098	10	0340	0325	0300	0280	0290	0270	0270	0245	0250	0230	0220	0205	0100	0085	10
57	0097	0096	11	0310	0300	0260	0240	0250	0230	0220	0205	0210	0190	0190	0175	0070	0055	11
58	0094	0093	12	0285	0273	0220	0200	0210	0195	0190	0175	0170	0150	0160	0145	0040	0030	12
59	0091	0090	13	0260	0248	0180	0165	0180	0170	0160	0145	0130	0115	0130	0115	0020	0015	13
60	0088	0087	14	0235	0223	0150	0135	0160	0150	0130	0115	0100	0085	0100	0085	0010	0005	14
61	0085	0084	15	0210	0198	0120	0115	0140	0130	0100	0090	0070	0060	0070	0055	15
62	0082	0081	16	0185	0173	0110	0105	0120	0110	0080	0070	0050	0040	0040	0030	16
63	0079	0078	17	0160	0150	0100	0095	0100	0093	0060	0055	0040	0035	0020	0015	17
64	0076	0075	18	0140	0138	0092	0088	0085	0078	0050	0045	0030	0025	0010	0006	18
65	0073	0072	19	0125	0118	0084	0080	0070	0065	0040	0036	0020	0017	19
66	0070	0069	20	0110	0105	0076	0072	0060	0055	0032	0029	20
67	0067	0066	21	0090	0085	0068	0064	0050	0045	21
68	0064	0063	22	0080	0075	0060	0056	0040	0037	22
69	0061	0060	23	0070	0065	0052	0049	23
70	0058	0057	24	0060	0055	24
71	0055	0054	25	0050	0045	25
72	0052	0051																		
73	0049	0048																		
74	0046	0045																		
75	0043	0042																		
76	0040	0039																		
77	0037	0036																		
78	0034	0033																		
79	0031	0030																		
80	0028	0027																		
81	0025	0024																		
82	0022	0021																		
83	0019	0018																		
84	0016	0015																		
85	0013	0012																		
86	0010	0009																		
87	0007	0006																		
88	0004	0003																		
89	0001	0000																		

TABLE II.

VALUATION FACTORS.—Showing, at the specified ages at entry, and durations of assurance (reckoned from the end of the calendar year of entry), (1) The value of an ASSURANCE of 1 payable at death (with allowance for lapse); (2) The value of an ANNUITY of 1 per annum, payable so long as the assurance is subsisting.

FULL LAPSE RATE.

3 PER-CENT.

Duration of Assurance (n)	[x] = 5			[x] = 15			[x] = 25			[x] = 35		
	$d^{(w)} \Lambda_{x +n}$	$w d \bar{A}_{x +n}$	$w d \bar{A}_{x +n}$	$d^{(w)} \Lambda_{x +n}$	$w d \bar{A}_{x +n}$	$w d \bar{A}_{x +n}$	$d^{(w)} \Lambda_{x +n}$	$w d \bar{A}_{x +n}$	$w d \bar{A}_{x +n}$	$d^{(w)} \Lambda_{x +n}$	$w d \bar{A}_{x +n}$	$w d \bar{A}_{x +n}$
0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9	9
10	10	10	10	10	10	10	10	10	10	10	10	10
11	11	11	11	11	11	11	11	11	11	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13
14	14	14	14	14	14	14	14	14	14	14	14	14
15	15	15	15	15	15	15	15	15	15	15	15	15
16	16	16	16	16	16	16	16	16	16	16	16	16
17	17	17	17	17	17	17	17	17	17	17	17	17
18	18	18	18	18	18	18	18	18	18	18	18	18
19	19	19	19	19	19	19	19	19	19	19	19	19
20	20	20	20	20	20	20	20	20	20	20	20	20
21	21	21	21	21	21	21	21	21	21	21	21	21
22	22	22	22	22	22	22	22	22	22	22	22	22
23	23	23	23	23	23	23	23	23	23	23	23	23
24	24	24	24	24	24	24	24	24	24	24	24	24
25	25	25	25	25	25	25	25	25	25	25	25	25
26	26	26	26	26	26	26	26	26	26	26	26	26
27	27	27	27	27	27	27	27	27	27	27	27	27
28	28	28	28	28	28	28	28	28	28	28	28	28
29	29	29	29	29	29	29	29	29	29	29	29	29
30	30	30	30	30	30	30	30	30	30	30	30	30
31	31	31	31	31	31	31	31	31	31	31	31	31
32	32	32	32	32	32	32	32	32	32	32	32	32
33	33	33	33	33	33	33	33	33	33	33	33	33
34	34	34	34	34	34	34	34	34	34	34	34	34
35	35	35	35	35	35	35	35	35	35	35	35	35
36	36	36	36	36	36	36	36	36	36	36	36	36
37	37	37	37	37	37	37	37	37	37	37	37	37
38	38	38	38	38	38	38	38	38	38	38	38	38
39	39	39	39	39	39	39	39	39	39	39	39	39
40	40	40	40	40	40	40	40	40	40	40	40	40
41	41	41	41	41	41	41	41	41	41	41	41	41
42	42	42	42	42	42	42	42	42	42	42	42	42
43	43	43	43	43	43	43	43	43	43	43	43	43
44	44	44	44	44	44	44	44	44	44	44	44	44
45	45	45	45	45	45	45	45	45	45	45	45	45

(reckoned from the end of the calendar year of entry), (1) The value of an ASSURANCE of 1 payable at death (with allowance for lapse); (2) The value of an ANNUITY of 1 per annum, payable so long as the assurance is subsisting.

FULL LAPSE RATE.

3 PER-CENT.

Duration of Assurance (n)	[x] = 45		[x] = 55		[x] = 65		[x] = 75	
	$d(w)\bar{A}_{x n}+n$	$wd\bar{q}_{x n}+n$	$d(w)\bar{A}_{x n}+n$	$wd\bar{q}_{x n}+n$	$d(w)\bar{A}_{x n}+n$	$wd\bar{q}_{x n}+n$	$d(w)\bar{A}_{x n}+n$	$wd\bar{q}_{x n}+n$
0	12442	4163	21478	4680	33636	4327	50463	3506
1	24666	7844	34996	7152	48878	5861	63039	4500
2	30221	9106	40751	7837	54480	6119	73353	4500
3	33976	9707	44892	8131	58527	6159	76170	4333
4	36963	10028	48444	8271	62064	6125	78555	4262
5	39666	10227	51680	8324	65132	6033	80658	4117
6	42265	10358	54472	8325	68069	5923	82592	3972
7	44692	10428	57712	8282	70866	5796	84341	3823
8	46935	10441	60478	8191	73440	5649	85950	3673
9	49212	10410	63081	8071	75787	5486	87400	3524
10	51364	10342	65516	7916	77937	5310	88647	3375
11	53461	10242	67740	7736	79920	5129	89604	3223
12	55541	10120	69787	7520	81698	4942	90569	3077
13	57577	9974	71667	7291	83253	4747	91092	2928
14	59566	9804	73426	7052	84597	4548	91714	2789
15	61496	9612	75086	6806	85739	4349	92336	2650
16	63346	9397	76638	6556	86698	4149	92958	2511
17	65101	9161	78071	6302	87566	3955	93580	2372
18	66759	8906	79402	6048	88333	3766	94202	2233
19	68380	8643	80617	5794	89069	3587	94824	2094
20	69900	8365	81741	5542	89782	3408	95446	1955
21	71354	8082	82782	5296	90482	3229	96068	1816
22	72754	7795	83738	5055	91174	3050	96690	1677
23	74093	7507	84603	4817	91858	2871	97312	1538
24	75364	7217	85384	4581	92533	2692	97934	1399
25	76570	6928	86084	4349	93208	2513	98556	1260

TABLE III.

VALUATION FACTORS.—Showing, at the specified ages at entry, and durations of assurance (reckoned from the end of the calendar year of entry), (1) The value of an ASSURANCE of 1 payable at death (with an allowance for lapse); (2) The value of an ANNUITY of 1 per annum, payable as long as the assurance is subsisting.

3 PER-CENT.

HALF LAPSE RATE.

Duration of Assurance (<i>n</i>)	<i>[x] = 5</i>				<i>[x] = 15</i>				<i>[x] = 25</i>				<i>[x] = 35</i>				Duration of Assurance (<i>n</i>)
	<i>d</i> (<i>w</i>) <i>A</i> _{<i>x</i> +<i>n</i>}		<i>w</i> <i>t</i> _{<i>d</i>} <i>A</i> _{<i>x</i> +<i>n</i>}		<i>d</i> (<i>w</i>) <i>A</i> _{<i>x</i> +<i>n</i>}		<i>w</i> <i>t</i> _{<i>d</i>} <i>A</i> _{<i>x</i> +<i>n</i>}		<i>d</i> (<i>w</i>) <i>A</i> _{<i>x</i> +<i>n</i>}		<i>w</i> <i>t</i> _{<i>d</i>} <i>A</i> _{<i>x</i> +<i>n</i>}		<i>d</i> (<i>w</i>) <i>A</i> _{<i>x</i> +<i>n</i>}		<i>w</i> <i>t</i> _{<i>d</i>} <i>A</i> _{<i>x</i> +<i>n</i>}		
1	12150	11374	07764	8192	12922	10120	17091	9974	20546	9052	0						
2	14368	14332	11187	11474	17851	13340	23177	13028	28246	11903	1						
3	15455	15413	13198	13126	20451	14648	26211	14234	32292	13067	2						
4	16389	16467	14684	14172	22182	15305	28298	14861	31929	13590	3						
5	17196	17037	15867	14845	23611	15742	30020	15256	36931	13825	4						
6	17978	17442	16920	15322	24934	16109	31613	15548	38760	13961	5						
7	18728	17734	17937	15705	26193	16440	33111	15764	40462	14025	6						
8	19427	18109	18966	16027	27406	16718	34532	15914	42070	14030	7						
9	20126	18468	20031	16307	28607	16962	35917	16021	43635	13998	8						
10	20841	18843	21120	16554	29790	17172	37278	16093	45153	13930	9						
11	21578	19279	22207	16764	30921	17328	38590	16119	46636	13832	10						
12	22354	19675	23294	16954	32010	17439	39852	16103	48117	13714	11						
13	23175	20035	24367	17125	33056	17504	41083	16054	49577	13573	12						
14	24025	20369	25419	17283	34049	17521	42266	15968	51013	13409	13						
15	24879	20679	26440	17427	34995	17495	43399	15845	52427	13224	14						
16	25725	20961	27418	17555	35902	17432	44473	15685	53814	13017	15						
17	26555	21216	28348	17669	36778	17335	45530	15504	55190	12793	16						
18	27360	21445	29265	17755	37634	17212	46557	15299	56529	12561	17						
19	28137	21644	30164	17813	57847	12314	18						
20	28881	21819	31046	17843	59142	12053	19						
21	29583	21971	31924	17853	60407	11776	20						
22	30279	22101	32779	17832	61610	11485	21						
23	30976	22212	33618	17786	62854	11183	22						
24	31678	22302	34447	17716	64049	10871	23						
25	32391	22372	35262	17623	24						

TABLE IV.

VALUATION FACTORS.—Showing, at the specified ages at entry, and durations of assurance (reckoned from the end of the calendar year of entry), the value, ${}^{w(d)}(IA)_{[x]+n}$, of a SURRENDER ALLOWANCE of 1 in respect of each year of duration completed at exit, with proportionate allowance for fractional durations, payable (after 5 years' duration) on lapse, with allowance for mortality.

FULL LAPSE RATE.

3 PER-CENT.

Duration (<i>n</i>)	AGE AT ENTRY = [<i>x</i>]									Duration (<i>n</i>)
	1	5	15	25	35	45	55	65	75	
0	1.958	.825	1.097	.951	.781	.726	.640	.439	.142	0
1	3.476	2.460	2.546	2.135	1.829	1.582	1.145	.718	.232	1
2	4.445	3.768	3.479	2.883	2.528	2.057	1.429	.883	.288	2
3	5.330	4.957	4.212	3.486	3.064	2.438	1.680	1.045	.352	3
4	6.142	6.053	4.903	4.015	3.526	2.786	1.930	1.223	.430	4
5	6.933	7.132	5.623	4.608	3.987	3.136	2.191	1.420	.528	5
6	7.295	7.537	5.814	4.689	4.023	3.111	2.104	1.329	.454	6
7	7.611	7.874	5.949	4.724	4.023	3.058	1.987	1.208	.388	7
8	7.886	8.150	6.032	4.719	3.990	2.983	1.854	1.078	.310	8
9	8.122	8.371	6.070	4.680	3.933	2.890	1.706	.942	.168	9
10	8.322	8.542	6.078	4.620	3.853	2.778	1.557	.812	.144	10
11	8.488	8.665	6.064	4.545	3.748	2.654	1.409	.667	.093	11
12	8.623	8.747	6.034	4.460	3.628	2.511	1.271	.528	...	12
13	8.731	8.790	6.003	4.373	3.496	2.358	1.143	.427	...	13
14	8.817	8.800	5.974	4.295	3.358	2.196	1.019	.318	...	14
15	8.886	8.782	5.953	4.236	3.216	2.035	.894	.228	...	15
16	8.940	8.743	5.939	4.183	3.071	1.876	.778	.153	...	16
17	8.983	8.684	5.936	4.141	2.922	1.731	.674	.112	...	17
18	9.013	8.614	5.944	4.112	2.778	1.592	.579	.092	...	18
19	9.034	8.534	5.956	4.085	2.638	1.457	.503	.058	...	19
20	9.054	8.446	5.959	4.051	2.512	1.336	.433	20
21	9.073	8.363	5.953	4.007	2.403	1.230	.363	21
22	9.087	8.284	5.933	3.954	2.310	1.125	.312	22
23	9.091	8.211	5.902	3.894	2.220	1.026	.273	23
24	9.086	8.148	5.859	3.826	2.152	.940	.236	24
25	9.076	8.102	5.807	3.753	2.050	.856	.204	25
26	9.061	8.070	5.748	3.669	1.967	.799	.154	26
27	9.044	8.042	5.681	3.582	1.880	.730	.150	27
28	9.020	8.006	5.605	3.489	1.794	.665	.138	28
29	8.985	7.960	5.526	3.392	1.703	.618	.115	29
30	8.942	7.908	5.436	3.287	1.610	.552	.073	30
31	8.890	7.846	5.340	3.180	1.517	.498	31
32	8.830	7.769	5.234	3.065	1.424	.436	32
33	8.762	7.680	5.121	2.949	1.329	.382	33
34	8.687	7.579	5.002	2.850	1.237	.322	34
35	8.603	7.469	4.876	2.704	1.144	.281	35
36246	36

TABLE V.

VALUATION FACTORS.—*Showing, at the specified ages at entry, and durations of assurance (reckoned from the end of the calendar year of entry), the value, $w^{(d)}(IA)_{[x]+n}$, of a SURRENDER ALLOWANCE of 1 in respect of each year of duration completed at exit, with proportionate allowance for fractional durations, payable (after 5 years' duration) on lapse, with allowance for mortality.*

HALF LAPSE RATE.

3 PER-CENT.

Duration (<i>n</i>)	AGE AT ENTRY = [<i>x</i>]									Duration (<i>n</i>)
	1	5	15	25	35	45	55	65	75	
0	2.204	1.844	1.597	1.252	1.026	.821	.584	.318	.099	0
1	2.996	2.853	2.287	1.779	1.480	1.160	.776	.452	.133	1
2	3.487	3.536	2.695	2.090	1.758	1.346	.888	.521	.159	2
3	3.914	4.107	3.010	2.337	1.970	1.497	.989	.591	.189	3
4	4.298	4.609	3.302	2.563	2.156	1.637	1.091	.668	.227	4
5	4.662	5.083	3.598	2.788	2.340	1.777	1.198	.754	.274	5
6	4.812	5.218	3.630	2.776	2.316	1.734	1.133	.697	.237	6
7	4.939	5.316	3.637	2.748	2.281	1.681	1.058	.629	.200	7
8	5.043	5.381	3.620	2.703	2.232	1.621	.977	.557	.157	8
9	5.125	5.416	3.583	2.644	2.175	1.554	.892	.484	.110	9
10	5.187	5.428	3.537	2.580	2.110	1.481	.806	.410	.075	10
11	5.229	5.417	3.486	2.513	2.034	1.404	.726	.337	.046	11
12	5.256	5.386	3.432	2.444	1.952	1.318	.652	.268	.016	12
13	5.268	5.337	3.383	2.380	1.868	1.231	.580	.207	...	13
14	5.271	5.274	3.342	2.323	1.782	1.141	.515	.155	...	14
15	5.267	5.202	3.309	2.280	1.697	1.050	.450	.119	...	15
16	5.258	5.123	3.284	2.241	1.612	.965	.390	.089	...	16
17	5.244	5.039	3.265	2.210	1.528	.886	.336	.070	...	17
18	5.226	4.954	3.256	2.187	1.446	.814	.291	.045	...	18
19	5.205	4.870	3.251	2.166	1.369	.743	.251	.023	...	19
20	5.186	4.783	3.241	2.141	1.299	.680	.216	20
21	5.168	4.705	3.226	2.111	1.240	.624	.182	21
22	5.148	4.634	3.205	2.079	1.186	.570	.162	22
23	5.126	4.569	3.178	2.042	1.139	.521	.129	23
24	5.100	4.514	3.146	2.001	1.094	.475	.121	24
25	5.072	4.468	3.108	1.957	1.052	.438	.105	25
26	5.041	4.432	3.066	1.911	1.006	.399	.078	26
27	5.013	4.399	3.022	1.862	.960	.366	.054	27
28	4.980	4.366	2.974	1.808	.914	.333	.041	28
29	4.945	4.328	2.924	1.753	.867	.302	.017	29
30	4.904	4.286	2.868	1.698	.818	.275	30
31	4.857	4.238	2.809	1.639	.768	.239	31
32	4.810	4.185	2.749	1.577	.723	.216	32
33	4.758	4.126	2.684	1.513	.671	.179	33
34	4.704	4.060	2.613	1.451	.622	.166	34
35	4.643	3.989	2.541	1.385	.579	.143	35
36531	.106	36

TABLE VI.

VALUATION DATA, formed by the combination of the records of two Industrial Offices as to Whole-Life Assurances, at actual office ages at entry, and durations reckoned from the end of the calendar year of entry.

(ORIGINAL FACTS.)

OFFICE AGES AT ENTRY													
Duration	1		2		3-7		8-12		13-17		18-22		Duration
	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	
0	241,088	5,196.5	61,540	1,313.4	253,445	5,392.9	219,481	4,681.3	206,719	4,672.5	204,663	5,342.2	0
1	143,160	3,093.8	20,495	439.4	79,845	1,706.7	77,116	1,651.4	88,968	2,018.8	96,500	2,526.4	1
2	112,549	2,438.6	12,545	271.7	51,399	1,113.4	53,340	1,158.9	61,360	1,413.9	67,428	1,793.9	2
3	99,615	2,158.4	13,325	288.6	44,550	965.2	45,435	986.4	52,790	1,214.5	58,031	1,540.3	3
4	86,615	1,876.8	11,905	258.0	42,255	915.2	42,484	922.3	48,694	1,120.2	51,383	1,360.4	4
0-4	683,027	14,704.1	119,810	2,571.1	471,494	10,093.4	437,856	9,400.3	458,531	10,439.9	478,005	12,563.2	0-4
5	73,970	1,602.7	8,680	188.1	32,185	697.3	34,460	743.4	35,678	822.8	38,265	1,011.2	5
6	67,774	1,468.4	8,575	185.6	32,123	696.2	32,879	713.9	34,193	786.5	35,715	942.9	6
7	60,736	1,315.8	7,780	168.6	27,465	595.4	29,060	631.3	28,861	663.7	33,024	869.0	7
8	55,675	1,206.2	7,725	167.3	25,148	545.1	28,594	623.5	27,876	640.1	27,865	734.3	8
9	46,932	1,016.8	5,415	117.2	18,232	395.2	18,061	392.7	18,467	425.2	18,489	490.8	9
5-9	305,087	6,609.9	38,175	826.8	135,153	2,929.2	143,054	3,104.8	145,075	3,338.3	153,358	4,048.2	5-9
10	40,365	874.7	4,485	97.3	13,480	292.3	13,097	284.8	13,310	306.0	16,072	423.3	10
11	38,198	827.7	3,950	85.6	13,097	283.9	11,885	237.0	12,916	296.9	14,808	389.2	11
12	37,361	809.5	4,150	89.9	13,702	296.7	13,569	295.4	13,246	307.0	13,514	361.7	12
13	36,090	783.8	4,295	93.2	15,023	325.7	13,892	291.7	13,542	292.3	13,444	325.8	13
14	33,027	715.6	3,980	86.2	13,910	301.4	13,358	277.6	12,326	260.8	12,872	303.0	14
10-14	185,041	4,011.3	20,860	452.2	69,212	1,500.0	65,801	1,386.5	65,340	1,463.0	70,710	1,803.0	10-14
15-19	159,172	3,196.7	21,972	439.8	71,607	1,430.8	58,589	1,164.9	47,996	1,003.2	49,715	1,160.7	15-19
20-24	92,793	1,731.2	15,519	292.0	50,718	954.2	39,765	747.0	30,025	597.6	27,669	618.2	20-24
25-29	25,533	481.7	6,492	122.6	15,387	290.7	16,780	325.2	11,138	223.6	9,342	208.3	25-29
30-36	4,851	101.1	1,319	27.8	3,890	83.9	8,457	178.4	5,132	116.0	3,797	90.8	30-36
Total	1,455,504	30,896.0	224,147	4,732.3	817,461	17,282.2	770,302	16,307.1	763,237	17,181.6	792,596	20,492.4	Total

TABLE VII.

VALUATION DATA, as above, adjusted (as explained on pp. 553-555) for valuation at individual durations, and central ages at entry.
(ADJUSTED FACTS.)

Duration	VALUATION AGES AT ENTRY										Duration
	1		5		10		15		20		
	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	
0	302,628	6,509.9	253,445	5,392.9	220,309	4,656.9	205,891	4,694.8	201,930	5,141.6	0
1	163,655	3,533.2	79,845	1,706.7	75,087	1,607.2	90,898	2,041.4	95,072	2,448.4	1
2	125,004	2,710.3	51,399	1,113.4	51,261	1,112.6	63,319	1,444.8	66,135	1,729.5	2
3	112,940	2,447.0	44,550	965.2	44,309	953.2	53,916	1,245.5	57,285	1,483.6	3
4	98,520	2,134.8	42,255	915.2	41,243	894.1	49,935	1,138.8	51,042	1,327.4	4
0-4	802,837	17,335.2	471,494	10,093.4	432,209	9,224.0	463,959	10,565.3	471,464	12,130.5	0-4
5	82,650	1,790.8	32,185	697.3	33,364	723.9	36,639	838.7	38,221	994.1	5
6	76,349	1,654.0	32,123	696.2	31,887	691.8	35,087	799.4	35,791	929.3	6
7	68,516	1,484.4	27,465	595.4	28,530	614.8	29,399	678.8	33,465	861.8	7
8	63,825	1,382.7	24,723	535.9	27,532	599.4	28,894	657.8	28,056	727.7	8
9	52,417	1,135.6	18,162	393.6	17,781	383.5	18,747	433.6	18,495	479.9	9
5-9	343,757	7,447.5	134,658	2,918.4	139,094	3,013.4	148,826	3,408.3	154,028	3,992.8	5-9
10	45,187	979.3	13,143	285.0	12,895	280.1	13,418	305.9	15,778	410.9	10
11	42,395	918.6	12,850	278.6	11,701	232.7	13,038	297.4	14,622	377.3	11
12	41,852	906.8	13,361	289.3	13,328	288.0	13,487	313.9	13,325	347.9	12
13	40,843	885.2	14,565	315.7	13,732	288.4	13,702	295.6	13,267	314.6	13
14	37,414	810.5	13,503	292.7	13,220	275.3	12,464	263.1	12,529	290.4	14
10-14	207,691	4,500.4	67,422	1,461.3	64,876	1,364.5	66,109	1,475.9	69,521	1,741.1	10-14
15-19	182,781	3,669.3	70,475	1,406.1	58,316	1,159.1	47,746	999.2	48,829	1,127.1	15-19
20-24	110,932	2,072.9	48,710	915.2	39,379	738.1	29,777	592.5	27,011	602.1	20-24
25-29	32,792	618.5	14,649	276.7	16,853	323.6	11,038	225.1	9,108	200.9	25-29
30-36	6,305	134.4	3,734	80.0	8,384	176.2	5,303	120.6	3,756	90.1	30-36
Total	1,687,095	35,778.2	811,142	17,151.1	759,111	15,998.9	772,758	17,386.9	783,717	19,884.6	Total

TABLE VI—continued.
VALUATION DATA, formed by the combination of the records of two Industrial Offices as to Whole-Life Assurances,
at actual office ages at entry, and durations reckoned from the end of the calendar year of entry.
(ORIGINAL FACTS.)

OFFICE AGES AT ENTRY													
Duration	23-27		28-32		33-37		38-42		43-47		48-52		Duration
	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	
0	238,993	7,110.9	210,575	7,262.2	166,790	6,730.6	172,192	8,227.8	188,119	10,925.4	264,574	18,750.2	0
1	113,405	3,391.1	91,850	3,195.2	72,847	2,963.8	76,585	3,702.2	93,363	5,464.1	137,020	9,832.5	1
2	78,840	2,402.1	61,164	2,190.4	47,723	1,984.8	51,850	2,561.0	64,579	3,861.2	90,019	6,604.6	2
3	64,944	1,977.2	50,201	1,778.9	37,270	1,550.5	41,476	2,013.7	50,948	3,050.0	66,889	4,872.2	3
4	54,848	1,672.2	41,914	1,485.0	32,025	1,330.9	35,088	1,728.0	42,280	2,495.2	61,628	4,406.9	4
0-4	551,030	16,553.5	455,704	15,911.7	356,655	14,560.6	377,191	18,262.7	439,289	25,795.9	620,130	44,466.4	0-4
5	43,942	1,326.2	32,470	1,141.8	25,167	1,037.8	28,365	1,386.9	35,137	2,062.2	46,219	3,272.7	5
6	38,639	1,173.7	29,656	1,048.9	23,826	983.0	24,604	1,205.1	32,436	1,904.0	42,197	2,996.3	6
7	33,151	1,004.7	24,126	851.9	19,761	814.0	21,165	1,037.7	24,390	1,422.9	32,321	2,306.6	7
8	28,136	852.2	19,951	706.4	16,952	699.3	17,696	864.5	20,822	1,224.6	25,503	1,829.4	8
9	19,832	602.9	15,906	561.5	11,411	469.5	13,474	656.7	14,970	877.3	17,751	1,270.4	9
5-9	163,700	4,959.7	122,109	4,310.5	97,117	4,003.6	105,304	5,150.9	127,755	7,491.0	163,991	11,675.4	5-9
10	16,418	497.2	11,712	414.6	9,535	392.0	10,949	531.5	11,865	690.1	14,212	1,003.3	10
11	16,762	499.9	12,299	419.2	10,162	403.4	10,483	493.2	12,731	720.4	14,164	1,013.8	11
12	15,392	469.1	10,806	386.2	8,596	352.3	11,561	560.2	12,770	734.5	14,568	1,012.2	12
13	15,158	406.8	12,095	360.7	12,092	400.2	15,918	587.5	17,463	745.5	20,308	1,001.6	13
14	14,900	385.7	11,973	343.8	11,449	363.6	15,316	537.1	17,640	707.8	17,969	847.8	14
10-14	78,630	2,258.7	58,885	1,924.5	51,834	1,911.5	64,227	2,709.5	72,469	3,598.3	81,221	4,878.7	10-14
15-19	53,857	1,376.2	48,326	1,358.0	47,975	1,501.4	57,665	1,996.7	55,693	2,181.7	52,177	2,419.0	15-19
20-24	32,318	766.1	30,915	820.2	28,405	825.2	34,668	1,104.9	31,274	1,102.5	24,564	1,030.1	20-24
25-29	10,860	258.4	11,054	286.4	9,500	274.8	10,041	326.6	8,190	300.7	4,555	189.6	25-29
30-36	3,879	98.2	3,092	86.6	2,074	66.6	1,980	71.3	1,182	49.5	571	25.4	30-36
Total	894,274	26,270.8	730,085	24,697.9	593,560	23,143.7	651,076	29,622.6	735,852	40,519.6	947,209	64,684.6	Total

TABLE VII—continued.
VALUATION DATA, as above, adjusted (as explained on pp. 553-555) for valuation at individual durations, and central ages at entry.
(ADJUSTED FACTS.)

Duration	VALUATION AGES AT ENTRY										50		Duration
	25		30		35		40		45		Sum Assured	Premium	
0	247,127	7,413.4	206,719	7,162.4	165,215	6,707.1	172,128	8,128.5	185,368	10,709.2	268,877	18,669.1	0
1	117,392	3,533.3	89,762	3,152.6	72,475	2,942.0	75,958	3,610.8	91,760	5,332.1	139,030	9,807.4	1
2	82,159	2,526.4	59,310	2,145.9	47,671	1,958.8	51,744	2,547.8	63,510	3,771.7	90,941	6,577.8	2
3	67,724	2,087.7	48,699	1,727.3	36,738	1,547.5	41,038	1,996.6	50,106	2,978.4	68,287	4,947.5	3
4	56,480	1,743.1	40,750	1,456.7	31,898	1,318.3	34,645	1,696.3	42,134	2,446.3	61,588	4,369.8	4
0-4	570,882	17,303.9	445,240	15,644.9	354,027	14,473.7	375,513	18,010.0	433,178	25,257.7	628,723	44,367.6	0-4
5	45,091	1,374.4	31,772	1,119.3	24,835	1,037.1	27,920	1,348.1	34,926	2,031.7	46,721	3,274.1	5
6	39,016	1,197.1	29,582	1,048.3	23,545	981.0	24,387	1,179.9	31,996	1,865.2	42,488	2,990.0	6
7	33,480	1,030.6	23,600	834.6	19,509	813.6	21,097	1,008.2	24,458	1,447.4	32,403	2,255.6	7
8	28,631	880.0	19,487	691.6	16,774	637.4	17,320	836.2	20,845	1,216.1	26,301	1,846.9	8
9	20,220	622.2	15,787	558.5	11,136	464.9	13,322	610.3	15,122	880.9	17,936	1,262.7	9
5-9	166,438	5,104.3	120,228	4,252.3	95,799	3,994.0	104,046	5,012.7	127,347	7,441.3	165,849	11,629.3	5-9
10	17,041	515.8	11,477	413.2	9,423	382.4	10,970	529.3	11,862	685.9	14,550	1,010.9	10
11	17,370	523.0	11,988	411.8	10,113	399.2	10,380	485.7	12,583	706.9	14,278	1,026.6	11
12	15,953	492.5	10,514	377.1	8,522	350.7	11,555	555.7	12,713	721.0	14,730	1,024.8	12
13	15,579	422.4	11,851	356.3	11,929	391.5	15,754	579.0	17,983	750.3	20,286	1,014.0	13
14	15,500	402.7	11,806	330.4	11,359	363.2	15,206	529.2	17,457	692.8	18,763	871.9	14
10-14	81,443	2,356.4	57,636	1,897.8	51,346	1,887.0	63,865	2,678.9	72,598	3,556.9	82,607	4,948.2	10-14
15-19	55,086	1,412.3	48,150	1,353.2	47,742	1,495.7	57,788	1,987.0	56,030	2,180.3	53,284	2,482.2	15-19
20-24	33,203	787.4	30,774	810.0	27,931	814.9	35,778	1,130.8	31,305	1,109.1	24,598	1,043.3	20-24
25-29	11,236	266.8	11,105	280.4	9,628	277.7	10,039	326.9	7,997	297.1	4,507	189.1	25-29
30-36	3,961	99.9	3,011	86.1	2,122	65.8	1,973	71.2	1,118	47.7	553	25.7	30-36
Total	922,249	27,331.0	716,174	24,333.7	588,595	23,008.8	649,002	29,217.5	729,573	39,890.1	960,121	64,685.4	Total

TABLE VI—continued.
VALUATION DATA, formed by the combination of the records of two Industrial Offices as to Whole-Life Assurances,
at actual office ages at entry, and durations reckoned from the end of the calendar year of entry.
(ORIGINAL FACTS.)

OFFICE AGES AT ENTRY												
Duration	53-57		58-62		63-67		68-72		73-77		78-82	
	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium
0	237,331	20,865.9	239,813	26,945.3	198,412	29,017.7	126,111	21,315.7	68,890	17,226.5	18,775	6,476.4
1	129,980	11,515.8	137,417	15,565.2	107,529	15,645.6	68,730	13,299.6	36,171	9,090.2	8,739	3,023.4
2	87,921	7,966.2	88,811	10,332.2	65,022	9,824.9	39,103	7,913.3	19,392	5,216.2	3,815	1,395.1
3	68,035	6,143.6	66,755	7,635.3	50,637	7,547.4	31,066	6,001.3	14,272	3,580.1	2,870	990.4
4	60,372	5,317.2	59,222	6,542.5	47,413	6,773.5	29,618	5,365.5	13,828	3,242.1	2,377	834.4
0-4	583,639	51,808.7	592,018	67,020.5	469,013	68,809.1	294,628	56,895.4	152,553	38,355.1	36,576	12,719.7
5	46,963	4,090.0	44,582	4,889.6	35,346	4,977.3	22,670	4,017.9	10,516	2,390.9	1,857	604.4
6	40,251	3,525.6	38,001	4,182.1	28,467	4,029.1	16,325	2,923.0	7,105	1,623.0	850	280.4
7	31,764	2,796.7	27,232	3,024.2	17,733	2,547.8	10,486	1,898.0	4,440	1,012.9	501	166.6
8	24,703	2,174.1	20,653	2,302.5	14,103	2,009.3	5,891	1,086.8	1,491	365.4	140	46.8
9	15,395	1,348.3	12,059	1,346.1	7,346	1,019.8	2,604	506.7	423	110.1	54	17.1
5-9	159,076	13,934.7	142,527	15,744.5	102,995	14,613.3	57,976	10,432.4	23,975	5,502.3	3,402	1,115.3
10	11,849	1,022.1	10,317	1,121.8	5,817	811.2	1,956	360.6	197	49.1	32	9.1
11	11,405	953.0	10,316	1,070.0	5,876	781.1	2,030	344.3	431	93.8	3	.9
12	12,639	1,066.4	10,823	1,145.5	5,582	757.7	2,489	436.3	491	108.0	33	9.5
13	17,807	1,084.7	13,398	1,085.7	7,373	752.1	2,849	334.5	387	69.1	63	17.1
14	16,288	932.8	12,004	896.1	5,669	551.6	2,447	312.2	308	54.1	3	.9
10-14	69,988	5,059.0	56,858	5,319.1	30,317	3,653.7	11,771	1,787.9	1,814	374.1	134	37.5
15-19	38,328	2,168.1	21,536	1,589.4	9,077	865.5	2,291	281.9	338	57.8	8	2.6
20-24	13,605	670.5	6,843	428.0	1,219	103.6	32	5.8	4	.9	4	1.3
25-29	1,889	100.5	698	43.6	72	5.5	8	.6
30-36	120	7.8	31	3.4	6	.8	9	.8
Total	866,645	73,749.3	820,511	90,148.5	612,699	88,051.5	366,707	69,404.2	178,692	44,290.8	40,124	13,876.4

TABLE VII—continued.
VALUATION DATA, as above, adjusted (as explained on pp. 553-555) for valuation at individual durations, and central ages at entry.
(ADJUSTED FACTS.)

VALUATION AGES AT ENTRY													
Duration	55		60		65		70		75		80		Duration
	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	Sum Assured	Premium	
0	235,843	21,238.3	247,831	26,995.4	193,262	29,057.1	127,484	24,969.3	67,690	17,381.1	15,734	5,626.4	0
1	130,200	11,655.4	140,519	15,518.0	106,099	15,815.8	69,826	13,780.0	34,894	9,017.1	7,248	2,593.8	1
2	88,365	8,014.3	90,971	10,394.5	64,251	9,989.3	39,123	8,084.4	18,508	5,133.5	3,099	1,161.4	2
3	68,723	6,134.7	67,613	7,804.1	50,415	7,657.5	30,655	6,032.0	13,536	3,520.2	2,275	796.0	3
4	61,968	5,335.9	59,871	6,717.4	47,500	6,888.6	29,126	5,370.0	13,076	3,191.0	1,918	682.6	4
0-4	585,099	52,408.6	606,805	67,429.4	461,527	69,408.3	296,214	58,235.7	147,704	38,242.9	30,274	10,860.2	0-4
5	47,373	4,106.2	45,485	4,948.5	35,341	5,104.4	22,517	4,045.6	9,812	2,314.6	1,560	514.4	5
6	40,855	3,518.3	38,875	4,313.7	28,347	4,094.9	16,089	2,937.9	6,481	1,529.2	718	241.5	6
7	32,096	2,844.2	27,591	3,079.6	17,534	2,571.2	10,402	1,918.7	4,049	954.3	402	134.6	7
8	24,685	2,195.3	21,498	2,399.7	13,677	1,994.5	5,525	1,012.2	1,331	333.7	120	40.7	8
9	15,300	1,368.8	12,545	1,391.2	7,054	1,048.2	2,384	478.9	378	100.4	35	11.1	9
5-9	160,309	14,032.8	145,694	16,132.7	101,953	14,813.2	56,917	10,423.3	22,051	5,232.2	2,835	942.3	5-9
10	11,759	1,030.5	10,782	1,186.0	5,484	792.3	1,716	326.1	162	41.7	21	5.7	10
11	11,669	959.0	10,460	1,142.6	5,877	754.4	1,824	318.0	366	81.0	2	.5	11
12	12,981	1,094.0	10,916	1,181.8	5,337	744.7	2,266	406.5	426	95.9	26	7.5	12
13	18,225	1,086.3	13,342	1,124.8	7,081	739.7	2,686	329.0	329	61.5	43	11.8	13
14	16,113	938.8	12,166	924.5	5,481	549.2	2,226	293.0	230	40.7	2	.7	14
10-14	70,747	5,108.6	57,666	5,559.7	29,260	3,580.3	10,718	1,664.5	1,513	320.8	94	26.2	10-14
15-19	37,641	2,153.0	21,662	1,633.1	8,512	833.2	1,983	250.7	291	50.8	6	2.1	15-19
20-24	13,403	670.4	6,544	418.7	957	86.4	30	5.3	4	.9	4	1.3	20-24
25-29	1,888	102.3	613	40.0	48	4.1	8	.6	25-29
30-36	95	5.8	30	3.3	6	.8	9	.8	30-36
Total	869,182	74,481.5	839,044	91,216.9	602,263	88,726.3	365,871	70,580.3	171,571	43,848.2	33,213	11,832.1	Total

TABLES showing the detailed Valuation Results, for each duration, in respect of
ALL AGES AT ENTRY COMBINED, on the undermentioned basis.

TABLE VIII.

FULL LAPSE RATE.

3 PER-CENT

Curate Duration	PRESENT VALUE OF					LIABILITY ON BASIS OF		Curate Duration
	Sums Assured	Surrender Allowances	Office Premiums	$\frac{3}{4}$ Office Premiums	Net Premiums	$\frac{3}{4}$ Office Premiums	Net Premiums	
0	459,732·4	32,079·6	879,143·3	586,095·4	469,500·8	94,283·4	22,311·2	
1	389,557·5	31,542·0	712,490·5	474,993·5	378,506·4	53,894·0	42,593·1	
2	293,323·7	28,008·7	527,052·9	351,368·6	271,399·2	30,036·2	49,933·2	
3	255,023·0	26,919·7	432,489·6	288,326·4	224,741·9	(7,783·5)	57,200·8	
						(1,399·8)		
4	246,485·6	27,366·1	388,062·1	258,708·2	206,028·7	(4,510·3)	67,823·0	
						(19,653·8)		
0-4	1,644,122·2	145,916·1	2,939,238·4	1,959,492·1	1,550,177·0	(190,507·4)	239,861·3	0-
						(21,053·6)		
5	203,182·4	24,240·9	302,516·3	201,677·5	161,849·5	(3,014·3)	65,573·8	
						(28,760·1)		
6	185,601·9	21,585·0	267,010·6	178,006·9	141,451·6	(2,456·6)	65,735·3	
						(31,636·6)		
7	150,611·5	17,342·8	211,030·9	140,687·2	110,634·6	(1,818·8)	57,319·7	
						(29,085·9)		
8	127,332·3	14,580·6	174,082·7	116,055·4	90,316·2	(1,336·0)	51,596·7	
						(27,193·5)		
9	88,452·2	10,057·4	118,256·3	78,837·7	61,012·3	(849·0)	37,497·3	
						(20,520·9)		
5-9	755,180·3	87,806·7	1,072,896·8	715,264·7	565,264·2	(9,474·7)	277,722·8	5-
						(137,197·0)		
10	73,975·4	7,877·6	94,147·8	62,765·2	49,027·9	(499·7)	32,825·1	1
						(19,587·5)		
11	76,741·4	7,377·0	90,990·5	60,660·4	48,471·7	(219·7)	35,646·7	1
						(23,677·7)		
12	80,269·1	7,282·1	92,230·5	61,486·9	48,247·3	26,064·3	39,303·9	1
13	101,598·6	6,971·1	89,686·1	59,790·8	57,172·8	48,778·9	51,396·9	1
14	96,967·4	6,205·6	79,884·7	53,256·5	52,385·4	49,916·5	50,787·6	1
10-14	429,551·9	35,713·4	446,939·6	297,959·8	255,305·1	(719·4)	209,960·2	10-
						(168,024·9)		
15	76,073·0	5,017·7	62,264·7	41,509·7	40,112·2	39,581·0	40,978·5	1
16	65,837·6	4,509·6	54,288·3	36,192·2	34,039·7	34,155·0	36,307·5	1
17	62,608·4	4,361·5	50,424·6	33,616·5	31,719·3	33,353·4	35,250·6	1
18	63,934·4	4,596·9	50,973·3	33,982·2	31,949·1	34,549·1	36,582·2	1
19	73,817·8	4,819·0	54,022·6	36,014·9	35,099·3	42,621·9	43,537·5	1
15-19	342,271·2	23,304·7	271,973·5	181,315·5	172,919·6	184,260·4	192,656·3	15-
20	67,152·9	3,929·2	44,877·7	29,918·4	30,346·3	41,163·7	40,735·8	20
21	45,494·6	2,740·9	30,366·9	20,244·4	20,069·4	27,991·1	28,166·1	21
22	36,774·0	2,266·5	24,485·5	16,323·5	15,894·9	22,717·0	23,145·6	22
23	32,989·9	2,108·6	22,505·7	15,003·8	13,935·0	20,094·7	21,163·5	23
24	25,492·2	1,436·9	15,622·8	10,415·3	10,148·2	16,513·8	16,780·9	24
20-24	207,903·6	12,482·1	137,858·6	91,905·4	90,393·8	128,480·3	129,991·9	20-
25	22,571·2	1,209·8	13,269·7	8,846·4	8,614·1	14,934·6	15,166·9	25
26	17,128·6	981·0	10,387·9	6,925·3	6,452·4	11,184·3	11,657·2	26
27	11,955·2	677·8	7,066·8	4,711·1	4,350·0	7,921·9	8,283·0	27
28	9,583·0	581·1	5,760·6	3,840·5	3,469·4	6,323·6	6,694·7	28
29	6,411·1	376·1	3,788·8	2,525·9	2,201·5	4,261·3	4,585·7	29
25-29	67,649·1	3,825·8	40,273·8	26,849·2	25,087·4	44,625·7	46,387·5	25-
30	4,585·1	250·4	2,533·8	1,689·3	1,488·4	3,146·2	3,347·1	30
31	4,022·3	225·2	2,287·8	1,525·2	1,268·7	2,722·3	2,978·8	31
32	4,737·8	235·1	2,546·6	1,697·6	1,413·9	3,275·3	3,559·0	32
33	1,406·6	101·2	953·2	635·5	433·8	872·3	1,074·0	33
34	2,680·1	144·0	1,499·2	999·4	743·1	1,824·7	2,081·0	34
35	3,265·9	191·3	1,865·5	1,243·6	918·4	2,213·6	2,538·8	35
36	15·6	4	5·1	3·4	3·4	12·6	12·6	36
30-36	20,713·4	1,147·6	11,691·2	7,794·0	6,269·7	14,067·0	15,591·3	30-
Total	3,467,391·7	310,196·4	4,920,871·9	3,280,580·7	2,665,416·8	(200,701·5)	1,112,171·3	Total
						(697,708·9)		
Modification where Surrender Allowance is excluded		Nil	2,462,096·1	186,811·0	1,005,295·6	

Negative values are throughout printed in *italics*.

TABLES showing the detailed Valuation Results, for each duration, in respect of ALL AGES AT ENTRY COMBINED, on the undermentioned basis.

TABLE IX. HALF LAPSE RATE. 3 PER-CENT.

PRESENT VALUE OF					LIABILITY ON BASIS OF		Curative Duration
Sums Assured	Surrender Allowances	Office Premiums	$\frac{3}{4}$ Office Premiums	Net Premiums	$\frac{3}{4}$ Office Premiums	Net Premiums	
856,600·8	35,633·9	1,554,217·7	1,036,145·1	877,364·1	143,910·4	14,870·6	0
549,068·7	23,936·8	952,404·0	634,935·9	535,566·2	61,930·4	37,439·3	1
389,422·3	19,174·3	664,676·3	443,117·5	363,796·9	34,520·9	44,799·7	2
327,002·2	17,394·0	529,785·0	353,189·9	292,688·2	10,344·1	51,708·0	3
306,375·7	16,878·6	464,704·4	309,803·0	261,968·5	1,550·4	61,285·8	4
					6,593·9		
					20,045·2		
2,428,469·7	113,017·6	4,165,787·4	2,777,191·4	2,331,383·9	257,299·7	210,103·4	0-4
					21,595·6		
247,099·5	14,384·6	356,264·8	237,509·9	202,393·0	4,667·2	59,091·1	5
					28,641·4		
222,755·7	12,691·4	311,034·2	207,356·1	175,513·1	3,885·1	59,934·0	6
					31,976·1		
179,310·7	10,143·8	244,106·9	162,737·0	136,743·0	2,977·9	52,711·5	7
					29,694·5		
150,649·7	8,492·3	200,248·1	133,498·6	111,393·2	2,256·7	47,748·8	8
					27,990·1		
103,993·3	5,839·0	135,275·1	90,183·6	74,876·2	1,424·7	34,956·1	9
					21,078·4		
903,808·9	51,551·1	1,246,929·1	831,286·1	700,918·5	15,211·6	254,441·5	5-9
					139,285·5		
85,747·7	4,533·0	106,580·7	71,054·0	59,442·1	946·4	30,838·6	10
					20,173·1		
87,614·4	4,208·9	101,854·3	67,902·9	58,091·3	592·4	33,732·0	11
					24,512·8		
90,508·5	4,129·3	102,280·7	68,187·4	57,327·5	266·0	37,310·3	12
					26,716·4		
112,390·8	3,930·9	98,757·7	65,838·5	66,942·7	50,483·2	49,379·0	13
106,550·4	3,481·4	87,544·9	58,363·2	61,084·0	51,668·6	48,947·8	14
482,811·8	20,283·5	497,018·3	331,346·0	302,887·6	1,804·8	200,207·7	10-14
					173,554·1		
83,584·3	2,812·7	68,220·3	45,480·3	46,799·2	40,916·7	39,597·8	15
72,418·4	2,518·6	59,406·8	39,604·4	39,805·9	35,332·6	35,131·1	16
69,002·1	2,435·3	55,198·2	36,798·8	37,176·9	34,638·6	34,260·5	17
70,760·8	2,563·0	55,827·6	37,218·6	37,602·0	36,105·2	35,721·8	18
81,107·3	2,670·7	58,852·5	39,235·0	41,232·4	44,543·0	42,545·6	19
376,872·9	13,000·3	297,505·4	198,337·1	202,616·4	191,536·1	187,256·8	15-19
73,166·9	2,163·6	48,615·2	32,410·1	35,450·0	42,920·4	39,880·5	20
49,530·5	1,508·9	32,893·3	21,928·8	23,357·1	29,110·6	27,682·3	21
40,091·2	1,242·9	26,493·0	17,661·8	18,576·5	23,672·3	22,757·6	22
35,949·2	1,152·7	24,299·8	16,199·8	16,272·8	20,902·1	20,829·1	23
27,547·9	782·0	16,791·2	11,194·1	11,781·6	17,135·8	16,548·3	24
226,285·7	6,850·1	149,092·5	99,394·6	105,438·0	133,741·2	127,697·8	20-24
24,300·6	656·0	14,209·9	9,473·3	9,988·9	15,483·3	14,967·7	25
18,493·0	530·3	11,126·3	7,417·6	7,514·6	11,605·7	11,508·7	26
12,896·6	365·6	7,560·0	5,040·1	5,079·3	8,222·1	8,182·9	27
10,379·3	313·0	6,173·7	4,115·8	4,067·3	6,576·5	6,625·0	28
6,913·2	201·5	4,043·7	2,695·7	2,586·1	4,419·0	4,528·6	29
72,982·7	2,066·4	43,113·6	28,742·5	29,236·2	46,306·6	45,812·9	25-29
4,921·5	133·8	2,696·7	1,797·7	1,754·8	3,257·6	3,300·5	30
4,309·7	119·8	2,428·0	1,618·6	1,497·8	2,810·9	2,931·7	31
5,036·7	124·2	2,682·4	1,788·1	1,656·6	3,372·8	3,504·3	32
1,518·3	53·8	1,014·0	676·0	518·8	896·1	1,053·3	33
2,848·4	75·9	1,579·0	1,052·9	878·1	1,871·4	2,046·2	34
3,488·9	101·1	1,970·3	1,313·6	1,083·9	2,276·1	2,506·1	35
16·2	·2	5·4	3·5	3·5	12·9	12·9	36
22,139·7	608·8	12,375·8	8,250·4	7,393·5	14,498·1	15,355·0	30-36
4,513,371·4	207,377·8	6,411,822·1	4,274,548·1	3,679,874·1	274,316·1	1,040,875·1	Total
					720,517·2		
Location where Allowance excluded	Nil	3,513,303·2	238,823·3	1,000,068·2	...

Negative values are throughout printed in *italics*.

TABLES showing the detailed Valuation Results, for each duration, in respect of ALL AGES AT ENTRY COMBINED, on the undermentioned basis.

TABLE X.

EXCLUDING LAPSE RATE.

3 PER-CENT.

Currate Durations	PRESENT VALUE OF				LIABILITY ON BASIS OF		Currate Durations
	Sums Assured	Office Premiums	Office Premiums	Net Premiums	Office Premiums	Net Premiums	
0	1,594,001·4	2,675,710·1	1,783,806·7	1,589,243·8	189,505·3	4,757·6	0
1	798,901·2	1,301,254·6	867,503·1	767,771·7	<i>63,025·0</i> 423·1	31,129·5	1
2	531,392·3	855,686·6	570,457·7	492,336·2	<i>39,848·0</i> 782·6	39,056·1	2
3	429,984·1	661,900·3	441,266·9	384,144·7	<i>13,409·5</i> 2,126·7	45,839·4	3
4	389,605·7	566,416·4	377,611·0	334,570·9	<i>9,065·9</i> 21,060·6	55,034·8	4
0-4	3,743,884·7	6,060,968·0	4,040,645·4	3,568,067·3	<i>321,153·7</i> 24,393·0	175,817·4	0-4
5	306,788·4	426,008·1	284,005·5	253,649·6	<i>6,654·2</i> 29,437·1	53,138·8	5
6	272,602·0	367,502·0	245,001·1	217,769·3	<i>5,534·8</i> 33,135·7	54,832·7	6
7	217,536·8	286,230·4	190,820·2	168,699·7	<i>4,264·8</i> 30,981·4	48,837·1	7
8	181,516·6	233,342·2	155,561·5	136,858·0	<i>3,291·1</i> 29,246·2	44,658·6	8
9	124,426·2	156,670·7	104,447·1	91,437·6	<i>2,115·2</i> 22,084·3	32,988·6	9
5-9	1,102,870·0	1,469,753·4	979,835·4	868,414·2	<i>21,860·1</i> 144,894·7	234,455·8	5-9
10	101,015·6	122,032·6	81,355·0	71,649·8	<i>1,408·4</i> 21,069·0	29,365·8	10
11	101,504·1	115,200·0	76,800·0	69,135·8	<i>959·6</i> 25,663·7	32,368·3	11
12	103,467·0	114,514·1	76,342·6	67,572·5	<i>569·0</i> 27,693·4	35,894·5	12
13	125,751·9	109,735·5	73,157·0	77,627·6	<i>175·0</i> 52,709·9	48,124·3	13
14	118,306·8	96,749·8	64,499·3	70,447·9	<i>53,806·9</i>	47,858·9	14
10-14	550,045·4	558,232·0	372,154·5	356,433·6	<i>3,112·0</i> 181,002·9	193,611·8	10-14
15	92,791·1	75,357·9	50,238·5	53,976·1	42,552·6	38,815·0	15
16	80,443·5	65,500·3	43,666·9	45,953·5	36,776·6	34,490·0	16
17	76,818·7	60,866·4	40,577·7	43,002·3	36,241·0	33,816·4	17
18	79,126·9	61,578·2	41,052·1	43,650·6	38,074·8	35,476·3	18
19	89,937·8	64,530·7	43,020·4	47,660·3	46,917·4	42,277·5	19
15-19	419,118·0	327,833·5	218,555·6	234,242·8	200,562·4	184,875·2	15-19
20	80,364·8	52,969·4	35,312·9	40,707·1	45,051·9	39,657·7	20
21	51,364·9	35,834·9	23,889·8	26,765·4	30,475·1	27,599·5	21
22	44,039·6	28,814·2	19,209·4	21,324·3	24,830·2	22,715·3	22
23	39,456·0	26,364·7	17,576·4	18,664·9	21,879·6	20,791·1	23
24	29,964·1	18,130·1	12,086·9	13,426·8	17,877·2	16,537·3	24
20-24	248,189·4	162,113·3	108,075·4	120,888·5	140,114·0	127,300·9	20-24
25	26,317·1	15,280·9	10,187·3	11,357·4	16,129·8	14,959·7	25
26	20,076·2	11,962·5	7,975·0	8,570·5	12,101·2	11,505·7	26
27	13,985·7	8,116·8	5,411·3	5,799·1	8,574·4	8,186·6	27
28	11,299·6	6,638·8	4,425·7	4,654·1	6,873·9	6,645·5	28
29	7,488·0	4,328·9	2,885·8	2,960·7	4,602·2	4,527·3	29
25-29	79,166·6	46,327·9	30,885·1	33,341·8	48,281·5	45,824·8	25-29
30	5,304·2	2,877·9	1,918·6	2,004·7	3,385·6	3,299·5	30
31	4,633·1	2,583·2	1,722·2	1,706·3	2,910·9	2,926·8	31
32	5,367·5	2,830·8	1,887·2	1,879·7	3,480·3	3,487·8	32
33	1,643·4	1,080·6	720·4	596·7	923·0	1,046·7	33
34	3,035·8	1,666·0	1,110·6	996·0	1,925·2	2,039·8	34
35	3,737·7	2,085·4	1,390·2	1,233·8	2,347·5	2,503·9	35
36	16·9	5·5	3·6	5·1	13·3	11·8	36
30-36	23,738·6	13,129·4	8,752·8	8,422·3	14,985·8	15,316·3	30-36
Total	6,167,012·7	8,638,357·5	5,758,904·2	5,189,810·5	<i>346,125·8</i> 754,234·3	977,202·2	Total

Negative values are throughout printed in *italics*.

TABLES showing the detailed Valuation Results, for each duration, in respect of ALL AGES AT ENTRY COMBINED, on the undermentioned basis.

TABLE XI. EXCLUDING LAPSE RATE. 4 PER-CENT.

Duration	PRESENT VALUE OF				LIABILITY ON BASIS OF		Duration
	Sums Assured	Office Premiums	Office Premiums	Net Premiums	Office Premiums	Net Premiums	
0	1,336,854.7	2,396,664.6	1,597,776.5	1,332,212.3	260,891.8	4,672.4	0
1	677,361.2	1,171,750.6	781,167.0	649,242.8	103,805.8	28,118.4	1
2	449,692.1	769,711.5	513,140.9	414,570.2	63,448.8	35,121.9	2
3	363,594.2	594,525.1	396,350.1	322,423.6	32,815.7	41,170.6	3
4	331,079.8	508,950.0	339,300.1	281,387.6	17,758.7	49,692.2	4
0-4	3,158,612.0	5,441,601.8	3,627,734.6	2,999,836.5	478,720.8	158,775.5	0-4
5	261,142.2	383,129.6	255,419.7	213,194.7	16,755.5	47,947.5	5
6	231,507.0	329,970.2	219,950.1	182,149.8	15,873.9	49,357.2	6
7	183,784.6	258,194.1	172,129.2	140,051.7	15,423.1	43,732.9	7
8	152,547.6	208,195.0	138,796.5	112,726.1	13,404.4	39,821.5	8
9	104,079.0	139,368.3	92,912.2	74,857.0	15,189.3	29,222.0	9
5-9	933,060.4	1,318,857.2	879,237.7	722,979.3	89,701.5	210,081.1	5-9
10	84,773.3	108,642.0	72,427.9	58,709.5	11,294.6	26,063.8	10
11	85,682.7	102,780.7	68,520.4	56,823.2	10,588.0	28,559.5	11
12	87,817.5	102,404.1	68,269.4	55,675.0	11,473.2	32,142.5	12
13	108,134.9	98,249.3	65,469.1	64,609.6	14,063.5	43,525.3	13
14	101,904.2	86,606.6	57,737.7	58,588.1	15,188.5	43,316.1	14
10-14	468,312.6	498,682.7	332,454.5	294,405.4	145,557.8	173,907.2	10-14
15	79,612.7	67,277.2	44,851.6	44,624.5	15,888.2	34,988.2	15
16	68,645.3	58,336.9	38,891.3	37,730.6	16,058.1	30,914.7	16
17	65,260.9	54,086.8	36,058.1	35,079.6	16,020.8	30,181.3	17
18	66,790.6	54,391.6	36,394.4	35,327.1	16,036.2	31,463.5	18
19	76,298.7	57,331.5	38,221.0	38,671.4	16,077.7	37,627.3	19
15-19	356,608.2	291,624.0	194,416.4	191,433.2	163,118.0	165,175.0	15-19
20	68,657.6	47,184.9	31,456.7	33,176.3	17,200.9	35,481.3	20
21	46,454.6	31,899.0	21,265.9	21,773.6	15,188.7	24,681.0	21
22	37,500.7	25,634.6	17,089.8	17,269.7	14,010.9	20,231.0	22
23	33,568.8	23,471.1	15,647.5	15,086.9	14,921.3	18,481.9	23
24	25,730.9	16,193.9	10,795.9	10,919.2	14,935.0	14,811.7	24
20-24	211,912.6	144,383.5	96,255.8	98,225.7	115,656.8	113,686.9	20-24
25	22,678.0	13,681.9	9,121.3	9,252.4	13,556.7	13,425.6	25
26	17,224.0	10,700.6	7,133.7	6,943.0	10,090.3	10,281.0	26
27	12,007.3	7,265.9	4,843.9	4,692.5	7,163.4	7,314.8	27
28	9,640.2	5,930.1	3,953.3	3,745.5	5,686.9	5,894.7	28
29	6,423.2	3,878.8	2,555.9	2,386.9	3,837.3	4,036.3	29
25-29	67,972.7	41,457.3	27,638.1	27,020.3	40,334.6	40,952.4	25-29
30	4,573.5	2,584.2	1,722.8	1,622.0	2,850.7	2,951.5	30
31	3,998.3	2,324.4	1,549.6	1,383.7	2,448.7	2,614.6	31
32	4,669.3	2,563.3	1,709.1	1,532.0	2,960.2	3,137.3	32
33	1,406.1	970.1	646.7	478.5	759.4	927.6	33
34	2,646.5	1,509.7	1,006.4	809.2	1,640.1	1,837.3	34
35	3,234.7	1,883.6	1,255.6	996.1	1,979.1	2,238.6	35
36	15.2	5.1	3.4	3.4	11.8	11.8	36
30-36	20,543.6	11,840.4	7,893.6	6,824.9	12,650.0	13,718.7	30-36
Total	5,217,022.1	7,748,446.9	5,165,630.7	4,340,725.3	525,225.5	876,296.8	Total

Negative values are throughout printed in *italics*.

TABLE XII.

TABLES showing the detailed Valuation Results, for each Central Age at Entry, in respect of ALL DURATIONS COMBINED on the undermentioned basis.

FULL LAPSE RATE.

3 PER-CENT.

Central Age at Entry	PRESENT VALUE OF					NET LIABILITY ON BASIS OF					3 PER-CENT.	
	Sums Assured	Surrender Allowances	Office Premiums	Net Premiums		3 Office Premiums			Net Premiums		Without Surrender Allowances	Central Age at Entry
				With Surrender Allowances	Without Surrender Allowances	+	—	Balance	With Surrender Allowances	Without Surrender Allowances		
1	227,132.6	54,066.4	427,431.3	221,559.4	187,503.5	26,430.6	30,185.8	3,755.2	59,639.6	39,629.1	1	
5	82,861.0	20,274.9	141,244.3	56,075.7	46,082.4	16,606.8	7,633.6	8,973.2	47,060.2	36,778.6	5	
10	99,847.8	17,459.8	147,654.1	62,919.0	52,684.0	21,457.9	5,586.2	18,871.7	54,388.6	47,163.8	10	
15	122,433.2	16,618.1	172,605.4	91,215.7	80,638.3	27,430.9	3,449.7	23,981.2	47,835.6	41,794.9	15	
5-15	305,142.0	54,352.8	461,503.8	307,668.7	179,404.7	68,495.6	16,669.5	51,826.1	149,284.4	125,737.3	5-15	
20	145,061.0	17,197.3	200,517.8	114,605.8	103,171.6	31,482.2	2,902.3	28,579.9	47,652.5	41,880.4	20	
25	192,721.9	20,224.4	267,035.7	148,112.9	134,593.2	41,195.3	6,272.6	34,922.7	61,833.4	58,128.7	25	
30	168,338.1	15,582.0	222,865.9	123,287.2	112,492.2	41,538.7	6,195.9	35,342.8	60,632.9	55,845.9	30	
35	158,801.2	13,674.2	199,748.0	100,836.6	100,836.6	44,955.9	5,646.1	39,309.8	62,331.7	57,965.2	35	
40	206,486.0	15,493.9	244,180.7	138,152.3	127,199.6	65,703.2	6,510.4	59,192.8	83,827.6	79,286.4	40	
20-40	871,408.2	82,171.8	1,134,348.1	634,298.9	578,292.6	224,875.3	27,527.3	197,348.0	319,281.1	293,115.6	20-40	
45	250,808.3	18,782.6	318,343.6	212,229.0	180,882.5	76,319.0	9,957.1	66,361.9	97,708.4	92,092.2	45	
50	372,614.7	22,914.6	484,929.2	270,431.8	253,684.0	88,523.7	16,280.4	72,243.3	125,097.5	118,930.7	50	
55	374,622.2	23,958.9	511,942.5	341,295.0	281,039.1	79,319.2	22,033.1	57,286.1	117,541.7	110,422.7	55	
60	395,941.9	23,233.5	562,267.6	374,845.2	293,675.0	64,939.9	20,609.7	44,330.2	100,296.8	102,266.9	60	
45-60	1,402,987.1	88,889.6	1,877,482.9	1,042,232.3	979,274.6	309,101.6	68,880.3	240,221.5	449,644.4	423,712.5	45-60	
65	313,154.0	17,614.6	477,359.2	318,239.5	256,301.3	38,793.0	2,663.9	12,529.1	74,467.3	68,268.4	65	
70	212,057.2	9,966.5	328,130.9	218,754.0	174,694.1	21,662.6	18,392.9	3,269.7	41,235.2	37,363.1	70	
75	112,233.4	2,759.5	175,633.1	117,088.7	98,796.5	7,631.5	9,730.3	2,095.8	16,196.4	15,193.5	75	
80	23,277.2	375.2	38,982.6	21,229.5	21,001.1	715.5	3,051.5	2,336.0	2,422.9	2,276.1	80	
65-80	660,731.8	30,715.8	1,020,105.8	557,115.8	537,620.7	68,805.6	57,438.6	15,798.8	134,321.8	123,101.1	65-80	
Total	3,467,391.7	310,196.4	4,920,871.9	3,280,580.7	2,462,096.1	697,708.9	200,707.5	497,007.4	1,112,171.3	1,005,295.6	Total	

showing the detailed Valuation Results for each Central Age at Entry, in respect of ALL DURATIONS COMBINED, on the undermentioned basis.

3 PER-CENT.

HALF LAPSE RATE.

Central Age at Entry	PRESENT VALUE OF				NET LIABILITY ON BASIS OF							
	Sums Assured	Surrender Allowances	Office Premiums	Office Premiums	Net Premiums			Office Premiums.			Net Premiums	
					With Surrender Allowances	Without Surrender Allowances	Balance	With Surrender Allowances	Without Surrender Allowances	With Surrender Allowances	Without Surrender Allowances	
1	327,175.1	36,703.8	578,522.6	385,681.8	270,598.0	27,380.9	49,183.8	58,506.7	65,396.7	56,577.1	1	
5	134,313.6	15,754.4	224,372.9	149,581.8	95,041.3	17,464.5	16,978.3	15,268.2	42,563.7	39,272.3	5	
10	152,237.1	12,874.8	221,235.3	147,490.0	106,905.9	26,702.3	9,080.4	4,747.1	47,579.9	45,331.2	10	
15	179,781.0	11,786.9	217,320.9	161,880.8	138,363.5	30,807.9	4,120.8	14,900.2	43,621.9	41,417.5	15	
5-15	466,331.7	40,416.1	692,929.1	461,952.6	340,310.7	74,974.7	30,179.5	44,795.2	133,765.5	126,021.0	5-15	
20	206,288.9	11,874.5	279,619.4	186,412.8	163,821.1	34,321.9	3,471.3	19,876.1	44,526.6	42,467.8	20	
25	268,497.3	13,858.2	369,376.5	246,251.0	210,903.1	44,169.7	8,065.2	22,246.3	60,104.2	57,536.2	25	
30	230,373.7	10,666.2	308,551.6	205,701.2	184,508.9	43,849.5	8,510.8	24,672.5	56,531.0	54,855.9	30	
35	212,143.2	9,248.2	273,288.4	182,192.1	155,974.2	47,020.5	7,821.2	29,951.1	57,596.0	56,169.0	35	
40	266,433.5	10,224.1	325,858.3	217,238.8	189,925.9	68,225.7	8,806.9	49,194.7	77,944.6	76,507.6	40	
20-40	1,183,736.6	55,871.2	1,556,693.9	1,037,795.9	896,142.1	238,187.3	36,375.4	201,811.9	296,702.4	287,594.5	20-40	
45	330,241.3	12,143.8	416,224.6	277,482.8	249,128.5	78,435.7	13,533.4	52,758.5	93,256.6	91,302.2	45	
50	470,078.1	14,608.1	623,392.6	415,595.2	369,066.7	89,906.2	20,814.9	69,091.3	115,619.8	113,478.8	50	
55	465,914.5	15,002.3	613,566.1	429,044.2	371,433.2	80,161.6	28,289.0	51,872.6	109,783.6	106,843.1	55	
60	486,988.6	14,272.3	632,897.5	461,931.5	399,903.6	64,543.4	25,214.0	39,329.4	101,357.3	98,478.3	60	
45-60	1,753,222.8	56,026.5	2,376,080.8	1,584,053.7	1,389,290.2	313,046.9	87,851.3	225,195.6	420,017.3	410,102.4	45-60	
65	378,952.8	10,654.8	577,139.9	384,760.2	320,480.3	37,889.1	33,041.7	4,847.4	69,127.3	66,308.2	65	
70	250,183.5	5,900.1	386,650.9	257,767.3	213,715.0	29,967.0	22,650.7	7,683.7	38,315.9	36,468.5	70	
75	127,701.4	1,597.9	200,083.4	133,388.9	114,024.4	7,400.2	11,489.8	4,089.6	15,274.9	14,786.5	75	
80	26,067.5	207.4	43,721.5	29,147.7	23,999.8	671.1	3,543.9	2,872.8	2,275.1	2,210.0	80	
65-80	782,905.2	18,360.2	1,207,595.7	805,064.1	676,272.2	66,927.4	70,726.4	8,646.1	124,993.2	119,773.2	65-80	
Total	4,513,371.4	207,377.8	6,411,822.1	4,274,548.1	3,679,874.1	720,517.2	274,316.4	446,201.1	1,040,875.1	1,000,068.2	Total	

Negative values are throughout printed in italics.

TABLES showing the detailed Valuation Results, for each central Age at Entry respect of ALL DURATIONS COMBINED, on the undermentioned basis.

TABLE XIV.

EXCLUDING LAPSE RATE.

3 PER-CE

Central Entry Ages	PRESENT VALUE OF				LIABILITY ON BASIS OF		
	Sums Assured	Office Premiums	$\frac{2}{3}$ Office Premiums	Net Premiums	$\frac{2}{3}$ Office Premiums		Net Premiums
					+	-	
1	498,020·7	816,954·5	544,636·4	426,985·1	29,687·8	76,303·5	71,035·6
5	251,851·8	396,699·3	264,466·3	212,070·9	19,157·8	31,772·3	39,780·9
10	257,989·7	357,787·2	238,524·8	214,502·3	30,339·8	10,874·9	43,487·4
15	283,311·9	373,228·3	248,818·7	243,138·2	36,901·3	2,408·1	40,173·7
5-15	793,153·4	1,127,714·8	751,809·8	669,711·4	86,398·9	45,055·3	123,442·0
20	311,016·5	406,733·8	271,155·8	269,110·0	40,499·6	638·9	41,906·5
25	395,038·5	531,013·3	354,008·6	339,120·0	48,192·9	7,163·0	55,918·5
30	333,420·5	443,776·3	295,850·9	280,913·3	46,500·8	8,931·2	52,507·2
35	299,137·3	387,295·7	258,196·8	245,453·1	49,554·4	8,613·9	53,684·2
40	360,216·5	447,559·5	298,373·1	286,900·3	71,238·3	9,394·9	73,316·2
20-40	1,698,829·3	2,216,378·6	1,477,585·2	1,421,496·7	255,986·0	34,741·9	277,332·6
45	435,248·1	555,363·2	370,242·0	347,801·8	80,894·8	15,888·7	87,446·3
50	609,249·0	812,540·3	541,693·4	501,859·8	91,470·2	23,914·6	107,389·2
55	591,278·8	818,410·2	545,606·6	489,426·3	80,753·7	35,081·5	101,852·5
60	607,547·6	858,549·4	572,366·4	513,492·6	64,174·1	28,992·9	94,055·0
45-60	2,243,323·5	3,044,863·1	2,029,908·4	1,852,580·5	317,292·8	103,877·7	390,743·0
65	462,752·1	700,311·7	466,874·6	399,316·5	36,966·7	41,089·2	63,435·6
70	296,450·3	455,598·1	303,731·8	261,274·1	20,141·7	27,423·2	35,176·2
75	145,322·8	227,624·3	151,749·5	131,371·8	7,134·7	13,561·4	13,951·0
80	29,160·6	48,912·5	32,608·5	27,074·4	625·7	4,073·6	2,086·2
35-80	933,685·8	1,432,446·6	954,964·4	819,036·8	64,868·8	86,147·4	114,649·0
Total	6,167,012·7	8,638,357·6	5,758,904·2	5,189,810·5	754,234·3	346,125·8	977,202·2

TABLE XV.

EXCLUDING LAPSE RATE.

4 PER-CE

Central Entry Ages	PRESENT VALUE OF				LIABILITY ON BASIS OF		
	Sums Assured	Office Premiums	$\frac{2}{3}$ Office Premiums	Net Premiums	$\frac{2}{3}$ Office Premiums		Net Premiums
					+	-	
1	371,310·6	684,545·2	456,363·7	317,925·7	15,752·0	100,805·1	53,384·9
5	187,783·4	333,516·6	222,344·5	155,138·6	10,779·4	45,340·5	32,644·8
10	195,459·5	303,218·6	202,145·9	158,952·5	17,807·7	24,494·1	36,507·0
15	218,206·9	318,613·3	212,408·7	184,599·0	19,956·6	14,158·4	33,607·9
5-15	601,449·8	955,348·5	636,899·1	498,690·1	48,543·7	83,993·0	102,759·7
20	243,571·1	350,153·6	233,435·6	208,453·3	23,246·8	13,111·3	35,117·8
25	314,516·2	461,043·0	307,362·0	266,574·4	31,593·2	24,439·0	47,941·8
30	270,521·7	388,714·1	259,142·5	224,543·4	33,678·4	22,299·2	45,978·3
35	247,913·1	343,166·6	228,777·5	199,900·0	38,133·0	18,997·4	48,013·1
40	305,267·7	400,930·4	267,287·0	238,525·7	57,798·2	19,817·5	66,742·0
20-40	1,381,789·8	1,944,007·7	1,296,004·6	1,137,996·8	184,449·6	98,664·4	243,793·0
45	375,688·4	502,594·6	335,062·8	295,034·7	67,257·9	26,632·3	80,653·7
50	534,289·7	744,568·8	496,379·2	434,194·3	75,721·3	37,810·8	100,095·4
55	528,288·5	756,072·6	504,048·2	432,356·4	69,663·9	45,423·6	95,932·1
60	552,208·6	801,491·9	534,328·0	462,761·0	56,578·6	38,698·0	89,447·6
45-60	1,990,475·2	2,804,727·9	1,869,818·2	1,624,346·4	269,221·7	148,564·7	366,128·8
65	427,677·7	660,334·8	440,223·4	366,915·3	33,132·5	45,678·2	60,762·4
70	278,177·7	433,671·2	289,114·0	244,258·2	18,380·8	29,317·1	33,919·5
75	138,128·2	218,511·5	145,674·3	124,610·5	6,559·1	14,105·2	13,517·7
80	28,013·1	47,300·1	31,533·4	25,982·3	577·5	4,097·8	2,030·8
65-80	871,996·7	1,359,817·6	906,545·1	761,766·3	58,649·9	93,198·3	110,230·4
Total	5,217,022·1	7,748,446·9	5,165,630·7	4,340,725·3	576,616·9	525,225·5	876,296·8

Negative values are throughout printed in *italics*.

TABLE XVI.

LE showing the NET LIABILITY on the several Valuation bases specified, at the undermentioned Central Entry Ages and Durations.

CENTRAL AGE AT ENTRY, 1.

VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curate Duration
Net Premiums			$\frac{2}{3}$ Office Premiums			Net Premiums	$\frac{2}{3}$ Office Premiums	
Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
2,376	2,776	3,189	3,715	9,006	18,229	3,132	21,453	0
388	552	651	4,855	7,598	11,741	420	13,751	1
229	372	446	4,486	6,514	9,396	99	11,118	2
476	597	663	4,195	5,960	8,326	189	10,067	3
824	909	953	3,503	5,013	6,935	398	8,614	4
4,293	5,206	5,902	20,754	34,091	54,627	4,238	65,003	0-4
1,168	1,207	1,224	2,620	3,876	5,403	638	6,943	5
1,423	1,505	1,575	2,188	3,266	4,533	922	6,077	6
1,603	1,718	1,829	1,720	2,616	3,639	1,141	5,131	7
1,823	1,971	2,114	1,336	2,104	2,951	1,383	4,441	8
1,793	1,944	2,093	849	1,425	2,038	1,422	3,340	9
7,810	8,345	8,841	8,713	13,287	18,564	5,503	25,932	5-9
1,819	1,977	2,136	500	946	1,408	1,489	2,597	10
1,985	2,159	2,336	220	592	960	1,667	2,138	11
2,263	2,459	2,654	56	266	569	1,936	1,792	12
2,520	2,734	2,945	338	67	175	2,190	1,424	13
2,603	2,818	3,031	588	378	205	2,287	992	14
11,190	12,147	13,102	720 982	1,804 445	3,112 205	9,569	8,943	10-14
16,538	18,101	19,738	9,527	9,778	10,538	15,358	926 4,234	15-19
13,300	14,586	15,934	10,594	11,466	12,763	12,635	7,433	20-24
5,160	5,587	6,020	4,299	4,618	5,050	4,850	3,303	25-29
1,348	1,425	1,498	1,029	1,074	1,132	1,229	783	30-36
59,639	65,397	71,035	30,187 26,431	49,182 27,381	76,303 29,688	53,385	100,804 15,753	Total

Negative values are throughout printed in *italics*.

TABLE XVI—continued.

TABLE showing the NET LIABILITY on the several Valuation bases specified, at the undermentioned Central Entry Ages and Durations.

CENTRAL AGES AT ENTRY, 5, 10, 15.

Curtate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curtate Duration
	Net Premiums			$\frac{2}{3}$ Office Premiums			Net Premiums	$\frac{2}{3}$ Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	3,302	2,576	1,562	6,455	15,592	25,651	1,543	42,038	
1	3,682	3,086	2,438	4,149	5,540	6,873 292	2,051	13,021	
2	4,324	3,631	2,968	2,701	3,335	4,240 476	2,448	8,257	
3	5,285	4,441	3,707	1,594	2,204 189	2,564 691	3,044	6,226	
4	6,420	5,397	4,559	1,007 540	1,581 778	2,131 1,310	3,731	4,809	
0-4	23,013	19,131	15,234	15,906 540	28,255 967	41,759 2,769	12,817	74,351	0
5	6,131	5,132	4,340	395 873	791 975	1,252 1,425	3,555	3,017	
6	6,794	5,775	5,014	268 1,316	619 1,491	1,002 1,998	4,103	2,442 212	
7	6,633	5,704	5,048	99 1,498	361 1,688	626 2,157	4,136	1,636 278	
8	7,067	6,140	5,513	153 1,956	340 2,148	540 2,655	4,527	1,157 598	
9	5,246	4,596	4,170	1,625	1,683	77 2,003	3,425	633 566	
5-9	31,871	27,347	24,085	762 7,268	1,924 7,985	3,297 10,238	19,746	8,885 1,654	
10	4,148	3,659	3,353	1,475	1,557	1,774	2,756	352 632	
11	4,249	3,784	3,517	1,838	2,008	2,326	2,894	237 1,047	
12	4,971	4,432	4,114	2,053	2,195	2,471	3,390	133 985	
13	5,522	4,966	4,661	2,716	2,958	3,356	3,845	34 1,589	
14	5,506	4,971	4,691	2,907	3,174	3,587	3,878	1,844	
10-14	24,396	21,812	20,336	10,989	11,892	13,514	16,763	756 6,097	
15-19	29,327	26,966	25,938	18,540	20,389	22,938	21,511	14,165	
20-24	24,027	22,565	22,096	17,941	19,641	21,799	18,473	15,159	
25-29	10,913	10,420	10,285	8,660	9,309	10,089	8,722	7,504	
30-36	5,736	5,527	5,468	4,559	4,788	5,052	4,727	3,964	
Total	149,283	133,768	123,442	16,668 68,497	30,179 74,971	45,056 86,399	102,759	83,992 48,543	Total

Negative values are throughout printed in *italics*.

TABLE XVI—continued.

TABLE showing the NET LIABILITY on the several Valuation bases specified, at the undermentioned Central Entry Ages and Durations.

CENTRAL AGES AT ENTRY, 20, 25, 30, 35, 40

Curtate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curtate Duration
	Net Premiums			Office Premiums			Net Premiums	Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	4,861	2,802	...	12,846	21,839	23,537	8	52,563	0
1	8,147	6,928	5,340	8,404	8,694	6,872 131	4,489	20,389	1
2	10,290	8,885	7,325	4,965	4,785	3,946 307	6,171	13,115	2
3	12,206	10,605	9,027	1,312 157	418 1,058	387 1,050	7,619	7,565	3
4	13,766	12,001	10,362	1,727	2,187	3,010	8,860	3,971	4
4-4	49,270	41,224	32,054	27,527 1,884	36,376 2,605	34,742 4,498	27,150	97,603	0-4
5	13,505	11,791	10,260	3,997	4,380	5,274	8,718	993 197	5
6	14,253	12,644	11,273	5,259	5,742	6,670	9,606	69 1,020	6
7	13,915	12,512	11,369	6,196	6,773	7,728	9,708	2,587	7
8	13,191	11,978	11,020	6,470	7,021	7,867	9,471	3,404	8
9	10,627	9,732	9,046	5,700	6,158	6,800	7,781	3,498	9
5-9	65,491	58,657	52,968	27,631	30,074	34,339	45,284	1,062 10,706	5-9
10	9,585	8,844	8,294	5,606	6,026	6,598	7,156	3,798	10
11	10,459	9,727	9,204	6,891	7,410	8,077	7,969	5,146	11
12	10,761	10,037	9,520	6,880	7,299	7,831	8,278	5,113	12
13	13,357	12,673	12,272	12,496	13,482	14,642	10,752	11,031	13
14	13,938	13,298	12,942	13,770	14,791	15,971	11,375	12,333	14
10-14	58,100	54,579	52,232	45,643	49,008	53,119	45,530	37,421	10-14
15-19	66,532	64,078	62,744	66,773	70,532	74,766	55,720	60,121	15-19
20-24	51,622	50,349	49,727	54,072	56,294	58,726	44,824	49,520	20-24
25-29	21,026	20,666	20,490	21,692	22,341	23,044	18,722	20,028	25-29
30-36	7,240	7,150	7,110	7,177	7,331	7,493	6,562	6,654	30-36
Total	319,281	296,703	277,334	27,527 224,872	36,376 238,188	34,742 255,985	243,792	98,665 184,450	Total

Negative values are throughout printed in italics.

TABLE XVI—continued.

TABLE showing the NET LIABILITY on the several Valuation bases specified, at the undermentioned Central Entry Ages and Durations.

CENTRAL AGES AT ENTRY, 45, 50, 55, 60.

Curative Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curative Duration
	Net Premiums			$\frac{3}{4}$ Office Premiums			Net Premiums	$\frac{3}{4}$ Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	7,862	4,342	18	36,996	53,192	66,719	...	86,241	
1	18,662	16,328	13,492	21,044	22,817	24,114	12,377	35,187	
2	22,756	20,491	17,932	10,158	10,974	11,890	16,484	19,433	
3	25,403	23,160	20,532	(682	867	1,154)	18,919	7,340	
				(254	236	69)			
4	29,700	26,996	24,328	9,113	9,190	9,341	22,484	(364) (3,567)	
0-4	104,383	91,317	76,392	(68,880	87,850	103,877)	70,264	(148,565) (3,567)	0-
				(9,367	9,495	9,410)			
5	28,670	26,004	23,483	13,580	13,463	13,443	21,751	8,359	
6	29,458	27,046	24,821	15,717	15,718	15,791	23,040	11,127	
7	25,383	23,509	21,823	14,530	14,555	14,628	20,301	9,678	
8	22,646	21,127	19,789	13,893	13,931	14,069	18,446	11,089	
9	16,248	15,261	14,398	10,701	10,796	10,911	13,437	8,885	
5-9	122,405	112,947	104,314	68,421	68,493	68,842	96,975	49,138	5
10	14,374	13,577	12,902	10,264	10,374	10,507	12,074	8,828	
11	15,571	14,788	14,131	11,832	11,951	12,149	13,254	10,416	
12	17,688	16,868	16,184	13,940	14,097	14,276	15,221	12,431	
13	25,211	24,334	23,665	27,832	28,557	29,331	22,295	26,292	
14	24,781	23,991	23,396	28,216	28,881	29,585	22,084	26,722	
10-14	97,625	93,558	90,278	92,084	93,890	95,848	84,928	84,689	10-14
15-19	74,325	72,292	70,726	82,839	84,246	85,716	67,005	78,259	15-19
20-24	40,401	39,568	38,920	45,175	45,642	46,127	37,144	42,868	20-24
25-29	9,246	9,098	8,979	9,929	9,989	10,053	8,625	9,463	25-29
30-36	1,253	1,239	1,226	1,288	1,291	1,296	1,187	1,235	30-36
Total	449,643	420,019	390,745	(68,880	87,850	103,877)	366,128	(148,565) (269,219)	Total
				(309,103	313,046	317,292)			

Negative values are throughout printed in *italics*.

TABLE XVI—continued.

TABLE showing the NET LIABILITY on the several Valuation bases specified, at the undermentioned Central Entry Ages and Durations.

CENTRAL AGES AT ENTRY, 65, 70, 75, 80.

Curtate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curtate Duration
	Net Premiums			Office Premiums			Net Premiums	Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	3,910	2,375	...	34,271	44,281	55,669	...	58,597	0
1	11,714	10,545	9,209	15,442	17,281	19,425	8,781	21,458	1
2	12,333	11,421	10,385	7,726	8,910	10,376	9,917	11,526	2
3	13,827	12,903	11,910	989	(254)	(678)	11,399	(1,618)	3
4	17,113	15,983	14,832	8,274	(647)	(316)	14,218	(60)	4
					7,881	7,399		5,971	
0-4	58,897	53,227	46,336	(57,439)	70,726	86,148	44,315	(93,199)	0-4
				(9,263)	8,528	7,715		(6,031)	
5	16,100	14,957	13,833	10,310	9,824	9,295	13,286	8,120	5
6	13,806	12,963	12,150	9,345	9,024	8,676	11,685	7,755	6
7	9,785	9,269	8,769	6,862	6,678	6,469	8,447	5,880	7
8	6,869	6,533	6,222	4,875	4,769	4,656	5,994	4,259	8
9	3,583	3,423	3,275	2,486	2,436	2,381	3,158	2,190	9
5-9	50,143	47,145	44,249	33,878	32,731	31,477	42,570	28,204	5-9
10	2,899	2,782	2,680	2,243	2,216	2,190	2,589	2,038	10
11	3,384	3,274	3,180	3,117	3,114	3,111	3,076	2,927	11
12	3,621	3,516	3,422	3,135	3,126	3,115	3,317	2,944	12
13	4,785	4,673	4,580	5,397	5,418	5,441	4,444	5,182	13
14	3,960	3,870	3,800	4,435	4,445	4,459	3,692	4,259	14
0-14	18,649	18,115	17,662	18,327	18,319	18,316	17,118	17,350	10-14
15-19	5,934	5,820	5,730	6,581	6,591	6,604	5,581	6,339	15-19
20-24	641	631	624	699	698	699	611	677	20-24
25-29	43	42	42	45	45	45	33	37	25-29
30-36	14	14	14	14	14	14	14	14	30-36
Total	134,321	124,994	114,657	(57,439)	70,726	86,148	110,242	(93,199)	Total
				(68,806)	66,926	64,870		(58,652)	

Negative values are throughout printed in *italics*.

TABLE XVI—continued.

TABLE showing the NET LIABILITY on the several Valuation bases specified, at the undermentioned Central Entry Ages and Durations.

ALL AGES AT ENTRY COMBINED.

Curtate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curtate Duration
	Net Premiums			Office Premiums			Net Premiums	Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	22,311	14,571	4,769	94,283	143,910	189,805	4,683	260,592	0
1	42,593	37,439	31,130	53,594	61,930	69,025	28,118	103,506	1
2	49,932	41,800	39,056	30,036	34,521	39,848	35,122	63,449	2
3	57,202	51,709	45,839	7,783	10,343	13,409	41,170	32,816	3
4	67,823	61,286	55,034	1,400	1,550	2,126		60	4
				4,510	6,594	9,066	49,691	17,758	
				19,654	20,045	21,060		9,538	
0-4	239,861	210,105	175,828	190,506	257,298	321,153	158,784	478,721	0-4
				21,054	21,595	24,392		9,598	
5	65,574	59,091	53,140	3,015	4,067	6,655	47,948	10,953	5
				28,760	28,642	29,437		16,676	
6	65,734	59,933	54,833	2,456	3,885	5,535	49,356	8,588	6
				31,637	31,975	33,135		20,114	
7	57,319	52,712	48,838	1,819	2,977	4,265	43,733	6,767	7
				29,086	29,694	30,982		18,423	
8	51,596	47,749	44,658	1,336	2,257	3,291	39,821	5,598	8
				27,194	27,899	29,247		19,350	
9	37,497	34,956	32,988	849	1,425	2,115	29,223	3,973	9
				20,521	21,073	22,035		15,139	
5-9	277,720	254,441	234,457	9,475	15,211	21,861	210,081	35,879	5-9
				137,198	139,283	144,896		89,702	
10	32,825	30,839	29,365	500	946	1,408	26,064	2,949	10
				19,588	20,173	21,069		15,296	
11	35,648	33,732	32,368	220	592	960	28,860	2,375	11
				23,678	24,513	25,663		19,536	
12	39,304	37,312	35,894	266	569	869	32,142	1,925	12
				26,717	27,693	28,715		21,473	
13	51,395	49,380	48,123	48,779	50,482	52,770	43,526	1,458	13
								44,094	
14	50,788	48,948	47,860	49,916	51,669	53,807	43,316	992	14
								45,158	
10-14	209,960	200,211	193,610	720	1,804	3,112	173,908	9,699	10-14
				168,025	173,554	181,002		145,557	
15-19	192,656	187,257	184,876	184,260	191,536	200,562	165,175	926	15-19
								163,118	
20-24	129,991	127,699	127,801	128,481	133,741	140,114	113,687	115,657	20-24
25-29	46,388	45,813	45,825	44,625	46,305	48,281	40,952	40,335	25-29
30-36	15,591	15,355	15,316	14,067	14,498	14,987	13,719	12,650	30-36
Total	1,112,167	1,040,861	977,213	200,701	274,313	346,126	876,306	525,225	Total
				697,710	720,512	754,234		576,617	

Negative values are throughout printed in *italics*.

NOTE.—Owing to the omission, throughout the above Table, of the decimals, the figures do not in all cases, quite agree with those given, under the heading of "Liability", in Tables VIII to XV.

TABLE XVII.

Net yearly Premiums for the Assurance of 1, computed on the undermentioned bases.

Age at Entry	INTEREST AT 3 PER-CENT				4 PER-CENT.		INTEREST AT 3 PER-CENT				Age at Entry
	Full Lapse rate		Half Lapse rate		Excluding Lapse rate	(6)	Full Lapse rate		Half Lapse rate		
	With Provision for Surrender Allowance	(2)	Without Provision for Surrender Allowance	(3)			With Provision for Surrender Allowance	(4)	Without Provision for Surrender Allowance	(5)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	-010930	-009250	-010918	-009898	-011100	-009860	-00168	15.371	-00102	9.342	1
5	-008305	-006825	-010092	-008922	-011324	-009850	-00148	17.821	-00117	11.593	5
10	-008914	-007464	-011171	-010161	-012660	-011066	-00145	16.267	-00101	9.041	10
15	-011811	-010444	-013432	-012562	-014681	-013056	-00137	11.596	-00087	6.477	15
20	-014433	-012993	-015743	-014853	-016828	-015136	-00144	9.977	-00089	5.653	20
25	-016323	-014833	-017821	-016911	-019013	-017201	-00149	9.128	-00091	5.106	25
30	-018616	-016986	-020316	-019326	-021694	-019761	-00163	8.756	-00099	4.873	30
35	-021308	-019508	-023454	-022334	-025005	-023017	-00180	8.448	-00112	4.775	35
40	-025227	-023227	-027591	-026371	-029366	-027216	-00200	7.928	-00122	4.422	40
45	-030912	-028662	-033005	-031655	-034930	-032698	-00225	7.279	-00135	4.090	45
50	-037624	-035294	-040262	-038902	-042279	-039991	-00233	6.493	-00136	3.378	50
55	-047065	-044245	-049815	-048225	-052057	-049743	-00282	5.992	-00162	3.250	55
60	-060239	-057089	-063190	-061390	-065566	-063277	-00315	5.229	-00180	2.849	60
65	-079258	-075728	-082195	-080185	-084603	-082425	-00353	4.454	-00201	2.445	65
70	-106483	-102893	-109067	-107037	-111247	-109242	-00359	3.371	-00203	1.861	70
75	-144008	-141448	-145995	-144575	-147944	-146170	-00256	1.778	-00142	0.973	75
80	-194153	-192063	-195764	-194601	-197482	-195969	-00209	1.076	-00116	0.593	80

TABLE XVIII.

TABLE showing, in respect of Uniform Rates of Expenditure of 33 $\frac{1}{3}$, 40, and 45 per-cent upon all premiums, the equivalent outlay, in terms of the first t years' premiums (less provision for current death and surrender claims), and a reduced percentage on subsequent premiums.

No. of full Pre- miums ex- pended (<i>t</i>)	UNIFORM RATE OF EXPENDITURE ON ALL PREMIUMS												No. of full Pre- miums ex- pended (<i>t</i>)	
	33½ per-cent				40 per-cent				45 per-cent					
	Interest 3 %		4 %	Interest 3 %		4 %	Interest 3 %		4 %					
	Full lapse rate	Half lapse rate	Excluding lapse rate	Full lapse rate	Half lapse rate	Excluding lapse rate	Full lapse rate	Half lapse rate	Excluding lapse rate					
	REDUCED PROVISION FOR EXPENSES, ON PREMIUMS PAYABLE AFTER <i>t</i> YEARS													
AGE AT ENTRY, 10.														
1	20.1	28.9	31.6	31.2	29.1	36.3	38.5	38.2	35.9	41.9	43.7	43.5	1	
2	11.9	24.9	29.6	28.9	22.4	33.0	36.9	36.2	30.2	39.0	42.3	41.8	2	
3	4.0	21.0	27.5	26.3	15.7	29.6	35.1	34.0	24.5	36.1	40.7	39.9	3	
4	...	17.0	25.2	23.5	9.1	26.2	33.1	31.7	18.8	33.2	39.1	37.8	4	
5	...	12.9	22.8	20.6	2.3	22.8	31.1	29.2	13.0	30.2	37.3	35.7	5	
6	...	9.3	20.4	17.7	...	19.8	29.1	26.7	8.4	27.6	35.6	33.5	6	
7	...	5.6	18.0	14.6	...	16.8	27.0	24.2	4.0	25.1	33.8	31.3	7	
AGE AT ENTRY, 30.														
1	24.6	29.3	31.4	31.0	33.0	36.7	38.4	38.1	39.2	42.3	43.6	43.4	1	
2	19.0	25.9	29.3	28.6	28.4	33.9	36.7	36.1	35.5	39.9	42.2	41.8	2	
3	13.6	22.6	27.1	26.1	24.1	31.2	34.9	34.1	32.0	37.7	40.7	40.0	3	
4	8.4	19.2	24.8	23.4	19.8	28.5	33.0	31.9	28.4	35.4	39.2	38.2	4	
5	3.0	15.7	22.4	20.5	15.5	25.6	31.1	29.5	24.8	33.1	37.6	36.3	5	
6	...	12.5	19.8	17.5	11.9	23.1	29.0	27.1	22.1	31.1	35.9	34.3	6	
7	...	9.1	17.1	14.3	8.3	20.5	26.8	24.5	19.3	29.0	34.1	32.2	7	
AGE AT ENTRY, 50.														
1	24.4	28.4	30.4	30.0	32.7	35.9	37.5	37.3	38.9	41.5	42.9	42.7	1	
2	18.0	24.0	27.2	26.6	27.5	32.3	35.0	34.4	34.6	38.5	40.7	40.3	2	
3	11.8	19.5	23.9	22.8	22.4	28.6	32.2	31.4	30.3	35.5	38.5	37.8	3	
4	5.2	14.8	20.3	18.8	17.1	24.8	29.3	28.1	25.9	32.3	36.1	35.1	4	
5	...	9.7	16.5	14.5	11.4	20.7	26.2	24.6	21.3	29.0	33.6	32.2	5	
6	...	4.8	12.3	9.7	6.5	16.9	22.9	20.8	17.5	25.9	30.9	29.2	6	
7	7.9	4.6	1.2	12.8	19.4	16.8	13.5	22.7	28.1	25.9	7	
AGE AT ENTRY, 70.														
1	25.2	27.3	28.7	28.5	33.7	35.3	36.4	36.2	40.2	41.4	42.2	42.0	1	
2	17.5	21.1	23.6	23.0	28.1	30.7	32.6	32.1	36.0	37.9	39.3	39.0	2	
3	9.3	14.3	18.0	17.0	22.0	25.7	28.5	27.7	31.6	34.3	36.4	35.8	3	
4	...	6.7	11.7	10.2	15.4	20.3	24.1	23.0	27.0	30.6	33.4	32.5	4	
5	4.7	2.5	7.9	14.5	19.4	17.8	22.1	26.8	30.4	29.3	5	
6	9	8.6	14.3	12.1	18.4	23.6	27.5	26.0	6
7	2.3	8.8	5.9	14.6	20.6	24.8	22.8	7	

ADDENDUM.

It has been suggested by Mr. R. P. Hardy and others that it would be of advantage to add to the preceding paper a comparative Table of policy-values, computed on each of the bases employed, and tabulated, in respect of different entry ages and durations, so as to show the reserves for £100 assured. We have accordingly had these values computed, and append the results in Tables XIX to XXIII, for ages at entry 1, 15, 35, 55, and 75, respectively, and curtate durations 0, 1, 2, . . . 13, 14, 15, 20, 25, 30, 35 years, with a maximum attained age of 85 years.

In computing the reserves on a net premium basis, the assumptions made in the paper have been to some extent departed from, in order to eliminate a possible source of error, referred to in the paper, and also in the discussion which followed its reading. The net premiums given in Table XVII were computed on the assumption that the office age (next birthday) coincided with the true age at entry; whilst the valuation factors employed were computed at the office age at entry, *plus* the curtate duration. This introduced a certain anomaly, especially in the first year of duration, where the assumed entry age and valuation age would thus be identical; whilst the rate of lapse employed in the valuation factors was that observed from the end of the calendar year of entry, that is, after the expiration of an average period of about six months from entry.

For the calculation of the comparative reserves now submitted, the net premiums given in Table XXIV have accordingly been re-computed, upon the assumption throughout that the true age at entry is half a year less than the office age at entry; the valuation age being taken, as before, as the sum of the office age at entry and the curtate duration.

The net premiums with allowance for lapse have been independently computed on this basis; but, where the element of lapse is excluded, the net premium (at all ages above 1) for age $(x-\frac{1}{2})$ has been taken as the mean between those at age $(x-1)$ and x .

In view of the rapid movement in the rate of mortality during the first year of life, the net premium for office age 1 was specially computed, at the assumed entry age of six months, by the interpolated Table of numbers living deduced by Dr. Farr, and published in the English Life Tables, for quarterly intervals

of life, on pp. 146, 147, and for monthly intervals, during the first year of life only, on p. xxiii of the Introduction to those Tables.

Whilst it will be found that the results given in Tables XIX to XXIII present, on the whole, the same characteristics, and lead to the same conclusions, as those previously arrived at, the modifications introduced vary to some extent the relative reserves at individual durations and entry ages.

This is more particularly the case at age at entry 1, negative values being now introduced in the valuations at curtate durations 1 to 5 on a net premium basis, whilst the net reserves previously given in Table XVI at entry age 1 were throughout positive.

In view of these modified results the remarks made in the paper under the heading "Infantile Reserves" require some qualification, although, on the whole, they are still applicable.

In conclusion, it may be well to point out that undue weight should not in our opinion be given to the results brought out by the *net premium* valuations with allowance for lapse, as a valuation on such a basis is hardly likely to be made in actual practice, and the net premiums used can only be regarded as approximate, owing to the rapid change in the rate of exit in the first few years after entry, and the consequent difficulty of arriving at the true continuous annuity on which to base such net premiums.

It may also be that (as pointed out by Mr. Anderson in the discussion) there was some loss of accuracy in computing the net premiums with allowance for lapse by an assumed lapse rate which, perhaps, followed a slightly different curve (α , Table I) from that assumed in the calculation of the valuation factors (β , Table I).

TABLE XIX.

NET LIABILITY *per* £100 Assured.

AGE AT ENTRY, [1].

Curtate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curtate Duration
	Net Premiums			$\frac{3}{4}$ Office Premiums			Net Premiums	$\frac{3}{4}$ Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	·019	·014	·246	1·283	3·091	6·262	·201	7·287	0
1	·971	·806	·453	3·013	4·719	7·306	·622	8·512	1
2	1·224	·954	·514	3·592	5·217	7·526	·821	8·900	2
3	1·122	·789	·294	3·719	5·285	7·386	·744	8·925	3
4	·802	·442	·083	3·558	5·093	7·049	·513	8·751	4
5	·298	·062	·594	3·175	4·700	6·549	·149	8·411	5
6	·096	·551	1·178	2·876	4·291	5·955	·288	7·975	6
7	·529	1·069	1·785	2·518	3·830	5·326	·747	7·502	7
8	1·009	1·634	2·436	2·100	3·310	4·642	1·250	6·973	8
9	1·539	2·249	3·130	1·627	2·729	3·904	1·800	6·385	9
10	2·111	2·905	3·861	1·107	2·099	3·121	2·386	5·751	10
11	2·742	3·619	4·651	·525	1·405	2·275	3·031	5·053	11
12	3·436	4·394	5·492	·126	·644	1·372	3·733	4·292	12
13	4·175	5·210	6·369	·825	·162	·431	4·474	3·489	13
14	4·946	6·048	7·265	1·562	·997	·529	5·234	2·666	14
15	5·728	6·893	8·161	2·316	1·842	1·490	5·997	1·840	15
20	9·421	10·853	12·316	5·905	5·835	5·948	9·498	1·954	20
25	13·118	14·756	16·375	9·589	9·841	10·301	12·917	5·657	25
30	17·355	19·156	20·894	13·887	14·413	15·148	16·837	9·904	30
35	22·214	24·130	25·943	18·862	19·615	20·564	21·355	14·799	35

Negative values are throughout printed in *italics*.

TABLE XX.

NET LIABILITY *per* £100 Assured.

AGE AT ENTRY, [15].

Curtate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curtate Duration
	Net Premiums			$\frac{3}{4}$ Office Premiums			Net Premiums	$\frac{3}{4}$ Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapses rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	·467	·439	·484	·929	1·408	·827	·394	3·814	0
1	1·671	1·523	1·445	1·136	·912	·147	1·196	2·978	1
2	2·905	2·600	2·396	·541	·073	1·110	1·987	2·154	2
3	4·094	3·639	3·325	·304	·816	2·052	2·759	1·350	3
4	5·266	4·655	4·220	1·229	1·782	2·958	3·499	·578	4
5	6·462	5·660	5·084	2·208	2·720	3·833	4·206	·159	5
6	7·364	6·499	5·905	2·902	3·499	4·666	4·877	·858	6
7	8·273	7·348	6·742	3·623	4·297	5·514	5·559	1·569	7
8	9·198	8·217	7·596	4·373	5·121	6·379	6·258	2·297	8
9	10·142	9·102	8·465	5·154	5·968	7·259	6·975	3·045	9
10	11·091	10·001	9·353	5·966	6·838	8·159	7·712	3·812	10
11	12·047	10·914	10·260	6·806	7·731	9·078	8·469	4·602	11
12	13·005	11·843	11·186	7·672	8·648	10·016	9·247	5·412	12
13	13·969	12·786	12·136	8·570	9·589	10·978	10·048	6·247	13
14	14·936	13·744	13·105	9·495	10·551	11·960	10·871	7·105	14
15	15·903	14·715	14·095	10·442	11·534	12·963	11·718	7·988	15
20	20·896	19·851	19·397	15·547	16·800	18·335	16·337	12·802	20
25	26·455	25·578	25·279	21·343	22·704	24·295	21·624	18·313	25
30	32·651	31·912	31·727	27·850	29·250	30·827	27·597	24·538	30
35	39·471	38·836	38·711	35·053	36·416	37·903	34·261	31·487	35

Negative values are throughout printed in *italics*.

TABLE XXI.
NET LIABILITY *per* £100 Assured.
AGE AT ENTRY, [35].

Curate Duration	VALUATION AT 5 PER-CENT. ON BASIS OF						AT 4 PER-CENT. ON BASIS OF		Curate Duration
	Net Premiums			Office Premiums			Net Premiums	Office Premiums	
	Full Lapse-rate	Half Lapse-rate	Excluding Lapse-rate	Full Lapse-rate	Half Lapse-rate	Excluding Lapse-rate	Excluding Lapse-rate		
0	·695	·649	·669	<i>1·616</i>	<i>2·855</i>	<i>3·542</i>	·575	<i>6·398</i>	0
1	2·413	2·215	2·062	<i>2·243</i>	<i>2·393</i>	<i>2·089</i>	1·765	<i>5·124</i>	1
2	4·313	3·849	3·485	<i>1·452</i>	<i>1·209</i>	<i>·607</i>	2·991	<i>3·813</i>	2
3	6·204	5·492	4·934	<i>·132</i>	<i>·231</i>	<i>·904</i>	4·247	<i>2·469</i>	3
4	8·075	7·138	6·410	1·421	1·786	2·443	5·538	<i>1·088</i>	4
5	10·017	8·839	7·918	3·128	3·434	4·014	6·858	·326	5
6	11·592	10·369	9·454	4·529	4·939	5·615	8·213	1·776	6
7	13·184	11·930	11·015	5·997	6·499	7·243	9·600	3·260	7
8	14·806	13·519	12·605	7·526	8·100	8·900	11·021	4·781	8
9	16·454	15·136	14·223	9·112	9·744	10·587	12·471	6·332	9
10	18·119	16·780	15·861	10·742	11·426	12·298	13·956	7·921	10
11	19·826	18·457	17·532	12·429	13·148	14·037	15·477	9·549	11
12	21·558	20·160	19·231	14·143	14·906	15·807	17·025	11·206	12
13	23·314	21·890	20·951	15·941	16·700	17·600	18·609	12·901	13
14	25·089	23·645	22·697	17·757	18·526	19·420	20·229	14·634	14
15	26·884	25·425	24·471	19·613	20·386	21·269	21·880	16·401	15
20	35·969	34·485	33·521	29·211	29·927	30·703	39·442	25·563	20
25	45·162	43·826	42·961	39·220	39·867	40·544	39·628	35·394	25
30	54·405	53·235	52·471	49·372	49·908	50·456	49·129	45·561	30
35	63·170	62·116	61·466	59·035	59·431	59·833	58·318	55·395	35

Negative values are throughout printed in *italics*.

TABLE XXII.
NET LIABILITY *per* £100 Assured.
AGE AT ENTRY, [55].

Curtate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Curtate Duration
	Net Premiums			$\frac{3}{4}$ Office Premiums			Net Premiums	$\frac{3}{4}$ Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	1.288	1.314	1.347	<i>4.176</i>	<i>5.677</i>	<i>7.009</i>	1.238	<i>9.036</i>	0
1	4.503	4.291	4.103	<i>3.847</i>	<i>3.995</i>	<i>4.020</i>	3.776	<i>6.234</i>	1
2	7.715	7.287	6.889	<i>1.435</i>	<i>1.248</i>	<i>.998</i>	6.357	<i>3.384</i>	2
3	11.045	10.540	9.692	1.551	2.014	2.043	8.968	<i>.502</i>	3
4	14.493	13.462	12.517	4.836	4.960	5.107	11.612	2.417	4
5	18.050	16.636	15.357	8.331	8.245	8.188	14.282	5.365	5
6	20.649	19.487	18.194	10.929	11.236	11.265	16.970	8.333	6
7	23.834	22.331	21.036	14.164	14.244	14.347	19.669	11.313	7
8	26.717	25.169	23.863	17.150	17.270	17.414	22.375	14.299	8
9	29.567	27.990	26.680	20.144	20.297	20.470	25.080	17.286	9
10	32.394	30.779	29.467	23.151	23.306	23.493	27.771	20.257	10
11	35.155	33.543	32.228	26.129	26.304	26.488	30.443	23.207	11
12	37.871	36.257	34.948	29.090	29.263	29.439	33.095	26.135	12
13	40.530	38.912	37.622	32.017	32.167	32.339	35.712	29.024	13
14	43.121	41.519	40.248	34.888	35.027	35.186	38.287	31.867	14
15	45.645	44.065	42.816	37.698	37.825	37.973	40.818	34.661	15
20	57.128	55.728	54.636	50.657	50.713	50.794	52.608	47.677	20
25	66.522	65.411	64.538	61.415	61.472	61.535	62.642	58.756	25
30	74.098	73.092	72.472	70.114	70.026	70.140	70.804	67.767	30

Negative values are throughout printed in *italics*.

TABLE XXIII.

NET LIABILITY *per* £100 Assured.

AGE AT ENTRY, [75].

Contate Duration	VALUATION AT 3 PER-CENT, ON BASIS OF						AT 4 PER-CENT ON BASIS OF		Contate Duration
	Net Premiums			Office Premiums			Net Premiums	Office Premiums	
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Full Lapse rate	Half Lapse rate	Excluding Lapse rate	Excluding Lapse rate		
0	2·324	2·490	2·324	5·213	10·122	12·397	2·244	12·822	0
1	7·635	7·313	6·936	5·896	6·485	7·091	6·656	7·695	1
2	12·246	11·868	11·373	1·285	1·588	1·984	10·981	2·739	2
3	16·967	16·354	15·638	3·758	3·381	2·923	15·130	2·050	3
4	21·682	20·759	19·729	8·867	8·296	7·631	19·116	6·651	4
5	26·438	25·062	23·647	14·059	13·113	12·139	22·941	11·066	5
6	29·928	28·705	27·392	17·985	17·266	16·449	26·619	15·310	6
7	33·341	32·187	30·965	21·846	21·242	20·560	30·135	19·368	7
8	36·551	35·468	34·380	25·507	25·011	24·490	33·490	23·240	8
9	39·180	38·569	37·639	28·584	28·580	28·240	36·713	26·960	9
10	42·358	41·541	40·726	32·210	32·012	31·793	39·777	30·496	10

Negative values are throughout printed in *italics*.

TABLE XXIV.

NET YEARLY PREMIUMS *for the Assurance of 1, computed on the undermentioned bases.*

Age next Birthday at Entry	INTEREST AT 3 PER-CENT			AT 4 PER-CENT	Age next Birthday at Entry
	Full Lapse rate	Half Lapse rate	Excluding Lapse rate		
1	·012243	·011720	·01147	·01032	1
15	·011557	·013225	·014472	·012848	15
35	·021032	·023129	·024695	·022658	35
55	·046104	·048805	·050958	·048645	55
75	·139872	·141883	·143806	·142008	75

ABSTRACT OF THE DISCUSSION.

Mr. W. S. ANDERSON, in opening the discussion, said the question of applying a rate of lapse to the valuation of assurances had already been dealt with in the *Journal of the Institute*, but he thought the present was the first occasion upon which the disturbing element of the payment of a surrender value upon discontinuance had been brought into the calculations. The writers had gone into the matter with great thoroughness, and had set out their results in an admirable series of tables. The variety of methods in which the figures had been grouped, and the clear line of demarcation preserved throughout between the positive and negative values, would enable a great deal of information as to the effect of introducing a lapse rate to be gleaned by anyone making a study of the figures. The writers described in the earlier pages of the paper the method in which the rate of lapse had been investigated. The figures obtained showed the rate measured from the end of the year of entry, and consequently related to the years of assurance one, two, &c. Upon the basis of those rates factors had been computed for use in the valuation. On page 547 the authors said—"The next step was to deduce the graduated rates of lapse, as arising in successive policy years, which were required for the calculation of the net premiums to be employed in the valuation"; and they proceeded to relate that, by the application of the graphic method, a fresh set of rates were obtained, applicable to policy years, upon the basis of which net premiums were computed for use in the valuation. The rates were shown side by side in Table I. An alternative method of finding the net premiums would have been, simply to estimate the rate of lapse for the one half-year from the moment of entry to the end of year of assurance 0, and then to graft on to the values obtained by means of the lapse rates originally deduced, the experience of that first half-year. Thus the annuity measured from the point of entry would be a term annuity for half a year, subject to the double decrement of mortality and lapse, together with the factor $\frac{D_{[x-\frac{1}{2}]+\frac{1}{2}}}{D_{[x-\frac{1}{2}]}}$ multiplied into the annuity-value at the end of the assurance year 0, which had already been deduced. By that means the labour of computing an alternative series of commutation columns would have been avoided, and the net premiums obtained would be for the exact entry age, in lieu of the age at the end of the first year of assurance. There was one possible danger in making use of two sets of lapse rates. On turning to the chart on page 569, one could appreciate the difficulty of correctly interpolating the rates for policy years during the early years of assurance, and more particularly in carrying the curve back from the end of year of assurance 0 to the exact point of entry. The interpolation would require to be very accurately done, in order that the net premiums based upon lapse rate α and the valuation factors on rate β should be absolutely harmonious. When one was dealing merely with aggregate reserves, the difference was not likely to be of much moment, but when viewing separately the reserves in the early years of assurance, any slight discrepancy might introduce a considerable disturbing element. On pages 553 and 554 the authors

had described a method of adjusting the quinquennial groups in order to obtain greater accuracy. As the question had been dealt with at some length, he thought it would be of interest to test the exact effect of making the adjustment referred to. By the courtesy of Mr. Ackland, he had had access to the necessary data, and had taken out the value of the sums assured and the value of the office premiums in respect of all entry ages from 8 to the end of life, for the separate curtate durations 0, 5, 10, and 20 years. He had valued the sums assured and office premiums by three different methods—(1) exact ages, (2) ordinary quinquennial groups, and (3) groups adjusted by the method described in the paper. He had been compelled to use the ordinary valuation factors, based upon mortality without lapse. The results obtained in each year of duration were very similar. The error involved in using ordinary quinquennial groups was extremely small, being only about 1 in 2,000 in the value of sum assured, and 1 in 4,000 in the value of office premiums. By introducing the adjustment referred to in the paper the amount of the error was reduced to about one-fourth of that previously obtained. But it was unfortunate that, whereas in the case of the ordinary groups the error was on the side of safety—in that the value of sum assured was over-estimated, and the value of office premiums under-estimated—the effect of employing the adjustment was to turn the error in the opposite direction. He did not know how far these conclusions would be borne out by further investigations, but, as each of the four years tested had given similar results, he thought it probable that they yielded a fair indication of the effect which would follow a wider application of the suggested method. In statement B, on page 560, it would be seen that the effect of applying the lapse rate in the case of a net premium valuation was to increase the reserve in respect of all cases except those entering at age 1. The writers had raised the question as to the reason for the irregularity at that particular age, and he thought he was able to give some explanation of it. On turning to the net premiums in Table XVII, it would be seen, on comparing column 2 and column 6, that the net premium, including the lapse element, was, in all cases, considerably below the net premium, excluding lapse, except in the case of age 1, where they were almost identical. After a number of years, therefore, when the lapse rate had become almost a negligible quantity, in the case of most ages the effect of valuing with a lapse rate was to commute a much lower net premium, and therefore to bring out a higher reserve than would be obtained by valuing without lapse. But in the case of entrants at age 1, when applying the lapse rate one practically valued the same net premium as before, and that accounted for the relatively smaller reserve when compared with higher entry ages. Still, that only pushed the question one stage further back, and he would now pass to the real secret of the discrepancy. It had been mentioned by the authors that, in computing the net premiums, the integral age attained at the end of the year 0 of assurance had been used throughout. That, of course, over-estimated the age by half a year, and accounted for the apparent anomaly in Table XVI, where the reserve value at the expiration of the assurance year 0 upon a net basis was given as

nil. But in the case of age 1 at entry a further complication was introduced, as the method altered the incidence of the graduated sum assured, set out on page 551. He thought it would be found, when exact effect was given to the valuation assumption that the lives entered at age 6 months, and when allowance was made for the precise scale of increasing assurance previously referred to, a strict net premium valuation without lapse would yield negative reserves during the first few years of assurance; that is, the current risk during those years would be greater than the net premiums received. When, therefore, one came to apply a lapse rate to the valuation, it would be found, on viewing the question by the retrospective method, that the fund was burdened with the task of making up the negative values of those who withdrew in the first few years. Moreover, the surrender values during the early years were considerably in excess of the reserve, and the cases surrendering would therefore form a further burden on the fund. He thought it would be agreed that, in view of the negative values to which it gave rise in the early years, a strict net premium method of valuation was not applicable in the case of age 1 at entry. It followed that the writers might draw a more general conclusion than that set down, and state that in all cases, so far as their figures showed, the effect of applying a lapse rate to a net premium valuation was to increase reserves. With regard to the office premium valuations in the tables shown on page 561, it appeared to him that, in considering the reserves excluding the lapse element, one must eliminate negative values, but in the case of a reserve on a lapse basis, the negative values could be included, subject to the necessary safeguards. Viewed in that way, one saw that the effect of bringing in the rate of lapse, while maintaining the same net premiums, was to yield a lower reserve in all cases. Those appeared to him to be the two main conclusions of the paper, though it was conceivable that either might be reversed if a different rate of lapse and scale of surrender were adopted. Before closing he would turn to pages 541 and 542 of the paper, on which the authors had referred to the importance of paying regard to the lapse rate, and had instanced certain cases to which they thought it particularly applicable. In the case of an office in a weak financial condition, where it became a question of valuing for solvency, they considered that the rate of lapse might legitimately be introduced, and that wherever negative values arose they might be included. He wished to associate himself entirely with that expression of opinion, it being understood that the circumstances in connection with the office were such that there was no reason to anticipate any material alteration in the lapse rate. He considered that a second case to which the method was especially applicable was that of an office with a guaranteed scale of surrender values, which, generally speaking, were in excess of the office reserves. Such a case might arise where an office, owing to financial reasons, had been compelled to weaken its valuation basis. There, it would be advisable to test the sufficiency of its reserves by introducing the lapse element, and it was quite conceivable that, even working with the same net premiums, a higher reserve might be found necessary. However, taking the more typical case of an office which had ample funds to

permit of a valuation by ordinary methods, and where the scale of surrenders was at all points within the reserves—assuming further that the office premiums had been obtained by adding a sufficient loading to the ordinary net premium, excluding the lapse element—it was clear that in such a case the application of a lapse rate, if the same net premiums were employed, would have the effect of diminishing the reserve. He did not gather clearly from the paper whether the authors would, under these circumstances, advocate the adoption of a lapse rate in the general valuation, and members would, he thought, welcome a further expression of opinion from Mr. Ackland upon this phase of the subject.

Mr. W. R. DOVEY desired to offer one or two observations rather from the practical side. The authors had given much food for thought, and in some respects the suggestions made appeared to be almost revolutionary. In many parts of the paper it was said that lapse must be taken into account in order to arrive at a true estimate of the liabilities. But he, Mr. Dovey, presumed that, in making valuations of ordinary offices, as a general rule it was not the object purely to arrive at a true estimate in the sense of a test of solvency, but rather to take other considerations also into account. He could not follow the authors in their contention that, lapses being much more numerous than deaths, it was therefore essential that they should be taken into account. Mortality was the cause of the contract, but lapses were accidental to it, however important they might be in the actual course of the business. It seemed to him that, if lapses were very numerous, it only made it the more necessary for the treatment of retiring policyholders to be very carefully considered without the element being introduced into the valuation. He thought that the principles upon which industrial valuations were made should be essentially the same as those of an ordinary office, and that in general the conduct of the business of an industrial office should follow that of an ordinary one as far as possible, and that any differences should be purely due to the exigencies of practice. On page 542 of the paper there appeared a brief abstract of the various plans adopted for the valuation of industrial offices. The reserves of these companies were in course of transition at the present time, and since the Act of 1870 came into force, there had been a very decided growth in their reserves. It seemed undesirable therefore to form conclusions on methods of valuation which were transitional. Departures had taken place from recognised standards to meet the necessities of the cases. With regard to the remark that, in conducting a valuation, the surrender values of the office should be taken into account, and should be made an element in the basis of the calculations, it appeared to him that the surrender values should be fixed with regard to the reserves, rather than that the reserves should be determined by the surrender values of industrial offices. Industrial offices were as a rule considerably behind ordinary ones in the subordinate benefits attaching to the policies, and, the greatest difficulty in the way of improvement was, that the reserves were not as strong as they might be. Notwithstanding the most interesting and valuable investigations before them, he would still prefer to pin his faith to the sweet simplicity of an unalloyed net premium valuation.

Mr. H. W. MANLY said that, as he had been for many years connected with the valuation of industrial businesses, he might be expected to say a few words on the most interesting paper that had been read. It showed an amount of hard work and industry which all must admire. He regarded the paper as one which should give one pause before attempting to make any valuation upon the system which it advocated of providing for lapses. He confessed he had some little difficulty in following Mr. Ackland through the paper. With regard to the data used for obtaining the valuation factors, he understood it was not one table, but two or three tables; and he could not see why that should have been necessary. If it was intended to make an investigation of that kind, one should set out with a table which would represent the progress of the office from the commencement of the insurance to the end, whether it was by death or by withdrawal. All that was wanted was three columns, representing those entering in the year, those who died in the year, and those who withdrew in the year. In fact, the claims were actually in respect of deaths which occurred without the allowance for lapse. If the members remembered his Table 3 in his paper on the Valuation of Pension Funds, they would know there was a column of existing from which the value of the contributions could be found; there was a column of deaths from which to find the value of the claims from death, and there was the column of withdrawal from which could be found the value of a return of part of the premiums on that withdrawal. Turning now to the formula for the value of the pure premium; if the pure premiums were multiplied by the denominator of the fraction to represent the value of the premiums, and the numerator were treated as the value of the sum assured, the value of the insurance at its inception would be 0. It rather mystified him, therefore, to find in the summary of valuation for the year 0, that there was an actual value set down. In considering how to value this class of business, one had to keep in mind the actual cost of the business. The cost of obtaining the business was very considerable, and the first premium certainly did not contribute much more than the office expenditure; and what little claims might arise had to be provided out of that. For the first two or three years the expenditure was so great that if a company could manage to provide its claims and expenditure and have something left, it had done extremely well. The object of the paper, he understood, was not to advocate a pure premium valuation, but to apply the annuity and assurance values to make a valuation by gross premiums, deducting something from the premiums for the expenditure. He had not studied it very closely, but he understood it was suggested that in such a case negative values might be included. If that were done, one must not deduct some imaginary rate of expenditure from the premiums, but the actual expenditure which was at present incurred for the whole of the business. There must not be such a thing as setting aside a sum as an asset to represent the value of the business obtained. Anything of that kind was a charge on capital—if there were any capital—and not an asset in the assurance fund. When making a gross premium valuation, one must be careful that the actual expenditure was deducted from the premiums before valuing them.

Mr. Anderson had said something of which he (Mr. Manly) strongly approved, namely, that in deducing values by means of quinquennial groupings, there was no necessity for ascertaining all the functions for each age at entry; it was only necessary to group the contracts for valuation in the groups which had been selected for ascertaining the valuation factors. For the last thirty years he had valued in groups by a table which he based in a similar way to that—it was for sickness business—and it had been a very great success. Year after year one could see exactly how the reserve was coming out; or, if there was anything abnormal, it could be traced to some source or other. Therefore, if one intended to make a valuation of that kind, it was as easy, and quite as correct, to value them in groups of ages at entry, as to distribute them into separate ages. Then, with regard to the valuation of policies under the infantile table, explained on page 551, the only way in which those contracts could be valued properly was by the retrospective method, and if that were adopted he thought it would be found that at each succeeding age there would be nothing left for reserve. The premium table gave at each succeeding age the full assurance for the premium which was charged after allowing for expenses. He could not see that there was any necessity whatever to bring in any reserve in such a case as that. If the premium covered the expenses and the claims on that scale, he thought the company had done very well. In the early years of an industrial company, it was impossible to make anything like a recognised net premium valuation and pay the large expenses of the business. He thought the method which had been pursued by others and by himself for a great many years, of assuming that during the first five years of the insurance the premiums received would only provide for the expenses and the claims during that time, was justified. He thought it was a correct method to assume that, after five years, the insurances should be treated as entering an ordinary assurance company at the net premium for the increased age. From that date there would be a net premium reserve to provide for all those contracts which became permanent. The conclusion he had come to after reading the paper was that the method he had mentioned was the best and simplest, and enabled one to know where he was.

Mr. E. C. THOMAS said that the authors referred to the fact that very little had been contributed in the *Journal* on the subject of their paper, and he thought it would be conceded that at least one of the reasons was its extreme complexity. Another reason was the danger of the subject. It seemed to him that the question of the lapse element in assurance business was a very dangerous one, and for that reason actuaries in the past had decided to steer clear of it. In the special circumstances mentioned in the paper, one would agree with the authors that such a basis of valuation would be a legitimate and proper one to use; such, for instance, as when an office was in a very bad way, and might have to be wound up on the basis of an ordinary valuation, and as a test of solvency one introduced the rate of lapse in order to see really whether the office ought to be allowed to continue operations; and, if so, whether it would be able to pay twenty shillings in the pound. These were entirely different circumstances from those which arose in the ordinary way, when a company was

valued with a view of declaring a bonus. One must join issue with Mr. Ackland when, according to his paper, it was left almost open as to whether it would be justifiable to use a rate of lapse when valuing with the idea of obtaining a surplus. As Mr. Anderson had already said, the ordinary case was that of a company whose surrender values were less than reserve values. In that case, if an office could make a strong net premium valuation, surely it was far more desirable in every way to leave the rate of lapse severely alone, and value by the ordinary net premium method. There was another set of circumstances under which Mr. Ackland appeared to think the method would be justifiable, namely, when the assets and liabilities of one company were being transferred to another company. That seemed to him (the speaker) a dangerous expedient to adopt, because the very fact of the transfer being made might have a very material effect, in one direction or the other, upon the rate of lapse which would be experienced by the company after the transfer had been effected. That might mean that the company taking over the business had paid much too dearly for it. There was one point which struck him in connection with these tables. A net premium valuation, in which all the factors, including the net premium, were based upon the combined elements of mortality and lapse, was compared with an ordinary net premium valuation based only on mortality. It appeared to him (Mr. Thomas) that the fact that one was going to assess the annuities and single premiums by factors which gave effect to the experience one was likely to have in the way of decrement, bringing in both mortality and lapse, surely would not affect one way or another the amount of premium which would be received each year; it would not result in either more or less being received. Therefore he would like to see a comparison of two valuations by the net premium method, where the net premium was the same in both cases. It seemed that that would show better what the effect of introducing a rate of lapse was. The net premium based upon mortality and lapse had no bearing on the matter, unless the office premiums were calculated in the same way, and that was very unlikely. He thought the general danger of the subject was best exemplified in the figures of Statement A on page 559. It would appear from that statement that if an office was in a strong position, and able to make a net premium valuation, and if it wanted to be perfectly correct, it should introduce a rate of lapse which would increase the reserve from £977,000 to £1,112,000. But if the funds were in such a weak position that the company had to value the office premiums less 33 per-cent, they might legitimately introduce a rate of lapse, and, by including negative values, thus reduce their liabilities from £754,000 to £497,000, a decrease of over 30 per-cent.

Mr. A. W. WATSON in closing the discussion, said the present paper, so thoroughly and completely prepared, brought to the test of discussion a subject which had occasioned considerable difficulty to actuaries in practice—not the smallest part of that difficulty arising from the fact that there existed no instrument by which one could test the validity of the results brought out by this special method of valuation. For his own part, therefore, he welcomed the production of the paper which gave such an instrument. He desired to associate

himself with the authors in the contention that it was not scientific to ignore the element of lapse in industrial valuations. He would go so far as to say that the ordinary method of valuation by the English Life Table No. 3, without considering the question of lapse, or without special consideration of the question of mortality, was a purely conventional basis. Whilst it was justified by the regularity of the results which it brought out, and by its known sufficiency in the case of an office which was not troubled by any abnormal conditions, he did not think it could be held necessarily, or even probably, to give a close approximation to the actual value of the liabilities. In that connection he regretted that the authors had not devoted a little space to the question of differing rates of mortality. They had assumed that the English Life Table No. 3 was obtaining—an assumption which, he thought, would hardly be borne out by the facts. As was well known, the English Life Table of 1881-90 gave a widely different curve to that of the Life Table No. 3, the rate of mortality being much lower at the younger ages and greater at the higher ages. One knew that would result in an increased reserve on the net premium basis, and it was possible—though one should speak with hesitation on the point—it would involve an increased reserve on the lapse-included basis. He was somewhat surprised to find that in Table 1 the rates of lapse were so very high in the first 10 or 15 years. There was a very interesting table on page 317, vol. xxvii of the *Journal*, which gave the lapse rates prevailing in the period 1874-8 in a certain large collecting friendly society, and the rates given in the present paper were widely different from those. He believed that the lapse rates given by Messrs. Ackland and Bacon represented a considerably closer approximation to the rates prevailing to-day than those of Dr. Sprague's table to which he had referred, particularly in regard to the assurances of greater duration. In Dr. Sprague's very interesting table, a constant rate of lapse was given, about 1.1 per-cent, after varying durations of membership, and that was assumed to be a minimum rate of lapse, which would be attained by all policies sooner or later. As a matter of fact, the business of that society, at the time Dr. Sprague investigated it, contained practically no policies of a greater duration than 30 years, and very few of a duration exceeding 25 years. The rate of lapse in the quarter of a century which had subsequently been added to the experience of that society, was altogether different from the assumed minimum of 1.1 per-cent. It had fallen to something like one-third of that rate, and practically disappeared at the greatest durations of the assurance. That of course was one point in favour of the present paper, because no minimum rate of lapse was assumed; but, on the contrary, it was supposed that the lapse rate continued until it ran out at 89 years of age. He noticed that the authors had assumed that the business entering at age 2 might be valued by the factors obtained for age 1. He was rather sorry that, small as that business might be in proportion, they had not investigated the lapse rate appertaining to age at entry 2. The authors would probably have found that the lapse rate appertaining to age 2 was about double, at the earliest durations, to that appertaining to age at entry 1. It

would be observed from Table 1 that the lapse rates at age at entry 5 were very much greater than those at age at entry 1; and he thought that if Mr. Ackland had taken out the business appertaining to age at entry 2, he would have found that, so far as the rates were concerned, it would be more satisfactory to value it by the age 5 factors, than by the age 1 factors. With regard to the adjustment of the sums assured to overcome the objection of grouping, which had been dealt with at some length by Mr. Anderson, he wished to ask Mr. Ackland whether, admitting the rapidity with which the adjustment could be made, he considered it really essential. There was one adjustment not referred to in the paper, but which would be in some circumstances, he thought, considerably more important. In the case of a business which was valued by its own mortality experience it became necessary to consider the question of the deductions from the sums assured, when the claims came in, by reason of the under-statement of the ages on admission, then discovered. The table of new business given in the paper for specimen entry ages showed how certain decennial ages aggregated enormous quantities of the business, and thus indicated that there was a good deal of loose statement, and probably much under-statement, of age on admission. In one particular case within the speaker's knowledge, the deductions from the sums assured on this account were considerably greater than 1 per-cent of the whole amount of claims; and those deductions, appertaining as they did to the highest entry ages, where the effect of discount was of relatively small importance, represented an appreciable percentage upon the net liability. Of course, as the authors assumed that the standard mortality was prevailing, the excessive mortality arising from the under-statement of age might be assumed to be set off by the deductions themselves; but if Mr. Ackland had taken the true experience in respect of mortality, he would probably agree that the deduction described was an adjustment which it was important should be made. For that reason he suggested that the adjustment the author had made of the sums assured was hardly necessary. When an annual valuation was made—and annual valuations in industrial business were becoming the fashion—if one made it on the select basis, there was quite sufficient work to do in the time at disposal without introducing refinements which were not necessary to the reliability of the valuation. There were many other points of great interest in the paper, but he would not, at that hour, prolong the discussion by dealing with them, save that he felt compelled to say a word on negative values. One might admit, he thought, that, when they were valuing on the lapse experience of the society, the negative values which arose on the valuation were something entirely different, in essence, from the negative values which actuaries were accustomed to in the ordinary way of business. But he was not one of those who, for that reason, would include them in the valuation; they did not represent profits which might, or might not, accrue, because the lapse element having been discounted, they might be said to represent profits which in all probability would accrue, but, nevertheless, they were profits not yet realised, and it seemed to him to be objectionable to bring in a future profit against a present liability. It had been

shown in Statement A that a lapse-excluded valuation gave a reserve of £754,000, but that the lapse-included reserve was £697,000, or if negative values were included it was £200,000 less. He could very well understand that it might be desirable to adopt a with-lapse basis, when the difference between that and without-lapses was £60,000, but he would strongly demur to adopting a basis which would give a reserve of £300,000 less in the one case than in the other. Even though actuaries themselves might consider that such a reserve was entirely a safe one, one had to bear in mind what public opinion might have to say about the proposal, and in the matter of exclusion of negative values public opinion had had a long start. Some years ago the question came to the front in an interesting manner. A large collecting friendly society had been valued on a with-lapse basis, its own lapses and its own mortality having been used, and the actuary, an eminent member of the profession, had given a valuation result on the basis proposed in the present paper, negative values not being excluded. The society's affairs came into public notice, not in any way in connection with the valuation, but the valuation was dragged in. An enquiry was held, and, amongst other things, the eminent lawyer conducting it considered the valuation. He understood the nature of negative values as ordinarily arising, and upon this knowledge he dealt with the question of the valuation in his report, which was published as a State paper. He remarked that "There was an apparent surplus shown of more than £50,000, and of that £38,800 was made up of negative values. The real solid surplus was therefore under £12,000." He recited the instructions to public valuers, and went on to say that the actuary concerned "was not a public valuer, and did not consider himself bound by those instructions. But if they were sound they must be sound for all valuers." Other somewhat unfavourable remarks followed. Whether they were or were not erroneously conceived, such comments could not be very pleasant to the actuary who made the valuation. It might be that duty required an actuary to take the risks of criticism, and, if it did, they must not shrink from it. But in the present case he was not satisfied that the perilous course of inclusion of the negative values was the right one. They represented a future profit, and as such he preferred to treat them. Policyholders would clamour for bonuses, or for exemption from the payment of premiums in old age, as was becoming common. They would thus find means of utilising the surplus year after year as it was declared. His conclusion was that one should not anticipate a future surplus, but recognize the positive liabilities found, and as the negative values came in they would be in a position, not having anticipated them, to deal with them as surplus. That was the line he was himself prepared to take up.

Mr. ACKLAND, in reply, said he did not think he need occupy the time of the Institute very much, except to express on behalf of Mr. Bacon and himself their obligations to the members for the kind things which had been said about the paper, and also for the criticisms, both favourable and unfavourable, which had been offered, which no doubt would receive due consideration. The authors had no anticipation that the paper would have an entirely favourable reception, and therefore were not surprised that, in the remarks

of many speakers, the danger signal and the red lamp had been prominently held forth. Dealing briefly with the remarks of individual speakers, he expressed obligations to Mr. Anderson for the thorough way in which he had gone into the paper, and dealt with the methods employed. No doubt there was a great deal in what he said about the alternative method of obtaining net premiums as at entry. The authors had not very much time to deal with the very large volume of work contained in the paper, and the tables appended to it, and therefore preferred to adopt the method set out in the paper, because it appeared to them that that would produce a fairly accurate result, and they did not go into the other method, although he thought it might have been worthy of consideration. It might at once be admitted that the age at entry was over-stated in deducing the net premium, but he did not think the authors would admit that it would have any material effect on the general bearing of the results, and he would be rather surprised to find that the abnormality shown in respect of age 1 arose from that error in the age at entry. That error was, it is true, at a maximum at age 1, when the incidence of mortality was very rapidly changing; but it was also to be noted that the sum assured was at a minimum in the first year of assurance, and that indeed in the first three months there was no sum assured at all payable, and therefore practically no mortality in operation. The figures given by Mr. Anderson, with regard to the adjustment for groups, were very interesting, showing the marked measure of greater accuracy obtained by the grouping. Mr. Watson had referred to the fact that the grouping was comparatively unimportant, but anything which saved labour and gave more accuracy must be of considerable value. He thought Mr. Anderson seemed in general agreement with the conclusions in the paper. Mr. Dovey appeared, however, entirely opposed to those general conclusions, and to be very sensitive as to the question of lapse being introduced in valuations. No doubt that was a subject which would require careful consideration, and on which different opinions were held. But he submitted, first, that there was no real indication that had come under the notice of the authors, that there was any material movement in the direction of tightening reserves in industrial assurance companies since the Act of 1870, or that the methods adopted during that period were only transitional. Secondly, whilst theoretically it was true that surrender values should be fixed in relation to the reserves, and not *vice versa*, the difficulty was that one could not fix the surrender value in relation to reserves until the true liability had been ascertained, and in arriving at this true liability the authors contended that lapses should be brought into account. Mr. Manly appeared to have been in some difficulty about the formulas and methods employed in the paper, and no doubt the short time the paper had been in the hands of the members would, in a measure, account for that difficulty. The method adopted was substantially that indicated by Mr. Manly himself. The authors had proceeded with due regard to the numbers living year by year, and the separate decrements from death and lapse, but they had preferred to get out formulas based on the normal commutation columns, instead of deducing a separate mortality table. Mr. Manly advocated what

had been described as an $(x+5)$ valuation, on the ground that one "knew where one was." But he (Mr. Ackland) would briefly submit that it was precisely when valuing on those lines that the objection held, that one did not know where one was. Mr. Thomas advocated the use of the same net premium in the comparisons. There might be a good deal in that suggestion, but it seemed to him that practically what was required was available in the paper, where a uniform percentage was taken off the office premium, and the same reduced premium—two-thirds of the office premium—was employed in both sets of values. He was disposed to agree with Mr. Watson that, if one took the age at entry 2, the lapse rate would be found to be greater than that at age 1, but it was very convenient to arrange the groups so as to have central ages which were multiples of five; and, moreover, there was only the small proportion of $\frac{1}{5}$ of the whole cases in the first group, which arose at age 2, and that could not disturb the results very much. Mr. Watson spoke of negative values, and objected to their inclusion, on the ground that they were "profits not yet realised." He agreed that where a valuation was made without allowance for lapses, negative values were in the nature of profits not yet realised, and they might not be realised at all, if extinguished by lapses. But he thought that where allowance had been made for lapses, negative values could only be regarded as profits not yet realised, in precisely the same sense in which the whole of the premiums valued might be so regarded.

On Life Premium Book-keeping. By JAMES CHATHAM, F.I.A.,
F.F.A., *Secretary and Actuary to the Scottish Life Assurance
Company.*

[Read before the Institute, 25 April 1904.]

THE subject to which I invite your attention is one which has never been directly submitted to the Institute of Actuaries, although reference has on various occasions been incidentally made to it. In the discussion which followed the reading of my essay of 1895, to which was awarded the Messenger Prize, it was suggested that I should give a whole set of forms of books necessary for conducting the business of an office, and a classification book for valuation purposes, showing how the information in each would afford a perfect check in the end, and this paper is, to some extent, the outcome of that suggestion. While this is the position of matters in the Institute, it is otherwise with the Faculty, which has now undertaken the functions hitherto performed by the Actuarial Society of Edinburgh. There the subject has frequently been brought before them, more particularly by Mr. McLauchlan, and, on the last occasion, reference was made to the suggestion I have referred to, and it was added that it was a point to which attention might profitably be given. In these circumstances, I do not offer any apology for bringing the matter before you, but I shall not attempt, on this occasion, to give a whole set of forms of books. I shall confine my remarks chiefly to premiums, as I think that in this way the paper will be more easily followed, while it will be possible, within this compass, to show the application of the various checks by means of the Valuation Books described in my essay.

The present is an opportune time for discussing the subject. Many changes have taken place in recent years in the practice of Life Assurance Offices, necessitating corresponding alterations in the book-keeping. For instance, policies used generally to be issued before the first premium was paid, but the case of *Roberts v. The Security Co., Ltd.*, has altered the practice in this respect in many offices, and now it is the custom to issue interim

receipts for collection, and it is only after the premium has been paid that the policy is written. Again, there has been a growing desire for simplicity in working arrangements, and great improvements have been introduced. For example, even in so simple a case as the receipt for a renewal premium, a considerable change has been made. Many years ago these receipts used to be filled in with very full particulars, and they were signed by an official, and, sometimes, also by one or more directors. Gradually the form has been shortened, the signature or signatures lithographed, the amounts stated in figures instead of words, and the particulars typewritten. Recently another advance has been made, and the particulars can be printed on receipts by means of a machine at the rate of about 1,000 per hour.

Considerable difficulty is occasionally experienced by some in grasping the principles upon which life assurance book-keeping is based, and, for the benefit of students, I propose to commence with a simple statement of the objects I have in view, and the methods by which I hope to attain them, assuming only a knowledge of the elements of book-keeping and of the ordinary routine of a life office.

One main object to be kept in view in arranging premium books is the reconciliation of the various items of premium income. There is, first of all, the reconciliation of the premiums received during the year, in other words, the premium income for the year with what, according to the books, was to fall due during the year. There is also the reconciliation of what was in force at the beginning of the year with what is in force at the end of it, the latter forming the starting-point for the reconciliation of the premiums in the succeeding year. The books should also permit of the sums assured in force at the end of the year being similarly reconciled.

To commence at the beginning, when a proposal is accepted by an office, particulars of the premium, etc., are either written upon it or upon a sheet attached for the purpose, and the interim receipt is prepared from these particulars. That interim receipt I propose to make the foundation of my book-keeping system. In building upon it, I lay down two principles (1) that first premiums are kept quite distinct from renewal premiums and dealt with by themselves, and (2) that what is "written on" to the credit of premiums shall, if not paid, be "written off" in identical terms, and not by a mere adjustment of differences, such as might be done in the case of a half-yearly premium being

changed into yearly. The second principle may appear, in some cases, to be an unnecessarily long way of making entries, but, on the other hand, it has the merit of simplicity, and for that reason I have adopted it.

The interim receipt is entered in a book called the "First Premiums Written On Book", is posted from that book to the debit of the agent in the Agents' Ledger, and despatched to him with a counterfoil attached, which the agent returns along with a cheque for the amount. The cheque is entered in the Cash Book, and posted to his credit in the Agents' Ledger, and there is an end of the matter. If he does not send a cheque, but returns the interim receipt, it is entered in a book called the "First Premiums Written Off Book", exactly corresponding to the "First Premiums Written On Book." The totals of the Written On Book are, through the Journal, debited to the account Agents and credited to the account Premiums in the General Ledger, and the reverse is done in the case of the Written Off Book. The difference between the totals of the Written On and Off Books, is reconciled with the total of the premiums entered in the Register of Policies in the manner hereafter described, and the reconciliation may be made as often during the year as necessary.

When the first premium is paid, the policy is written and the particulars entered in the "Register of Policies", which forms the foundation for the renewal premium income. The renewal premiums are entered from the Register into the "Agency Renewal Premium Book", which is arranged according to months, subdivided according to agencies, and serves for five years, so that it is not necessary to re-write it every year, but only to extend the amount of the premium. The total of the premiums so entered should agree with the total of the premiums in the Register, and this ensures that provision is made for every premium being charged as it falls due. Renewal Premiums are debited from this book to the individual accounts in the Agents' Ledger, and, through the Journal, the total is debited to the account Agents and credited to the account Premiums in the General Ledger. One advantage of the Agency Renewal Premium Book is that it shows the quality of the business, and whether or where the business is running off quickly.

Changes very frequently occur among policies after payment of the first premium; some become claims, others are surrendered, many are forfeited through non-payment of the premium, while alterations are made in others; and provision must be made for

giving effect to all these. This is done by means of two books, called "Renewal Premiums Written Off Book" and "Renewal Premiums Written On Book", corresponding to the similar books for first premiums. For example, if a policyholder allows his policy to lapse, the receipt returned by the agent is written off in the Renewal Premiums Written Off Book; or, if he reduces the sum assured, the original premium is written off in that book and the altered premium written on in the Renewal Premiums Written On Book. The Renewal Premiums in the Written Off Book are credited to the individual accounts in the Agents' Ledger, and through the Journal the total is credited to the account Agents and debited to the account Premiums in the General Ledger. The reverse process is gone through in the case of the Written On Book.

I should explain that, in the office where the particular system I am describing is in use, premiums are accepted by half-yearly or quarterly instalments; that is to say, if death occurs before all the instalments in any policy-year are paid, the balance is deducted from the sum assured. Accordingly, all those instalments falling due beyond the financial year are properly included in the premium income for the year. Instalments falling due in the financial year are treated in the books as ordinary premiums, the term "instalment" being reserved exclusively for those falling due beyond it. In many offices, however, half-yearly or quarterly payments are true half-yearly or quarterly premiums; and, as any unpaid in the policy year are not deductible in the event of the policy becoming a claim, it is not correct to take credit for any falling due beyond the end of the year, and to include them in the premium income for the year. It will simplify matters, therefore, to give the forms first of all neglecting instalments, and to reserve for an appendix (A) the modifications necessary when it is desired to take them into account.

I should also explain that extra premiums are so comparatively few in number that no set of books is kept for them, but that they are treated in the same way as miscellaneous additions to the premium income. A simple record of the extra premiums falling due in each month is kept.

I should further explain that in direct Head Office cases the Cashier is treated for book-keeping purposes as if he were an ordinary agent, also that in all cases the rate of commission, if any, is shown on the receipt, and accounted for on payment of the premium.

Having now briefly described the process, it will be convenient to detail the means by which it is carried into effect. A specimen of each form will be given, and examples inserted to show the actual working of the scheme throughout a year.

Proposal Papers.—The following is a specimen of the sheet attached to the papers—

OUTSIDE.	INSIDE.
Policy No., 20001.	Age n.b.d., 40.
Name, <i>James Smith</i> .	Sum Assured, £1,000.
Sum Assured, £1,000.	Class, <i>Whole Life, With Profits</i> .
Agency, <i>Head Office</i> .	Annual Premium, £3. 4s. 6d. %
Commission, <i>None</i> .	„ „ £32. 5s. 6d.
Decision, <i>Accepted</i> .	Due Date, <i>1 Jan.</i>
Date, <i>1 Jan. 1903</i> .	Extra Premium, <i>None</i> .
Interim Receipt issued, <i>1 Jan. 1903</i> .	Destination <i>Ex'rs, &c.</i>
Policy despatched, <i>15th Jan. 1903</i> .	If World-Wide, <i>Yes</i> .
Reassurances, <i>None</i> .	
Non-forfeiture Debt	
Policy Loan £	
Assignment Nos.	

I should mention that in the office referred to all the subsequent correspondence is put up with the proposal papers; in fact, these papers contain a complete history of the case from its inception to its termination.

Interim Receipts.—These are prepared from the proposal papers, and the following is a specimen of the form used. Of the two counterfoils attached to the interim receipt, one is retained at Head Office and the other despatched to the agent along with the interim receipt.

SPECIMEN OF

THE LIFE ASSURANCE CO.

PARTICULARS OF ASSURANCE.

Life Assured, *James Smith*,
Sum Assured, . . . £1,000.
Class, *Whole Life, With Profits*.
Premium, . . . £32 5 0
Commission,
Balance, . . . £32 5 0
Payable, *1 January 1903*
Agency, *Head Office*.
Date of Issue, *1 January 1903*.
Debited by
Exd.
Interim Receipt No. 7146.

THE LIFE ASSURANCE CO.

PARTICULARS OF ASSURANCE.

Life Assured, *James Smith*.
Sum Assured £1,
Class, *Whole Life, With Profits*.
Premium, £32 5
Commission,
Balance remitted herewith . . . £32 5
Payable, *1 January 1903*.
Agency, *Head Office*.
Date of Receipt, 190
Signature,
(To be completed and, sent with remittance
Office immediately payment is received)
Interim Receipt No. 7146.

FIRST PREMIUMS.

First Premiums Written On Book.—The interim receipts are entered consecutively in this book. If every policyholder paid a full year's premium at the outset, and paid yearly thereafter, the premium book-keeping of a Life Office would be a very simple matter indeed; but some wish to pay by half-yearly or quarterly instalments, and others only a proportion at the outset, with the result that all the instalments may or may not fall due in the calendar year of entry. There are others who insure shortly after their birthday, and wish their policies dated back in order to secure the advantage of the lower premium, and this has the effect of sometimes causing two premiums to be payable in the same financial year. As the book-keeping of an office exists for the policyholders, and not the policyholders for the book-keeping, the forms must be applicable to cases like these, and it is this which makes the books more complicated than they otherwise would be; but the object should be to reduce them all, as far as possible, to the status of a yearly premium, and it is with this

INTERIM RECEIPT.

Interim Receipt.

THE LIFE ASSURANCE COMPANY.

PARTICULARS OF ASSURANCE.

Life Assured, *James Smith.*

Sum Assured, £1,000.

Class, *Whole Life, With Profits.*

Premium, £32 5 0.

Payable, *1 January.*

RECEIVED this day of 19 the Sum of
forty-two Pounds *five* Shillings and — pence, being the First
 Premium Payment for a Policy of Assurance to be issued immediately in
 pursuance of the above particulars.

This payment is received on the express stipulation that the person whose
 life is assured has had no illness since the date of the proposal to the Company
 for the Assurance.

This interim Receipt is issued subject to the same conditions as the Policy,
 and is valid for the Assurance only for fourteen days after its date; if the Policy,
 therefore, be not delivered to the Assured within that time, he should immediately
 write direct to the Head Office.

*(Signature)**Secretary.*

146.

*(Countersigned)**Agent.*

object that the First Premiums Written On Book shown on the
 next page has been prepared. The only heading calling for
 explanation is that of column (9). The $\frac{1}{2}$ or $\frac{1}{4}$ is intended to
 represent that the premium is payable by half-yearly or quarterly
 instalments.

In order to make matters clear, I have supposed three proposals
 to be accepted on 1 January 1903, the first by yearly premiums,
 the second by half-yearly instalments, and the third by quarterly
 instalments; also that five proposals are accepted on 1 July 1903,
 the first three similar to the above, a fourth by a proportion of
 the yearly premium at the outset, and a fifth by a single payment.

It will be observed that the form answers all of them. As
 Extras and Miscellaneous are not entered in the Premium column
 of the Register, the addition of columns (10), (11), (15), and (16)
 should agree with the total in the Register after deducting any
 premiums in the corresponding columns of the Written Off Book.
 This book I shall now proceed to describe.

FIRST PREMIUMS WRITTEN ON BOOK.

Date	Interim Receipt No.	Agency	Commission per-cent	Life Assured	Sum Assured	Class	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1903					£		
Jan. 1	7,146	Head Office	None	Smith	1,000	Whole Life	...
"	7,147	London	1 & 2½	Jones	500	Lim. Payt.	...
"	7,148	Glasgow	10 & 5	Brown	1,000	End. Assee.	Not taken u
July 1	7,149	Head Office	10 & 5	Clark	3,000	Lim. Payt.	...
"	7,150	"	1 & 2½	Baker	1,000	Lim. Payt.	...
"	7,151	London	1 & 2½	Young	2,000	End. Assee.	...
"	7,152	Glasgow	1 & 2½	Green	500	Whole Life	Proportion 1 Jan. 190
"	7,153	London	3	Reid	1,000	Whole Life Single Payt.	...
					£10,000	...	
		Deduct, per	Written Off Book	.	1,000
		New business for year	.	.	£9,000

(Instalments—see Appendix A.)

YEAR 1903.

FIRST PREMIUM PAYMENT				Agent's Ledger folio	OTHER PAYMENTS FALLING DUE		Checked by	Policy No.
Ordinary	Single (including 1-year term Policies)	Extra	Miscel- laneous		In Year	Beyond Year		
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
£ s. d.	£ s. d.	£ s. d.	£ s. d.		£ s. d.	£ s. d.		
32 5 0	40		20001
8 6 3	67	8 6 3	...		20002
9 0 10	35	27 2 6	...		—
93 5 0	40		20005
10 15 10	40	...	10 15 10		20006
14 15 0	67	14 15 0	29 10 0		20003
...	6 16 8	35	...	13 13 4		20004
...	421 10 0	67		20007
£168 7 11	£421 10 0	...	£6 16 8	...	£50 3 9	£53 19 2		
9 0 10	27 2 6	...		
£159 7 1	£421 10 0	...	£6 16 8	...	£23 1 3	£53 19 2		

SUMMARY FOR RECONCILEMENT OF

	Premiums in force at end of Year	Premiums received during Year
<i>First Premium Payment—</i>		
Ordinary	£159 7 1	£159 7 1
Single	421 10 0	421 10 0
Extra	—	0 0 0
Miscellaneous	—	6 16 8
<i>Other Payments falling due—</i>		
In Year	23 1 3	23 1 3
Beyond Year	53 19 2	—
	<u>£657 17 6</u>	<u>£610 15 0</u>

New Renewal Premium Income £236 7 6
 (£657 17 6 minus single premiums £421 10.)

FIRST PREMIUMS WRITTEN OFF BOOK.

Date	Interim Receipt No.	Agency	Life Assured	Sum Assured	Class	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1903 Jan. 20	7148	Glasgow	Brown	£ 1,000	End. Assee.	Not taken up

(Instalments—see Appendix A.)

First Premium Written Off Book.—The columns in this book are precisely the same as in the Written On Book, except that it is unnecessary to have those headed “Commission” and “Policy No.” Let us suppose that Interim Receipt No. 7148 is the only one returned unpaid, then it would be written off in the same way as it was written on, and would therefore appear in the book as shown above. If any interim receipt is returned for alteration, it would be dealt with in a similar way, and a fresh receipt written and entered in the First Premiums Written On Book. The practice of issuing an interim receipt has been advantageous from an office point of view, inasmuch as a larger proportion of proposals is now completed.

YEAR 1903.

FIRST PREMIUM PAYMENT				Agent's Ledger folio	OTHER PAYMENTS FALLING DUE		Checked by
Ordinary	Single (including 1-year term Policies)	Extra	Mis- cellaneous		In Year	Beyond Year	
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
£ s. d.	£ s. d.	£ s. d.	£ s. d.		£ s. d.	£ s. d.	
9 0 10	35	27 2 6

Register of Policies.—When a premium is paid, a Policy is written and entered in this Register before being despatched to the Agent. Policies therefore would be issued for all the Interim Receipts paid, and the necessary entries in the Register would be as shown on the next page.

The total of the premium columns gives the amount of single and annual premiums applicable to the New Business of the year. In the office referred to, the Register is kept up to date, all alterations being given effect to. It of course serves other purposes, but it is usual in offices to have it in this form, and it has therefore been adopted. The headings of the various columns explain themselves.

REGISTER OF POLICIES.

Policy No.	Date of Risk	Life Assured	Address	Date of Birth	Age at Entry	Years added or additional rate	Class	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
20001	1903 Jan. 1	Smith, James	15, St. Andrew Sq., Edinburgh, Solicitor.	1863, Jan. 15	40	...	Whole Life	...
20002	„	Jones, William	96, Gracechurch St., London, Banker.	1876, June 2	27	...	Whole Life 20 Payts.	...
20003	July 1	Young, Thomas	54, John Street, Dover, Merchant.	1877, Feb. 12	27	...	End. Assec. 65 or Death	...
20004	„	Green, Richard	40, West Street, Ayr, Agent.	1869, Oct. 17	34	...	Whole Life	Proportion 1 Jan. 1888 £6 16 3
20005	„	Clark, Joseph	65, Gt. King St., Edinburgh, Accountant.	1879, Jan. 3	25	...	Whole Life 20 Payts.	...
20006	„	Baker, Hugh	95, High Street, Lindlithgow, Jeweller.	1884, Aug. 12	19	5 years	Whole Life 30 Payts.	Rated under family ho
20007	„	Reid, John	53, Portman Sq., London, Gentleman.	1867, July 2	36	...	Whole Life Sing. Payt.	...

For summary of business in force at end of year—see page 644.

YEAR 1903.

WITH PROFITS				WITHOUT PROFITS				Date of Renewal	ENTERED IN		Date of last Payment	Destination	Agency	Alterations and Deductions	Assignment Nos.
Number	Premium			Sum Assured	Premium				A. R. P. Bk.	Links					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	£	s.	d.	£	£	s.	d.								
1000	32	5	0					Jan. 1	...			Exrs., &c.	Head Office		
500	8	6	3					Jan. 1			1922	Exrs., &c.	London		
	8	6	3					July 1			July				
1000	14	15	0					July 1			1941	Self, Exrs., &c.	London		
	14	15	0					Oct. 1			April				
	14	15	0					Jan. 1							
	14	15	0					Apl. 1							
500	13	13	4					Jan. 1				Wife, 1880 Act	Glasg'w		
1000	93	5	0					July 1			1922	Exrs., &c.	Head Office		
				1,000	10	15	10	July 1	...		1933	H. Thomson	Head Office		
					10	15	10	Jan. 1			Jan.				
				1,000	421	10	0					Exrs., &c.	London		
1000	£214	15	10	£2,000	£443	1	8								
1000	443	1	8												
500	£657	17	6												

* Agency Renewal Premium Book—see page 632.

† Links—see page 636.

RENEWAL PREMIUMS.

Renewal Premiums are based upon the Register of Policies and are at once entered from there into the Agency Renewal Premium Book under the month in which they are due, two half-years being entered in the case of Policies by half-yearly payments and four quarters in the case of Policies by quarterly payments. The correctness of the amounts so entered can be ascertained by summing the new premiums entered in that book and reconciling them with the amount in the Register, after deducting single premiums, the amount of which is obtained from the First Premium Books. This may be done at any time throughout the year.

In order to show the working of the various reconcilements, it will be necessary to assume a premium income at the beginning of the year. Let us suppose that it is payable during the year as shown below, also that the sums assured in force at the beginning of the year amount to £677,400 :

Month.	Amount.
January	£1,200 7 4
February	1,382 11 2
March	1,410 3 11
April	1,461 19 5
May	1,600 18 9
June	1,676 12 0
July	1,954 18 9
August	1,485 17 7
September	1,656 10 3
October	1,896 18 8
November	2,467 16 1
December	3,580 12 10
	<hr/>
	£21,775 6 9

Agency Renewal Premium Book.—All renewal premiums falling due in the ordinary course are charged through this book, and its full descriptive title might therefore be Agency Renewal Premium Written On Book (Ordinary). It is arranged according to months and sub-divided according to Agencies, and it will generally be found convenient to have a book for each month. It serves for five years; and once a premium or instalment is entered, all that is necessary is to extend it each year as it falls due, except of course in cases where the Policy has been surrendered or forfeited or otherwise dealt with. It is intended

that premiums when paid should be marked off in this Book. The following is a specimen of it for one month in the year, July, the other months being of course similar (see pages 634, 635). No provision has been made in the form for the addresses of the assured for the reasons explained afterwards.

All the headings of the various columns explain themselves. In column (9) the symbols $\frac{1}{2}$ and $\frac{2}{2}$ are used to designate the first and second half year's instalments, and in the same way the symbols $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$ to designate the first, second, third and fourth quarterly instalments.

Any alterations in premiums are intended to be made in red ink in the column headed "Premium per Register." The summation of this column at the end of the year shows the total amount in force. Alterations in the case of half premium and similar policies, and cessations of premium in the case of limited payment, endowment assurance, and like Policies, are intended to be obtained from a Premium Date Book (or Cards) which should be written up from the Register, and show the changes in premium occurring each month in any year.

When the premiums falling due in any month have all been extended, they are checked with the premium receipts in the manner to be presently described. The total for each Agent is then summed, and these totals are again summed, the grand total forming the premium charge for the month. Accounts are then made out and dispatched along with the receipts to the respective Agents, each Agent being debited in the Agents' Ledger with the total for his agency.

It will be seen that the Agency Renewal Premium Book is similar to the Monthly Renewal Premium Book kept in a number of Offices, in which the policies are arranged in numerical order. The main differences are that in the former the policies are sub-divided according to agencies, and the premiums in force are entered each year they fall due instead of those going off the books being shown as deductions in the year they are cancelled. The former seems the best way to keep a permanent record in book form of the monthly debits to Agents, when that is desired. One objection to it is that alterations are frequently necessary, such as changes in agencies, but this can be met by means of recently devised books, under which leaves can be inserted or removed at pleasure. Another objection to it is that it requires to be re-written every five years. Both these can be overcome by the use of cards if a permanent record in book form

AGENCY RENEWAL PREMIUM BOOK

AGENCY		Apts. Ldg. Fo.	Comm. per-cent	Due Date	Policy No.	Life Assured	Remarks			
District and Town	Agent									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)			
Head Office	...	40	—	2	1269	J. McGregor	...			
			5	31	1980	W. Wilson	...			
			—	30	2740	H. Stewart	...			
			—	4	7420	J. Carter	Died, 23/1/03			
			5	31	10823	A. Inglis	Surrendered, 3/8/03			
			5	6	12671	J. Robertson	...			
			2½	9	14933	P. MacLean	...			
			2½	31	15067	H. Cowan	...			
			2½	12	18611	T. Bell	...			
			5	20	19392	R. Johnston	...			
			2½	1	20005	J. Clark	...			
			—	1	20006	H. Baker	...			
			London	...	67	5	1	3521	H. Ward	...
						—	7	4136	J. Watts	...
—	31	8633				L. Cliff	Premis. paid up, 1901			
5	25	10972				E. Abbott	...			
2½	31	12543				F. Law	...			
5	12	13022				W. Thompson	Lapsed, 1901			
5	15	15134				C. Wilkins	...			
2½	18	15221				A. Moore	...			
2½	7	17011				H. Mason	...			
—	9	17622				W. Collins	...			
2½	31	18739				M. Harper	...			
—	30	18780				A. Davies	...			
2½	1	20002				W. Jones	Altered to yearly, 1901			
2½	1	20003				T. Young	Lapsed, October 1901			
Totals for Year										

FOR THE MONTH OF JULY.

Premium per Register	PREMIUMS DEBITED TO AGENTS IN YEAR				
	1901	1902	1903	1904	1905
(10)	(11)	(12)	(13)	(14)	(15)
£ s. d.	£ s. d.	£ s. d.	£ s. d.		
47 12 4	47 12 4	47 12 4	47 12 4		
342 6 9	342 6 9	342 6 9	342 6 9		
95 12 2	95 12 2	95 12 2	95 12 2		
7 13 4	7 13 4	7 13 4	—	—	—
32 16 1	32 16 1	—	—	—	—
72 1 0	72 1 0	72 1 0	72 1 0		
84 7 3	84 7 3	84 7 3	84 7 3		
243 7 7	243 7 7	243 7 7	243 7 7		
£925 16 6	£925 16 6				
£44 2 2	...	44 2 2	44 2 2		
63 7 8	...	£937 2 7	63 7 8		
£107 9 10			£992 16 11		
£93 5 0					
10 15 10					
£104 0 10					
£104 0 0	104 0 0	104 0 0	104 0 0		
37 2 9	37 2 9	37 2 9	37 2 9		
46 1 11	46 1 11	46 1 11	—	—	—
78 13 4	78 13 4	78 13 4	78 13 4		
53 5 0	53 5 0	53 5 0	53 5 0		
63 12 8	63 12 8	—	—	—	—
174 3 2	174 3 2	174 3 2	174 3 2		
130 6 8	130 6 8	130 6 8	130 6 8		
£687 5 6					
£90 16 9	90 16 9	90 16 9	90 16 9		
65 4 2	£778 2 3	65 4 2	65 4 2		
£156 0 11	...	£779 13 9			
£124 3 4	124 3 4		
96 13 4	96 13 4		
£220 16 8					
£8 6 3	8 6 3		
14 15 0	£962 14 9		
£23 1 3					
...	1901 £1,703 18 9	1902 £1,716 16 4	1903 £1,955 11 8		

is not considered essential. It will be seen from what follows, however, that neither books nor cards are absolutely necessary.

Renewal Premium Receipts.—The writing of premium receipts and notices used to occupy a very considerable time, and not only had they to be written, but they had also to be separately compared. In the system I am describing, however, the particulars are printed by a machine with endless belts formed of links filled with indiarubber type, by means of which the impression is printed off. The particulars are taken from the Register of Policies, and the links are arranged according to agencies in each month. It is unnecessary in this case to compare the receipts or the notices. After the receipts have been printed off, it is obvious that they must correspond with the premiums extended for that month in the Agency Renewal Premium Book. The addresses may be set up in links in a similar way and used for the dispatch of the Renewal Premium Notices, &c. If these are not set up, it will be necessary to insert the addresses in the Agency Renewal Premium Book.

As the links are filled with indiarubber type in separate letters, alterations are easily made upon them. It is intended that the particulars of any alterations of premium should be taken from the Renewal Premiums Written On and Off Books to be presently described. Before any alteration is made upon a link, however, an impression is taken of it and this forms a deduction; another impression is taken after the alteration has been made, and this forms an addition. By this means a perfect record is kept of all changes. It is obvious that all blanks in the Agency Renewal Premium Book must have a corresponding entry in the list of impressions of deductions. Separate lists of the new premiums and alterations, the latter divided into additions and deductions, are kept for each month, and the difference between the totals added to, or deducted from, the amount in force for the month at the end of the previous year, gives the amount falling due in the current year, as shown below for the month of July. These lists also show the amount remaining in force at the end of the year.

July Premiums in force at 31 Dec. 1902 (p. 632)	£1,954	18	9
Add, new Premiums (p. 631)—			
Policy No. 20002 Jones		8	6 3
	£1,963	5	0
Deduct, Alterations, &c. (p. 639)—			
Policy No. 7420 Carter, Dead		7	13 4
Charge per Agency Renewal Premium Book (p. 635)	£1,955	11	8

It will be seen that the links are simply duplicates of the Agency Renewal Premium Book, and therefore the latter is, as already stated, not absolutely necessary. If it is dispensed with, it would be necessary to print for office use a list of the premiums falling due in any month, but there is no difficulty in this, as it can be rapidly done. This list could be used to mark off whether the premiums are paid or not, and would thus afford a ready means of reference. Agents' Accounts and Branch Debit Notes may also be printed by means of the machine, the same particulars as for the premium receipts and notices being used. A further description of the method, and of the forms to which it is adapted, is given in Appendix B.

As the links are, as stated above, duplicates of the Agency Renewal Premium Book, it is not essential to the system proposed that they should be introduced, because the receipts and notices could be written in the old way from the book itself. In that case it would be necessary to insert the address in that book.

Renewal Premiums Written Off Book.—In the great majority of cases alterations are made after the premium or instalment has been charged for the year, and it is part of the object of this book to write off the premium or instalment so charged along with any instalments necessary to make up the annual premium. The entries therefore naturally divide themselves into two principal sections, "Premiums or Instalments Charged" and "Premiums or Instalments Not Charged." The entries in the former section are made from the returned receipts. The latter section is also used to give effect to other alterations in the premium income where the premium has not been charged.

These two principal sections require to be sub-divided. In addition to premiums falling due in the current year, we have premiums due before the current year requiring to be written off in it, and similarly the premium first affected by the alteration may fall due beyond the current year. We therefore require to provide for premiums falling due before, in, and beyond, the current year. The first two (before, in) apply to the Charged section, and the last two (in, beyond) to the Not Charged section.

The form of book with specimen entries, which will be explained later, is as follows:

RENEWAL PREMIUMS WRITTEN OFF BOOK.

Date	Policy No.	Agency	Life Assured	Sum Assured	Remarks	Due Date
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1903				£		1902
Jan. 15	19431	Glasgow	John Macfarlane	300	Surrendered . . .	Dec. 30
Feb. 2	3860	Head Office	James Carter .	500	Died 23 January . . .	1903
"	7420	"	"	1,000	"	Jan. 29
July 10	20002	London	William Jones	Altered to yearly from July	July 4
Oct. 1	16422	Head Office	Samuel Barker	...	Half Premium increased	July 1
Nov. 1	20003	London	Thomas Young .	2,000	Forfeited . . .	Oct. 17
				£3,800		

(Instalments—see Appendix A.)

The totals for each month are in practice inserted at the end of the book, and show the amount of premiums to be written off from the monthly charge. They are also used for reconciliation purposes.

It may be desirable in some offices to divide this book into sections, such as Claims, Surrenders, Lapses, and Alterations. If it is desired to ascertain the actual premiums received on new Policies issued during the year, that is to say, the new premium income for the year, it would be necessary to have a section for alterations on these Policies, as the second half year, or the second, third, or fourth quarter, might be affected.

YEAR 1903.

PREMIUMS CHARGED				Agent's Ledger Folio	PREMIUMS NOT CHARGED		ALTERED IN		Checked by
Due in rent Year	Extra	Mis- cellaneous	Due before Current Year		Due in Current Year	Due beyond Current Year	A.R.P. Book	Links	
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
£ s. d.	£ s. d.	£ s. d.	£ s. d.		£ s. d.	£ s. d.			
...	5 8 3	35	5 8 3
10 0	40
...	7 13 4	7 13 4
6 3	67	...	8 6 3
...	20 10 0
15 0	67	...	44 5 0
11 3	£5 8 3	...	£33 11 7	£60 4 7

SUMMARY FOR RECONCILEMENT OF

	Premiums in force at end of Year	Premiums received during Year
<i>Premiums Charged—</i>		
Due in Current Year	£35 11 3	£35 11 3
Extra	—	0 0 0
Miscellaneous	—	0 0 0
Due before Current Year	5 8 3	5 8 3
Do. do.	—	5 8 3
<i>Premiums not Charged—</i>		
Due in Current Year	£33 11 7	£33 11 7
Due beyond Current Year	60 4 7	—
	<u>£134 15 8</u>	<u>£79 19 4</u>

Renewal Premiums Written On Book.—Sometimes additions are made to the Premium Income, and this necessitates a book for writing on, similar to the one for writing off, the headings for which are almost exactly the same. For instance, if a premium is changed from yearly to half-yearly, the yearly premium is written off, and the two half years written on. It would no doubt be easier in some cases to insert the difference, but the method proposed has the merit of simplicity. The entries here consist principally of alterations, including revived policies, and the full descriptive title of the book might therefore be Renewals Premiums Written On Book (Alterations).

RENEWAL PREMIUMS WRITTEN ON BOOK.

Date	Policy No.	Agency	Life Assured	Sum Assured	Remarks	Debit Date
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1903				£		1903
July 10	20002	London	William Jones	...	Altered to yearly from July	July 10
Oct. 1	16422	Liverpool	Samuel Barker	...	Half Premium increased	Oct. 1

(Instalments)—see Appendix A.

In order to show how the Renewal Premiums Written On and Off Books work, we may now suppose a number of alterations to be made on Policies. In the great majority of cases the entries are very simple, but the more difficult ones have been chosen to show the use of the various columns. It should be mentioned that the alterations have not been shown in the Register of Policies nor in the premium column of the Agency Renewal Premium Book, in order to keep the printing clear.

Policy No. 19431 is presumed to be effected in December of a previous year by half-yearly instalments of £5. 8s. 3d., and to be surrendered within the days of grace of the December payment. £5. 8s. 3d. is therefore written off in the Charged section of the

YEAR 1903.

PREMIUMS CHARGEABLE				Agent's Ledger folio	PREMIUMS NOT CHARGEABLE		ALTERED IN		Checked by
Due in Current Year	Extra	Mis- cellaneous	Due before Current Year		Due in Current Year	Due beyond Current Year	A. R. P. Bk.	Links	
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
£ s. d.	£ s. d.	£ s. d.	£ s. d.		£ s. d.	£ s. d.			
16 2 11	67			
...	41 0 0	...			
£16 2 11	£41 0 0	...			

SUMMARY FOR RECONCILEMENT OF

Premiums Chargeable—

	Premiums in force at end of Year	Premiums received during Year
Due in Current Year	£16 2 11	£16 2 11
Extra	—	0 0 0
Miscellaneous	—	0 0 0
Due before Current Year	0 0 0	0 0 0
Do. do.	—	0 0 0

Premiums Not Chargeable—

Due in Current Year	41 0 0	41 0 0
Due beyond Current Year	0 0 0	—
	<u>£57 2 11</u>	<u>£57 2 11</u>

* Not chargeable through this book, because they will in ordinary course be charged in the Agency Renewal Premium Book as they fall due.

Written Off Book as due before current year, and a similar sum in the Not Charged section as due in the current year. The accounts for the previous year having been closed and the sum of £5. 8s. 3d. taken credit for in the outstanding premiums of that year, the amount, not having been afterwards received, must form a special deduction from the premiums received during the current year, as shown in the reconciliation at the foot of the form on page 639, where it is entered twice.

Policy No. 3860, effected by yearly premiums of £12. 10s. payable 29 January, is presumed to have become a claim on 23 January, and the premium therefore falls to be written off in the Charged section, the receipts for the month having of course already been debited.

Policy No. 7420 on the same life is supposed to have been effected in July of a previous year by half-yearly premiums of £7. 13s. 4d., payable 4 July and January. As death occurred on 23rd January, the premium due in that month would be payable. The two half-yearly payments therefore require to be written off in the Not Charged section, one as being due in the current year, namely in July, and the other as due beyond it, namely in January.

Policy No. 20002 was taken out on 1 January 1903, by half-yearly premiums of £8. 6s. 3d., which are changed to yearly after the second half-year's premium has fallen due. As the second half-year's premium has been charged, £8. 6s. 3d. is written off in the Charged section, and £8. 6s. 3d. in the Not Charged section, and as the latter is payable in January 1904, it falls to be written off in "Due beyond current year." The new yearly premium of £16. 2s. 11d. is inserted in the Chargeable section of the Written On Book.

Policy No. 16422 is supposed to have been effected on the half premium scheme by an annual payment of £20. 10s., to be increased in 1903 to £41. The former amount requires to be written off in the Not Charged section, and the latter to be written on in the Not Chargeable section of the Written On Book, as it will be debited for the year in the ordinary course in the Agency Renewal Premium Book.

Policy No. 20003, taken out on 1 July 1903, by quarterly premiums of £14. 15s., is supposed to have become forfeited by non-payment of the second quarterly premium falling due in October. As this premium has been charged, it is written off in the Charged section, while the other three premiums falling due beyond the year are written off in the Not Charged section.

SUMMARY OF PREMIUMS WRITTEN ON AND OFF.

I have described all the books necessary for writing premiums on and off, and may now summarize the results. The ordinary charge for the month, taken from the Agency Renewal Premium Book, with the addition of the Chargeable premiums in the Renewal Premiums Written On Book, and the deduction of the Charged premiums in the Written Off Book, constitute the renewal premium charge for the month. The inclusion of the monthly totals of the First Premiums Written On and Off Books gives the total premium charge for the month. The total for the 12 months gives the premium income for the year, £22,363. 5s. 4d., as shown on page 643. This Summary may be conveniently inserted at the end of the Renewal Premiums Written On Book.

SUMMARY OF PREMIUMS WRITTEN ON AND OFF, 1903.

Month	WRITTEN ON			WRITTEN OFF		
	Agency Renewal Premium Book	Renewal Premiums Written On Book (Alterations)	Total Renewal Premiums	First Premiums Written On Book	Renewal Premiums Written On Book	First Premiums Written Off Book
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Jan.	1,200 7 4	...	1,200 7 4	49 12 1	5 8 3	9 0 10
Feb.	1,382 11 2	...	1,382 11 2	...	12 10 0	...
Mar.	1,410 3 11	...	1,410 3 11
April	1,461 19 5	...	1,461 19 5
May	1,600 18 9	...	1,600 18 9
June	1,671 3 9	...	1,671 3 9
July	1,955 11 8	16 2 11	1,971 14 7	517 2 6	8 6 3	...
Aug.	1,485 17 7	...	1,485 17 7
Sept.	1,656 10 3	...	1,656 10 3
Oct.	1,932 3 8	...	1,932 3 8
Nov.	2,467 16 1	...	2,467 16 1	...	14 15 0	...
Dec.	3,575 4 7	...	3,575 4 7
	£21,800 8 2	£16 2 11	£21,816 11 1	£596 14 7	£49 19 6	£9 0 10
			596 14 7	...	9 0 10	
			22,413 5 8		£50 0 4	
			59 0 4			
			£22,363 5 4			

Agents' Ledger.—It has been stated that the premiums are debited to the Agents in a Ledger, and it will be convenient now to give a specimen of this book, and the entries in it relating to one Branch. It seems unnecessary to give any explanation regarding them.

AGENTS' LEDGER.

Agent : London.

Commission Terms :

Special Notes :

D.R.

('R.

Date	Number		Com- mission	Premiums	Date	Number		Premiums
1903			£	£ s. d.	1903			£ s. d.
Jan. 1	1 7147	Jones .	1 %	8 6 3	Jan. 15	...	Cash .	8 6 3
July 1	1 7151	Young .	1 %	14 15 0	July 10	...	Cash .	436 5 0
" 1	1 7153	Reid .	3 %	421 10 0	" 20	20002	Jones .	8 6 3
" 1	...	Renewals	...	962 14 9	" 31	...	" .	359 0 0
" 20	20002	Jones .	1 %	16 2 11	Aug. 31	...	" .	611 11 5
Oct. 1	20003	Young .	1 %	14 15 0	Nov. 10	20003	Young .	14 15 0
				£1,438 3 11				£1,438 3 11

This is the last book for which I shall give forms, and, so far as life premiums are concerned, it may be regarded as the principal one, as all the premiums falling due are accounted for here; but in the general book-keeping system of an office, it really forms a subsidiary book, as it merely shows in detail what appears in the General Ledger in the aggregate under the account Agents.

RECONCILEMENTS OF SUMS ASSURED AND PREMIUM INCOME.

Reconcilement of Sums Assured and Premiums in Force at End of Year.—We are now in a position to ascertain the sums assured and premium income in force at the end of the year. To the amounts in force at the end of the previous year is added the difference between the sums assured and relative premium income in the First Premiums Written On and Off Books, in other words, the new business for the year; also the amounts in the Renewal Premiums Written On Book. The sums assured and premium income in the Renewal Premiums Written Off Book are then deducted, and the result gives the sums assured and premium income in force at the end of the year, as shown in the following statement. The manner in which the amounts are arrived at is shown at the foot of the respective forms for these books, where “0” is used to designate that figures would be inserted if there were any, and “—” to show that they would not.

	Sums Assured.	Premiums.
Amounts in force as at 31 December 1902 (p. 632) .	£677,400	£21,775 6 9
Add, Difference between First Premiums Written On and Off Books (p. 627) . . .	9,000	236 7 6
Add, per Renewal Premiums Written On Book (p. 641) .	0	57 2 11
	£686,400	£22,068 17 2
Deduct per Renewal Premiums Written Off Book (p. 639) .	3,800	134 15 8
Amounts in force as at 31 December 1903	£682,600	£21,934 1 6

This reconcilement may in practice be most appropriately inserted in the Register of Policies at the close of each financial year. It will be found that it agrees with the amounts shown in the Valuation Register on page 647.

Reconcilement of Premiums received during Year.—We come now to the last and perhaps the most important part of the work, namely, the reconcilement of the premiums received during the year; in other words, the premium income for the year, with what, according to the books, was to fall due during the year. This is shown in the following statement:—

Premium Income in force at 31 December	
1902 (p. 632)	£21,775 6 9
Add, Difference between First Premiums	
Written On and Off Books (p. 627)	610 15 0
Add, per Renewal Premiums Written on	
Book (p. 641)	57 2 11
	£22,443 4 8
Deduct, per Renewal Premiums Written Off	
Book (p. 639)	79 19 4
Premiums received during year (p. 643)	£22,363 5 4

Let me briefly recapitulate the forms of books I have introduced. They are as follows:—

First Premiums—

Written On Book.

Written Off Book.

Register of Policies.

Renewal Premiums—

Agency Renewal Premium Book.

Written Off Book.

Written On Book.

Agents' Ledger.

The difficulty of describing any method of book-keeping for life premiums, however simple, is considerable, and I have not attempted any other forms. No doubt it may be convenient in some offices to sub-divide the books, but they would only be particular examples of the general forms given above.

RECONCILEMENT WITH VALUATION REGISTERS.

Having described the forms of books for life premiums, it will be convenient now to show how they are reconciled with the classification books for valuation purposes. I assume a knowledge of my essay on the subject (*J.I.A.*, xxxii, 393).

Cards are written from the policies themselves as they are

issued ; and not only do they form the basis of the records for valuation purposes, but they serve as a valuable check upon the particulars inserted in the Policy, as any discrepancy is likely to be noticed when writing the card. Alterations on Policies are dealt with in a similar manner. The necessary particulars are inserted from the cards in the Valuation Registers under the appropriate class ; and as these particulars are the same as the corresponding ones in the Register of Policies, it is obvious that the total of the Valuation Registers must agree with the total of the Register of Policies.

The simplest way to ascertain whether they agree is to take out the totals at the end of each year in each class in the Valuation Registers ; and, as stated above, the grand total should agree with the sums assured and premiums in force, as shown in the Register of Policies. If they do not agree, then assuming the Register of Policies to be correct, the error must lie either in the business written into the Valuation Registers or in the business written off. The new business is easily checked by summing the new Policies entered during the year, or, what is more quickly done, taking the difference between the total at the end of the year and what was in force at the beginning of the year, allowance being made of course for any transfers that may have taken place. The division of the Register of Policies into With and Without Profits sections facilitates this work. The grand total of the new business for the year should then agree with the total in the Register of Policies. If it does not, the individual items must be gone over and the error located.

If this does not set matters right, then the totals of the business written off in the Claims Valuation Registers must be taken down and compared with the amount in the Renewal Premiums Written Off Book ; and if they do not agree, the individual items must be compared as before and the error located.

As the totals of the various columns are supposed now to have been made correct, it is obvious that the premiums received during the year according to both sets of books must agree, bearing in mind that payments of an incidental nature are not inserted in the Valuation Register, but that a separate note is kept of them.

The forms of the Valuation Register and the Claims Valuation Register, so far as they relate to the subject on hand, are here given with the relative entries and reconcilements.

VALUATION REGISTER.

			OFFICE YEARLY PREMIUM											
Date	Policy No.	Sum Assured	Ordinary				Extra				Other Receipts			
			Due			Not Due	Due		Not Due					
		£	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
1903		£												
Brt. Fd.	...	677,400	21,775	6	9	...								
From Whole Life Section—														
Jan. 1	20001	1,000	32	5	0	...								
July 1	20004	500	...			13	13	4	...			6	16	8
Oct. 17	16422	1,000	41	0	0	...								
From Limited Payment Section—														
Jan. 1	20002	500	16	12	6	...								
July 1	20005	3,000	93	5	0	...								
" 1	20006	1,000	10	15	10	10	15	10						
" 1	20007	1,000						421	10	0
" 10	20002	...	16	2	11	...								
From Endowment Assurance Section—														
July 1	20003	2000	29	10	0	29	10	0						
		£687,400	£22,014	18	0	£53	19	2				£428	6	8
			74	11	1	29	10	0				5	8	3
			£21,940	6	11	£24	9	2				£422	18	5
		4,800	30	14	7	...								
			£21,900	12	4	...								
			24	9	2	...								
		£682,600	£21,934	1	6									

CLAIM VALUATION REGISTER.

			OFFICE YEARLY PREMIUM								
Date	Policy No.	Sum Assured	Ordinary			Extra		Other Payments			
			Paid	Unpaid		Paid	Unpaid				
		£	£	s.	d.	£	s.	d.	£	s.	d.
1903		£									
From	Whole Life Section—										
Feb. 2	3860	500	...			12	10	0	..		
„ 2	7420	1,000	7	13	4	7	13	4			
From	Limited Payment Section—										
July 10	20002	...	8	6	3	8	6	3	..		
From	Half Premium Section—										
Oct. 1	16422	1,000	...			20	10	0			
From	Endowment Assurance Section—										
Jan. 15	19431	300	...			10	16	6	..		
Nov. 1	20003	2,000	11	15	0	14	15	0	..		
						29	10	0			
		£4,800	£30	14	7	£74	11	1	..		
						29	10	0			

I should mention that the treatment here of cases where a proportion is paid at the outset and the first yearly premium falls due beyond the end of the year, is somewhat different from that adopted in my essay. There the proportion was entered in the Due column, and the balance of the yearly premium in the Not Due column, while here the first proportion is in all cases treated as an Other Receipt. The change has been made as it simplifies the reconcilements.

For convenience the miscellaneous receipts and payments have been shown in an additional column. The miscellaneous receipts are the first proportion of the year's premium on Policy No. 20004 and the single premium for Policy No. 20007, and the miscellaneous payment is the sum of £5. 8s. 3d. applicable to the income of the previous year already referred to in the paper.

The totals in these books agree with the sums assured and premiums in force, as shown in the reconciliation inserted in the Register of Policies (p. 644); but if they did not, then we would proceed to divide the entries in the Valuation Register into new business and transfers. This is done only for the Due column in the following statement, as none of the transfers happen to affect the Not Due column :

Total of Due column	£22,014	18	0
Total at beginning of Year	21,775	6	9
	<hr/>		
Entries for Year	£239	11	3
Of which transfers—			
Policy No. 16422	£41	0	0
.. .. 20002	16	2	11
	<hr/>		
New Premiums in Due column	£182	8	4
	<hr/>		

Transfers being excluded, the total for the year in the Due column in the Valuation Registers corresponds with the Ordinary and Other Payments Falling Due in Year in the First Premiums Written On and Off Books, as shown below, and that of the Not Due, £53. 19s. 2d., corresponds with the Other Payments Falling Due Beyond Year, in those books.

First Premiums Written On and Off Books—			
Ordinary	£159	7	1
Other payments falling due in Year	23	1	3
	<hr/>		
Total of New Premiums in Due column of Valuation Register, as above	£182	8	4
	<hr/>		

The transfers in the Valuation Register, in which are included revived policies, &c., may be similarly checked with the Renewal Premiums Written On Book. The total of those in the Due column corresponds with those of the columns Due in Current Year in the Chargeable and Not Chargeable sections, together with the Due before Current Year of that book, as follows:

Renewal Premiums Written On Book—

Chargeable, due in current year	£16 2 11
Due before current year	0 0 0
Not chargeable, due in current year	41 0 0

Total of Transfers in Due column of Valuation Register	£57 2 11
--	----------

The total of the transfers in the Not Due column corresponds with that of the Due beyond Current Year of the Renewal Premiums Written On Book.

Coming now to the Claim Valuation Registers, the Paid column, together with the sum in brackets in the Unpaid column, corresponds with the Due beyond Current Year column in the Not Charged section of the Renewal Premiums Written Off Book, and the balance of the Unpaid column in the former book corresponds with the columns Due in Current Year in the Charged and Not Charged sections, together with the Due before Current Year column of that book, as follows:

Claim Valuation Register—

Paid	£30 14 7
Unpaid (in brackets)	29 10 0

Total of Not Charged, due beyond current year, in Renewal Premiums Written Off Book	£60 4 7
---	---------

Renewal Premiums Written Off Book —

Charged, due in current year	£35 11 3
Due before current year	5 8 3
Not Charged, due in current year	33 11 7

Total of Unpaid column (excluding brackets) in Claim Valuation Register	£74 11 1
---	----------

The deduction of the Unpaid column of the Claim Valuation Register, £74. 11s. 1d., from the total of the Due column in the Valuation Register, £22,014. 18s., gives the ordinary premium income of the year, £21,940. 6s. 11d. To this

must be added the difference between Other Receipts and Other Payments £422. 18s. 5d., making the total premium income £22,363. 5s. 4d.

Deducting now the Paid column of the Claim Valuation Register, £30. 14s. 7d., from £21,940. 6s. 11d., we get £21,909. 12s. 4d., and finally adding £24. 9s. 2d. in the Not Due column, as explained in my essay (page 410), we get £21,934. 1s. 6d., as the premium income in force at the end of the year.

The deduction of the sums assured in the Claim Valuation Register, £4,800, from the total of the Valuation Register, £687,400, leaving £682,600, gives the sums assured in force at the end of the year. The £4,800 deducted is £1,000 less than the amount shown in the Renewal Premiums Written Off Book, owing to its being unnecessary to write off the sum assured in the latter book in a case where the low premium in a half premium Policy is being increased to the high premium, as in the example shown. As the sum of £1,000 is similarly entered in the Valuation Register and not in the Renewal Premiums Written On Book, the differences between the two sets of books will of course agree.

SPECIAL CASES.

Re-assurances.—These have not been specially referred to, as it is obvious they can be dealt with in a similar manner to ordinary premiums.

Dated Back Policies.—It is the practice of some offices to date back Policies after the books are closed for the year, and no difficulty need be experienced in dealing with them, provided a space is left for them at the beginning of the new year in the First Premiums Written On and Off Books, and they are entered separately. In the reconciliation of the premium income of the year, it is necessary to take into account, in addition to the usual entries, a full year's premium. It is common for offices to close their books before all the interim receipts issued are paid, in order to save undue delay in the preparation of the accounts. Such cases must be treated in all respects as if Policies had been issued.

Additions to Monthly Charge.—Sometimes a renewal premium on a new Policy falls due after the Agency Renewal Premium Book has been made up for the month. For instance, if a Policy is taken out by quarterly premiums and dated back say three months to

save a birthday, the second quarterly premium would be due immediately, and could not be entered into the Agency Renewal Premium Book, as that book would be closed for the month, and the receipts despatched. The best way to treat such a case is still to enter the quarterly payment in the Agency Renewal Premium Book, and treat it as an additional charge for the month.

Half-Credit Premiums.—It is proposed that an account should be opened for these in the Agency Renewal Premium Book, and the amounts so advanced each month carried to investments.

Deferred Premiums.—Sometimes premiums are not payable until a certain event happens, such as the expiry of a commutation period, and in this case also the best course is to open an account for the full premium in the Agency Renewal Premium Book, and write it off as a miscellaneous deduction from the premium income.

Reduction of Premium.—A temporary reduction of premium may be dealt with in a similar way. As only the reduced premium appears in the Agency Renewal Premium Book, the amount of the reduction would be entered in an account, and a receipt printed for it, the receipt being written off through miscellaneous deductions, as above.

Premiums Paid in Advance.—A premium is sometimes paid before the month's charge is made up, and in that case the best plan is to enter it in the Miscellaneous Column of the Renewal Premiums Written On Book on the date on which it is paid, at the same time making a note in the Agency Renewal Premium Book. This may be done by entering the amount in red ink under the proper year, which would at once direct attention to it, and prevent the receipt from being issued. It is not intended that any alteration should be made in the premium links. The amount would be included in the Agent's debit for the month in which it falls due, and the receipt then written off to his credit through the Miscellaneous Column of the Renewal Premiums Written Off Book.

APPENDIX A.

As previously mentioned, the system I am describing was originally applied to premiums payable by instalments, and the treatment in their case has been reserved until now, so as not to interfere with the clearness of the general system, as their introduction requires the forms to be more elaborate. The necessary alterations for each book are stated below.

It will, as before, be necessary to assume a premium income at the beginning of the year, and the same total has been used, but divided into premiums and instalments. It will be observed that no instalments fall due in the last three months of the year. This follows from the definition of the term instalment given at the commencement of the paper. The extreme case in a half-yearly Policy would be one effected in December, and consequently the second instalment would fall due in June; similarly in a quarterly Policy, the last instalment would fall due in September.

	Premiums.	Instalments.
January . .	£1,156 4 2	£44 3 2
February . .	1,320 17 11	61 13 3
March . .	1,260 0 7	150 3 4
April . .	1,410 12 2	51 7 3
May . .	1,525 11 0	75 7 9
June . .	1,359 14 4	316 17 8
July . .	1,884 13 10	70 4 11
August . .	1,445 1 6	40 16 1
September . .	1,563 3 1	93 7 2
October . .	1,896 18 8	—
November . .	2,467 16 1	—
December . .	3,580 12 10	—
	<hr/> £20,871 6 2	<hr/> £904 0 7

First Premiums Written On Book.—If instalments are kept separate, the column “Beyond Year” must be sub-divided into “Premiums” and “Instalments”, and the two items of £10. 15s. 10d. and £29. 10s. transferred to the latter.

First Premiums Written Off Book.—In the same way the column “Beyond Year” must be sub-divided into “Premiums” and “Instalments.”

The following would be the summary for the reconcilements at the end of the year, taking the difference between these two books :—

	PREMIUMS IN FORCE AT END OF YEAR		PREMIUMS RECEIVED DURING YEAR	
	Premiums	Instalments	Premiums	Instalments
<i>First Premium Payment—</i>	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Ordinary	159 7 1	—	£159 7 1	—
Single	421 10 0	—	421 10 0	—
Extra	—	—	0 0 0	—
Miscellaneous	—	—	6 16 8	—
<i>Other Payments Falling Due—</i>				
In Year	23 1 3	—	23 1 3	—
Beyond Year	13 13 4	40 5 10	—	40 5 10
	£617 11 8	£40 5 10	£610 15 0	£40 5 10
New Renewal Premium Income	£196 1 8	£10 5 10		

Renewal Premiums Written Off Book.—If instalments are kept separate, the columns headed due “In” and “Beyond” current year, (9), (14) and (15), require to be sub-divided into “Premiums” and “Instalments.” It should be remembered that the instalments in any year really relate to the premium income of the previous year. The items of £5. 8s. 3d. in column (14) and £7. 13s. 4d. in column (15) will be transferred to “Instalments”, and of the item of £44. 5s. in column (15), £29. 10s. will be similarly transferred.

The following is the summary for the reconcilements at the end of the year:—

	PREMIUMS IN FORCE AT END OF YEAR		PREMIUMS RECEIVED DURING YEAR	
	Premiums	Instalments	Premiums	Instalments
<i>Charged—</i>	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Due in Current Year	35 11 3	0 0 0	35 11 3	0 0 0
Do. do.	—	—	—	0 0 0
Extra	—	—	0 0 0	—
Miscellaneous	—	—	0 0 0	—
Due before Current Year	5 8 3	—	5 8 3	—
Do. do.	—	—	5 8 3	—
<i>Not Charged—</i>				
Due in Current Year	28 3 4	5 8 3	28 3 4	5 8 3
Do. do.	—	—	—	5 8 3
Due beyond Current Year	23 1 3	37 3 4	—	37 3 4
	£92 4 1	£42 11 7	74 11 1	£47 19 10

Renewal Premiums Written On Book.—If instalments are kept separate, the columns headed due “In” and “Beyond” current year require, as in the Written Off Book, to be sub-divided into “Premiums” and “Instalments.”

The following is the summary for the reconcilements at the end of the year:—

	PREMIUMS IN FORCE AT END OF YEAR		PREMIUMS RECEIVED DURING YEAR	
	Premiums	Instalments	Premiums	Instalments
<i>Chargeable—</i>	£ s. d.	£ s. d.	£ s. d.	£ s. d.
Due in Current Year	16 2 11	0 0 0	16 2 11	0 0 0
Do. do.	—	—	—	0 0 0
Extra	—	—	0 0 0	—
Miscellaneous	—	—	0 0 0	—
Due before Current Year	0 0 0	—	0 0 0	—
Do. do.	—	—	0 0 0	—
<i>Not Chargeable—</i>				
Due in Current Year	41 0 0	0 0 0	41 0 0	0 0 0
Do. do.	—	—	—	0 0 0
Due beyond Current Year	0 0 0	0 0 0	—	0 0 0
	£57 2 11	£0 0 0	£57 2 11	£0 0 0

As instalments are assumed to be debited to agents only when they fall due, and not at the beginning of the policy year, the ordinary monthly entries in the Summary of Premiums Written On and Off are identical with those in the form previously shown on page 643. But as any instalments due beyond the end of the financial year are included in the revenue for the year, two Journal entries require to be made, the one writing on the instalments outstanding at 31 December 1903, and the other writing off the instalments outstanding at 31 December 1902, these having already been taken credit for in the premium income for 1902, although not actually receivable until 1903. The amount outstanding at 31 December 1903 is £901. 14s. 10d., as shown later, and the adjustments to obtain the premium income for the year are as follows:—

Premiums received during Year, as per Summary (p. 643)	£22,363 5 4
Add, Instalments as at 31 Dec. 1903 (p. 655)	901 14 10
	£23,265 0 2
Deduct, Instalments as at 31 Dec. 1902 (p. 652)	904 0 7
Premiums received during Year, under Instalment System	£22,360 19 7

Reconcilement of Premiums in Force at end of Year.—

The form of reconcilement is similar to that applicable to premiums only, and is as follows:—

	Premiums.	Instalments.
Amounts in force as at 31 December 1902 (p. 652) .	£20,871 6 2	£904 0 7
Add, difference between First Premiums Written On and Off Books (p. 653) .	196 1 8	40 5 10
Add, per Renewal Premiums Written On Book (p. 654)	57 2 11	0 0 0
	£21,124 10 9	£944 6 5
Deduct, per Renewal Premiums Written Off Book (p. 653)	92 4 1	42 11 7
Amounts in force as at 31 December 1903 .	<u>£21,032 6 8</u>	<u>£901 14 10</u>

Reconcilement of Premiums received during Year.—This reconcilement is again similar to that applicable to premiums only, and the details are as follows:—

	Premiums.	Instalments.
Amounts in force as at 31 December 1902 (p. 652) .	£20,871 6 2	£904 0 7
Add, difference between First Premiums Written On and Off Books (p. 653) .	610 15 0	40 5 10
Add, per Renewal Premiums Written On Book (p. 654)	57 2 11	0 0 0
	£21,539 4 1	£944 6 5
Deduct, per Renewal Premiums Written Off Book (p. 653)	74 11 1	47 19 10
	£21,464 13 0	896 6 7
	896 6 7	
Premiums received during Year (p. 654) .	<u>£22,360 19 7</u>	

Reconcilement with Valuation Registers.—It is hardly necessary to give these, as the modifications are obvious.

APPENDIX B.

As the method of writing premium receipts and notices referred to in the foregoing paper has only recently been devised, and may not be generally known, I think it will be desirable to give a brief description of it, in order that my remarks may be followed.

Some time ago I saw a machine called the "Addressograph", which prints addresses by means of an endless chain consisting of links filled with indiarubber type. Into its merits for the ostensible object for which it was invented I do not intend to go, but it occurred to me that the machine might be adapted for the filling in of the particulars necessary for the writing of premium receipts. For many years these particulars were written, but latterly in a number of offices it has been usual to typewrite them, and the use of the "Addressograph" for the purpose is a further development of the latter method. Just as a typed receipt was an advance upon the written one, so is the printed receipt an advance on the typed one.

The operation is simplicity itself. The particulars necessary for the premium receipts and notices are filled into each link by means of indiarubber type. These links are joined together in a moment and form an endless chain which revolves upon a drum as each impression is printed off. These chains are arranged first according to months, and second according to agencies. The following is a specimen of the receipt as printed:—

THE LIFE ASSURANCE COMPANY.

Policy No. 20002, *William Jones.*

Premium, £s 6 3, due 1 July 1903

Agency, *London Office.*

RECEIVED, this day of 190 the above noted
Premium payable in terms of the Policy.

Countersigned

Manager.

The rapidity with which the machine prints is practically limited only by the rate at which it can be supplied. Two thousand

addresses per hour are stated to have been done by it, but for the purpose to which I put it, a thousand may be looked upon as a fair number. Even at this reduced rate the receipts for a month of the largest insurance office in the City could be disposed of in a few hours. The machine is very simply operated, and the work may very well be done by lady typists.

Another advantage of the method is that the same particulars serve for the renewal premium notices which it is usual to issue, as will be seen from the following specimen:—

Renewal Premium Notice.

THE LIFE ASSURANCE COMPANY.

Policy No., 20002, *William Jones.*

Premium, £8 6 3, *due 1 July 1903.*

Payable to *London Office.*

The Manager begs leave to remind you that the Premium becomes due on the above-mentioned date, and requires to be paid within one calendar month thereafter. Remittances should be made by Cheque, Bank Draft, or Post-Office Order, free of expense to the Company.

The Company is not bound to send this Notice, nor can the want of it be admitted as a reason for non-payment.

* * Please produce this Notice on payment.

MEMORANDUM TO BE USED WHEN REMITTING.

I send you herewith remittance for £ : : in payment of the amount above stated, and have to request that you will send me the Official Receipt.

Date

190 .

Signature

Any permanent change of address should be noted here:—

New Address

There is the further advantage that no comparison is required either for the receipts or the notices, as the particulars in both cases must be accurate, assuming them to be correct at the outset. A mark may be put on the receipt and the notice to indicate the rate of commission payable, such as a period to represent $2\frac{1}{2}$ per-cent, and a colon 5 per-cent.

A still further advantage is that the same particulars can be used for the making out of accounts for agents, and for the debit notes which it is the custom in a number of Offices to send to the Branches. This has been rendered possible by the insertion of a stout pin in the first link of each agency, which rings a bell when the last of the previous agency has been printed. A permanent record of all the receipts sent out for collection can also be kept for use at the Head Office.

It is obvious that the same particulars can, with the addition of the date of birth, be used for a Life Index by being printed on cards, the date of birth being added with the pen.

The names and addresses of policyholders can of course be set up also, and they would serve not only for the addressing of the notices and the corresponding envelope, but also for other purposes, such as the despatch of annual reports, bonus notices, &c.

If any changes in the particulars require to be made, they are easily given effect to, as there is a rubber type for each separate letter; and when the particulars are no longer required, the link containing them is easily detached from the rest of the chain. These changes may be made either from the books described or from a book specially written up for the purpose.

No exception can be taken to the accuracy of the proposed method, as, the type once properly set, the particulars must be correctly printed each time the link is used. Not only are the particulars more accurate than by the ordinary system, but a very considerable saving is effected if the proposed method is adopted.

[The machine described above by Mr. Chatham was exhibited at the Meeting, and its working illustrated at the close of the discussion.]

ABSTRACT OF THE DISCUSSION.

THE PRESIDENT (Mr. W. Hughes) said they were pleased to welcome a fair number of visitors interested in accountancy, and other matters connected with life assurance not purely actuarial, and would be glad to hear any remarks they had to make. The machine exhibited would be open to the inspection of the members, but he hoped the owners or inventors of the machine would find some other name for it than the dreadful one which had been given to it. He thought the word "Addressograph" was neither Greek, French, Latin, Scandinavian, nor anything else.

Mr. T. J. SEARLE was glad to see the author dealing with so practical a subject. He felt somewhat diffident about discussing the paper; for one reason, that he had not meddled with the particular matter for some years, and for another, that from long experience he was very shy of interfering with another man's way of doing his work. There were very many ways of obtaining the same object, and to each man that way was best with which he was most familiar. It should not be supposed that any particular method or set of books would keep the accounts of a company right; it was not the books that would do it, but the people who had the working of the books. He was glad the author kept general principles in view, and started with an enunciation of them. The paper stated:—"One main object to be kept in view in arranging premium books is the reconciliation of the various items of premium income. There is, first of all, the reconciliation of the premiums due during the year, in other words, the premium income for the year, with what, according to the books, was to fall due during the year. There is also the reconciliation of what was in force at the beginning of the year, with what is in force at the end of it, the latter forming the starting point for the reconciliation of the premiums in the succeeding year. The books should also permit of the sums assured in force at the end of the year being similarly reconciled." If those two principles could be kept in mind they would do a great deal towards keeping straight the accounts of any company. But as might be gathered, he preferred his own way of doing the work. Some ten or twelve years ago he had the honour of putting before the Institute a sketch of a premium book. He liked that book, because he thought it attained the object in view in a very simple straightforward manner. He liked a final premium book, what he called the actuary's premium book, which set forth every individual policy in its numerical order; for this reason, that if every number was in order there was only one particular place for a particular entry. If one took a book which was divided into months and agencies, a policy might be in one place or it might be in another, or it might be in both, or it might be in neither, while it ought to be in one; but if there was only one place for it in the whole book anybody knew where to look for it. He liked the one straightforward order of numbers. Then he liked the premiums in force at the beginning of the year in one column, the premiums in force at the end of the year in another column, and in between those the amounts paid, posted for each individual policy. He found no difficulty in making such a book to balance, and it showed very plainly that the whole of the premium

income had been accounted for. The author, in his opinion, had mixed that up with some things intended for a very different purpose. The author had made the renewal premium register serve so many purposes, that he (Mr. Searle) was afraid he was trying to make it serve too many, to be satisfactory in any of them. It tried to serve the purpose of an ultimate premium book, and also what he used to call the agency debit and credit journals. For his part he very much preferred the latter system. He did not much like posting the totals from the so-called Agency Renewal Premium Book; he would rather that, after the receipts were prepared, a record should be made of all the receipts sent out for collection. He thought that record could most conveniently be obtained in a book called the Agency Debit Journal. It did not require the name of the life assured—it did not require a lot of other information given for other purposes; it simply required the agent's name and the number, and the amount debited to him. It was a plain Agency Debit and Credit Journal. The practice which the author indicated of treating the cashier as if he were an agent was perfectly right, because it made matters uniform; but the new system of printing the manager's signature did not commend itself to him at all. The receipts should not be signed by the manager, still less by a director, because they knew nothing about it; but they should be signed by the accountant. He then sent them to the agent or cashier to be collected, and they countersigned them; but the man who signed them in the first instance should, in his opinion, be the accountant, and he should actually sign them, and not put a stamp on them. He should be responsible for seeing that everything he signed was debited either to the agent or to the cashier in his debit journal. He wished to pass on to the question of the Renewal Premium Register; it was one book, but the author tried to make it serve several purposes. The Renewal Premium Book was referred to in the paper in the following terms:—"It will be seen that the Agency Renewal Premium Book is similar to the Monthly Renewal Premium Book kept in a number of offices, in which the policies are ranged in numerical order." He did not quite gather whether the author meant that he would have the Monthly Renewal Premium Book kept in addition to the Agency Book.

Mr. CHATHAM replied in the negative.

Mr. SEARLE, continuing, said the author further stated: "Another objection is that it requires to be re-written every five years. Both these can be overcome by the use of cards if a permanent record in book form is not considered essential. It will be seen from what follows, however, that neither books nor cards are absolutely necessary." The Premium Renewal Book was the same thing as the Agents' Renewal Book, but where would they be if they did away with that book, and all the purposes the author suggested it should serve? He thought there was something there that needed a little explanation. His idea about the Renewal Register was a book divided into months, not divided into agencies. Some years ago the late Mr. Sunderland pointed out to him an improvement which was perhaps worth mentioning, as it might be of convenience to some members. The book was not divided into separate months, but into three volumes—

one called January, April, July, and October, another February, May, August, and November; another March, June, September, and December. No matter whether the policy was yearly, half-yearly, or quarterly, it was entered in its proper order in that book. There were four columns headed with the names of the months, and the day of the month was put in one, two, or four of them, according as the premiums were payable yearly, half-yearly, or quarterly, so that if the day of the month occurred four times, when one was going through for any month they took it, but if the day of the month only occurred in certain columns they did not take it, except in the particular month in which it occurred. That plan saved a good deal of re-writing. He liked a Renewal Premium Book apart from all the other work which the author had combined with it. He, nevertheless, agreed with Mr. Chatham that probably such a Renewal Premium Book could be swept away when the links were used, because it was not an account book; it was what he should call a memorandum book, which should be kept clearly and properly all the same, but it was no portion of the accounts. If the receipts and notices could be written without it, we might do away with it. The date at which a premium was to cease by efflux of time could not be put on the link, but provision must be made for it in some other way. Then the Renewal Premium Register might be done away with altogether, it being well understood that it was not intended to do away with the author's Agency Renewal Premium Book, which was wanted for a good many other purposes. He thought the new system of links would be one of the most interesting portions of the evening's proceedings.

Mr. JOHN ANDERSON said the office with which he was connected had had an opportunity of judging of the machine shown, and had found it extremely useful in preparing the receipts and notices. It was found that, in addition to printing the receipts and notices, by adding a link between the particulars of the policies not only could the particulars of the policies be printed, but the address added upon the same sheet of paper. By one operation upon the machine the particulars of the policy could be obtained, and immediately below, in the proper space allotted for the purpose, the address to which the notice was to be sent was added by another operation. That was a development which the author had not mentioned, but which he thought would be extremely useful. By a special arrangement upon the chains, the address links could be used for circularizing the policyholders. His company had not had a great deal of experience with the machine, but, so far, it had been extremely useful, and it had been found that one of their lady typewriters had been able to set up the links for the new policies. The Addressograph Company set up the original entries, and sent the links in the form of chains, and one was then in a position to print off perhaps a whole month's receipts and notices in a couple of hours' time. He came to the meeting more particularly to see whether there were any other members who had had any experience of the machine, and if he could obtain any hints. He was extremely obliged for the suggestion made in regard to the printing of the agents' account forms; that was quite a new idea to him. He had not used the machine in that form, at present writing all the account forms by hand; but the idea was an extremely good one, which he thought might be taken advantage of.

Mr. J. HOWARD BARNES thanked the author for the immense amount of pains he had taken in placing before the members some of the modern improvements invented for the preparation of renewal notices and receipts. The speaker would like to know whether the type in the machine was easily kept clean throughout the year, or whether it required special cleansing. He presumed special arrangements for storage were necessary. There was one point, however, which presented itself to his mind, quite apart from the machine, namely, a remark the author made on page 622 with regard to instalment premiums. The author had found it necessary in his paper to provide separate forms where half-yearly or quarterly payments were regarded as instalments instead of premiums, because, he said "all those instalments falling due beyond the financial year are properly included in the premium income for the year." He did not know whether there had been any recent expression of opinion at the Institute upon the point, but it seemed to him that the premium instalments falling due beyond the year should not be included in the premium income of the year. It was, of course, a temptation to bring a full year's premium income under new business into the revenue account, but the author's difficulties with instalment premiums would apparently be got over if with a new premium of say £10, by four quarterly instalments of £2. 10s. each, only one, two or three of which were due to be paid in the first financial year, only £2. 10s., £5, or £7. 10s., were brought into the revenue account. The practical difficulty in the way of treating instalments due beyond the year as belonging to the revenue of the year was, that if once done it would be increasingly hard to make a change, while the continuance of the practice meant a vast amount of labour, an enormously inflated amount of premiums outstanding at the end of the year in the balance sheet, and, most important of all, it seemed to him, that they were bringing in as one of the assets, debts which, in the case of new policies, were unsecured if the policy lapsed, and which could not always be attached for the premium account when the policy had a surrender-value. He ventured to suggest that the proper way of treating the instalments was to treat them only as assets in the event of a claim arising, or possibly when a surrender had to be calculated, and that they ought to include in the revenue account only premiums that fell due in the year.

Mr. A. E. MOLYNEUX thought that, while the whole of the members were indebted to the author for his paper, the younger members of the Institute were particularly under obligation to him. Many of them were in large life assurance offices where the work was very much sub-divided, and, while naturally anxious to learn the details of their business, they often found it difficult, and sometimes impossible, to get any practical acquaintance with the work of other departments than their own. The present paper, and that of Mr. Smith Nicol in 1897, were specially helpful, in that they not merely gave a verbal description of the various books, but also furnished actual specimen pages with a complete copy of the headings of the various columns. The author went further still, and gave specimen figures illustrating the posting from one book to another, and the reconcilements which had to be made of the sums assured and

premiums in successive years. He thought the author was mistaken in saying that the subject had never been directly submitted to the Institute of Actuaries, because he found in *J.I.A.*, ii, 199, a memorandum by the late Mr. Porter on book-keeping by double entry, in which he mentioned that he had seen the notes of a lecture by Mr. Jellicoe on the "Principles and Philosophy of Account Keeping", which was read before the Institute of Actuaries. The date of the lecture was not given, and the lecture itself did not seem to have found its way into the *Journal*, but it appeared to have been read before the Institute. Mr. Porter showed that, among other things, Mr. Jellicoe dealt with the subject of life assurance companies' accounts. The date of the lecture would be somewhere about 1850. He had thought it might be of use to some of the younger members if he compiled a list of the references which comprised the literature of life assurance book-keeping so far as he had been able to trace it. The references that he had found were thirteen in number, as follows:—

- (1.) *J.I.A.*, ii, 199. Mr. H. W. Porter refers to a Lecture by Mr. C. Jellicoe, on the "Principles and Philosophy of Account-Keeping", read before the Institute (Lecture not in *J.I.A.*).
- (2.) *J.I.A.*, xxix, 481. Mr. George King's Paper on the Life Assurance Companies Acts, 1870-1872.
- (3.) *J.I.A.*, xxx, 493. Mr. T. J. Searle's Paper on the "Progress of Profit in a Life Assurance Fund."
- (4.) *J.I.A.*, xxxii, 393. Mr. J. Chatham's Messenger Prize Essay.
- (5.) *J.I.A.*, xxxiv, 346. Mr. W. R. Dovey's Presidential Address to the Actuarial Society of New South Wales, 1898.
- (6.) *Transactions of the Actuarial Society of Edinburgh*, vol. iv, No. 6. Mr. W. Smith Nicol's Paper on "Forms of Life Office Books."
- (7.) The same *Transactions*, vol. iv, No. 7. Mr. A. D. L. Turnbull's Paper, "Some suggestions in regard to Life Office Accounting."
- (8.) *Transactions of the Faculty of Actuaries*, vol. i, No. 7. Mr. J. J. McLauchlan's Paper, as revised, 1902; the Paper was first read before the Actuarial Society of Edinburgh, 1886.
- (9.) Dr. Sprague's book, "Life Insurance Accounts", 1874.
- (10.) *The Incorporated Accountants' Journal*, vol. i, 1889-90. Mr. T. E. Young's Lecture, delivered in 1889, before the Society of Accountants and Auditors, "The Finance of Life Assurance in its relation to the work of Accountants and Auditors."
- (11.) *Transactions of the Chartered Accountants' Students' Society of London*, 1887. Mr. Lowes Dickinson's Paper on Assurance Accounts.

- (12.) The same *Transactions*, 1892. Mr. John Gane's Paper on Life Assurance Accounts.
- (13.) Book (recently published) by Mr. T. E. Young and Mr. R. Masters, "Insurance Office Organization, Management, and Accounts."

The papers were not exclusively concerned with the subject of life assurance book-keeping, but they all more or less touched on it, some of them exclusively, and some of them only by the way, in dealing with other subjects. As Mr. Chatham remarked in his essay, "The systems in vogue in different offices were so varied that no general rule could be laid down." As an instance of that, he might mention that in his own office there was no such thing as the "interim receipt", which the author, and also Mr. McLauchlan, made the very foundation of their system. When a proposal came to the office through an agent and was accepted, there was no question of an interim receipt at all; the agent was sent an acceptance letter, which quoted the amount of premium to be paid; and if the agent advised the office that the premium had been paid to him, then he was sent a receipt, and the agent's account was debited, the receipt giving cover until the actual policy was sent. He noticed that in the discussion at the Faculty on Mr. McLauchlan's paper, Mr. John Lamb advocated making the Policy Register rather than the interim receipt the starting point in the system of book-keeping, and Mr. Young in his lecture referred to the Register of Policies as "The basis of the operations and liabilities of a company", so that he probably agreed also. On page 623 a specimen of the sheet attached to the proposal papers was given. He did not know whether that was intended to be a complete statement of the figures, but it occurred to him that perhaps it might be worth while to have a proposal number as well as a policy number, to identify the cases that were declined or not taken up, or for any other reason not carried through. It was a good thing to have a number to identify them by, as well as those where policies were issued. On the same page he noticed that the author said that in the office referred to "All correspondence was put up with the proposal papers." He did not know whether that included letters asking for surrender-values, or applying for loans.

Mr. CHATHAM replied in the affirmative.

Mr. MOLYNEUX thought that in some offices some of the papers would get rather bulky before much time had elapsed. On pages 630 and 631 a specimen Register of Policies was given, in which there was a column, No. 20, for alterations and deductions. It sometimes happened that four or five alterations might occur in the history of one individual policy. It was not every company which insisted on proof of age being furnished before the policy was issued, so that an alteration might occur from a difference of age; then the assured might want to alter his policy from whole life to endowment assurance; or the premiums might be commuted; so that it seemed to him there ought to be a large space left in the column for each policy. On page 633 the author spoke of the use of the Agency Renewal Premium Book as being "the best way of keeping a permanent record in book form of the monthly debits to agents"; and then he added, that to meet the case of changes in agencies they could use "recently devised

books, under which leaves can be inserted or removed at pleasure." If these were books of account, he ventured to suggest it was rather a dangerous thing to use a book whose leaves could be removed at pleasure. Did not the fact of the leaves being capable of removal at pleasure prevent one calling it a "permanent record"? because if an agent ceased to act, and the whole of the leaves relating to the agency were removed, it could hardly be called a permanent record. With regard to the new "links" system, he was old-fashioned enough, to prefer receipts and notices of premium on which some human hand-writing appeared. The links were very clever, but they were not human. It seemed to him that when a man had once paid his premium it did not so much matter what receipt was sent him, but with a premium notice it was very likely to happen that if a man saw it in print he would say, "This is only printed matter", and throw it aside. On the last page of the paper the author said, "There is the further advantage that no comparison is required either for the receipts or the notices, as the particulars in both cases must be accurate, assuming them to be correct at the outset." If by any chance an error occurred at the outset, the mere fact of the machine not being human prevented the error being put right. A clerk in the office might be wrong this time and right next time, but the "link", if once wrong, would be always wrong, unless the assured found out the mistake.

The **PRE-IDENT** asked the members to pass a cordial vote of thanks to Mr. Chatham for his paper. He thought the paper very valuable, containing as it did a number of ingenious devices and useful hints.

MR. CHATHAM, in reply, thanked the members very much for the kind manner in which the paper had been received. With regard to the remarks made by the various speakers, Mr. Searle had referred to a *Renewal Premium Book* which was arranged in numerical order. He was not aware of any office in which there was a *Renewal Premium Book* in numerical order, without being divided according to months. The proposed system was simply a further development of that book, into agencies, which enabled them to debit the agents at once, instead of writing out debit notes. Although he thought it was original at the time he introduced it some years ago, he believed several offices had adopted it. He noticed, for instance, that Messrs. T. E. Young and R. Masters in their book "*Insurance Office Organization, Management, and Accounts*", published two or three days ago, gave the same kind of form for their *Renewal Premium Book*. With regard to debit and credit books, he resolved to dispense, if possible, with technical language, and put the names in a plain simple fashion, so that a clerk could write up the books and understand what he was doing. He thought Mr. Searle would find on examination that the forms given practically served the purpose he aimed at, but were simpler. With regard to the permanent record to which he referred, it applied only to the addressograph, as would be found in the appendix relating specially to that machine. The receipts were not intended to be written up in another book in a permanent form. One could run off so very quickly on the machine the particulars of the premiums falling due, that it was practically no

extra work at all to get a permanent record. In that event it would really supersede the Agency Renewal Premium Book, of which it was practically a duplicate. He was wondering, when Mr. Searle objected to the manager's signature being lithographed on the receipts and that the accountants should sign them, if he (Mr. Searle) had ever signed receipts himself. He (Mr. Chatham) had had that misfortune a number of years ago, and it was such a laborious work, and took up so many hours' time, that he strongly advocated the change to the lithographed signature, in order that the work might be done quicker. The forms were of course countersigned by the agents on the actual receipt of the money. Mr. Anderson referred to having adopted the same system of printing the renewal premium receipts and notices, and said he had put the address of the policyholder alongside. That prevented one from doing the agents' accounts, because he did not think they could be well done with the address appearing. He therefore resolved to have the address separate, but to have the two, though on separate chains, hanging together, so that any change upon the one could quickly be made upon the other. Mr. Barnes asked whether the type kept clean. It did; only a very small amount of ink was taken off upon the india-rubber type, and any left after printing quickly evaporated. There was no difficulty in that connection. It might be of interest to the members to know that the machines had been at work in the office of a very large publishing firm in London for several years. They were used every morning for the addressing of wrappers, and no difficulty had been experienced. With regard to the premium instalments included in the year, he thought it was quite legitimate to take the instalments due beyond the year into account, because a contract had been made, and the company was entitled to deduct the premium in the event of death taking place. Mr. Molyneux referred to a memorandum having been submitted to the Institute so far back as vol. ii, in which there was a reference to a lecture on accounts. He thought that lecture referred more to the philosophy of the subject; he had gone in for the practical rather than the philosophical side of the question. With regard to the interim receipt, Mr. Molyneux sent out the acceptance notice first, and then, when the money was paid, he sent the receipt. He thought that was practically equivalent to his interim receipt, but he thought his system was rather better, because the receipt was waiting for the money, and it was always convenient to hand it over when the proposer paid. The proposal papers were kept according to the policy number, and there was a separate series of numbers for declined proposals. He had had experience of both methods of keeping correspondence—of keeping it with the proposal papers and of not so keeping it, and he had no hesitation in saying that the method of keeping the correspondence with the proposal papers was much the better plan. In his company, all the correspondence, whether it was about an assignment, or a loan, or a surrender-value, was put with the proposal papers. In a very few cases the correspondence was somewhat bulky, but in the vast majority no difficulty was experienced. Practically the same thing applied with regard to alterations and deductions in the register of policies; in very

few cases were there a large number of alterations and deductions. The system of books to which he referred when he said that the leaves could be removed at pleasure, was that known as the Dade system, but when he said the leaves could be removed at pleasure he did not mean that they could be taken out or put in just as one pleased; they were under lock and key, and the book must be unlocked before a single leaf could be taken out. The abstracted leaves could also be put under lock and key, and that constituted a permanent record. Mr. Molyneux referred to an error made at the outset being perpetuated, but that was not possible, because the links were summed as they were set up from the register, and must agree with the register, there being a perfect check upon every step in the process. Before a link was removed and any change made, an impression was taken, and then after the change was made another impression was taken. The difference between the additions of the changes, and the deductions, added to the amount in force in the previous year, gave the renewal charge for the month. In addition, there was the check by means of valuation books. The system of links had not been for long in operation, only about a year, but its progress had been much greater than he anticipated. One office in London had already adopted it, he understood successfully. An office in Edinburgh had also adopted it, and another very large office there had now finally decided to so, after a very careful examination of it. So far as he knew, one office had adopted it in New York, and another in Melbourne, so that it had spread in a manner he had never anticipated. In the working of the system, he had been greatly aided by his friend and assistant, Mr. Alexander Fraser, a Fellow of the Institute, and he wished to acknowledge his indebtedness to that gentleman for the help he had given.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, APRIL 1904.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART I).*Examiner*—PROF. S. L. LONEY, M.A.*Supervisors*—MESSRS. S. G. WARNER, and L. F. HOVIL.*First Paper.**A Short Collection of Actuarial Tables (containing a table of logarithms)
will be supplied for use in answering Question 8.*

1. Without using logarithms, multiply 4.38729 by 463.91 correct to two places of decimals, and divide 1 by 3.14159265 correct to four places of decimals. [Contracted methods preferred.]

2. A man bought 40 horses at £30 each; he sold a certain number at a profit of 20 per-cent, and the rest at eleven-twelfths of the first selling price. If his total profit is £165, find how many horses he sold at a profit of 20 per-cent.

3. Solve the equations—

$$(i) \quad \left. \begin{array}{l} ax + by = b^2, \\ bx - (a + b)y = a^2 \end{array} \right\};$$

$$(ii) \quad 16x(x+1)(x+2)(x+3) + 15 = 0.$$

4. Find the condition that one of the roots of the quadratic equation $ax^2 + 2bx + c = 0$ may be double the other.

Find the quadratic equation such that the sum of its roots is 11, and the square of the difference of its roots is 17.

5. If the fractions $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$ are all equal, prove that each of them is equal to

$$\left(\frac{pa_1^n + qa_2^n + ra_3^n}{pb_1^n + qb_2^n + rb_3^n} \right)^{\frac{1}{n}}$$

where p, q, r, n are any numbers whatever.

If $a : b :: c : d$, prove that

$$\frac{ab^3 - c^3d}{b^2c^2 - c^3d} = \frac{b^3}{ad^2} + 1.$$

6. Find n from the equation ${}_nP_4 = \frac{90}{7} \times {}_nC_6$.

There are ten candidates for three vacancies on a committee; but one candidate A will only serve if B is elected, and another candidate C will not serve if B is elected. In how many ways may the three vacancies be filled?

7. Establish the truth of the binomial expansion for $(x+a)^n$ in the case when n is a positive integer.

Find in its simplest form the $(r+1)$ th term in the expansion of $(1-px)^{\frac{a}{p}}$.

8. Explain why, in an ordinary table of logarithms to base 10, only the decimal parts of the logarithms are given.

What is the least integral value of x for which $(1.05)^x > 100$?

9. Sum the following series:

(i) $1.2.4 + 2.3.5 + 3.4.6 + \dots$ to n terms;

(ii) $1 + 4x + 18x^2 + 86x^3 + \dots$ to n terms, and, when possible, to infinity.

10. Explain what is meant by the words Chance and Probability, and show how the latter is measured.

n persons are sitting at a round table, and from them three are selected at random; show that the chance no two of those selected are sitting next one another is

$$\frac{(n-4)(n-5)}{(n-1)(n-2)}.$$

11. A heap of playing cards contains 6 hearts, 5 spades, and 4 clubs. A card is chosen at random 9 times in succession, and is not replaced. Find the chance

(i) that there are no hearts left, and

(ii) only hearts are left.

12. Show that, with a certain reservation, the operators E and Δ satisfy the ordinary fundamental laws for algebraic quantities, and hence that they can be combined with one another as if they were symbols of quantity.

Prove that

$$\begin{aligned} u_0 + u_1 \cdot x + \frac{u_2 \cdot x^2}{2} + \frac{u_3 \cdot x^3}{3} + \dots \\ = e^x \left[u_0 + x \Delta u_0 + \frac{x^2}{2} \Delta^2 u_0 + \dots \right] \end{aligned}$$

Second Paper.

A Short Collection of Actuarial Tables (containing a table of logarithms) will be supplied for use in answering Question 19.

13. A man invests half his money in $2\frac{1}{2}$ per-cent Consols at $90\frac{1}{2}$, one-third in a $4\frac{1}{2}$ per-cent Debenture at $104\frac{1}{2}$, and the remainder in a Gas Stock paying 6 per-cent at 120. Neglecting brokerage, find to the nearest shilling the average interest he gets on his money.

14 Express $\frac{1}{x^2-5x+6} + \frac{3}{x^2-x-2}$ as a single fraction in its lowest terms.

Extract the square root of $4x^6 - 4x^5 + x^4 + 12x - 6 + \frac{9}{x^4}$

15. If $x=3-\sqrt{5}$, express in its simplest form the fraction $\frac{x^2-2x-3}{2x^2-3x-4}$.

If $x=5$ when $y=8$, and $x=10$ when $y=1$, and the relation between x and y is $x^ny=C$, find the values of n and C .

16. Insert two arithmetic means between two quantities x and y , and prove that their product is greater than xy .

17. Find the number of combinations of n things, taken r at a time, without assuming the formula for permutations.

As r increases from 1 to n , show that ${}_nC_r$ increases up to a certain value and then diminishes; and, when n is odd, find what value of r makes ${}_nC_r$ a maximum.

18. When n is a positive integer, find the sum of the coefficients in the expansion of $(1+x)^n$.

When x is a small fraction, show that

$$\sqrt[4]{\frac{1+x}{1-x}} = 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{3x^3}{16} \text{ approximately.}$$

19. Making use of the logarithmic tables, find the value of

$$(1) (3.07) - 1.346;$$

$$(2) \left[\frac{\sqrt{5.55} \div (6.432)^{\frac{3}{5}}}{17.89 \times \sqrt[3]{.000374}} \right]^{\frac{1}{2}}.$$

20. Obtain the expansion of $\log_e(1+x)$ in a series of powers of x , when $x < 1$.

What is the corresponding expansion when $x > 1$?

Find the sum to infinity of the series

$$\frac{x^2}{1.2} - \frac{x^3}{2.3} + \frac{x^4}{3.4} - \dots$$

where $x < 1$.

21. Prove that the product of any r consecutive integers is divisible without remainder by r .

If n be any integer, prove that $n^{13} - n$ is divisible by 2,730

22. Explain what is meant by Inverse Probability.

A bag contains six balls, each of which is known to be red or white, either colour being equally likely. Two balls are then drawn and found to be one red and the other white; these are then replaced and two others drawn. Show that the chance they are both white is $\frac{7}{30}$.

23. Show that

$$\phi(0) + \phi(1) + \dots + \phi(n) \\ = (n+1)\phi(0) + \frac{(n+1).n}{1.2} \Delta \phi(0) + \frac{(n+1).n.(n-1)}{1.2.3} \Delta^2 \phi(0) + \dots$$

Hence find the sum of n terms of the series whose n th term is $n^4 + 3n$.

24. If u_0, u_1, \dots, u_6 be consecutive terms of a series, prove that

$$u_3 = \frac{1}{20}(u_0 + u_6) - \frac{3}{10}(u_1 + u_5) + \frac{3}{4}(u_2 + u_4).$$

Supply the missing term in the following table:

$u_0 = 72,795$	$u_4 = 67,919$
$u_1 = 71,651$	$u_5 = 66,566$
$u_2 = 70,458$	$u_6 = 65,152$

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART II).

Examiners—MESSRS. T. G. ACKLAND, H. J. BAKER, A. G. HEMMING,
and J. SPENCER.

First Paper.

A Short Collection of Actuarial Tables will be supplied for use in answering Questions 1, 2, and 8.

1. Explain how a table of values of the function $\log \left(\frac{e^x - 1}{x} \right)$, for positive and negative values of x , affords the means of finding the amounts and present values of continuous annuities certain, at all rates of interest.

Find approximately the value of a_{30} at 5.1 per-cent, from the 5 per-cent values of a_n .

2. A debenture of £100, with interest at 5 per-cent per annum, payable on 1 January and 1 July, and redeemable at par on 1 July 1928, is bought on 1 April 1904, for £112. 5s. Find approximately the rate of interest realized by the purchaser, the premium being written off by 49 equal half-yearly instalments, commencing 1 July 1904.

3. Deduce the probability (i) that (y) will be alive t years after the death of (x); (ii) that (y) will be alive at the end of the t th year succeeding the year in which (x) shall die.

4. Deduce an expression for the amount to which an annuity-due will accumulate at the death of (x):

(i) on the assumption that the amount payable is uniform, whenever death shall happen;

(ii) on the assumption that at death the accumulated amount of the particular annuity is payable.

5. Define the expression $a_{\overline{(xn)(ym)}}^{\overline{}}$, and deduce its value when $m \geq n$.

6. Show how tables of the function $\frac{a_x}{a_x}$, where the numerator and denominator are based upon different tables of mortality at the same rate of interest, can be employed for comparing whole-life policy-values at different ages and durations, deduced from the two mortality tables.

How would you determine, by the above method, whether ${}_{15}V_{20}$ is greater or less by the O^M Table than by the combined H^M and $H^{M(5)}$ Tables?

7. Show that, if $n < t$,

$$P_{[x]t} - P_{[x]n}^1 = P_{[x]n}^1 \cdot nV_{[x]t}$$

and explain the result verbally.

8. Find the annual premium (assuming a loading of 10 per-cent of the office premium, and a constant of 2*s.* 6*d.*) which should be charged for the following benefit on a life now aged 30:

(i) £200 to be payable, should the assured be living at the end of 20 years;

(ii) £100, together with all premiums paid (without interest), to be payable should the assured die during the term of 20 years.

9. How would you proceed to construct a table of values of A_{xy}^1 , without using commutation columns?

10. What data would you require in order to construct a table showing the mortality experience of a large town?

Why is it impossible to construct a true mortality table solely from the records of a census, or from an enumeration of deaths?

11. Having given that $l_x = ks^x g^{c^x}$, show that μ_x may be written in the form $A + Bc^x$, and find $\text{colog}_{ep} p_x$ in terms of the same constants.

12. Establish the formula

$$\bar{A}_x = \mu_x \bar{a}_x + \frac{1}{2}(\bar{a}_{x-1} - \bar{a}_{x+1})$$

What does this become when interest is eliminated?

Second Paper.

13. Show that the relation between the value, at a given effective rate of interest, of an annuity-certain payable p times a year, and an annuity-certain payable yearly, is independent of n , the term of the annuity. Deduce an expression for the relation when $p = \frac{1}{r}$.

14. Find the present value of an annuity of the n th order, to yield interest at the rate j per annum on the whole capital for the entire term of the annuity, the capital to be replaced by means of a sinking fund accumulating at the rate i per annum.

15. Given the following mortality table, deduce, in respect of a life aged 82, (a) the expectation of life, (b) the "probable lifetime", (c) the age at which it is most probable that he will die. Find also the average age at death of the 186 lives aged 95.

x	l_x	d_x	x	l_x	d_x
80	15,530	2,150	92	756	263
81	13,380	2,007	93	493	183
82	11,373	1,847	94	310	124
83	9,526	1,674	95	186	79
84	7,852	1,493	96	107	49
85	6,359	1,308	97	58	28
86	5,051	1,122	98	30	15
87	3,929	943	99	15	8
88	2,986	773	100	7	4
89	2,213	617	101	3	2
90	1,596	480	102	1	1
91	1,116	360

16. In connection with a loan, repayable during n years by equal monthly instalments of principal and interest, an Insurance Company issues to the borrower, aged x , a policy securing the balance of principal outstanding at his death, should that event occur during the n years. Obtain a formula for the net single premium for the assurance. What objection is there to the quotation of a uniform annual premium in such a case?

17. Find an expression for the office annual premium for a reversionary annuity to (x) after the death of (y), the premiums to be returnable if (x) die before (y), the loading being a percentage and a constant.

18. Find the values of ${}_nV_x^{(m)}$, ${}_n\ddot{V}_x^{(m)}$ and ${}_n\ddot{V}_x^{(m)} \frac{t - \frac{1}{s}}{n + \frac{1}{nb}}$, in terms of ${}_nV_x$ and $P^{(m)}$.

19. A half-yearly annuity of 1 per annum is to be paid to (x) on his attaining age 50, and on his death, either before or after that age, is to become immediately payable to his wife, now aged y , during the remainder of her life. Find the annual premium for this benefit; and also for a similar annuity which, however, in the event of (x) dying before age 50, is not to be payable until (y) attains age 50.

20. A whole-life policy for £1,000, taken out n years ago, on a life then aged x , is entitled to reversionary bonus additions of £300. It is desired to apply the bonus to convert the policy into an endowment assurance. Show how to find the age at which the endowment assurance will mature.

21. Having computed tables of $\log D_x$ and $\log C_x$, for all ages, at two different rates of interest, how would you verify the accuracy of the results; and given values of M_x and N_x for all ages, at a single rate of interest, how would you verify their accuracy?

22. Given $n_{[x]}$, the number entering at age x ; $e_{[x]+r}$, the number existing, after exactly r years' duration; and $d_{[x]+r}$, the number of deaths having a curtate duration of r years; for all values of r ; and assuming that there are no withdrawals included in the experience;—deduce a formula for the number exposed to risk at age $[x] + t$.

23. Show that

$$\frac{1}{l_x} \int_0^1 l_{x+t} \cdot \mu_{x+t} \cdot dt = q_x,$$

and give a verbal explanation of the formula.

24. Deduce, by means of Finite Differences, the value of the complete expectation of two joint lives (x) and (y) .

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III).

Examiners—MESSRS. A. D. BESANT, B.A., H. FOOT, B.A., R. TODHUNTER, M.A.,
and J. D. WATSON.

First Paper.

1. Explain the nature of a life table. Show generally that such a table may be obtained from (a) population statistics, (b) the records of a special class such as the peerage, (c) the experience of life offices, pointing out the essential differences of procedure entailed by the different characteristics of the three classes of data.

2. State fully the methods you would adopt to extract the mortality experience of an old-established life office. Having regard to the various practical uses to which such an experience could be put, discuss the relative advantages of dividing the data into two or more sections, either as regards date of entry or otherwise.

3. Explain clearly what is meant by a "select life table."

Write down the expressions that have been given for the number of select lives and the number of impaired lives among the survivors after t years of $l_{[x]}$ persons, and state with reasons whether you consider them accurate.

4. Draw up a complete memorandum of instructions for the classification of whole-life with-profit policies (subject to premiums payable throughout life) for the purposes of a net premium valuation, and state how the valuation ages would be determined and what valuation functions you would use.

5. An old-established office which formerly valued its liabilities by the combined H^M and $H^{M^{50}}$ Tables at $2\frac{1}{2}$ per-cent interest with special reserves to increase the loading on the profit and non-profit policies now desires to adopt the new British Offices Experience. Discuss the general question of the most suitable table or tables which might be adopted and the probable effect upon the reserves.

6. Give a short account of the methods by which endowment assurances may be valued when grouped according to (a) the year of maturing, (b) the year of birth.

Discuss the practicability of applying these methods of group valuation to double endowment assurances.

7. What are the relative advantages of meeting the extra risk in the case of an under-average life (1) by charging the premium as on a normal life of a higher age, and (2) by way of a fixed uniform debt on the policy for a limited period? Assuming the former to be a company's usual practice, how would you determine the equivalent debt?

8. Explain the considerations which would guide you in forming tables of non-profit rates. Would your view as to the basis to be adopted be influenced by the circumstances of the particular company for which the rates were required? Consider, for example, the case of a company which had during a long period experienced a very favourable mortality, maintained a high rate of bonus on its participating policies, and concurrently strengthened its reserves.

9. On what basis would you calculate rates for immediate annuities? State your reasons and give the formula employed.

Give the formula, with necessary explanations, for fixing the prices of annuities subject to the condition that if the annuitant die within the first n years after purchase a proportion of the purchase money shall be returned.

10. An old-established firm, which has for many years employed a staff of about 500 men, desires you to advise what annual sum, measured as a percentage of the salaries payable to future employees, it would have to set aside to accumulate a fund to provide (a) to those of them who remain in the service at age 65 a pension from that age of as many sixtieths of their last year's salary as there have been years of service, and (b) in respect of those who, after having been employed at least 10 years, die in the service, a payment equal to their current annual salary. For what information would you ask the firm, and how would you proceed to ascertain the required percentage?

11. On what principles would you determine the surrender-value to be allowed for a double endowment assurance policy?

12. A, who is assurable at the ordinary rate for his age, is entitled contingently on his surviving his mother, aged 50, and to her having no further issue, to a one-third share in trust funds, at present invested as follows:—

(a) Various leasehold properties with unexpired terms of from 30 to 40 years, at present producing about £300 per annum net.

(b) £1,200 Consols; £1,500 Western Australia 4 per-cent Inscribed Stock, 1934; £200 Great Indian Peninsula Railway B Annuity.

State how you would estimate the present market value of his share, indicating any points upon which you would ask for further information.

Second Paper.

13. What is the theoretical basis of the "Final Series" method, and how would you apply the method if you wished to use it in the construction of a mortality table? Why was it used in the case of the American (30 Offices) Table?

14. Specify the several methods employed in the graduation of the British Offices Life Tables (without discussing the reasons for their adoption or the processes used in their application), and state in which cases the law of uniform seniority obtains.

Investigate the question whether a joint-life annuity-value could be calculated from the $O^{(M)}$ Table as from the $O^{(M,5)}$ by substitution of a single life and alteration of the rate of interest.

15. Describe a method of obtaining a graduated mortality table from a limited experience by reference to a standard table, and mention any incidental advantages of the method.

16. What are the main sources from which the surplus of a life office is derived? By what methods would you ascertain in practice how much of the surplus was derived from each source?

17. Prove that the expected death strain under a whole-life policy, subject to an annual premium payable throughout life, increases with the duration of the assurance, if at all ages on the basis of the valuation mortality table and rate of interest Δ^2a is algebraically $> i\Delta a$.

18. Mention the principal systems at present in use for the distribution of the surplus of a life assurance company among the participating policyholders.

State, with reasons, which system you would consider the most suitable for adoption by a new company.

19. Compare the incidence on policies of different durations, under the various principal methods of distribution, of a reduction in the divisible surplus of an old-established company, owing to the depreciation of recently purchased Stock Exchange securities below cost, and state which method appears to you to apportion a loss of this nature most equitably.

20. A company distributes its surplus quinquennially on the compound reversionary bonus plan; bonuses vest five years after the date of assurance, and claims are payable immediately on proof. Explain in detail how, given the valuation schedules, you would estimate the cost of a bonus of 1 per-cent per annum.

21. How would you calculate (*a*) the office premium for a term assurance of less than one year's duration; (*b*) the net annual premium for a 5-year term policy convertible, at the option of the holder, at the end of the fourth year into a whole-life policy at the ordinary premium for the life assured's then age?

*22. A, aged 45 last birthday, at present has a life interest in, and should he live 20 years, will then become absolutely entitled to, a trust fund now represented by the following investments:

£860 Consols;

£500 on mortgage at $4\frac{1}{2}$ per-cent of property of ample value;

Freehold ground rents of £56 amply secured with an unexpired term of 80 years.

What is the maximum amount that could be lent on security of his interests?

*23. At what do you estimate the market value of the following life policy?

Whole-life policy for £2,500 with profits effected on 1 October 1873, on a male life born 24 July 1849, at an annual premium of £60. 2s. 1d., payable on 1 October. The bonuses already added amount to £980, and the office distributes its profits quinquennially (31 December 1908, &c.) in the form of a simple reversionary bonus, the last declared being at the rate of 30s. per-cent per annum, with an intermediate bonus of £1 per-cent per annum.

*24. A fund of £10,000 Great Western Railway 4 per-cent Debenture Stock is divisible at the death of a man aged 50 equally among such (if any) of three males now aged 18, 20, and 22 respectively as shall survive him. How would you determine the market values of their respective interests?

** In answering these questions the Candidate is to set out his work as in an actual numerical valuation, but to exhibit the final results in terms of the actuarial functions involved, without inserting the numerical values of the functions, stating, however, the mortality tables and rates of interest which he would use.*

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART IV).

Examiners—Messrs. J. BLAKEY, J. E. FAULKS, B.A., W. P. PHELPS, M.A., and
E. A. RUSHER.

First Paper.

1. Give the provisions of the Judicature Act, 1873, relating to the assignment of choses in action.

On the occasion of a mortgage of a reversionary interest the proposed mortgagee is asked to dispense with (a) making enquiries as to previous charges from, (b) giving notice to, a particular one of the three trustees of the fund. Discuss the position fully.

2. A woman who married in 1890 died intestate in 1896, leaving her husband and an only child, a daughter, surviving. At the time of her death she was owner of the following property :

- (a) An estate in fee simple.
- (b) An estate in tail male with remainder to A in fee simple.
- (c) A leasehold house.
- (d) Furniture.
- (e) Consols.

Who becomes entitled to the property ? Is it necessary to take out letters of administration ?

3. A proposes to effect a whole-life policy, and desires that it should be "for the benefit of wife and children under the 'Married Women's Property Act.'" Mention what points you would put before him for consideration, dealing particularly with the question of second marriages.

What, if any, additional points would you raise if the proposal were for an endowment assurance ?

4. What are contained in (a) the Memorandum of Association, (b) the Articles of Association, and how does the Act of 1900 regulate the contents of either ? To what extent, and how, may either be subsequently modified ?

5. A large and old-established bank has a pension fund to provide pensions for its employees on retiring at the age of 60. Every employee must at entry be between 18 and 21 years of age, and must undergo medical examination. A fixed percentage of his salary is compulsorily applied each year towards the pension fund, and the pension is a percentage of the salary at the date of retirement for each year of service. An employee leaving the service prior to age 60 receives no compensation for his contributions, but in the event of death a payment is made to his widow of the total contributions accumulated at $2\frac{1}{2}$ per-cent simple interest. How would you proceed to value such a fund, and what particulars would you call for ?

6. What is meant by a Collecting Friendly Society, and in what respects does the law relating to such a society differ from that governing other friendly societies?

How does a Collecting Friendly Society differ from an Industrial Assurance Company?

7. A life office distributes its surplus on the contribution system (Dr. Sprague's). Draft a form of book for recording in the case of whole-life policies the necessary particulars and calculations for valuation and distribution of profits, adding any explanatory notes that may be desirable.

8. A life office buys in London at 108 American Railroad 6 per-cent Gold Bonds, repayable 1908 at par, principal and interest being payable in New York. Assuming that the bonds are held until maturity, discuss the various modes of keeping the Investment Ledger account, dealing particularly with (a) accrued interest (b) exchange.

9. Describe how tables of index numbers for comparing the value of money in different periods are obtained, and state the objections which may be urged against their use for that purpose.

10. What are the principal conditions regulating the borrowing of money from the Government by local authorities upon the security of the local rates?

11. If the $2\frac{1}{2}$ per-cents, redeemable in 1905, and Consols, redeemable in 1923, could be bought under par at the same price, state, with your reasons, which stock you would recommend to be purchased.

What charges, other than brokers' commission, are incurred in purchasing Consols, Bank of England Stock, British Corporation Stocks, and British Railway Debenture Stocks respectively?

12. Discuss the advisability of a life office investing at the present time in

(a) Bombay, Baroda and Central Indian Railway Guaranteed Stock.

(b) East Indian Railway Annuities (A, B, C and D).

(c) Canadian Pacific Railway Debenture Stock.

(d) The Ordinary Stocks of Gas Companies.

Second Paper.

13. Discuss shortly the circumstances in which a contract otherwise valid may be set aside on the ground of mistake.

The sale of a life policy has been duly completed, and it has subsequently been discovered that, unknown to the vendor, the life assured has died before formation of the contract for sale. Has the vendor any, and, if so, what remedy?

Indicate any bearing which your answer to the last question may have on the completion of a proposal for life assurance.

14. What is the difference between a cheque and a bill of exchange? What remedy has the owner of a bill of exchange who has lost it before it is overdue?

A by fraud obtains a cheque from B and endorses it to C for value. Can C successfully sue B on the cheque?

Can the finder of a lost cheque enforce its payment?

15. Give the substance of the Life Assurance Companies (Payment into Court) Act, 1896, and of the Rules of Court relating thereto.

Discuss the question as to whether advantage can be taken of the provisions of the Act where the only defect in title is that the policy itself is not forthcoming.

16. State in detail the changes that have been made since the year 1880 in the law with regard to payment of claims under Colonial Probates or Letters of Administration.

17. Draw up a schedule showing how, from the Registrar of Friendly Societies' Tables, monetary and other tables may be obtained in the most suitable form for general use in the valuation of friendly societies.

18. State the steps to be taken by a friendly society to obtain registration under the Friendly Societies Acts. What are the respective advantages and disadvantages of so registering?

Are there any, and, if so, what restrictions upon the investment of the funds of a friendly society?

19. Discuss the advantages and disadvantages of the card system in connection with the non-actuarial departments of life assurance business. Draft forms of card suitable for (a) an index of assured lives, (b) a record, for all office purposes, of loans repayable by terminable annuities. Give a specimen entry in each case.

20. Draft a form of claim register suitable for arriving with a minimum of trouble at the information usually given as to claims in Annual Reports, and also at any information required in connection with an estimate of profit or loss from mortality by the death-strain method.

21. How are the note issues of country banks in England and Scotland respectively regulated? What is the position of the holders of such notes in the event of the failure of the banks issuing them, and what alteration, if any, of the law as to security for payment of the notes do you consider to be desirable?

22. What securities constitute the Unfunded Debt of this country? Discuss the policy of borrowing large amounts in this form of debt.

23. What would you consider at the present time to be the most suitable investments for the funds of a life office? Give reasons for your answer, and state what are the leading principles to be observed in the choice of securities. In what respects do these principles differ from those applicable to Banking Finance?

24. The following stocks and bonds appear in the Stock Exchange Official List :—

Atchison Topeka & Santa Fé Ry. *General Mortgage* 4 per-cent
100 year Gold Coupon Bonds.

Cleveland, Cincinnati, Chicago, and St. Louis *First Collateral Trust Mortgage (St. Louis Division)* 4 per-cent Gold Bonds.

Northern Pacific *Prior Lien Railway and Land Grant* Gold Bonds.

Erie R.R. Coy. *Equipment Trust* Bonds.

Reading Co. *Common Stock Voting Trust Certificates.*

State clearly what information is given as to the various securities by the words printed in italics.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1903-1904.

First Ordinary Meeting, 30 November 1903.

The first ordinary meeting of the Session 1903-1904 was held at the Hall of the Institute, on the 30th day of November 1903.

The President (Mr. W. HUGHES) in the Chair.

A paper entitled "On the Comparative Mortality among Assured Lives of Abstainers and Non-Abstainers from Alcoholic Beverages", by Mr. R. M. Moore, was read in abstract by the author.

The following gentlemen took part in the discussion :—Messrs. H. W. Manly, J. H. Barnes, A. R. Barrand, G. King, D. C. Fraser, P. J. Whittaker, M.P. (a visitor), and the President.

Second Ordinary Meeting, 21 December 1903.

The President (Mr. W. HUGHES) in the Chair.

A paper entitled "The Income Tax as affecting Life Offices, with special reference to some Recent Decisions", by Mr. J. E. Faulks, B.A., was read in abstract by the author.

The following gentlemen took part in the discussion :—Messrs. A. D. Besant, A. H. Bailey, J. B. Gillison, J. D. Watson, C. D. Higham, H. W. Manly, A. Levine, J. H. Barnes, H. E. W. Lutt, F. B. Wyatt, and T. G. Ackland.

Third Ordinary Meeting, 25 January 1904.

The President (Mr. W. HUGHES) in the Chair.

A paper entitled "A Comparison of the various methods of grouping Whole-Life Assurances for Valuation", by Mr. D. C. Fraser, M.A., was read in abstract by the author.

The following gentlemen took part in the discussion:—Messrs. H. T. Adlard, G. J. Lidstone, G. King, R. Todhunter, R. P. Hardy, and S. G. Warner.

Fourth Ordinary Meeting, 29 February 1904.

The President (Mr. W. HUGHES) in the Chair.

Mr. John Tatlock, M.A., F.R.A.S., was duly elected an Associate of the Institute.

A paper entitled "An Investigation into the Rates of Re-Marriage and Mortality among Widows in receipt of relief from the Patriotic (Russian War) Fund, 1854-1900", by Messrs. J. Burn and J. McDonald, was read by Mr. Burn.

The following gentlemen took part in the discussion:—Messrs. F. L. Collins, F. Schooling, T. G. Ackland, A. W. Watson, and the President.

Fifth Ordinary Meeting, 28 March 1904.

The President (Mr. W. HUGHES) in the Chair.

Mr. J. A. Rankin, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On the Valuation of Whole-Life Industrial Assurances, with allowance for Lapses", by Messrs. T. G. Ackland and J. Bacon, was read by Mr. Bacon.

The following gentlemen took part in the discussion:—Messrs. W. S. Anderson, W. R. Dovey, H. W. Manly, E. C. Thomas, and A. W. Watson.

Sixth Ordinary Meeting, 25 April 1904.

The President (Mr. W. HUGHES) in the Chair.

The President announced that, since the last meeting, two Fellows had died, namely, Mr. Edward Docker, and Mr. A. F. Burridge who was a Vice-President at the time of his decease.

A paper entitled "On Life Premium Book-Keeping", by Mr. J. Chatham, was read in abstract by the author.

The following gentlemen took part in the discussion:—Messrs. T. J. Searle, J. Anderson, J. H. Barnes, A. E. Molyneux, and the President.

The Fifty-seventh Annual General Meeting, 6 June 1904.

The President (Mr. W. HUGHES) in the Chair.

The proceedings at the Annual General Meeting will be found on page 689.

REPORT, 1903-1904.

The Council have the pleasure to report to the members upon the progress of the Institute during the session of 1903-1904, the fifty-sixth year of its existence.

There has been an *increase* of 28 in the number of members, as compared with the previous year. 133 candidates have been admitted as Probationers, and 53 as Students conditionally on their passing Part I of the Examination. At the end of the official year in which the Institute was incorporated by the Royal Charter the number of members was 434, while ten years later, at 31 March 1895, it was 775. Since that time the numbers have been as follows:

On 31 March 1896,	788,
..	1897, 826,
..	1898, 860,
..	1899, 834,
..	1900, 822,
..	1901, 818,
..	1902, 842,
..	1903, 828,
..	1904, 856.

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

Schedule of Membership, 31 March 1904.

	Honorary Members	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1903 .	1	225	262	316	24	828
ii. Withdrawals by						
(1) Death	6	3	...	1	48
(2) Resignation	2	16	...	
(3) Default in pay- ment of Sub- scriptions.	5	15	...	
	1	219	252	285	23	780
iii. Additions to Membership						
(1) By Election	2	76
(2) By Order of Council	60	...	
(3) By Re-instatement	6	8	...	
	1	219	260	353	23	856
iv. Transfers						
(1) By Examination:						
<i>from Associates</i>	5
<i>to Fellows</i>	5
	1	224	255	353	23	856
(2) By Examination:						
<i>from Students</i>	2
<i>to Fellows</i>	2
	1	226	255	351	23	856
(3) By Examination:						
<i>from Students</i>	16
<i>to Associates</i>	16
v. Number of Members in each class on 31 March 1904 .	1	226	271	335	23	856

The Council have, with great regret, to report the loss by death since the last Annual Meeting of seven Fellows, Mr. G. W. Berridge, Mr. A. F. Burrige, Mr. E. Docker, Mr. J. Duncan, Mr. J. Meikle, Mr. J. B. Tennant, and Mr. W. H. Tyndall; three Associates, Mr. John Gray, Mr. G. W. Kilford, and Mr. W. Marshall; and one Corresponding Member, Mons. Victor Senès.

The Annual Subscriptions, together with admission and other fees, amounted to £2,050. 6s., showing an increase of £152. 8s. 6d. as compared with those of the previous year. The total Income for the year was £2,721. 15s. 11d., and the total Expenditure £2,489. 5s. 8d. The Revenue Account and Balance Sheet are given herewith (p. 686).

The stock in hand of the Institute publications on 31 March was as follows :

No. of Copies	Description of Work
13,020	Parts of <i>Journal</i> .
507	Index to Vols. 1 to 10.
988	„ to Vols. 21 to 30.
1,124	<i>Text-Book</i> , Part I (New Edition).
1,486	„ Part II (Second Edition).
694	Government Joint-Life Annuity Tables.
771	Select Life Tables.
453	A Short Collection of Actuarial Tables.
251	Messenger Prize Essay (Friendly Societies).
30 in cloth } . . .	{ Lectures on Finance and Law (Clare and
2,930 in paper } . . .	
1,681	Lectures on the Companies Acts (A. C. Clauson).
1,638	Lectures on the Law of Mortgage (W. G. Hayter).
813	Lectures on the Measurement of Groups and Series (A. L. Bowley).
709	Transactions of the Second International Congress of Actuaries.

The following papers were submitted at the sessional meetings of the Institute, namely :

- 30 *November* 1903.—“On the Comparative Mortality among Assured Lives of Abstainers and Non-Abstainers from Alcoholic Beverages.”—Mr. R. M. Moore.
- 21 *December* 1903.—“The Income Tax as affecting Life Offices, with special reference to some Recent Decisions.”—Mr. J. E. Faulks.
- 25 *January* 1904.—“A Comparison of the various Methods of Grouping Whole-Life Assurances for Valuation.”—Mr. D. C. Fraser.
- 29 *February* 1904.—“An Investigation into the Rates of Re-Marriage and Mortality amongst Widows in receipt of relief from the Patriotic (Russian War) Fund, 1854-1900.”—Messrs. J. Burn and J. McDonald.
- 28 *March* 1904.—“On the Valuation of Whole-Life Industrial Assurances, with allowance for Lapses.”—Messrs. T. G. Ackland and J. Bacon.
- 25 *April* 1904.—“On Life Premium Book-keeping.”—Mr. J. Chatham.

For the Examinations held in the United Kingdom and the Colonies on 15, 16, 18, and 19 April last, 275 entries were received, namely:

106	for Part	I.
105	II.
45	III.
19	IV.

The results of the Examinations will be duly announced.* The Council warmly acknowledge the valuable services of the Honorary Examiners and Supervisors.

The several series of lectures which have been given during the winter months in recent years have been much appreciated, and, it is believed, have been of great value to students. The Council regret that it was not found possible to provide any such series during the past winter, but it is hoped that an arrangement may be made for another series at the end of the present year.

The Fourth International Congress of Actuaries was held in New York in September last. The President, three of the Vice-Presidents, and one of the Honorary Secretaries attended as Delegates of the Institute, together with a considerable number of other members. The number of the papers read at the Congress exceeded 70, of which about one-third were contributed by members of the Institute. The subjects treated covered a very wide field, and the papers were for the most part of a high degree of interest and importance. The collected transactions of the Congress will form a very valuable addition to our professional literature.

The Reports made by the Institute to the Royal Commissioners of the Patriotic Fund have been published in the Blue Books dealing with the subject. The administration of the Funds has now passed from the Commissioners to the newly incorporated Royal Patriotic Fund Corporation, who have asked the Institute for further advice as to certain particulars in connection with the valuations.

The Joint Committee of the Institute of Actuaries and the Faculty of Actuaries have published during the year two volumes completing the records of their labours. The first comprised the Mortality and Monetary Tables, based upon Mr. G. F. Hardy's Graduation of the Experience of Whole-life Participating Assurances on Male Lives, in the form of Select Tables ($O^{[M]}$); the second (which it was decided to publish as a separate and final volume) set out the detailed account of the Principles and Methods adopted in the Compilation of the Data, the Graduation of the Experience, and the Construction of Monetary Tables. Mr. G. F. Hardy has also recently completed the Graduation of the Non-participating Whole-Life Male Experience, in the form of Select Tables $O^{[NM]}$, and the results will, it is hoped, be available for the Members of the Institute in an early number of the *Journal*, and, in a separate form, for the use of the Offices which contributed to the Experience.

* These results for the United Kingdom and the Colonies, combined, are given on pp. 687, 688.

Revenue Account for the year ending 31 March 1901.

Dr.

Cr.

Amount of Funds at the beginning of the year—		£	s.	d.	£	s.	d.
General Fund.		7,603	6	7			
Messenger Legacy Fund		262	15	2			
Brown Prize Fund.		246	12	9			
Examination Fees for year 1903		109	1	0	8,411	15	6
Subscriptions—							
Fellows		635	5	0			
Associates		527	2	0			
Students		361	4	0			
Probationers		71	18	6			
		1,504	9	6			
One Annual Subscription Compounded for		10	10	0			
Fines for Re-instatement		2	16	0			
Application Fees—					1,698	15	6
Associates		4	4	0			
Students		41	12	6			
Probationers		37	16	0			
Examination Fees—					86	12	6
For year 1903		100	16	0			
For year 1904		254	2	6			
		354	18	0			
Class Fees		202	2	6			
Sales of Publications—							
Journal		214	8	5			
Text-Book, Part I (New edition)		44	9	3			
Text-Book, Part II (2nd edition)		119	8	1			
Government Annuity Tables		4	17	4			
Select Life Tables		9	17	7			
Short Collection of Actuarial Tables		10	12	10			
Hardy's Friendly Societies		1	4	2			
Legal, Financial, and Statistical Lectures		10	16	4			
Logarithm Cards		0	8	5			
Transactions of the Second International Congress		17	5	0	433	7	5
Dividends and Interest (less Tax)—							
General Fund.		223	10	6			
Messenger Legacy Fund		7	0	0			
Brown Prize Fund.		7	12	0			
					298	2	6
					£11,365	13	11

Balance Sheet, 31 March 1901.

LIABILITIES.		£	s.	d.	£	s.	d.
General Fund							
Messenger Legacy Fund		233	9	2			
Accumulated Dividends		136	6	0			
					369	15	2
Brown Prize Fund		200	0	0			
Accumulated Dividends		54	4	9			
Examination Fees for year 1904					254	4	9
					254	2	0
					£8,674	5	9

ASSETS.		£	s.	d.	£	s.	d.
Natal 3 per-cent Inscribed Stock (£3,000), cost					2,846	6	0
Metropolitan Railway 4 per-cent Debenture Stock (£1,050), cost					1,185	11	3
Great Eastern Railway 4 per-cent Debenture Stock (£1,000), cost					1,303	14	9
Great Northern Railway Preferred Ordinary Stock (£1,000), cost					1,142	11	9
Great Western Railway 4½ per-cent Debenture Stock (£800), cost					1,258	12	5
Cash on Deposit Account					100	0	0
Cash on Current Account					337	9	7
					£11,365	13	11

H. E. WILSON,
BERNARD WOODS,
W. M. MONILAWS, } Auditors.

Examined and found correct, 18 April 1901.

Expenditure on Valuations for the Commissioners of the Patriotic Fund

Amount of Funds at the end of the year, as per Balance Sheet

[The Institute also possesses certain copyrights and stocks of publications (see p. 634).]
Examined and found correct, 18 April 1901.

EXAMINATIONS, 1904.

Examinations were held on the 15th, 16th, 18th, and 19th of April, 1904, in the United Kingdom and the Colonies, at London, Edinburgh, Dublin, Adelaide, Melbourne, Sydney, Wellington, Montreal, Toronto, and Ottawa, with the following results, the names in each class being arranged in alphabetical order:—

PART I.

One hundred and six candidates sent in their names, of whom one hundred and one presented themselves, and sixty-one passed, namely:—

Class I:

✓ Dalrymple, A. G.	~McKechnie, J. B.
Falk, O. T.	• Parker, J. G.
✓File, L. K.	• Portch, A. G.
Fippard, R. C.	Rice, G. R.
Lewty, F. A.	Warren, C. F.

Class II:

Addey, L.	Marlin, J. H.
Boag, H.	Moran, A. J.
Brown, A. E.	Norris, I. T.
Cushing, R. M.	Reyner, H. F.
Defries, F.	Sharp, G. G.
Drake, C. C. H.	Sinclair, W. A.
Holness, A. S.	Sprules, A. M.
Latham, P. J.	Strong, G. G.
Lee, F. S.	Stuart, A. W.
Levey, R.	Underwood, R. E.
Linzmeier, L.	Warnock-Fielden, F. H.

Wenn, A. E.

Class III:

Allison, S. E.	Littlefair, J. T.
-Armstrong, C. H.	Macfarlane, E. S.
Ayscough, I.	Matheson, D.
Broad, L. C.	Monilaws, W. B.
Canter, H.	Mulcahy, F. B.
Cowdy, H. L.	Parker, W. M.
Eldridge, E. E. B.	Pearce, W. R.
Esler, J.	Phillips, W.
Fletcher, A. W. A. C.	Sadler, P.
Foot, A. H.	Spring, S. H.
Gray, M. D.	Sturt, A. J.
Humphreys, H. L.	Tamkin, W. E.
Johnson, F. H.	Thompson, W. G.
Lee, F.	Zumstein, H. C.

PART II.

One hundred and five candidates sent in their names, of whom ninety-one presented themselves, and twenty-three passed, namely:—

Class I:

Wolfenden, E. S.

Class II:

Blanchard, N.
Clinton, L. E.
Coates, F. G.

Ebihara, K.
Jackson, H. M.
Sugars, R. M.

Wilson, A. B.

Class III:

Ashley, C. H.
Atkins, L. G.
Barrett, W. G.
Collier, C. A.
Daman, G. W.
Ecroyd, C. W.
Goodman, G.

Hallett, W. S.
Harriss, W. J.
Mandling, R. G.
Rudd, A. J.
Sare, T. H.
Searle, A. J.
Sneddon, A. W.

Williams, F. A.

PART III.

Forty-five candidates sent in their names, of whom forty presented themselves, and twelve passed, namely:—

Class I:

None.

Class II:

†*Dawson, M. M.
†*Grant, M. D.

Milligan, C. L.
Robinson, H. T. K.

Thompson, T. P.

Class III:

Culley, A. B.
Curtis, W. A.
Gemmell, W.

Green, G.
Oakley, H. J. P.
Penman, W.

Sherriff, F. H.

PART IV.

Nineteen candidates sent in their names, all of whom presented themselves, and twelve passed, namely:—

Class I:

None.

Class II:

† Rietschel, H. J.

Class III:

† Brown, Henry
† Curjel, H. W.
†* Dawson, M. M.
† Diver, O. F.
† Galer, F. B.

†* Grant, M. D.
Hicks, A. J.
May, W. T.
Wares, H. W.
† Weatherill, H.

† Wood, A. B.

Those marked (†) have now completed the examination for the Class of Fellow, and (*) passed in both parts of the Final Examination.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The fifty-seventh Annual General Meeting of the members was held at Staple Inn Hall, on Monday, June 6, the President, Mr. William Hughes, in the chair.

The Report of the Council (given on p. 682) having been taken as read,

The PRESIDENT, in moving the adoption of the Report and accounts, said that there would be no need to detain the meeting very long, but it would be desirable to direct attention to a few points. In the first place, it was a matter of deepest regret that such a heavy death-roll among members had to be recorded. He believed it to be the largest in any one year of the Institute's existence, and it included some of the oldest and some of the most valued Fellows. Mr. Docker and Mr. Tyndall were not, he thought, very much known to present Actuaries, as they had long ceased active professional work, and had reached a very advanced age. Mr. Meikle was better known to members, and he was an example of how activity could be continued into advanced age. He was one of those who went over to America to the Congress, and there many Fellows had an opportunity of becoming even better acquainted with him than they were previously; and he contributed a paper to the proceedings of the Congress, and his mental and physical activity were surprising. Mr. Tennant, though resident at a distance from London, was well known to many members. Mr. Berridge had not been at the meetings for some little time; he suffered from asthma for some years, but his death was somewhat unexpected. Mr. Burridge was very well known to all, and in him the Institute had lost one of its most active and zealous members. There was little in connection with the Institute in which he did not take an active interest, and his death was specially deplored.

The number of members, it would be seen, was slightly increased. The Fellows remained at exactly the same figure, the increase being among the students. It was an evidence that the Institute Fellowship was valued by the younger men, not, as was the case in former days, because it was supposed to lead to immediate advancement to a high place in some Office, but, he thought, because the knowledge was appreciated for its own sake. The younger men felt that the better equipped they were for their business the better it was for them, irrespective of the prospects of advancement. The papers which were read during the session showed that the capacities of the Institute in the matter of subjects for discussion had not been exhausted. None of them, perhaps, could be called absolutely new, but they shed new light upon several interesting subjects. He would specially mention two of them: Mr. Moore's paper, on mortality among total abstainers, was very welcome as illuminating a hitherto somewhat dark subject. And Mr. Faulks' paper on the Income Tax also brought light to bear on another subject involved in some obscurity. The results of the examinations had again been productive of some disappointment, in consequence of the comparatively poor results, the proportion of passes to entrants being very small. But, as he remarked on a similar occasion last year, those figures must not be taken to indicate so bad a state of affairs as appeared at first sight. It arose partly, no doubt, from the very common practice among the younger members of the Institute of entering upon examinations with the idea of,

as had been said in expressive language, "Having a shot at it", whether there was any chance of getting through or not. He believed that tutors and lecturers had tried to persuade the students that it was better to wait until they were fairly sure of passing, before presenting themselves, because it was very undesirable to present to the public year by year the comparatively unsatisfactory figures which obtained at the present time. The public looked at the actual figures, and did not judge of the circumstances. The tutors and the Examination Committee were taking the matter into consideration, and would see if some expedient could not be devised to make the proportion of passes to entrants a little more satisfactory in appearance, as they were perhaps in reality, when all circumstances were considered. For various reasons, it was not found possible to arrange for any series of lectures last year. It was hoped that it could be arranged in the coming session. There were difficulties in deciding exactly the sort of lectures which were needed, whether they should be for students exclusively, or whether they should be of a more advanced character, so that, while being useful for students, they might serve for the information and instruction of those who had passed beyond the student stage. Lectures had been given on finance, law, and statistics, but there were probably many other subjects not yet touched upon, which could be profitably discussed if lecturers could be found willing to undertake them. The Council would gladly consider any suggestions from any member in that direction.

The International Congress at New York was, as was well known, a great success. Many accounts of it, of one kind and another, had appeared in the insurance press, and the matter had been referred to at the meetings several times, so that it was unnecessary to enter into it with any detail on the present occasion, except to state, as was done in the Report, that it was very successful in every respect. It was very satisfactory to notice that, although the Congress consisted of members of a large number of nationalities, yet about one-third of the papers were contributed by members of the Institute. They were very varied, and covered a great range of subjects. There was a good deal of enthusiasm and interest evinced in the Congress itself, and it had the effect, as was the case with all similar Congresses, of bringing together men who would not otherwise meet, and of thus promoting a feeling of fellowship among actuaries of various nations, which would do something, if only a little, to consolidate men of different nations in that way which was so desirable.

It would be seen that the Institute had again been doing something for the Patriotic Fund. The work which the Royal Commissioners of the Patriotic Fund had been doing was now in the hands of a new body, the Royal Patriotic Fund Corporation, and they had expressed the greatest gratitude for what the Institute had done, but they wanted just a little more from the same source, and that little more was forthcoming. He thought the Institute could congratulate itself on having done a public service in giving the assistance it had rendered to the Fund.

The last thing referred to in the Report was the very satisfactory announcement that at last the Institute had come to the end of that great work which had been occupying it, in co-operation with the Faculty, for so many years, and now he was able to announce the completion of the New Mortality Investigation. The last two volumes, issued during the present

year, completed the work, and he was sure that members would agree that those two volumes were of the highest possible value, and of permanent use, not only as working tools for practical men and scientific investigators, but also to students, as showing how the work had been done.

He would add a word on one or two matters not dealt with in the Report. A deputation of the Council had had the opportunity of meeting the President of the Local Government Board, together with the representatives of certain other societies, to urge upon the Government the desirability of taking the census quinquennially—adopting certain alterations and reforms in the manner of taking it—and of establishing a permanent census office. He was glad to be able to report that the President of the Local Government Board received the deputation with a great deal of sympathy, and he (Mr. Hughes) thought that, if that official were not hampered by financial considerations, he would at once have signified his acquiescence in the arguments of the deputation, and would have thrown the whole weight of his influence in obtaining what was so earnestly desired. But, as no reform could be carried out without the assistance of money, and as the Treasury was always disinclined to part with any money until obliged, members must not be too sanguine that the reform would come about quite yet; but the Institute had done what it could, and hoped for ultimate success. The other matter to which he would refer was perhaps a comparatively small one. It was known that there had been cases not only in connection with the Institute, but in other bodies, of people who improperly assumed the letters distinctive of membership or fellowship of those bodies. The Institute had not suffered much in this way, though there had been a few cases in which certain persons, not members of the Institute, had, nevertheless, described themselves as F.I.A. A Bill dealing with the matter had been introduced into Parliament (and there was every reason to think it would pass) to restrain unauthorized persons from using the professional designations and distinctive initials denoting membership of any chartered society, by making offenders subject to an injunction in Chancery, which would, no doubt, effectually put a stop to the practice complained of. He moved the adoption of the Report.

MR. T. G. C. BROWNE said it devolved upon him to second the motion which had been so ably placed before the meeting by the respected President. Mr. Hughes had so exhaustively referred to the various topics contained in the Report, that he (Mr. Browne) feared he had left little more to be said. He had referred to the statistics of increase, and had given the young members what would probably be regarded as poor comfort, in saying that knowledge was an admirable thing for its own sake, quite irrespective of any professional promotion which it might lead to. He (Mr. Browne) would, however, endeavour to remove such impression if the President's words had produced it. He thought the strenuous labours of the Institute during the last fifty years had not been in vain. They had undoubtedly placed the Institute in a position which had gone on increasing in importance in the public eye; and members must live in hopes that a statistical training would come to be valued outside the doors of insurance offices. Of the need of such a training for directing public opinion it was not necessary to go any further than the speeches, on both sides, on the Fiscal controversy. The President had referred to the very long death-roll among

the Fellows of the Institute. Some of them had passed away rich in years and honours, among them his old friend Mr. Meikle, with whom he had always had a certain personal connection which stamped itself upon the memory of every man in his career; for Mr. Meikle was his examiner when he passed the final examination of the Faculty of Actuaries, now some thirty-four years ago. The same enthusiasm which remained to the end characterized him at that period, and while the unfortunate candidates considered that the paper he set was by no means an easy one, they always received from him the greatest kindness and sympathy. He would be always a grateful memory to those who passed through his hands in the capacity of Examiner in the Faculty of Actuaries. Another loss which the Institute had sustained was that of one who had passed away in the prime of his business career, Mr. Arthur Francis BurrIDGE. He was sure Mr. BurrIDGE would be long missed, as his genial presence was always acceptable. The President had referred to the completion, or the approaching completion, of the great work which had been done, by the Joint Committee of the Institute and Faculty, in connection with the new mortality table. All must recognize the enormous amount of work which had been involved, and the wonderful results which had been obtained, particularly in regard to graduation, by Mr. G. F. Hardy. But he thought some of the members regretted that a book of monetary tables, on the model of Mr. R. P. Hardy's book on the *H^M* Table, was not issued by the Joint Committee. No doubt a committee had not a free hand; the results at which it arrived were compromises; in fact a committee could do very little good unless it approached its work in a spirit of compromise, and for that reason some people looked upon committees as an undesirable way of carrying out any object. But it was a necessary way, notwithstanding the saying attributed to Lord Grimthorpe, that he would not put himself into the hands of a committee, even if it were a committee of angels. He had always had such a great admiration for Mr. R. P. Hardy's book, as one of the most handy tools which actuaries could use in their daily work, that he regretted the Joint Committee had not as yet given them such a book on the new basis. But one must live in the hope that they had left something for private enterprise, and that before long a book would be published which would embrace the monetary functions given by Mr. Hardy; also the endowment assurance policy values. He would also express the hope that if such a work were undertaken by any individual, the Joint Committee would give them every facility for carrying it through in a complete form.

The adoption of the Report was then put, and carried unanimously.

The election of officers for the ensuing year was then proceeded with, Messrs. Levine and Kentish acting as scrutineers.

The PRESIDENT announced that the whole of the gentlemen recommended by the Council had been unanimously elected as follows:—

President.

HENRY COCKBURN.

Vice-Presidents.

THOMAS G. C. BROWNE.
GEORGE KING.

ERNEST WOODS.
FREDERICK SCHOOLING.

Council.

THOMAS GANS ACKLAND.	GEORGE KING.
*HENRY JAMES BAKER.	*GEORGE JAMES LIDSTONE.
ARTHUR DIGBY BESANT, B.A.	HENRY WILLIAM MANLY.
JAMES BLAKEY.	ARTHUR ERNEST MOLYNEUX.
THOMAS G. C. BROWNE.	*WILLIAM PEYTON PHELPS, M.A.
HENRY COCKBURN.	GERALD HEMINGTON RYAN.
GEORGE STEPHEN CRISFORD.	FREDERICK SCHOOLING.
GORDON DOUGLAS.	GEORGE TODD, M.A.
JOSEPH ERNEST FAULKS, B.A.	RALPH TODHUNTER, M.A.
DUNCAN CUMMING FRASER, M.A.	SAMUEL GEORGE WARNER.
*NIEL BALLINGAL GUNN.	ALFRED WILLIAM WATSON.
GEORGE FRANCIS HARDY.	JAMES DOUGLAS WATSON.
*RALPH PRICE HARDY.	ERNEST WOODS.
CHARLES DANIEL HIGHAM.	FRANK BERTRAND WYATT.
WILLIAM HUGHES.	THOMAS EMLEY YOUNG, B.A.

Treasurer.

GERALD HEMINGTON RYAN.

Honorary Secretaries.

GEORGE TODD, M.A.

| SAMUEL GEORGE WARNER.

* Not Members of the last Council.

Mr. COCKBURN returned the best thanks of the newly-elected Council and himself for the confidence expressed in the election, which, he felt sure, each and all would do their best to justify. Especially he wished to tender to the members his very sincere acknowledgment of the great honour they had paid to himself, in elevating him to the highest position in the Institute, a position which had been filled by a number of distinguished and successful men, in whose footsteps it was not easy to tread. He was conscious that that high place carried with it its own responsibilities. He could but hope to fulfil the duties, not with the brilliancy of some of his predecessors, yet in some degree which would tend to uphold the dignity of the Institute, and further its usefulness. He took encouragement when he looked at the names of those who had been elected to serve with him on the new Council, for there he saw not only many old friends, but the names of men whom he knew to be loyal and helpful, and who would always be ready to give their best thoughts and energies to anything which would promote the welfare of the Institute. Again he thanked sincerely all present for his election.

Mr. TAYLOR said he had much pleasure in proposing the re-election of Mr. Bernard Woods and Mr. W. M. Monilaws, and the election of Mr. G. A. Brown, to act as auditors. Those three gentlemen were sufficiently well known to nearly everyone in the room to recommend themselves, and, therefore, any comment with regard to them would be superfluous.

Mr. LAING cordially seconded the resolution, which was put to the meeting and carried unanimously.

Mr. A. H. BAILEY said he had great pleasure in proposing a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners, and Supervisors, for their services in the past year. He believed he was one of the oldest, if not indeed the oldest, member of the Institute, having been

admitted in the year 1848, the date of its establishment, and as he had also had some experience on the Council, he was perhaps able to appreciate the excellence of the work done during the past year by that body, as shown in the report presented to the meeting that day. The only matter to which it occurred to him to draw a little attention was, that he was rather struck with the accumulation of the funds, and he did not know whether that was necessary, or to be desired. Perhaps the Council would consider whether the subscriptions could be reduced. There seemed to him to be no object in having a general fund of something like £8,000, especially now that the heavy work connected with the mortality experience had been completed.

Mr. W. OSCAR NASH had much pleasure in seconding the vote of thanks moved by Mr. Bailey, and would only add that members should be especially grateful to the President and other officers who attended the American Congress and represented the Institute so successfully.

The motion was put, and carried unanimously.

The PRESIDENT, on behalf of the out-going Council, thanked the members very much for their vote of appreciation. With regard to Mr. Bailey's observations as to the accounts, he (Mr. Hughes) did not comment upon that, because he confessed it was a point on which he could say very little. All were conscious that the funds were large, but whether they were too large was a question yet to be considered. He would remind Mr. Bailey that the cost of the mortality investigation did not fall upon those funds, because it was contributed to by the Offices, and formed a separate fund altogether. With regard to the question of reducing the rate of subscription, or otherwise disposing of the supposed surplus, he was very glad to be able to transfer that to the shoulders of his successor. Perhaps he would take the matter into consideration.

Mr. A. W. TARN said that one more duty remained, namely, to propose a vote of thanks to Mr. Wilson, Mr. Bernard Woods, and Mr. Monilaws, for their services as auditors during the past year. He was sure that it would be passed unanimously, because it was well recognized that the services of auditors were quite necessary, even though they might not be very arduous. The Institute wished to thank them very heartily for those services.

Mr. A. R. BARRAND seconded the resolution with much pleasure.

Mr. WILSON thanked the members, on behalf of his colleagues and himself, for their vote. With regard to the funds, the system of book-keeping was so simple that the labours of the auditors were reduced to a minimum. He wished also to say that the Assistant-Secretary kept his books and papers in such excellent order, that there was very little indeed to do. But the work, though light, was performed with a great deal of pleasure.

The PRESIDENT, in adjourning the meeting, intimated that the next gathering of members would be held on November 28th, 1904, when those present would have the pleasure of listening to an address by the new President.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1903:

*By whom presented
(when not purchased).*

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- Local Loans Fund.
 Report and Accounts for 1902. }
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 Military Savings Banks. Account to 5 January }
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 Savings Banks and Friendly Societies. Account }
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 missioners. 1904. }
- Prevention of Corruption Bill.
 Report of Committee, with Proceedings.
- Registrar-General. England.
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- Workmen's Compensation.
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Toronto, Insurance Institute of

Proceedings, 1900-1, 1901-2, and 1902-3.	} <i>The Insurance Institute.</i>

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Mortality Experience of Thirty-four Life Companies upon Ninety-eight special classes of Risks.	{	<i>The Actuarial Society of America.</i>
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Walker (F. A.).		
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Money in its relations to Trade and Industry.		
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Die Lehre von der Mortalität und Morbilität.	{	<i>The National Mutual Life Assurance Society.</i>
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DCC Exercises, including hints for the solution of all the questions in "Choice and Chance."	}	<i>Purchased.</i>
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Another Copy.		<i>Purchased.</i>
The following additional copies of works already in the Library have also been added:—		
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Transactions, 1892.		<i>A. E. Molyneux.</i>
Hughes (W.).		
Practical Information for Life Assurance Agents. 1875.		<i>The Author.</i>
Karup (Prof. Johannes).		
Die Finanzlage der Gothaischen Staatsdiener-Wittwen- Societät am 31 December, 1890.	{	<i>The National Mutual Life Assurance Society.</i>

THE INSTITUTE OF ACTUARIES.

The Examinations of the Institute of Actuaries.

THE Council have been considering the Examination arrangements. At present two Papers are set in each of the four Parts of the Examinations, and four hours are allowed the students for answering each Paper, making, for each Part, eight hours in all. Seeing, however, that in Parts II, III and IV, it is not easy to limit the questions to those which can be answered in short space, the Council have decided to allow nine hours in all for the Examination in each of these Parts, without increasing the number of questions. In each Part three Papers will be set, and three hours will be allotted to each Paper. In this way, not only will the candidates have more time to answer the questions, but they will not be called upon to sit so long at a stretch, and thus the strain on them will be reduced. The change will take effect at the Examinations to be held in April 1905.

No alteration has been made in respect of Part I.

Opportunity is taken to reprint here the Rules prescribed by the Council to regulate the Examinations, and the Syllabus of Examinations. It will be noticed that a few verbal alterations have been made in the Syllabus, to remove ambiguity, and to make it quite clear to students which subjects they are expected to take up for each Part of the Examinations.

THE INSTITUTE OF ACTUARIES

(Established in 1848, and Incorporated by Royal Charter in 1884),

STAPLE INN HALL, HOLBORN, LONDON, W.C.

RULES prescribed by the Council of the Institute to regulate Examinations qualifying for admission to the Classes of Student, Associate, and Fellow, respectively.

GENERAL REGULATIONS.

1. The Examinations shall, until further notice, be conducted in writing, under such conditions as the Council may prescribe.

2. The Examiners shall place successful Candidates in three Classes, according to merit, and the names in each Class shall be arranged in alphabetical order.

3. No Candidate will be allowed to present himself for Examination until he has paid all Entrance Fees, Subscriptions, and Examination Fees that may be due.

4. Examinations will be held in April of each year, or at such other time as the Council may prescribe.

Examinations in the United Kingdom.

5. At least one month's public notice will be given of the days and hours, and of the place or places, of the Examinations.

6. Candidates for any Examination must give to the Honorary Secretaries fourteen days' notice in writing of their intention to present themselves for Examination, and must specify the particular Examination.

Examinations in the Colonies.

7. Public notice will be given, not later than in November of each year, by advertisement in at least one newspaper of the Colony in which an Examination is to be held, of the date and place of such Examination.

8. Candidates for any Examination in a Colony must give to the Honorary Secretaries notice in writing of their intention to present themselves for Examination, and must specify the particular Examination. Such notice must be posted in time to reach the Honorary Secretaries in London not later than on the 31st January preceding the date of the Examination.

CLASS OF STUDENT.

1. Candidates for admission to the Class of Student must, at the time of giving notice of their intention to present themselves for Examination, pay an Entrance Fee and an Examination Fee, each of £1. 1s.

2. Candidates who have paid the above Fees, and whose applications for admission have been approved by the Council, shall, unless exempted under the next clause, be examined in the subjects set forth in Part I of the annexed Syllabus. Application must be made on the form prescribed by the Council, which may be obtained from the Honorary Secretaries.

3. Graduates in Mathematical Honours of any University in the United Kingdom will be exempt from Examination in Part I of the Syllabus.

4. In the event of a Candidate passing the Examination, he will be admitted a Student after he has signed the Form of Obligation prescribed by the Council, and paid his Subscription of £1. 1s. for the current year.

5. In the event of a Candidate failing to pass the Examination, he shall, subject to the approval of the Council, and on payment of the Examination Fee, be permitted to be examined in a subsequent year.

CLASS OF ASSOCIATE.

1. Students shall be required to pass Parts I and II of the Syllabus to qualify for admission to the Class of Associate, and, subject to the exceptions contained in these Rules, no Student shall present himself for Examination in the second part unless he shall have previously passed the first part of the said Examination.

2. Candidates for Examination in either part must, at the time of giving notice of their intention to present themselves, pay a fee of £1. 1s.

3. Students of the Institute admitted after 1st October 1898, shall be required to pass only Part II of the Syllabus to qualify for admission to the Class of Associate.

4. Fellows (by Examination) of the Faculty of Actuaries in Scotland will be considered by the Council, other circumstances being in their view favourable, to be eligible as Associates, without passing the Examinations for admission to the Class of Associate.

CLASS OF FELLOW.

1. Students or Associates shall be required to pass Parts I, II, III and IV of the Syllabus to qualify for admission to the Class of Fellow, subject to the exceptions contained in these Rules. No Candidate shall present himself for Parts III and IV until the expiration of at least twelve months after passing Part II.

2. Candidates may present themselves for Parts III and IV in the same year, and in the event of a Candidate passing in Part IV and failing in Part III, he may present himself for Part III alone in a subsequent year. Otherwise Part III must be passed before Part IV.

3. Candidates for Examination in Parts III and IV must, at the time of giving notice of their intention to present themselves, pay a fee of £1. 1s. in respect of each part for which they enter.

4. Associates who, prior to the commencement of the *Bye-Laws* (26th February 1886), had passed Part I, or Parts I and II, shall be

required to pass only Parts II, III and IV, or only Parts III and IV, respectively, of the Examination to qualify for admission to the Class of Fellow.

5. Associates admitted after 26th February 1886, shall be required to pass only Parts III and IV of the Examination to qualify for admission to the Class of Fellow; and Students admitted after 1st October 1898 shall be required to pass only Parts II, III and IV of the Examination to qualify for admission to the Class of Fellow.

N.B.—Part III, Section A, of the Syllabus dated 9th January 1894 shall be held to be equivalent to Part IV; and Part III, Section B, to Part III of this Syllabus.

By order of the Council,

GEORGE TODD,	} <i>Hon. Secs.</i>
SAMUEL GEORGE WARNER,	

[Attention is called to the Regulations for Probationers printed on page 706.]

October, 1904.

SYLLABUS OF EXAMINATIONS

Referred to in the annexed Rules.

PART I.

- (1) Arithmetic and Algebra.
- (2) The Theory and use of Logarithms.
- (3) The Elements of the Theory of Probabilities.
- (4) The Elements of the Calculus of Finite Differences, including Interpolation and Summation.

PART II.

- (1) Compound Interest and Annuities-Certain.
- (2) The application of the Theory of Probabilities to Life Contingencies.
- (3) The Theory of Annuities and Assurances on Lives and Survivorships.
- (4) The elementary application of the Calculus of Finite Differences, and of the Differential and Integral Calculus, to Life Contingencies. Expressions for the Law of Mortality.
- (5) The principles (as distinguished from the methods) of the construction of Mortality Tables (excluding graduation); and the principles and methods of the construction of monetary and other Tables involving the Contingencies of Life.

PART III.

- (1) The methods of constructing and graduating Mortality, Sickness and other Tables.
- (2) The history and distinctive features of existing Tables.

- (3) The Valuation of the Liabilities and Assets of Life Assurance Companies.
- (4) The Distribution of Surplus.
- (5) The Calculation of Office Rates of Premium for Assurance, Annuity, Sickness and other risks, excluding Pension Funds, and Widows' and Orphans' Funds.
- (6) The practical valuation of Life Interests and Reversions, and of Policies for surrender or purchase.

PART IV.

- (1) The Elements of the Law of Real and Personal Property.
- (2) The Law relating to Life Assurance Companies and Life Assurance Contracts.
- (3) The Constitution, Valuation, and Calculation of Rates of Contribution, of Friendly Societies, Pension Funds, and Widows' and Orphans' Funds; and the Laws relating to such Institutions.
- (4) Life Assurance Book-keeping; preparation of Schedules, Statements and Reports.
- (5) The Principles of Banking and Finance, including a knowledge of the Constitution and Operations of the Bank of England, and of the National and Local Debts of the United Kingdom.
- (6) The Investments of Life Assurance Companies.

REGULATIONS FOR PROBATIONERS.

The Council has established a Class of Probationers who, while not being Members of the Institute, shall be allowed some of the privileges of the Members.

Persons desiring to become Probationers shall apply to the Council, and, if their applications are approved, shall become Probationers on payment of an entrance fee of 10s. 6d., but the Council may at any time withdraw their approval, and thereupon the person shall cease to be a Probationer. Should the Probationer subsequently be admitted a Member of the Institute, this fee of 10s. 6d. will be taken as paid on account of the entrance fee as Student.

The annual subscription for Probationers is 10s. 6d., payable on admission and on 1st October in each year. If the subscription for any year be not paid before the 31st December, then the defaulter shall no longer be a Probationer.

Probationers will be entitled to join the classes for Students, in accordance with the rules prescribed for such classes, and to attend the Ordinary General Meetings of the Institute, but not to vote or take part in the discussions thereat.

Probationers may borrow books from the Library for the purposes of their studies, but this privilege is subject to the discretion of the Librarians, and to the rules which the Council may from time to time prescribe.

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